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Bowing virtual strings with realistic haptic feedback

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ABSTRACT

We present a music interface implementing a bowed string. The bow is realised using a commercially available haptic device, consisting of a stylus attached to a robotic arm. While playing the virtual strings with the stylus reproducing the bow, users feel both the elastic force from the strings and the friction resulting from the interaction with their surfaces. The audio-haptic feedback is obtained by a physical model: four stiff strings are simulated using a finite difference time domain method, modelled as 1-Delements in the virtual 3-D space. The bow is simply modelled as a rigid cylinder that can move free in this space, and interact with the strings. Finally, the frictional interaction between such elements is modelled by a nonlinear friction model capable of reproducing the characteristic stick-slip phenomenon observed during string bowing. Moreover, the model can be dynamically controlled in one parameter so as to become more sticky or slippery. By turning on and off the frictional feedback, users can appreciate the significance of this interaction. A real-time visualisation of the bowed strings complements the audio-haptic display.

Keywords: Musical interface, Digital music instrument, Haptic feedback, Bowed string model, Finite difference

1 INTRODUCTION

In traditional musical performance, there exists a physical coupling between the musician and their mechanical instrument. This follows, as highlighted by Luciani et al. [1], from the energetic exchange between the human instrumental gestures and the sound being produced. According to O'Modharin and Gillespie [2], this energetic flow starts from the mechanical energy injected into the instrument by the performer, whose aim is to produce acoustic energy via the instrument's resonance. Some of this injected energy is stored in the instrument itself, which is to be reflected back to the performer at some point, e.g. in the form of haptic feedback. They conclude that this back and forth coupling of energy informs "the idea that an instrument becomes the extension of a player's body", a feeling that is shared by many musicians according to Nijs et al. [3].

In contrast to this energetic feedback loop interaction, the performance of digital musical instruments (DMIs) typically has a unidirectional energy path, namely from the musician towards the instrument. Such instruments are defined according to Miranda and Wanderley [4] as devices split between two independent units: a control surface and a sound generation unit, linked together by mapping strategies. However, this independence does not have to be the norm and it follows perhaps due to an aim toward generality in terms of possible sounds for a given control surface—at least for the case of software sound generating units, e.g. illustrated by the ubiquity of the MIDI keyboard as a possible controller for almost any software instrument.

DMIs are increasingly becoming a larger part of music production. In the case of pop music, the backbone of a music market sitting at an all time peak [5], synthesizers and samplers provide most if not all of the sounds on many recordings, according to Warner [6]. The accessibility of such instruments as software applications gives rise to large communities of learners and creators who share, play and teach music on web platforms further redefining notions of music making [7] and reinforcing the position of DMIs as part of the general musical canon. Therefore, one could argue that there should be a greater emphasis on DMIs in the field of musician-instrument interaction.

As part of a more general trend of haptic based applications in multimedia [8], audio-haptic musical applications based on interactions with virtual strings have shown a growing interest in the sound and music computing community. Willemsen et al. [9] used an older version of the Touch haptic device used in the current project, to control a physical model of a tromba marina—a bowed single-string instrument, in a virtual reality (VR) environment. The interaction included the elastic response of the string (through the collision between VR objects) but no frictional feedback during bowing was included. Keeping in the realm of VR, Fontana et al. [10] as well as Passalenti et al. [11] investigated the effect of haptic feedback with respect to plucked guitar string simulations. Concerning bowed strings, Sinclair et al. [12] included frictional force feedback in their digital waveguide based model, but however, it resulted from a different model than the one used for auditory feedback. Other custom-made haptic interfaces, including bowing, have been designed by researchers at ACROE in Grenoble to control physical models built from systems of interconnected mass-springs [13, 14, 15]. A project presenting a user study comparing the experience of rubbing virtual mass-spring-dampers by means of different control strategies, including the use of a 3D Systems haptic device, was carried out by Onofrei et al. [16]. The aim of the different controls was to mimic with increasing accuracy the gestural action of rubbing, culminating with the presence of frictional haptic feedback resulting from the actual simulation of the frictional interaction, i.e. the haptic friction force was the same as the force which excited the sound synthesis model. The music interface appearing in this paper is rooted on the design just described.

In this paper we present an implementation of a real-time physically-modeled music application simulating the bowing of strings, controlled via an interface which allows not only for intuitive instrumental gestures but also for realistic mechanical feedback as a result of the physical simulation. This is achieved by making use of the features of a 3D Systems Touch haptic device: in particular, by mapping the virtual bow-string interaction point on the tip of its stylus and by rendering both the elasticity and the frictional force of the strings, as resulting from the interaction of the bow with their surfaces. The strings are placed in a 3-D virtual space in a trapezoidal cross section, similar to how they would be placed on the neck of a violin. This allows for the bow to interact either with individual strings but also with pairs of strings at a time, as they lie on the same plane. Additionally, the displacement of the elements in the 3-D space is visualized at runtime, complementing the audio-haptic interaction. The setup aims at preserving a consistent link between the control and the synthesized sound, as suggested by Cadoz [17], with the energetic feedback loop of the instrumental gesture together with the haptic feedback giving rise to subtle nuances in the timbre and the dynamics of the sound. This link is greatly facilitated by the use of physical modelling synthesis, as all the different modalities—sonic, tactile and visual—result from the solution of the same mechanical system.

The rest of this paper is structured as follows: Section 2 presents the physical model behind the audio-haptic synthesis, Section 3 describes the control interface and introduces the haptic device, then Section 4 presents the real-time application illustrating the graphical user interface (GUI) and details the mapping between the app (particularly the physical model behind it) and the haptic device. A discussion follows in Section 5 focused on the playability of the app, while the final Section 6 concludes the paper and introduces possible future work.

2 PHYSICAL MODEL

This section describes the physical model used in this project and provides some considerations for its discretisation.

Consider a damped stiff string with a circular cross-section of length L (in m) defined for time $t \geq 0$ (in s) and space $x \in \mathscr{D}$ (in m) with domain $\mathscr{D} = [0, L]$. One can describe its transverse displacement by state variable $u = u(x,t)$ (in m), and after adding a bow excitation, its dynamics can be described by the following partial differential equation (PDE) [18, 19]

$$\partial_t^2 u = c^2 \partial_x^2 u - \kappa^2 \partial_x^4 u - 2\sigma_0 \partial_t u + 2\sigma_1 \partial_t \partial_x^2 u - \frac{F_{fr}}{\rho A} \tag{1}$$

$$F_{fr} = \delta(x - x_{\mathrm{B}}) f_B \Phi(v_{\mathrm{rel}}, a) \tag{2}$$

where ∂_t and ∂_x describe a derivative with respect to time and space respectively. The parameters are as follows: wave speed $c = \sqrt{T/\rho A}$ (in m/s) with tension T (in N), material density ρ (in kg/m^3), cross-sectional area $A = \pi r^2$ (in m^2), radius r (in m), stiffness coefficient $\kappa = \sqrt{EI/\rho A}$ (in m^2/s), Young's modulus E (in Pa), area moment of inertia $I = \pi r^4/4$, frequency-independent loss coefficient σ_0 (in s^{-1}) and frequency-dependent loss coefficient σ_1 (in m^2/s). The final term in Eq. (1) describes the bow frictional force excitation, F_{fr} (in N), using the following friction model [19]:

$$\Phi(v_{\text{rel}}, a) = \sqrt{2a} v_{\text{rel}} e^{-a v_{\text{rel}}^2 + 1/2}, \tag{3}$$

characterized by the friction parameter a (in s^2/m^2). Equation (3) is nonlinearly dependent on the relative velocity between the string—at externally supplied bow location $x_{\text{B}} = x_{\text{B}}(t) \in \mathscr{D}$ (in m)—and the bow:

$$v_{\text{rel}} = v_{\text{rel}}(t) = \partial_t u(x_{\text{B}}, t) - v_{\text{B}}, \tag{4}$$

where $v_{\text{B}} = v_{\text{B}}(t)$ is the externally supplied bow velocity (in m/s). The bow is located along the string using the spatial Dirac delta function $\delta(x - x_{\text{B}})$. Finally, the externally supplied bow force is $f_{\text{B}} = f_{\text{B}}(t)$ (in N). In this work, the boundary conditions of Eq. (1) are chosen to be simply supported such that

$$u = \partial_x^2 u = 0, \quad \text{at} \quad x = 0, L. \tag{5}$$

2.1 Discretisation

This work uses finite-difference time-domain (FDTD) methods to approximate the equations above. These methods discretize the continuous system into a grid in space and time according to $x = lh$ with spatial index l and grid spacing h (in m), and $t = nk$ with temporal index n and time step $k = 1/f_{\text{s}}$ (in s) and sample rate f_{s} (in Hz). The state variable $u(x,t)$ then becomes a grid function u_l^n which describes a grid point with spatial index l at temporal index n. A full discretisation of the bowed stiff string will not be given here for brevity, but is well covered in the literature (see e.g. [19]). A full derivation of the scheme used in this work can be found in [20, Sec. 8.4]. Instead, several considerations for discrete implementation will be given here. Notice that the various parameters receive a superscript n to indicate that they are time-varying.

For approximating first-order temporal derivatives there are various options, including the forward, backward and centred differences [19]. The latter can be proven to be second-order accurate and is defined as

$$\delta_{t\cdot} u_l^n = \frac{1}{2k}\left(u_l^{n+1} - u_l^{n-1}\right). \tag{6}$$

In this work, this approximation is used to discretise Equation (4) yielding

$$v_{\text{rel}}^n = I_l(x_{\text{B}}^n)\delta_{t\cdot} u_l^n - v_{\text{B}}^n \tag{7}$$

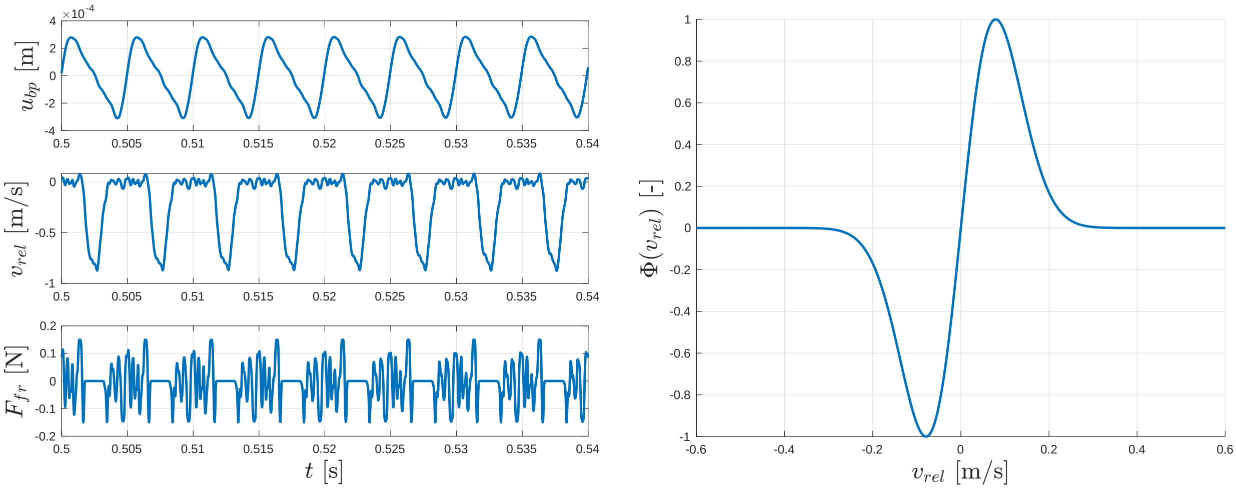
where $I_l(x_{\text{B}}^n)$ is an interpolation operator retrieving the state of the discrete string at location x_{B}^n, and is defined as [19]

$$I_l(x_{\text{i}}) = \begin{cases} -\alpha_{\text{i}}(\alpha_{\text{i}}-1)(\alpha_{\text{i}}-2)/6, & l = l_{\text{i}}-1, \\ (\alpha_{\text{i}}-1)(\alpha_{\text{i}}+1)(\alpha_{\text{i}}-2)/2, & l = l_{\text{i}}, \\ -\alpha_{\text{i}}(\alpha_{\text{i}}+1)(\alpha_{\text{i}}-2)/2, & l = l_{\text{i}}+1, \\ \alpha_{\text{i}}(\alpha_{\text{i}}+1)(\alpha_{\text{i}}-1)/6, & l = l_{\text{i}}+2, \\ 0, & \text{otherwise}, \end{cases} \tag{8}$$

with $l_{\text{i}} = \text{floor}(x_{\text{i}}/h)$ and $\alpha_{\text{i}} = x_{\text{i}}/h - l_{\text{i}}$. As v_{rel}^n is used in a nonlinear function (see Equation (3)), the scheme is now implicitly dependent on u_l^{n+1}. In this work, the iterative Newton-Raphson method is used at every sampling step to solve the nonlinear bow function with a maximum of 99 iterations per sample.

The output of the discrete system can be retrieved by selecting a grid point and 'listening' to this over time. As the displacement of a medium is related to sound pressure through a temporal derivative [21], the output is retrieved as follows:

$$\text{out}[n] = I_l(x_{\text{o}})\delta_{t\cdot} u_l^n \tag{9}$$



(a) (b)

Figure 1. (a) Simulation results for the bowing of a stiff string tuned to 196 Hz. Top: String displacement at bowing position, u_{bp} (in m). Middle: Relative velocity between the string and the bow at bowing position, v_{rel} in in ($\mathrm{m}^2/\mathrm{s}^2$). Bottom: Resulting frictional force at bowing excitation point, F_{fr} n in (N). (b) Shape of the dimensionless nonlinear friction characteristic, $\Phi(v_{rel})$ using the friction parameter $a = 80$ (in $\mathrm{s}^2/\mathrm{m}^2$)

where $x_\mathrm{o} = 0.7L$ is the output location (in m).

Finally, the spatial Dirac delta function in Eq. (1) is discretised using a cubic spreading operator $J_l(x_\mathrm{B}) = I_l(x_\mathrm{B})/h$ making changes in the bow location smooth.

2.2 Prototype model results

A prototype implementation of the physical model was first carried out in an offline setting using Matlab[1]. This allowed for more flexibility with respect to investigating the results and calibrating the physical parameters, with their choice based on [20].

Of notable interest was the behavior of the friction model and its capability of producing the stick-slip behavior typical of frictional interaction, which gives rise to the experimentally observed Helmholtz motion of bowed strings [22].

Figure 1a shows the displacement of the string at the bowing position, $u(x_B)$, with $x_B = 0.25L$, together with the relative velocity between the bow and the string v_{rel} as well as the resulting frictional force resulting from the bowing simulation of a steel string tuned to a fundamental frequency of 196 Hz, at a sampling frequency f_S of 44100 Hz. The tuning is achieved by setting the relevant string tension T in the model. Other physical parameters of the string are: $L = 0.5$, $\rho = 7850$, $r = 0.0005$, $E = 2 \cdot 10^{11}$, typical to a steel violin string. Furthermore, the damping parameters are set to zero and the friction parameter is taken as $a = 80$, resulting in the shape of the nonlinear friction characteristic $\Phi(v_{rel})$ shown in Figure 1b. The string is bowed with a constant bow velocity $v_\mathrm{B} = 0.2$ and bow force $f_\mathrm{B} = 0.15$. Units for these parameters were introduced in the previous section. The stick-slip behavior can be seen when looking at v_{rel}, with the value hovering over zero for the most part of the periodic motion, indicating that the bow and the string are stuck together, followed by an abrupt slip in which the magnitude of v_{rel} greatly increases. This results in a triangular wave shape in the displacement of the string. Additionally, during the slipping phase, the resulting frictional force can be observed to be zero, as there is essentially a loss of contact between the bow and the string, i.e. no interaction.

[1] https://www.mathworks.com/

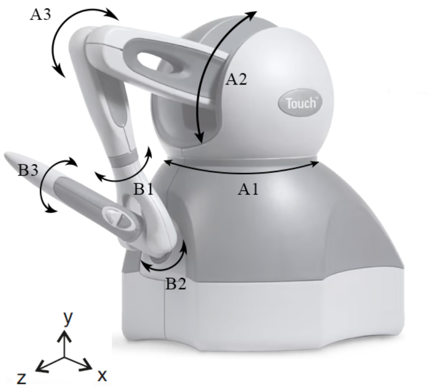


Figure 2. 3D Systems Touch haptic device.

3 CONTROL INTERFACE

To control the physical model system described in Section 2, a suitable interface needs to be found. The requirements of this interface are: 1) to enable flexibility in terms of instrumental gesture motion, and 2) to allow for the corresponding instantaneous haptic feedback.

3.1 Haptic Device

We found that the Touch professional haptic device manufactured by 3D Systems [23] is an ideal candidate for this task. It is a 6-degree of freedom haptic system equipped with silent internal motors which can provide force feedback in 3-degrees of freedom. The device is illustrated in Figure 2 together with its local coordinate system (CSYS) and the pivot joints B1, B2 and B3 which enable the translation in x, y, z directions of the gimbal joint (equivalent to pivot joint B2) and the 3 possible rotations of the stylus pen. This flexibility of motion in the 3-D space allows for replicating the gestural motion of bowing. In fact, one can hold the stylus in a similar fashion as they would hold an actual bow. Furthermore, the pivot joints A1, A2 and A3 provide force feedback, as opposed to the B joints which are only for rotational degrees of freedom. The combination of the force feedback in the A joints is enough to produce unique haptic response at the gimbal position in each of its three translation directions.

The maximum workspace dimensions of the device, resulting from its mechanical limits are [-210, 210] (in mm) in x direction (width), [-110, 205] (in mm) in the y direction (height) and [-85,130] (in mm) in z direction (depth). These represent the bounds of the haptic domain, in which the position of the gimbal joint will always lie. This positional information as well as the orientation of the stylus can be accessed from the device using the OpenHaptics API [24]. Another functionality of interest part of the API, relevant to the mapping strategy of the device to the physical model used for synthesis—described in more detail in the next section—is that haptic force feedback can be set at the location of the gimbal joint. This is achieved by the use of callback functions inside a high-priority scheduler thread. For the current application, the refresh rate of these actions is set to 1 kHz, by adjusting the number of times the scheduler ticks its callbacks every second.

4 REAL-TIME APPLICATION

The real-time application was developed in C++ using the JUCE framework[2]. It is open-source and available at [25] and a demo video can be found at [26].

A total of four simply supported damped stiff strings, modelled as per the details given in Section 2, are placed in a virtual 3-D space in a trapezoidal cross-section. By adjusting the tension parameter T the strings are tuned to the fundamental frequencies of the strings of a violin, i.e., 196.0, 293.6, 440.0 and 659.2 Hz, which correspond to the musical notes: G3, D4, A4 and E5 respectively. The approach for tuning as well as typical

[2]https://www.juce.com/

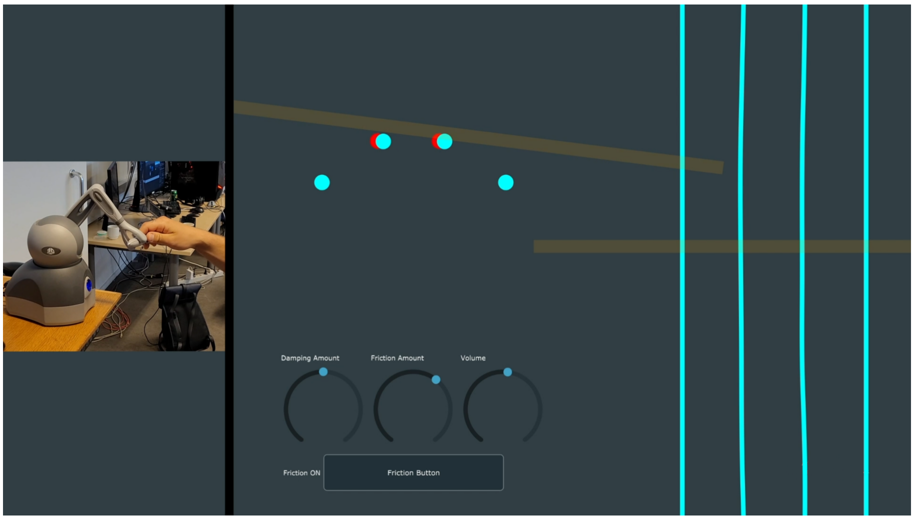


Figure 3. Snapshot from the demo video [26] of the digital music instrument showing both the software application window as well as the control via the haptic device.

parameters for such strings can be found in [20]. The strings can be excited by a perfectly rigid bow, with their interaction described by the friction model given in Equation (3). The CSYS of this virtual space is identical to the one of the Touch haptic device. Both the strings and the bow are modelled as infinitely thin lines.

4.1 Graphical user interface (GUI)

A snapshot of the application during use together with the its external control, taken from the demo video, can be seen in Figure 3. The window is divided into 3 areas of interest. On the right, a top view of the strings can be seen, i.e. the strings are projected onto the xz horizontal plane. The long transparent orange rectangle is the bow, whose color is grey when not in contact with any of the strings, e.g. when hovering above them in the 3-D virtual space. Its opacity is directly proportional to the externally supplied bow force, f_B, meaning that when the bow exerts more pressure onto the strings the transparency of its color is reduced, until becoming fully opaque when the force becomes largest. A copy of this same bow and strings can be seen on the top left part of the GUI window, viewed as projected in the frontal plane xy. The maximum displacement of the strings in this view is illustrated as red ellipses placed underneath the static cyan ones, which represent the "at rest" position. Finally, in the bottom left part of the application window, a number of knobs controlling various parameters in the physical model can be found as well as a button which turns on or off the frictional haptic feedback in the control interaction. Going from left to right the knobs control the: damping amount, which is a combination of the frequency in-dependent damping, $\sigma_0 \in [0.1, 10]$, and the frequency dependent damping, $\sigma_1 \in [0.0001, 0.015]$, introduced in Section 2. The choice of combining them is for ease of use for users who are unfamiliar with the details of the physical model underlying the simulation. Their ranges are chosen such that the control of the knob gives either a very large release of the sound or almost none at all. The second knob controls the friction parameter, $a \in [0.01, 15000]$, mapped logarithmically such that the friction interaction between the bow and the strings smoothly goes from "slippery" to "sticky". Lastly, a dimensionless global volume gain knob is included, controlling the master volume of the app.

Both the strings and the bow elements are depicted in the GUI as having some spatial volume: a width and height in addition to their segment length, forming for example the cross-section of the strings in the upper-left illustration. This is not the case for the virtual 3-D space of the physical model, where they are in fact infinitely thin (1-Delements), thus simplifying their interactions.

4.2 Haptic mapping

As previously mentioned, the stylus of the Touch device is an excellent control interface for a virtual bow. The position of the gimbal joint is mapped to the center of the bow in the virtual physical model space. Its orientation is extracted from the interface of the program with OpenHaptics API as the rotation angles of the stylus relative to the haptic CSYS, shown in Figure 2. For ease of control, the orientation of the bow in the physical model is fixed to be parallel with the frontal plane, i.e. xy, meanwhile allowed to rotate with respect to the horizontal plane xz. This means the bow is always perpendicular to the strings in the horizontal plane projection. Moving the stylus in the haptic space is equivalent to moving the bow in the physical model space.

In the haptic feedback callback function code, the shortest distance between the bow and each string is calculated at every haptic frame as the shortest line/segment perpendicular to both elements. The exact end point coordinates of this segment can also be found, keeping track of which point lies on the string and which one lies on the bow. Knowing this, the distance can be calculated relative to a direction vector going from the point on the string towards the point on the bow, therefore, when the distance is negative we know the bow is pushing onto the string. An elastic spring force equal to some heuristically chosen spring stiffness times the magnitude of this distance is then sent to the gimbal joint point, with an orientation given by the direction vector. This is how the user can feel each of the strings in the virtual space. When the bow is in contact with multiple strings, the reaction forces from all of them are summed up and sent as haptic feedback. Through linear mapping, this force is directly proportional to the externally supplied bow force in the physical model, f_{B}, with a maximum value $f_{\mathrm{B}} = 2$, hence the force with which the user presses onto the haptic strings gives the first audio synthesis model input. The second input, the externally supplied bow velocity v_{B} (in m/s), is similarly retrieved from the gestural motion of the user. It is again a linear mapping to some reasonable physical values: between -0.4 and 0.4 m/s, see [27], of the change in position of the gimbal joint in the 3-D haptic space between subsequent scheduler ticks. Essentially it is retrieved from the velocity of the stylus.

During the interaction of the bow with any of the strings, the frictional force resulting from the physical model is sent as haptic feedback to the gimbal joint, split in x, y, z components given by the opposite direction of motion of the bow. As for the case of the elastic feedback, frictional forces resulting from multiple synchronous interactions are summed up.

5 DISCUSSION

The DMI as prsened in the previous sections, i.e. the software application as sound generation unit and the Touch device as interface, is capable of producing "good" sounds without much practice. This is most likely due to the mapping choice of the gestural motion of the user to the input parameters of the friction model: the bow force f_{B} and the bow velocity v_{B}, which are limited in ranges that are relevant for the current physical model. Additionally, the haptic feedback, helps restrain the user from applying large bow forces that result in raucous motion of the string, corresponding to scratchy sound. This is achieved both by the elastic feedback of the strings, as well as the fact that the magnitude of the frictional force—and the haptic feedback it produces, increases with applied bow force. With some practice, the user can achieve also a "whistling" type of sound, resulting from multiple slipping of the bow, with two or more slipping phases per fundamental period [27].

What does take some practice is the ability to easily change from bowing individual strings to pairs of strings. With this, more interesting sounds can be achieved especially by modulating the applied bow force individually for different strings. The ability to bow the strings at a fixed location along their length also requires some rehearsing, as the user needs to get familiar with the mechanical response of the haptic device, independent of the haptic feedback. This reinforces the necessity to use the cubic interpolation operator given in Equation 8, used to place the bow on the string, in order to avoid clicks which would otherwise result from sudden jumps in the excitation location. The real-time visual feedback shown in the GUI helps the user better evaluate their movements in the 3-D virtual space.

Also, once the user becomes more accustomed to the control, variations in the attack and timbre of the sound can be achieved effortlessly. The modulation of the "damping amount" knob can be used to change the release of the sound in real-time and the "friction amount" knob can be used to increase the "stickiness" of the interaction and achieve a more plucked type sound.

6 CONCLUSION AND FUTURE WORK

This paper presents the implementation of a bowed strings musical interface. The audio-haptic synthesis is achieved by physical modelling, with the transverse displacement of a string being described by a PDE encompassing the string's physical properties and an external excitation. This excitation is provided by the contact with a perfectly rigid bow, and is described by a nonlinear frictional model, capable of exhibiting physically consistent behavior observed in bowed string experiments, particularly the stick-slip interaction between the two elements. The numerical simulation of this system is carried out using FDTD methods and is implemented in a real-time software application written in C++. This app is then controlled via an external haptic device, the 3D Systems Touch, which allows for the inclusion of the haptic feedback resulting from the simulation. The actual frictional force which is used to excite the strings, resulting from the physical model, is felt by the user when bowing the strings. Both the audio feedback and the frictional feedback are directly linked to the instrumental gesture, allowing for a more personal experience with the instrument as opposed to typical DMIs whose control is often disconnected from the underlying audio synthesis model.

Future work can involve including an additional control with the hand which is not holding the stylus of the haptic device. This control could be for instance a haptic glove with which the user could press the different strings against a virtual neck, changing the notes being played or slightly damping the strings and introducing harmonics. A sound-board could be added to the strings, simulating the body of the violin, or perhaps the output sound could be convolved in real-time with the impulse response of the body of a real violin, using partitioned convolution algorithms. Also, spring connections—potentially nonlinear—could be added between various strings, thus creating a more unphysical instrument which could be played in a physical manner. Also of great interest may be developing cheaper alternative haptic devices for the control, as the price of the Touch currently limits the accessibility of the current instrument.

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