

Outdoor activities to discover Medians and Centroid of Triangles

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The article describes those steps that led students to the discovery of properties regarding triangles, to the construction of median, perpendicular bisector (axis), and notable points. The approach used is that of Inquiry and Embodied cognition in outdoor context. The activity involved 15 students from a middle school in Trieste, Italy. The results were tested by proposing to the students 5 types of exercises on axis, medians, heights and bisectors and the notable points connected to them. Success rates were averaged and divided those that were subjected to outdoor activities (axis and median) compared to those that were carried out in the classroom (heights and bisectors). A further comparison was made between the success rates in drawing the objects under investigation. Finally, we investigated through Mentimeter the students' appreciation of the outdoor activity. 100% of students found the activity fun and helpful. Several used terms such as “beautiful, joy, happiness”. The 85% of students considered group work to be informative and useful, and the words “interesting, informative, learning” were reported by almost all students.

Introduction

The lack of cognitive prerequisites based on personal experience, sensory deprivation, the lack of direct experiences are elements of risk that we detect in today's young people and that lead them to have difficulties even when they need to analyse, deduce, abstract. The term “educate” derives from the Latin *ex ducere*, “to lead out”, in the sense of trying to get the best out of each student but it can also be interpreted as “to lead out” from the classroom. We are in a pandemic era. The need to move is even more important, like that of being together, collaborating, asking, and solving problems, breathing outdoors. Italy has had an extraordinary history of open-air public schools that began in the early 1900s for weak, fragile children who risked contracting TB (D’Ascenzo, 2018). After some experience it was noticed that these students learned more and better than the others, so they were also extended to “normal” children. The experience continued even during fascism. After the war it resumed strongly and ended in 1977 in Imola (Bo), when the last outdoor school closed, under the pressure of full-time and declining enrolments. In 2016, the “National Network of Outdoor Schools” was established in Italy, which groups and harmonizes the different experiences of a structured outdoor school system, where lessons outside the classroom can be taught, promoting students to rediscover a relationship with nature. More than 53 institutes currently belong to it. In the early 1900s, the outdoor school movement was widespread in many countries (Germany, England, France, Spain, United States, etc.). Recently, in some countries, there has been an increased interest in the development of outdoor and adventure education programmes (Fägerstam & Samuelsson, 2012). It would be an opportunity to use the virus catastrophe to change schools using the good practices of the past that have a lot to say even in the “digital” era. Never like now adolescents and young people need a true relationship (not only digital) with the teacher, to take responsibility, to learn from experimentation, to do

manual and artistic work, not to have the textbook and the program as the only obsessive reference.

The purpose of this article is twofold: on the one hand, it is intended to show how an outdoor activity should be presented with a view to the Embodiment, the Inquiry and with the aim of facilitating peer collaboration; on the other hand, test whether outdoor activities have been effective for understanding concepts and whether they have been appreciated by students. The activity involved 15 students from a middle school in Trieste, Italy.

Theoretical Background

Learning outside the classroom essentially can be defined as use of resources out of the classroom to achieve the goals and objectives of learning (Knapp, 2010; Smith & Walkington, 2020). The constant focus on textbooks and formal mathematical practice might invoke a view among students that mathematics is abstract, distanced and only useful in a classroom context working only in the textbook (Boaler, 1998). Existing research on outdoor learning in mathematics indicates positive affective outcomes and possible academic benefits from learning mathematics in an out-of-school context (Daher & Baya'a, 2012; Moffett, 2011; Noorani et al., 2010). Moreover, outdoor environments, are real-life contexts enabling children to internalise, transfer and apply mathematical ideas and provides direct experience, and the students need to be active in the learning process (Moffett, 2011). It lends itself to the Inquiry-based mathematics education a student-centered form of teaching whose guiding principle is that the students are supposed to work in ways like how professional mathematicians work (Artigue & Blomhøj, 2013; Council, 2000; Dorier & Maass, 2014). This means they must observe phenomena, ask questions, look for mathematical and scientific ways of answer these questions, interpret, and evaluate their solutions, and communicate and discuss their solutions effectively. The main feature of cooperative learning is the opportunity to discuss and reason with others and justify one's mathematical thoughts on how to solve different mathematical problems. Cooperative outdoor learning in mathematics gives the possibility to observe that a task at hand can be solved in more than one way and that more than one "right" solution to the problem may exist.

The sensorimotor experiences arising from the environment also play a paramount role in learning (Wilson, 2002). Embodied cognition is described as a bodily sense of knowing, expressed through physical movement and sensory exploration with environments (Kim et al., 2010; Merleau & Ponty, 2002; Smith & Gasser, 2005; Varela et al., 1991). There is complexity in the processes that may be involved in the development of embodied cognition as "knowledge depends on being in a world that is inseparable from our bodies, our language, our social history" (Varela et al., 1991, p. 173). According to the theoretical framework of embodied cognition, conceptual representations are grounded in different modalities, i.e., perceptual, motor, emotional (Barsalou, 2008). According to Glenberg (2010) perception and how memory works is affected by how people move their bodies. To that vein, Hu, Ginns et al. (2015) suggested that pointing and tracing gestures might enhance geometry learning by activating an "increased working memory channel". Current literature emphasises

the role of gestures as semiotic tools, contributing to deeper understanding of mathematical concepts (Arzarello et al., 2009).

Learning geometry can foster the ability to think logically, develop problem solving ability and reasoning, and can support many other topics in mathematics. Research has discussed geometrical properties of a figure (Duval, 1995; Fischbein, 1993; Laborde, 2005; Mariotti, 1995). According to Carden & Cline (2015) visualization refers to mental processes that describe visual or spatial information. Furthermore, for Arcavi (2003), visualization is a process that is the result of the creation, interpretation, reflection of images, diagrams with the aim of describing and communicating information on the development of an idea that is not known in advance to obtain a higher understanding.

The methodology

In the first part we describe those steps that led students to the discovery of properties regarding triangles, to the construction of median, perpendicular bisector (axis) and notable points. The approach used is that of Inquiry and Embodied cognition in an outdoor context. The activity takes place in the “Classroom under the sky”, so called the outdoor amphitheatre that was inaugurated this school year after more than a year of work. The environment is already welcoming in itself: a small pond right on the edge of a laurel grove, an open lawn that converges to the maple tree in the centre of the space, under which a blackboard and seats for students are placed. The students can also make use of portable shelves, to support books and notebooks (see https://www.youtube.com/watch?v=IGJbz_d7OU&t=80s for other examples). The activity involved 15, six grade students, four boys and eleven girls, from a middle school in Trieste, Italy. Students were randomly divided into four groups of 4/5.

The results were tested by proposing, to the same 15 students, 5 types of exercises on axis, medians (which were taught outside the classroom), heights and bisectors (which were taught in the classroom), and the notable points connected to them. Success rates were averaged and divided those that were subjected to outdoor activities compared to those that were carried out in the classroom. A further comparison was made between the success rates in drawing the objects under investigation.

Finally, we investigated through Mentimeter the students’ appreciation of the outdoor activity.

The activities

Today is the day of the discovery of the rules of drawing, those rules that lead hand and body to trace elements that take position on the surface of the lawn, with established criteria and simple tools, ancestors of the modern squares and compasses that crowd our school desks. Ropes, broomsticks, wooden stakes are our tools and some plastic caps. The square is the same one used by the ancient Egyptians 5 thousand years ago: 1 piece of rope divided, by means of knots, into 12 equal pieces. Here is a short video of the activity carried out: tinyurl.com/outdoortriangles.

The perpendicular line and the axis

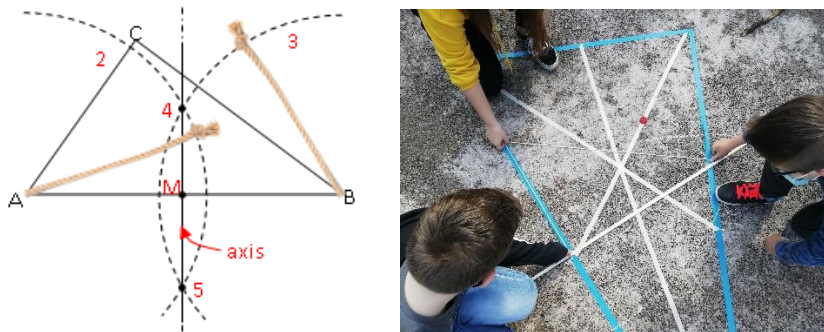
The activity starts by asking the kids to build a triangle by giving them 3 pieces of wooden planks deliberately of inappropriate lengths. They had to find out why it was not possible to do it and obtain so the fundamental property: The sum of the lengths of any two sides of a triangle is always greater than the length of the third side. The fact that they do not have a plane on the ground creates the first problems, which are solved by accepting that the built triangle is not perfectly parallel to the ground.

After having built a triangle with wooden planks, each group is asked to identify the midpoint of each side of the triangle. One group uses a piece of rope as a unit of measurement to measure one side of the triangle and then calculates the half dividing the result by two, another group uses a rope to measure one side, they join the two extremes obtained to have the half.

At this point the teacher's suggestion is needed, because the students still do not know certain techniques (Figure 1). By fixing one end of the string to the vertex of the triangle (A), a sufficiently wide arc is drawn (2), it is possible to identify all the points that are at the same distance from that vertex: we speak of an arc of circumference. We move to vertex (B), belonging to the same side (A and B are the ends of the same segment), repeating operation (3) thus identifying two crossing points of the arcs which are both at the same distance from the ends of 4 and 5. By joining these two points, a straight line is drawn (i.e., a segment, with a beginning and an end: axis), which marks its midpoint (M) on the side. Now it is their turn: Students understand that they must position themselves on the vertices of the triangle, so crouching on the ground, bent over their knees, they hold down the vertex of competence with their fingers on the ground. Another belonging to the group hands over a head of the rope to the companion who keeps it stuck on a vertex. Those who work on the flat surface have an advantage, but everyone succeeds. After leaving the vertex, as there is no other classmate to help, a student identifies another point of the rope so that the distance from the first extreme is more than half the side, which is carried out by estimating the position of the midpoint. She also lowers herself, captures between index, thumb and middle this extreme of the rope and, with some difficulty, also binds the chalk to proceed with the formation of the arc. At the teacher's suggestion, it is easier to form a buttonhole on the rope and insert chalk into it in such a way as to be freer in the movement. At this point, with the back bent but in an upright position, the pupil draws an arc: his task is to keep the rope always in tension, so that the radius remains constant.

It is time for the Egyptians! Some students suspect that this axis (and the other two on the other sides as well) form right angles to the side. Three students take the knots of our piece of rope divided into 12 equal pieces every 3, 4 and 5 segments of our and stretch it to obtain a right triangle. This impression is confirmed by positioning our large right-angled triangle of rope with the right angle placed on the identified midpoint (M). It is really true! The traced segment passes through the midpoint of the side perpendicularly. It is now up to the teacher to propose a name for this remarkable line: the axis.

Figure 1. The Axis



The Circumcentre and the circumference circumscribed

We note that in all the triangles of the groups the three axes cross in a single point. But “obtuse triangle group” has a strange design: did they do something wrong? Their point lies outside the triangle, on the longest side, the one opposite the obtuse angle. They explain how they proceeded, and the “acute triangle groups” confirm that the classmates followed the correct procedure. The peer review is a fundamental aspect in mathematics: it favors collaboration, the exchange of ideas, critical observation. It is precisely by leveraging on collaboration that students can build theorems themselves and then have the satisfaction that the theorem will not take the name of some important mathematician but will be reported in the notebook with the acronym of their names.

Let us go back to the obtuse triangle and the axis. The only thing that has changed compared to the other “experiments” is precisely the shape of the triangle, so we can argue that in the obtuse triangles the point of intersection of the axis ends outside the triangle. It is time to give a name to this center-point: we call it Circumcentre. At this point the students arrange themselves around the triangles drawn on the ground and look for some characteristic that may seem random but it is not. In the triangle shown below, which has the particularity of being a right triangle, the crossing point is neither “inside” nor “outside”, but it is “up”: it is in fact on one side; the longer one, facing the right angle. Young people do not yet know that we call it hypotenuse, there is time ... They are asked to observe some particularities in this and in the other triangles. Some suspect that the distances from the vertices to the circumcenter may matter. Let us check that's right! And here the circle comes into play, which although not treated in the sixth grade, always arouses its charm.

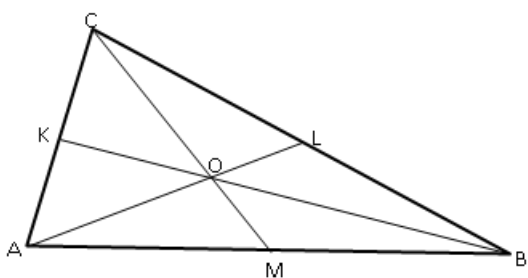
The end of the rope is now fixed on the circumcenter while a student stretches the rope until he leans on a vertex. He holds the mark between his index finger and thumb and verifies that rotating around the triangle for 360° touches all the vertices. The circumference is circumscribed and therefore the point is called circumcenter.

Galvanized by the result, we continue to take advantage of the fact that we have already identified the midpoints of the sides. What could be done with those points? Some propose to join them, others to join them at opposite summits. Both hypotheses are tested.

The centroid

The students join each vertex of the triangle to the midpoint of the opposite side: all these segments cross in a single point, O (Figure 2). The name of a geometric part is almost never accidental: since it arrives at the midpoint of the side, these segments are called medians. The meeting point is called centroid and it is the centre of gravity.

Figure 2. The Centroid



Once again, the challenge of identifying properties starts. The students soon discover that this point divides each median into two parts such that the one containing the vertex is longer than the other. Invited to measure better, the students do not take long to discover that the first part is exactly double the other, thanks to repeated tests with the rope on different triangles.

The test

The next day a test containing 5 exercises was administered to attest the knowledge learned on axis and medians experienced outdoors as well as in class, but also height and bisectors learned only in class.

Exercise 1. “Keywords table”: Write an “X” in the cell where there is a characteristic that serves to define the notable segment of the triangle (height, median, bisector or axis).

Exercise 2. “Definitions”: Define: The bisector is ... The height is ... The median is ... The axis is ...

Exercise 3. “Characteristics”: Report at least one characteristic of each segment and each notable point of the triangle: Medians ... Bisectors ... Axis ...

Exercise 4. “Remarkable points”: Fill in the table indicating in which triangles the notable points are internal (write “int”), external (write “ext”) or lie on one side or a vertex (write “up”).

Exercise 5. “Drawing”: Draw the required parts on the triangles, identify the crossing point; when you can, draw the inscribed or circumscribed circumference.

Results

Table 1 shows the averages of the success rates on all 5 types of exercises that concerned axis and medians (first row), topics addressed outdoors as well as in class, with the same ones on heights and bisectors (second row). The differences of the previous percentages have been added in the third row.

Table 1. Success rate

	Success rate														
Axis & Median	73	61	39	54	35	80	93	73	61	79	68	73	13	35	70
Height & Bisector	63	45	25	31	25	82	92	19	43	41	47	44	0	30	65
Differences	10	16	13	23	10	-1	1	54	18	37	21	29	13	6	5

We immediately notice a significantly higher score (except in one case) of the exercises on the topics covered outdoors. Overall, the results of the test on axis and medians, treated outdoors, is better than the part treated in the classroom. Only for a student, with excellent results, the results are comparable: correct answers “outdoors” 80%, “indoors” 82%. For 5 of the other 15 students, the outdoor activity resulted in an advantage of more than 20%. In outdoor activities, the results are higher and homogeneous: a percentage change coefficient of 50% for axis and 60% for medians. In indoor activities the results are lower and heterogeneous: C.V.% 103% for heights and 99% for bisectors.

Examining specifically the results obtained in Exercise 5 on drawings, we built a table like the previous one.

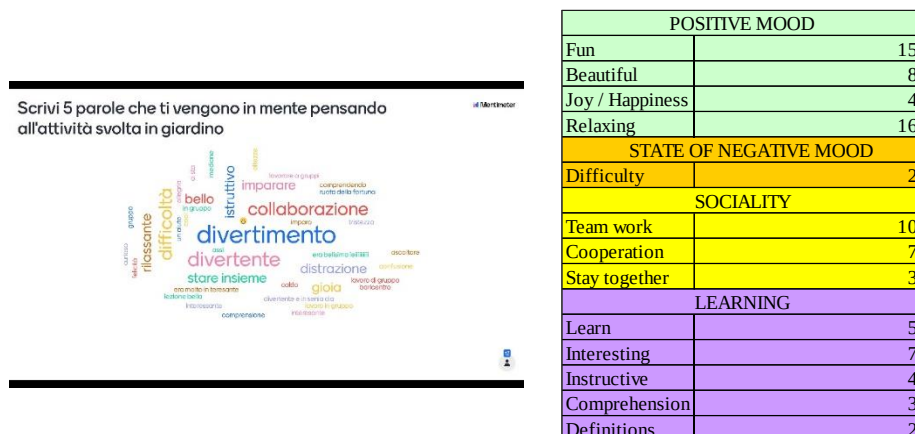
Table 2. Exercise 5: Success rate

	Success rate of Exercise 5 on Drawing														
Axis & Median	100	60	35	23	50	100	100	100	100	100	100	100	40	55	100
Height & Bisector	90	53	23	20	0	100	100	13	23	23	53	23	0	13	100
Differences	10	7	12	3	50	0	0	87	77	77	47	77	40	42	0

The difference between outdoor and indoor activities (see Table 2) is even more evident: the average success expressed as a percentage of the maximum score is for outdoor activity equal to 75% (axis) and 70% (median) while for indoor activity is equal to 38% (heights) and 41% (bisectors).

Finally, we wanted to investigate through Mentimeter the students’ appreciation of the outdoor activity. The students were asked to write the first 5 words that came to mind when thinking about the outdoor activity. The results are summarized in the Figure 3 (on the left) below together with the same request made the next day in the light of the previous answers (on the right). The expression “funny / fun” was the most voted followed by “collaboration / working with and in group / group / being together”.

Figure 3. Mentimeter



Conclusions

Covid-19 also launches a challenge to schools today in a strong crisis and that of outdoor schools is a real way that connects students with reality, nature, dexterity, art and a new responsibility towards creation, others, themselves. The reduction of socialization opportunities has brought several psychological miscomprehensiveness in adolescents: panic crises, cutting phenomena, anxiety. It should therefore come as no surprise that fun and socializing activities are of interest and approval. The survey carried out anonymously at the end of the outdoor lesson shows that 100% of students found the activity fun. Several used terms such as “beautiful, joy, happiness”. The 85% of students considered group work to be formative and useful, and the words “interesting, formative, learning” were reported by almost all students. Test results have also always been to the advantage of outdoor activity.

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