Mass from Classical Physics to Special Relativity: Learning Results

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Abstract

Mass is a fundamental multifaceted physical quantity, involved in courses from primary to secondary school. Its equivalence to rest energy is very important as well. Educational literature indicates remarkable problems in high school textbooks, misconceptions concerning $E_0=mc^2$, and a qualitative view of mass, with a teleological connotation. We therefore carried out a research into the learning paths of 42 skilled students attending a modern physics summer school, by means of an interactive tutorial. "Relativistic mass" conception was investigated too, as an important spin-off. Our main findings concerning the classical part of our working sheets were that 76% of students associated mass with mechanical phenomena and that the pre-theoretical conception *quantitas materiae* was rooted in some minds (between 12% and 15% of the sample); only 26% recognized mass explicitly as important in gravitational interaction between bodies, even if gravitational mass was considered by 50% as a parameter describing a generic interaction between bodies. Inertial mass was instead understood as given by Newton's second law by most students. As for relativistic part, mental representations of mass seemed to be related to students' learning environment. The young talents were very good at formalizing, mass-rest energy relationship being a striking exception: The "relational level of physical representation" prevails over other "levels". Eventually, no statistical significant correlation was found between the presence of the concept of mass as rest energy and the understanding level of "relativistic mass" (even if a sort of negative correlation between the former and the latter can be seen in their plot).

Keywords: mass-energy, rest energy, *quantitas materiae*, inertial and gravitational mass, "relativistic mass", skilled students, interactive tutorial, statistical correlation

Mass from Classical Physics to Special Relativity: Learning Results

Mass is a fundamental physical quantity, which is necessarily present in every physics course, in schools of all types and levels. According to the prominent historian and philosopher of physics Jammer (2000) «Next to space and time, mass is the most fundamental notion in physics, especially once its so-called equivalence with energy had been established by Albert Einstein. Moreover, it has even been argued repeatedly that "space-time does not exist without mass-energy"». Strictly speaking, equivalence between *rest energy* and mass was stated in Special Relativity. Burniston Brown (1959) defined mass as «the key term in dynamics». In 2005, Okun wrote: «There is no doubt that the problem of mass is one of the key problems of modern physics. Though there is no common opinion even among the experts what is the essence of this problem ».

This quantity shows a manifold character: Newton introduced the nonphysical *quantitas materiae* – measurable through the product $\rho*V$ – together with inertial mass (in $F=ma$) and gravitational mass (in universal gravitation law).

In 1905 Einstein showed a new relationship between Newtonian inertial mass and internal energy of a body (in the thermodynamic sense) – that will be called 'rest energy' later on – in the particular case of electromagnetic energy emission. Till 1907 he worked out *mass-energy equivalence* for a wider and wider range of phenomena. During a conference in Salzburg (Einstein, 1909) he decided to mention only the latter among consequences of the theory of relativity, « […] because it brings about a certain modification of the basic ideas of physics ». In general relativity momentum-energy density is the source of space-time geometry warping, where (rest) energy is equivalent to mass.

Eventually, "relativistic mass", a construct dependent on speed of a body in a reference frame, is sometimes used as a proper physical quantity, even if nowadays most of the scientific community considers it useless and misleading in terms of teaching (e.g. Fabri, 2007; Okun, 2001).

Educational researches (Lehrman, 1982) pointed out relevant problems in high school textbooks: Confusion between weight and gravitational mass, belief that equal arm scales measure weight instead of gravitational mass, operational definition of inertial mass as F/a without a non dynamic definition of force (so we are left with a circularity problem). Additional literature (Burniston Brown, 1959) indicates an increase of confusion about the concept of mass when a distinction between «inertial» and «gravitational» mass is made. This implies confusion about their proportionality in turn. Moreover, *quantitas materiae* has generated misconceptions concerning mass-energy equivalence: Mass is 'converted' into a generic 'energy' (the most frequent one); $E=mc^2$ represents 'conversion' of mass into energy; energy conservation and mass conservation laws are mixed up. Finally, an important research by Doménech, Casasùs and Doménech (1993) showed the presence of a qualitative view of mass in a group of 16- to 18-years old pupils, with a teleological connotation – encouraged by social view of science (Duschl, 1988) – instead of a scientific quantitative conception, where mass is an operative quantity (at least for educational purposes). This is due both to the belief that scientists describe objective reality and to «student bewilderment with the formal

[…] numerical reasoning used by scientists». Doménech et al. (1993) classified students' ways of looking at mass in five categories, being inspired by the solution models worked out

by Gorodetsky, Hoz and Vinner (1986). The categories were called *levels of 'physical representation'*:

1. *Ontological*: Mass as a general property of matter or even identified with matter/bodies/particles; it's considered a pre-theoretical definition (a theoretical framework is not developed). A typical example is *quantitas materiae.*
2. *Functional*: Mass identified with properties, tendencies or behaviours of the physical system. Ex. inertia on one hand, heaviness on the other hand.
3. *Translational*: Mass identified with another related quantity, such as density/volume or weight (pre-theoretical level)
4. *Relational*:* Mass clearly related with other concepts in a theoretical framework (also when not mathematically formalized).
5. *Operational*: Mass as numerical result to be obtained experimentally through «conceivable» and « explicit » operations. Ex. inertial mass as the measure of inertial scales.

In order to perform an inquiry about both the previous problems and, more generically, high-school pupils' learning of the fundamental but complex concept of mass, we decided to investigate lines of reasoning in 17 to 19 years-old students attending a modern physics summer school. Our research was essentially lead by two questions:

a. *How and in which contexts do our students relate themselves to the word "mass" and make use of it? What (mis)-conceptions can be found?*
b. *How do skilled students interpret the extension from the concept "mass" to "mass as rest energy" in the relativistic context (under the influence of our path)?*

Method

Participants

Students taking part in the school[1] were 23 boys and 18 girls (M_{age} = 18 years, age range: 17-19 years). They came from each Italian region, after a severe selection based on the arithmetic mean of their final marks in scientific subjects in the last two school years. Only one student was attending Liceo specializing in classical studies, the others in scientific studies, four of whom in scientific-technological studies.

1 IDIFO3 modern physics summer school was held at the University of Udine in July 2011.

There were five additional participants: University skilled students, one of whom took part in our activity freely.

Materials and Procedures

42 participants, namely 41 high-school students + 1 university student, attended our 90-minute interactive tutorial, including proposals for both individual reflections and group discussions. Each student read and filled in some worksheets (a little booklet) outlining our whole path. The sheets included inner (individual and "group") questions and a final questionnaire; the "group questions" were to be answered after a brief discussion in small groups.

Since we assert the importance of building a unitary theoretical framework which should account for all mass facets, we worked out a conceptual path on historical basis, starting from mass in some excerpts from Newton's *Principia*, going through Mach's (1883) considerations and criticism, in order to arrive to Einstein's revolutionary conception of mass: The module of momentum-energy four-vector.

The answers to internal questions that might be relevant, the group answers – recorded by a digital video camera – and the answers to the final test were analyzed, both in 'vertical' and 'horizontal' mode. The former type consists in examination of every answer *for all students*, first aiming at categorizing the replies on the basis of our research questions, then for finding their distribution in the students' sample. The latter type is instead a search for correlations among *each student's answers*: We wanted to find out their individual ways of 'looking at' mass, in order to recognize the profiles pointed out by Doménech et al. (1993).

Rationale. More precisely, our path began with Newton's operational definition Quantity of matter (see Burniston Brown, 1959). An analysis of inertial and gravitational mass concepts in Newtonian physics as well as a glance at the empirical and 'relational' Mach's definition of the former followed. An applet with a vertical light clock was useful to introduce proper time and relativistic time dilation quantitatively, having postulated the invariance of the speed of light in vacuum. Student visualized a particle world-line, as well as a photon world-line, (shown in Figure 1) on a screen. They solved a couple of problems on intervals in Minkowski space-time then, in order to familiarize with the latter, to understand four-vectors and to deduce quadrimomentum in analogy with its classic equivalent: *m* (Newtonian mass) times displacement four-vector, divided by proper time (a relativistic invariant), taking the limit $\Delta\tau \to 0$. So we obtain.

$$quadrimomentum = \lim_{\Delta T \to 0}\left[\mathrm{m}\left(\frac{\mathrm{c}\Delta t}{\Delta T}, \frac{\Delta x}{\Delta T}, \frac{\Delta y}{\Delta T}, \frac{\Delta z}{\Delta T}\right)\right] = (ymc, ymv_x, ymv_y, ymv_z)$$

We calculated a series expansion of the temporal component of quadrimomentum in the Newtonian limit afterwards, defining this new quantity the relativistic kinetic energy,

apart from an additive constant. Finally, we were able to infer and interpret the equation $E_0 = mc^2$, expressing mass-rest energy equivalence, where E_0 is the additive constant.

Classical inner questions. «Until the XVIII century mass was essentially considered as "quantity of matter", also by Isaac Newton who in his *Principia Mathematica* (1687) wrote [*first quotation*]. In the text below, which of the following concepts is prevalent in Newton? Mass, Body, Density or Volume? Why? ». After that we reconstructed the genesis of the famous formula $F_g = G\frac{m_1 m_2}{r^2}$ briefly, through further indirect and direct quotations, compared it with Coulomb's formula, and asked: « Observe that masses m_1, m_2 in the Universal Gravitation Law play the same role than electrical charges. On the basis of this analogy, can you tell what the meaning of the word **"gravitational mass"** is? ». The last Newton's quotation was about inertia, as well as the subsequent quotation by Mach (1883), who considered the former formulation as a vicious circle; we asked finally: «Here [*quotation*] the focus is that mass is no more the simple "quantity of matter" in Newton […]: It's a concept in evolution in his mind. Is there a difference between the mass in gravitation and this one? Explain». Group question: «What are the conceptual differences ultimately among the notions of mass examined so far? »

Relativistic inner problems. Students analyzed a nuclear fission process – of which we provided two examples – and tried to understand where the huge quantity of energy released comes from, if total energy has to be conserved. An analysis of a collision between two identical particles, creating a new rest particle, followed: We asked which forms of energy were changing.

Data and Findings

Classical part

Answers to the inner questions. The first answers show that students acknowledge the contents proposed through the reading, even if with some variations. Density is

considered related to mass by 43% of students, and to "quantity of matter" or "substance" – in their words – by 33%. For instance, Luca replies: «Because Newton uses it as reference point (valid for all bodies) to obtain the mass of every body»; Carmelo replies instead: «Because he speaks of quantity of matter in a volume, that is density or what he calls *no* of every body». From the second answers we found out that gravitational mass is considered as a parameter describing an attractive interaction between bodies; the emphasis is on the body in 69%, while 26% mention mass explicitly. Another 26% refer to universal gravitation law, but never using formalism. Third question: In 62% of cases the difference between inertial and gravitational mass was also expressed through a characterization of the latter, with respect to the former.

In regard to inertial mass, the category "Newton refers to inertial mass, which is the quantity governing the behaviour of bodies when accelerations/momentum variations (in collisions) are present" is prevalent: 55%. The concept is expressed in a variety of modalities, with most of the answers written in the form: "The ability/property of a body in contrasting a variation in its state of motion / state of rest". Other frequent answers are either "the ability/property of a body in contrasting a variation in its state of uniform linear motion / state of rest", or "the ability/ property of a body in contrasting the change of state".

Group question. The relative majority of answering students (12/28) try to give meaning to the concept of *quantitas materiae* in itself, whereas 8 fix their attention to the

circularity problem in Newton's definition and 8 (different) students just mention this facet of mass, without deepening its meaning. This data should be taken with a special care, because 33% of the whole sample didn't answer.

A precise distinction between the definitions of gravitational mass ("dynamic" quantity: A precise cause of motion is identified) and inertial mass ("kinematical" quantity: All interactions are considered) was found in 36% only, whilst confusion is present in 57%.

As for *inertial* mass (m_i) we grouped the answers in four not-exclusive categories, from the strictly scientific to the intuitive ones:

1. Constant / proportionality factor in second law of dynamics: 57%;
2. Operational concept, defined by symmetry in interactions, and inertial role of mass: 10%;
3. Concept extended from gravity to every interaction: 19%;
4. Property of the body, which 'resists'/ 'opposes' to something: 36%.

On the other hand, one-half (21/42) of the students consider *gravitational* mass (m_g) as (i) a property mediating/'permitting' the interaction between bodies or (ii) between masses (16 of these recognize gravitation as a two-body interaction explicitly). 26% highlight mass as (iii) source of interaction, that is active gravitational mass, typically writing 'property/capacity of generating a force'; 14% consider (iv) m_g involved in gravitational interaction only, while m_i in all physical interactions.

We deduced by variation analysis that categories (i) and (ii) become *less important* (26 → 18, 11 → 3 students respectively), while (iii) and (iv) become *more important* (3 → 12 and 0 → 6) passing from the second question to the group question.

Final test. Our final test was composed by four classical and two relativistic questions. C1: « When is mass involved in your everyday life? What are the phenomena in

which it is involved? » (see Figure 2), C2: « What physics theories study these phenomena? » (Figure 3), C3: « What do you mean by *quantity of matter*? » (Figure 4), C4: « What connotations and definitions of mass do you know? » (Figure 5)

Phenomena evoked in familiar contexts were in large part mechanical ones; some students referred to mechanical[2] *quantities* associated to mass instead. Besides, the most mentioned *theories* and physics sectors have been "dynamics" (52%), "mechanics" (40%) and "kinematics" (24%); "relativity" played an important role (36%) as well. On the other hand, there is awareness of the importance of mass in electromagnetism in few students

(3/42) and no one is able to contextualize it in familiar phenomena. It is worth noting that, in answers to C1, 7/42 indicated the *unique* mechanical phenomenon not depending upon mass (in vacuum): Free fall.

The results about *quantitas materiae* show that this pre-Machian conception of mass is rooted in some minds (6/42 for question C3; 5/42 for question C4). Nevertheless, it is not evoked by the oral answers to the first group question, as verified in the analysis of video recordings, so it's not *so much* rooted.

Relativistic part

Answers to the inner questions. From the analysis of the collision process it came out that 7/42 students thought that kinetic energy *and* rest / internal energy vary in the collision, whilst 4/42 mentioned kinetic energy *only* (Figure 6). Moreover, 15/42 followed this type of reasoning: Total energy, but not mass, is conserved and kinetic energy varies, so rest energy also do; when E_0 varies, mass varies in the same sense[3]: Mass-energy relationship

is valid in variation form as well. These results are however to be taken with a large grain of salt, because most of the students didn't answer (69% in the 1st case, 64% in the 2nd case).

Final test. R1 - « Does the inertial mass of a body change in function of its energy, apart from the kinetic energy? » (Figure 7)

We found no conceptual reference to the mass-rest energy equivalence in 40% of answering pupils, although our rationale had been brought on the ground of relativistic energy. Our aim was helping students to distinguish between mass as rest energy (its proper meaning) and 'relativistic mass'. The conceptual reference to mass-rest energy equation is present instead in 43% of cases, mainly implicit or explained in words. Fourteen percent of answers were uncertain, that is enunciations, invocations of a generic relation between mass and energy, not understandable sentences.

R2 -«*Relativistic mass* is mentioned in many textbooks. Explain what it is» (Figure 8).

A remarkable example of a fourth-category ("mass at relativistic speed") answer is « That means that mass in motion at very high speed can become energy and vice versa ». The most appropriate answer (III category, "mass depending on speed") is « Let's call the relativistic mass m_r. We want the classical expression of momentum to be valid with m_r instead of m. If we equal the expressions for p_{rel} we obtain $\gamma mv=m_r v$ where v is the particle velocity, and then $m_r = \gamma m$. » Twenty-six percent of the sample uses wrong terminology for this question.

Hypothesis testing. We performed a statistical analysis, namely the calculation of Spearman's rank correlation coefficient, in order to evaluate if our *null* associative hypothesis

"*There is no statistically significant correlation between the conceptual reference to mass- rest energy equivalence (mass is* $m=E_0/c^2$ *in SR) and the presence of the conception of*

relativistic mass (mass is $m_r = \gamma m_0$ *in SR)*" was supported. Procedures described by Cohen, Manion and Morrison (2007) were followed in measuring the association between two ordinal variables: Let's call them X and Y. Their values run from 1 to 5, according to the level of presence of conceptual reference to mass-rest energy equivalence (X) and the level of rooting and formalization of "relativistic mass" concept (Y); details are shown in Table 1. The level of significance (α) was set; there was *no statistical significant correlation* found between X and Y (ρ_s = -0.2126, α <.05, critical value: ρ_0 = .325 for N=37 couples of data; size effect: ρ^2= .0452). Moreover the probability that 37 measures of two uncorrelated variables yield a correlation coefficient[4] r if $|r| \geq .2$ is in the interval 22 – 25%.

The previous analysis is useful to clarify statements made in the poster presentation.

2 Only mechanical quantities were mentioned, apart from "quantity of matter", "force fields", "gravitational field".

3 Notice that these are the same word of Einstein's first article (1905).

4 Even if r is used properly in parametric statistics.

Students' profiles

The results concerning the five levels of physical representation by Doménech et al. (1993) are shown in Figure 10. *Relational* level of representation is prevalent: It affects 60% of the sample, as it was to be expected. We found no operational profiles and only one (partially) translational; to be noticed the not-negligible presence of ontological (4/42) and functional (3/42) profiles.

Discussion and conclusions

First Research Question

Students show good capability to understand historical physics texts.

Answer to the group question. As for *quantitas materiae,* or pupils focus their attention on circularity problem either try to give an (ontological) interpretation.

Nineteen percent seem to be more attracted by negative considerations than by the possibility of a personal revision. In regard to inertial mass, it is understood consistently with Newton's 2nd law by most students (57%, 24/42). When they have to compare the facets of mass synthetically, 15/42 (different) pupils tend however to fall in rigid patterns related to an action of the body in opposition to motion. Finally, the idea of gravitational reciprocal interaction seems to reduce in favour of an idea of force generated by a source.

Final test. The halving in the number of students with "holistic" vision of mass when changing from everyday phenomena to theories (questions C1 and C2) seems to indicate that "ubiquity" of mass was not rationalized by student having expounded it.

Results on "relativistic mass" (question R2) indicate that students expects a change in the meanings of many quantities in the passage from a theory to another; a conceptual revision is necessary, but it cannot be limited to the semantic aspects, like students do.

In the end, mental representations of mass seem to be strongly affected by learning areas, so it is important to design integrated teaching (Fabri 2007). We noted in particular a local view of the mass in special relativity (SR) in a context defined by speed and a grasping of the concept of mass in SR as limited to a "chapter" of physics.

Second Research Question

Final test. Our studentsare very good at formalizing, the relationship $E_0 = mc^2$ being a remarkable exception in this regard. The concept of "relativistic mass" given by γm_0 (speed -dependent mass) is integrated in Einsteinian paradigmin36% of the answering students

(14/39), but only 1 student gives the exact definition. This integration is absent instead in 31% (12/39). These results come from the answers to the VI question.

General discussion. Eventually, *terminology* plays an important role in the proper understanding of mass in relativistic context and in theoretical framing of its conceptual relations with total energy, rest energy and "relativistic mass". 7/39 wrong answers to the

question above are to be ascribed to terminology indeed: We found a mixing of (i) proper m_r terminological use, (ii)m_r reported to be equal to "the mass in relativity" (E_0/c^2 as well as "mass in $E = mc^2$ ") , (iii) mass at relativistic speed, (iv) *variation* of mass when energy varies.

Eventually, confusion between mass and mole is likely to be due to terminological use of the latter, both as physical quantity and as IS unit, in the right measure of quantity of matter.

Correlation test. The *null* hypothesis is supported by our ordinal data, so the *alternative* hypothesis of negative correlation is rejected for our sample. $P_{37}(|r| > r_0) = 22 - 25\%$ is too high for significance too. We are not allowed instead to say anything about causality. However, when you plot data (see Figure 9), it can be noticed that the top right part of the plot is scarcely populated: *The simultaneous understanding and mastery of the two ideas of mass seems not to occur effectively*.

Students' profiles.

First of all, you can see in Figure 10 that mixed categories were found, for students can never be subdivided into entirely separate groups. Most students proved good at understanding and using formal language, often expressing concepts by means of formulas.

However, 5/42 students does not refer to any theoretical framework. Functional level affects 31% of the sample: A theoretical framework is present, but in *implicit* form in their minds.

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Table 1. Ordinal variables – from R1 (left) and R2 (right) – for correlation analysis.

	Level of presence of conceptual reference to mass- rest energy equivalence (ordinal scale)	Level of presence of "relativistic mass" concept (ordinal scale)
Student	1 absent; 2 very low (implicit); 3 low (implicit); 4 high (explicit); 5 very high (explicit)	1 absent (rest energy/ internal energy); 2 weak (mass in Relativity/mass at relativistic speed); 3 medium (energy), 4 strong (mass generically depending upon velocity), 5 very strong ($m_r = \gamma m_0$ /varying with reference frame)
StudentA	5	4
StudentB	2	1
StudentC	5	1
StudentD	3	1
StudentE	1	2
StudentF	3	4
StudentG	4	1
StudentH	2	3
StudentI	1	3
StudentL	1	5
StudentM	4	1
StudentN	1	2
StudentO	3	2
StudentP	1	3
StudentQ	4	4
StudentR	3	4
StudentS	3	2
StudentT	1	5
StudentU	4	1
StudentV	1	5
StudentZ	1	3
StudentAA	4	3
StudentAB	1	1
StudentAC	4	3
StudentAD	3	2
StudentAE	4	3
StudentAF	3	5
StudentAG	5	3
StudentAH	2	2
StudentAI	3	3
StudentAL	1	1
StudentAM	3	1
StudentAN	2	3
StudentAO	2	5
StudentAP	1	3
StudentAQ	5	1
StudentAR	3	1

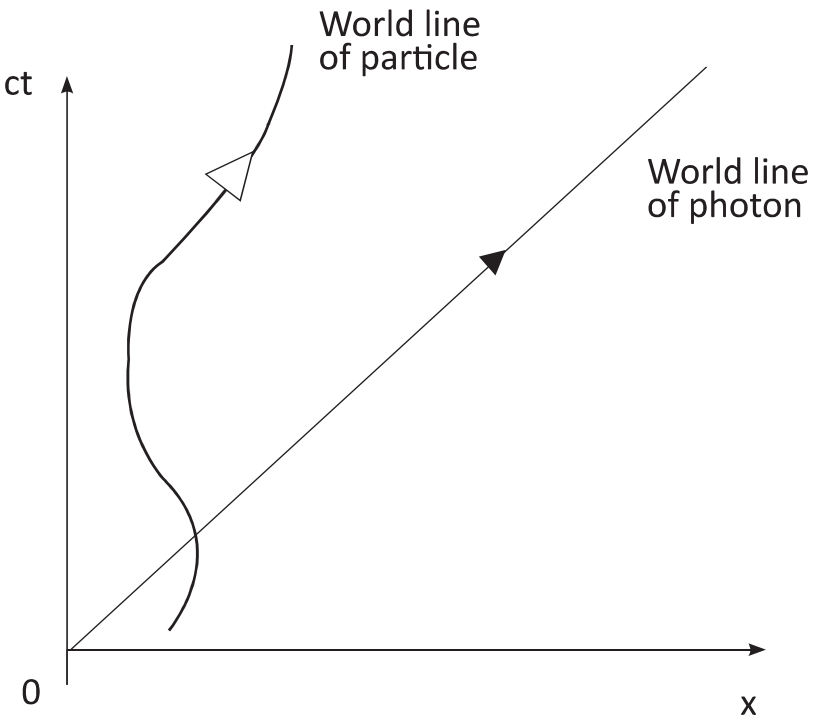

Figure 1. World-line of a generic particle and a light ray in the bidimensional Minkowski space-time (c is light speed in vacuum: Spatial distances are measured in time intervals on the ct axis).

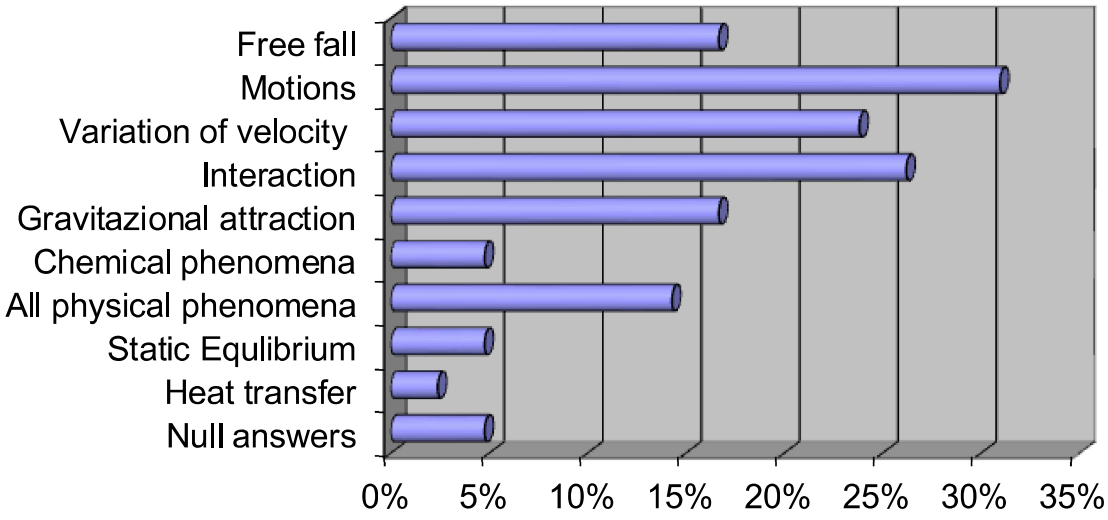

Figure 2. Mass in everyday life: Typologies of evoked phenomena (answers to C1 in the final questionnaire).

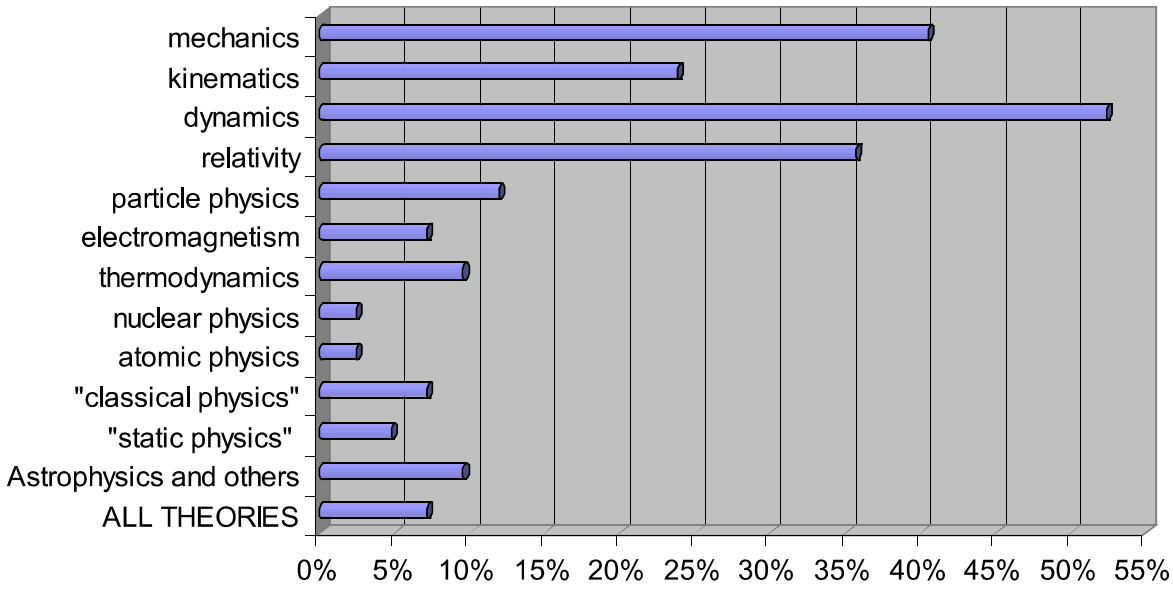

Figure 3. Physics theories and sectors concerning the phenomena previously recalled (answers to C2).

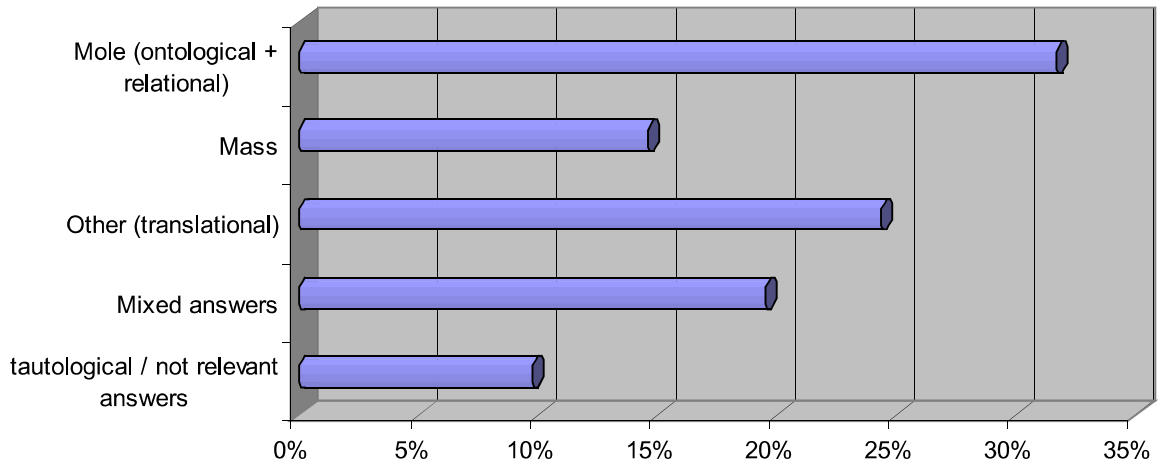

Figure 4. Students' conceptions of quantitas materiae; Other = N_A (Avogadro's number), density, "mass concentration in a given volume", number of molecules or atoms or particles in a body (answers to C3).

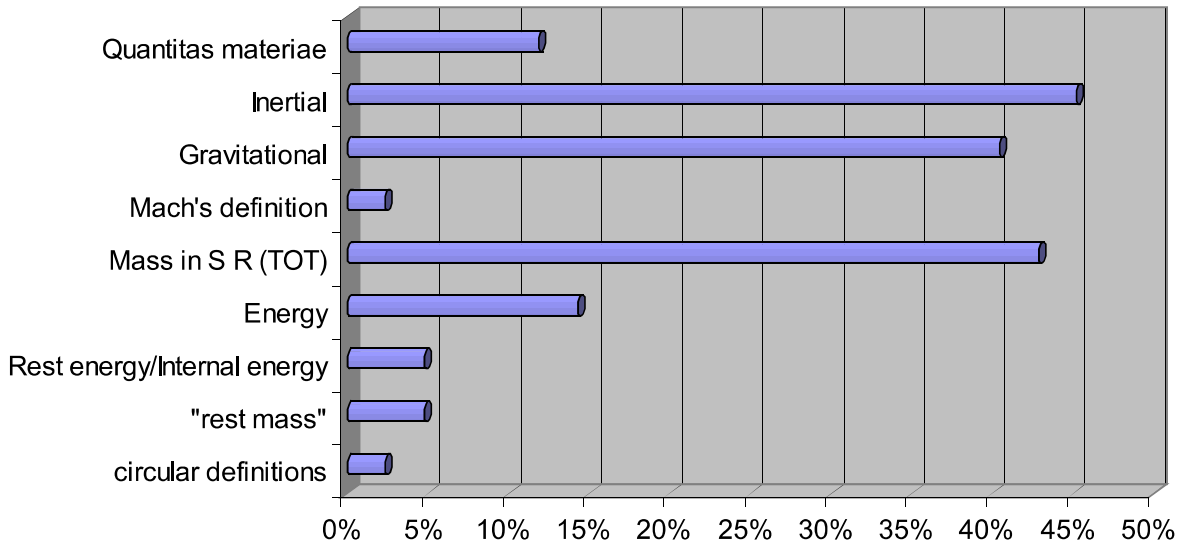

Figure 5. Facets of mass present in the answers to C4.

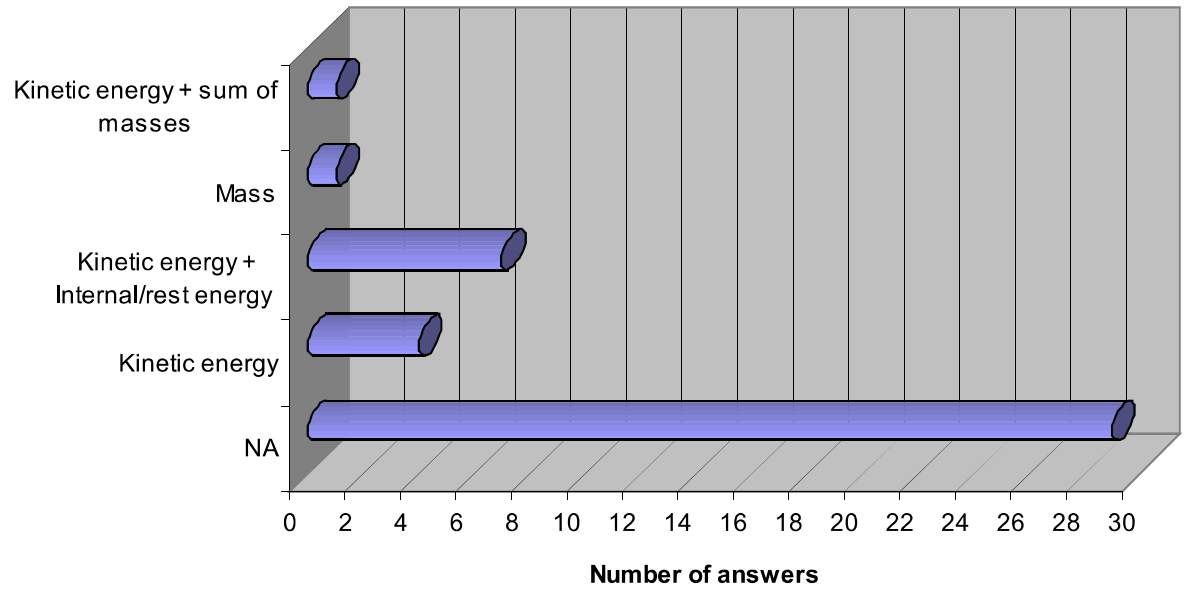

Figure 6. Forms of energy varying in a collision between two identical particles (inner relativistic question).

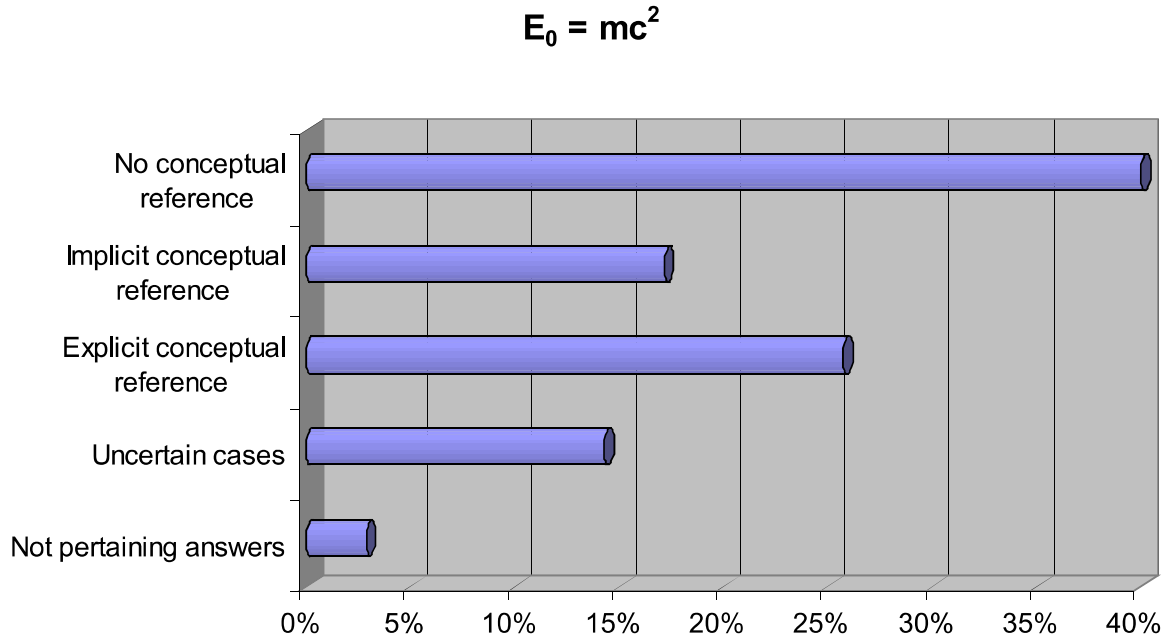

Figure 7. Conceptual reference to mass-rest energy equivalence $E_0 = mc^2$ (R1).

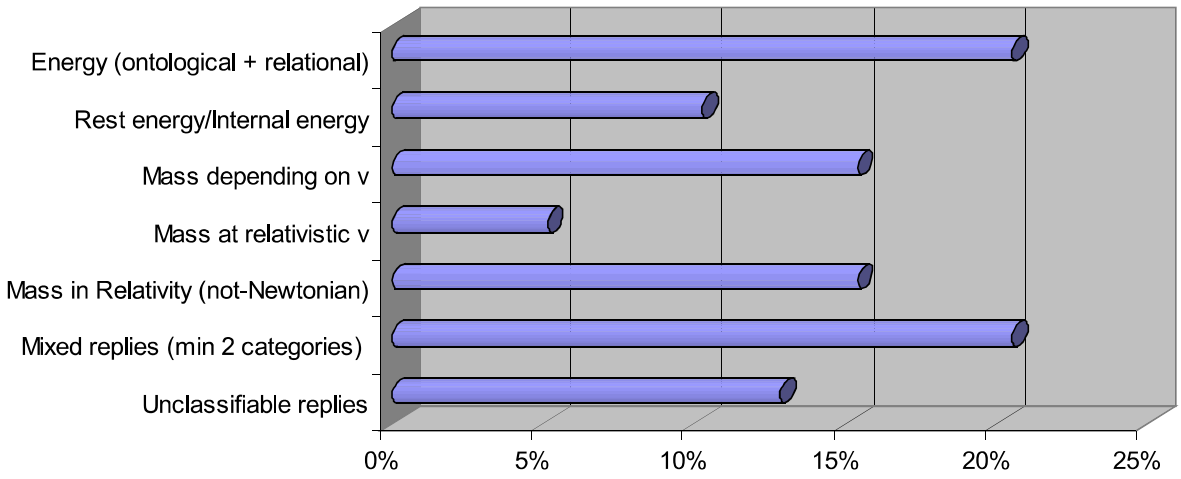

Figure 8. Conceptions of "relativistic mass" (answers to R2).

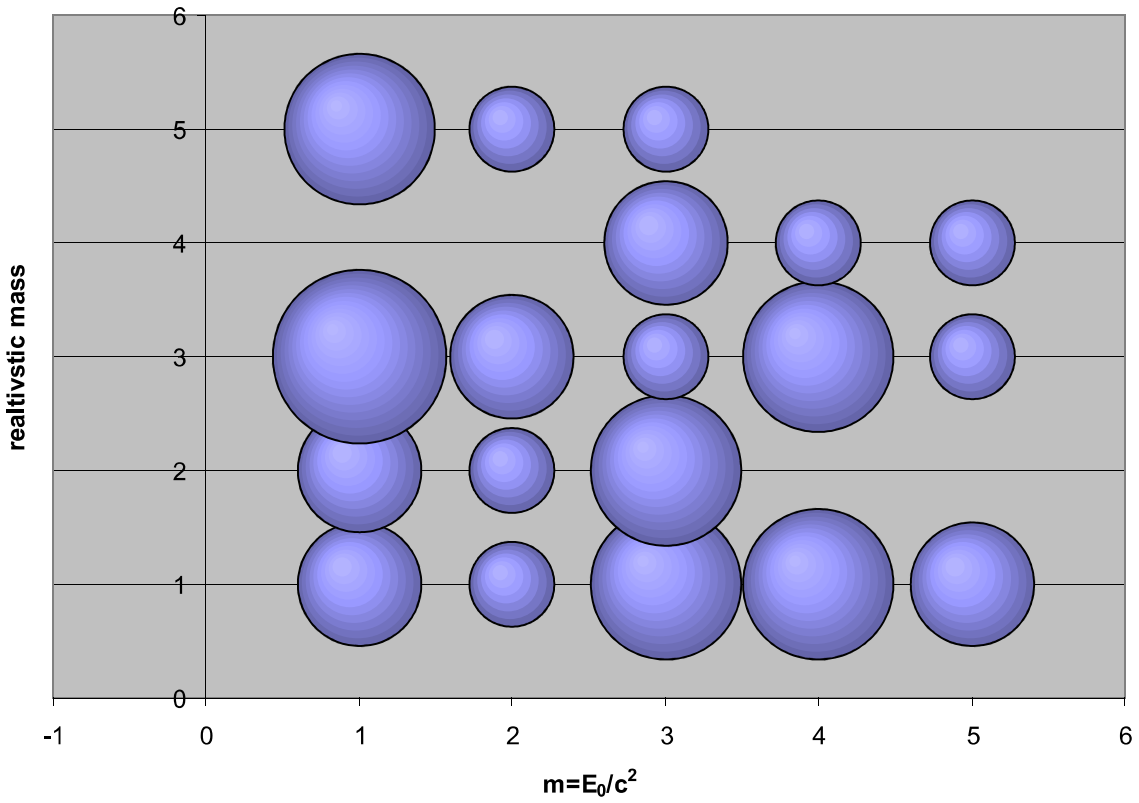

Figure 9. Ranks correlation (the dimension of each bubble is proportional to the frequency of the corresponding couple of data)

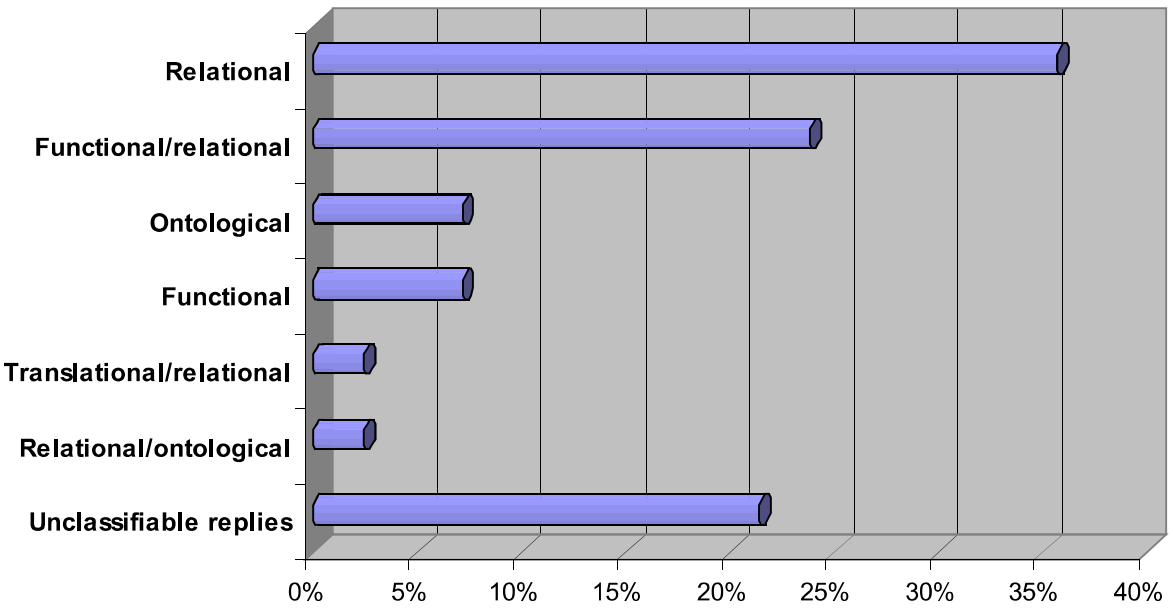

Figure 10. Students' profiles (worked out by 'horizontal' analysis).