

Minimum-Jerk Trajectory Planning in Robotic Manipulators With Kinematic Redundancy

Lorenzo Scalera¹

Polytechnic Department of Engineering and Architecture,
University of Udine,
33100 Udine, Italy
e-mail: lorenzo.scalera@uniud.it

Giuliano Fabris

Polytechnic Department of Engineering and Architecture,
University of Udine,
33100 Udine, Italy
e-mail: giuliano.fabris@uniud.it

Paolo Boscariol

Department of Management and Engineering,
University of Padova,
36100 Vicenza, Italy
e-mail: paolo.boscariol@unipd.it

Alessandro Gasparetto

Polytechnic Department of Engineering and Architecture,
University of Udine,
33100 Udine, Italy
e-mail: alessandro.gasparetto@uniud.it

Minimizing induced mechanical vibrations represents a challenging and critical issue in robotics, as they can degrade motion accuracy, reduce achievable performance, and accelerate mechanical wear. This article addresses the problem of minimum-jerk trajectory planning for redundant robotic manipulators by introducing an optimization approach that simultaneously exploits the structural redundancy of the robotic system and the functional redundancy of the assigned task. Unlike conventional redundancy-based approaches that typically rely on either kinematic redundancy alone or task redundancy properties, the proposed strategy leverages both types of redundancy to optimally plan the robot motion for the purpose of minimizing the end-effector jerk and, consequently, achieving smoother trajectories. In more detail, both the position of a selected redundant joint of the robot and one or more angles of the end-effector orientation are considered as optimization variables for each of the prescribed waypoints of the given path. The proposed strategy is verified on a robotic manipulator with seven degrees-of-freedom executing a pick-and-place operation. Extensive simulations and experimental tests demonstrate the effectiveness of the proposed strategy in reducing the end-effector jerk by more than 90% compared to a reference case.
[DOI: 10.1115/1.4072578]

Keywords: kinematics, jerk minimization, motion planning, redundancy, robotics

1 Introduction

Jerk minimization, defined as the third time derivative of position, is a central objective in robotic trajectory planning, as it is responsible for the continuity of acceleration profiles and plays a key role in limiting the excitation of structural vibrations and dynamic loads. Reducing jerk contributes to smoother motions, lower mechanical stress on actuators and transmission components, and improved tracking accuracy [1,2]. These requirements are critical in modern industrial robotics, where high-precision manufacturing tasks demand smooth trajectories, reduced wear, and reliable dynamic performance [3–5].

A wide range of methods has been proposed for the generation of smooth, minimum-jerk trajectories. In Ref. [1], minimum-jerk motion planning is addressed through cubic polynomial trajectories formulated within a global constrained minimax optimization. A different strategy is introduced in Ref. [6], where a cost function accounting for both execution time and jerk is optimized using fifth-degree B-spline curves. More recently, the authors in Refs. [7,8] investigate smooth trajectory planning for robotic manipulators by exploiting multiquadric radial basis functions. In Ref. [9], a jerk-segmented trajectory planning method based on continuous oscillatory type curves is presented. An algorithm for jerk-limited online trajectory generation with arbitrary target states is evaluated in Ref. [10]. Additional approaches, primarily developed for nonredundant manipulators, focus on time-jerk trade-off [11], adopt alternative motion laws, including sinusoidal jerk profiles [12], or employ heuristic optimization techniques such as particle swarm approaches [13]. However, these strategies typically assume a unique joint-space configuration of the robot imposed by the task, without leveraging additional degrees-of-freedom (DOFs).

In redundant manipulators, i.e., robots with more degrees-of-freedom than those strictly required to accomplish the task [14], redundancy can be leveraged as an additional variable for motion optimization, enabling further reductions in jerk. More specifically, a robotic system is said to be structurally redundant when the number of its degrees-of-freedom N exceeds the dimension of the operative space (planar or spatial). A robot is defined as functionally redundant when it has more degrees-of-freedom than the number R of operational-space variables required to fulfill a given task, i.e., when $N > R$. Redundancy enhances the flexibility of redundant robots compared to their nonredundant counterparts [15,16], as it can be leveraged to accomplish secondary objectives, such as collision avoidance [17,18], manipulability maximization [19], dexterity [20], or energy efficiency [21–23], without affecting the primary motion of the robot end-effector.

Recent research focusing on jerk minimization in redundant robotic systems includes the multilevel minimization approach proposed in Ref. [24], which addresses jerk-bounded trajectory planning for redundant manipulators. In Ref. [25], an approach for planning jerk-optimal trajectories with discrete time constraints for redundant robots in manufacturing tasks is presented. Additionally, the work in Ref. [26] introduces an optimization-based inverse kinematics method for a six-axis industrial robot that synthesizes joint trajectories following a specified tool path while ensuring smoothness and collision avoidance. More recently, the authors

¹Corresponding author.

Manuscript received February 13, 2026; final manuscript received August 12, 2026; published online August 26, 2026. Assoc. Editor: Nilanjan Chakraborty.

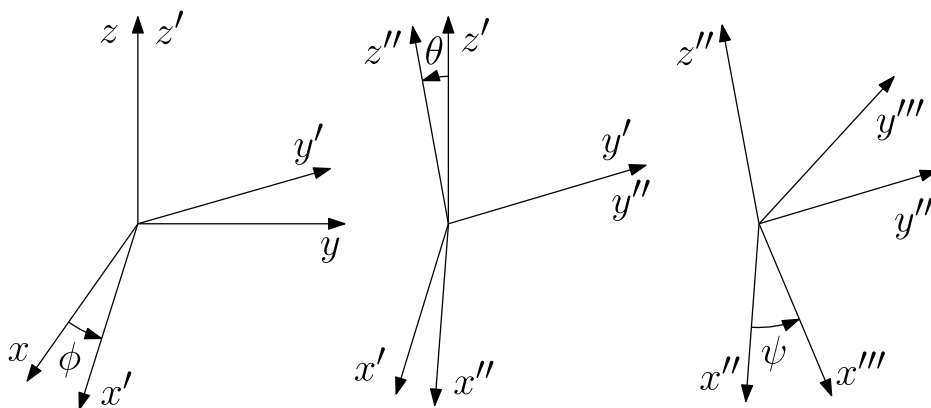


Fig. 1 Graphical representation of the ϕ, θ, ψ Euler angles in the ZYZ convention

in Ref. [27] present a minimum-jerk trajectory planning approach for redundant manipulators that simultaneously optimizes the time intervals between waypoints and the positions of a selected joint to minimize the end-effector jerk. However, in Ref. [27], only the robot structural redundancy is adopted to improve trajectory smoothness, without considering the possible functional redundancy of the assigned task.

By exploiting redundant degrees-of-freedom, these approaches enable smoother trajectories while keeping the robot end-effector on the desired path or guaranteeing its passage through specified waypoints. This results in reduced vibrations, decreased mechanical wear, and enhanced performance, which are especially critical in applications such as collaborative robotics [28] and precision manufacturing [29]. Consequently, redundancy-aware trajectory planning emerges as a promising approach to improve motion quality and system reliability. However, to the best of the author's knowledge, existing literature lacks trajectory planning strategies that simultaneously optimize both the position of a robot joint and the end-effector orientation angles under a fixed total execution time, leveraging both structural and functional redundancy to improve motion smoothness.

In this article, we introduce a redundancy-aware trajectory planning approach that simultaneously exploits the structural redundancy of the manipulator and the functional redundancy of the assigned task to generate minimum-jerk motions under a fixed execution time. Unlike existing approaches, the proposed formulation explicitly considers both the position of a selected redundant joint and one or more end-effector orientation angles as optimization variables at each prescribed waypoint along the task path, thus enabling a more effective exploitation of the available redundancy while preserving the geometric constraints of the task.

The proposed approach is validated on a robotic manipulator with seven DOFs performing a pick-and-place operation. Extensive numerical simulations and bespoke experimental tests demonstrate not only the feasibility of the approach but also its effectiveness in substantially improving motion smoothness, achieving a reduction of the end-effector jerk of up to 94.7% with respect to a reference scenario.

To summarize, the main contributions of this work are the following:

- a minimum-jerk trajectory planning strategy for redundant manipulators that jointly exploits both structural and functional redundancy under a fixed total execution time;
- an extensive numerical evaluation, also including a quantitative comparison with the state-of-the-art technique proposed in Ref. [27];
- an experimental validation on a seven-DOF redundant robotic manipulator, confirming the practical applicability and performance of the proposed approach.

The remainder of the article is organized as follows: Sec. 2 illustrates the proposed approach for smooth motion planning in redundant robots. Section 3 presents the case study, whereas the results of extensive numerical tests on a 7-DOF manipulator are presented in Sec. 4. Furthermore, Sec. 5 describes the results of bespoke experiments. Finally, Sec. 6 gives the conclusions of the work and outlines future research directions.

2 Proposed Approach for Smooth Motion Planning in Redundant Robots

In this article, we propose to exploit both the kinematic redundancy of the robotic system (*structural redundancy*) and the redundancy given by the difference between the robot degrees-of-freedom and those required by the prescribed task (*functional redundancy*) to minimize the jerk of the robot end-effector and to achieve smooth motion. In line with common practice in the literature [30], vibration reduction is achieved by minimizing the jerk of the robot end-effector.

The following four optimization approaches are compared to quantify the jerk and mechanical vibration reduction in redundant manipulators: Approach 1 is the state-of-the-art technique [27] used for comparison, whereas the proposed approach is subdivided into the incremental strategies named approaches 2, 3 and 4.

Approach 1 (for comparison): The reduction of the end-effector jerk $\mathbf{X}$ is carried out by optimizing the vector of joint positions $\mathbf{q}^*$ of a selected redundant joint at the waypoints defining the task. For each optimization iteration, the robot configuration at each waypoint is determined by solving the inverse kinematics problem, given the assigned value of the redundant joint $\mathbf{q}^*$. Then, the joint-space trajectory interpolating the configurations at the waypoints is planned, from which the end-effector jerk is evaluated. The optimization of a single joint variable at each waypoint, combined with the inverse kinematics resolution, leads to robot configurations that are selected to enhance trajectory smoothness, rather than being arbitrarily chosen. With respect to the minimum time-jerk formulation in Ref. [27], in this work the total execution time is fixed and defined a priori, and no optimization of the time intervals between successive waypoints is performed. This design choice allows us to isolate the contribution of jointly exploiting structural and functional redundancy without the additional optimization of the trajectory timing. If the generic trajectory $(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \ddot{\ddot{\mathbf{q}}})$ with the fixed total time T is considered, the optimization problem is formulated as follows:

$$\min_{\mathbf{q}^*} \sum_{n=1}^6 \int_0^T \mathbf{X}_n^2(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \ddot{\ddot{\mathbf{q}}}) dt \quad (1)$$

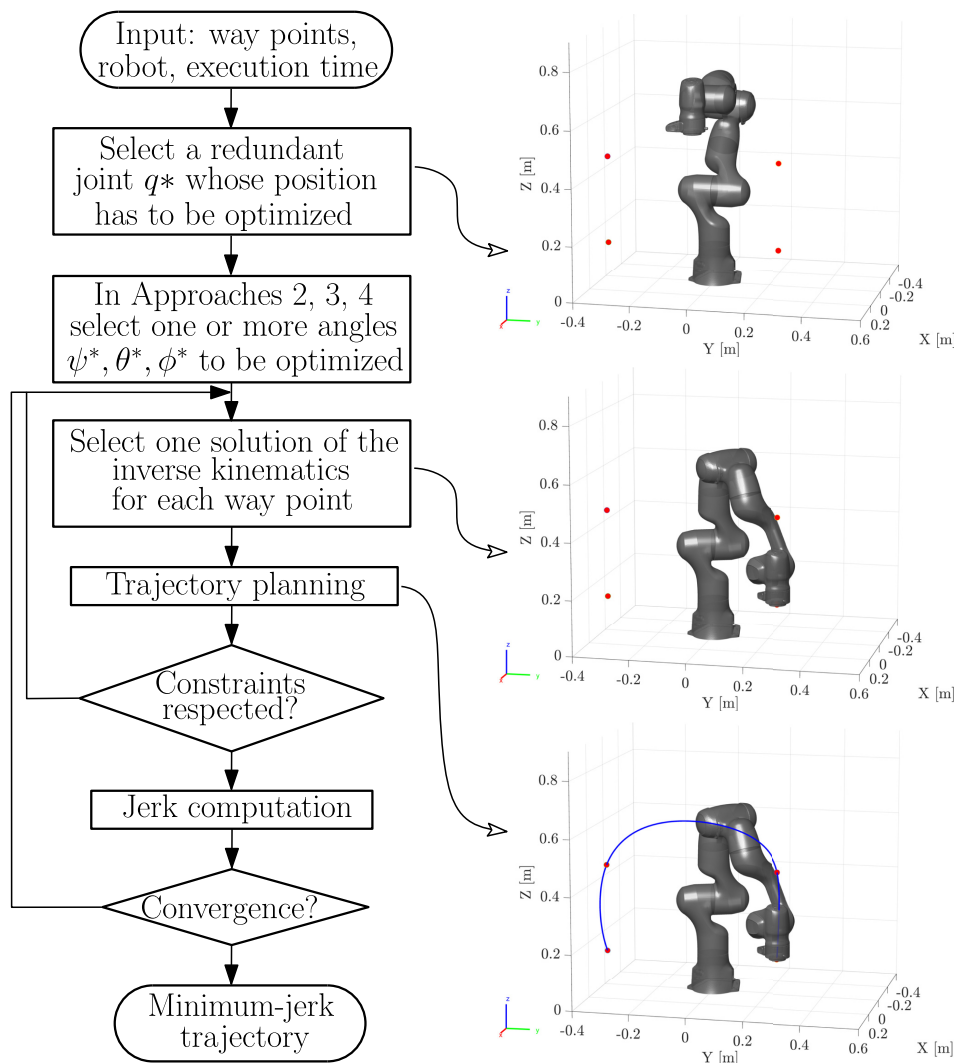


Fig. 2 Flowchart of the proposed minimum-jerk trajectory planning strategy based on kinematic redundancy for robotic manipulators

where the index n spans over the six components of the jerk of the end-effector. The problem (1) is subject to the following constraints:

$$\begin{cases} q_{\min} < q < q_{\max} \\ |\dot{q}| < \dot{q}_{\max} \\ |\ddot{q}| < \ddot{q}_{\max} \\ |\dddot{q}| < \dddot{q}_{\max} \end{cases} \quad (2)$$

where $q_{\min}$ and $q_{\max}$ indicate the lower and upper joint position limits, whereas $\dot{q}_{\max}$, $\ddot{q}_{\max}$, and $\dddot{q}_{\max}$ are the joint velocity, acceleration, and jerk limits, respectively. Additional constraints, such as obstacle avoidance and dynamics limits, are not considered in this work.

Table 1 Position and orientation of the robot end-effector in the four waypoints that define the considered pick-and-place task in the reference case

wp	x (m)	y (m)	z (m)	ϕ (rad)	θ (rad)	ψ (rad)
1	0.3	0.3	0.2	-1.5708	3.1416	3.1416
2	0.3	0.3	0.5	-1.5708	3.4034	3.1416
3	0.3	-0.3	0.5	-1.5708	2.8798	3.1416
4	0.3	-0.3	0.2	-1.5708	3.1416	3.1416

Approach 2 (proposed): Jerk reduction is obtained by jointly optimizing the vector of joint positions q^* of a selected redundant joint and the vector of angular positions ϑ^* that defines one of the orientation angles of the end-effector. The proposed approach 2 is incremental with respect to approach 1, since in this case the structurally redundant robotic system is also assumed to be functionally redundant with respect to the assigned task. In approach 2, a task with a fixed total time T is considered, and the following optimization problem is obtained:

$$\min_{q^*, \vartheta^*} \sum_{n=1}^6 \int_0^T X_n^2(q, \dot{q}, \ddot{q}, \ddot{q}) dt \quad (3)$$

The problem (3) is subject to the constraints in (2).

Table 2 Joint positions in the four waypoints in the reference case

wp	q_1 (rad)	q_2 (rad)	q_3 (rad)	q_4 (rad)	q_5 (rad)	q_6 (rad)	q_7 (rad)
1	0.388	0.137	0.371	-2.647	-0.138	2.772	2.455
2	0.201	-0.531	0.480	-2.391	0.499	2.026	2.015
3	-0.408	-0.491	-0.307	-2.401	-0.393	2.069	1.043
4	-0.348	0.140	-0.408	-2.647	0.153	2.771	0.675

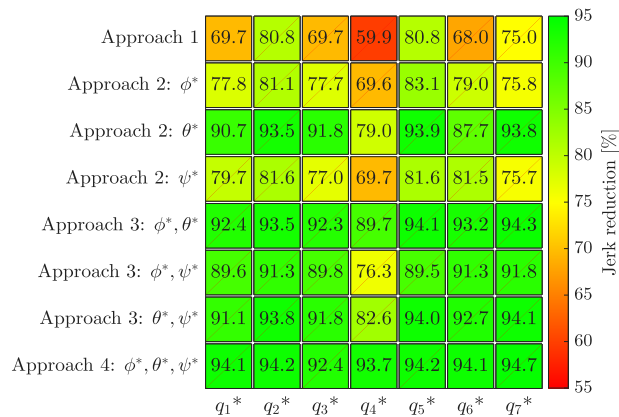


Fig. 3 Percentage jerk reduction in the compared approaches with respect to the reference case

Table 3 Joint positions in the four waypoints in approach 1 (q_5^*)

wp	q_1 (rad)	q_2 (rad)	q_3 (rad)	q_4 (rad)	q_5^* (rad)	q_6 (rad)	q_7 (rad)
1	-0.787	-1.125	1.714	-2.523	1.410	2.011	1.470
2	-0.484	-0.919	0.920	-2.333	0.947	1.673	1.679
3	0.485	-0.920	-0.920	-2.333	-0.948	1.672	1.463
4	0.786	-1.133	-1.712	-2.523	-1.414	2.003	1.672

Approach 3 (proposed): The number of optimization variables is further increased by considering the vector of joint positions $\mathbf{q}^*$ of a selected redundant joint and two vectors $\boldsymbol{\theta}_1^*$ and $\boldsymbol{\theta}_2^*$ of angular positions that define two orientation angles of the end-effector. The resulting optimization problem of the proposed approach 3,

subject to (2), is

$$\min_{\mathbf{q}^*, \boldsymbol{\theta}_1^*, \boldsymbol{\theta}_2^*} \sum_{n=1}^6 \int_0^T \dot{X}_n^2(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) dt \quad (4)$$

Approach 4 (proposed): We consider as optimization variables the vector of joint positions $\mathbf{q}^*$ of a selected redundant joint and all three vectors $\boldsymbol{\theta}_1^*$, $\boldsymbol{\theta}_2^*$, and $\boldsymbol{\theta}_3^*$ of the angular positions that define the three orientation angles of the end-effector. The proposed approach 4 considers the task orientation to be completely redundant from a functional point of view. The optimization problem results in

$$\min_{\mathbf{q}^*, \boldsymbol{\theta}_1^*, \boldsymbol{\theta}_2^*, \boldsymbol{\theta}_3^*} \sum_{n=1}^6 \int_0^T \dot{X}_n^2(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) dt \quad (5)$$

In (1), (3), (4), and (5), the vector $\mathbf{q}^*$ includes the positions of the selected joint in each of the waypoints. For example, if four waypoints (wp) define the robot path, $\mathbf{q}^* = [q_{wp1}^*, q_{wp2}^*, q_{wp3}^*, q_{wp4}^*]$. In a similar manner, in (3), (4), and (5), the vectors $\boldsymbol{\theta}^*$, $\boldsymbol{\theta}_1^*$, $\boldsymbol{\theta}_2^*$, and $\boldsymbol{\theta}_3^*$ include the angular positions of a selected Euler angle in each of the waypoints. The symbol $\boldsymbol{\theta}$ denotes one of the Euler angles in the ZYZ convention (ϕ, θ, ψ) that defines the end-effector orientation in the Cartesian space. A graphical representation of Euler angles is shown in Fig. 1.

The vector $\mathbf{X}$, computed in (1), (3), (4), and (5), includes the six components of the jerk of the end-effector. It is computed using the Jacobian matrix $\mathbf{J}$ of the redundant robot, its first $\dot{\mathbf{J}}$ and second $\ddot{\mathbf{J}}$ time derivatives, and the joint-space trajectory, as follows:

$$\mathbf{X} = \mathbf{J}\ddot{\mathbf{q}} + 2\dot{\mathbf{J}}\dot{\mathbf{q}} + \ddot{\mathbf{J}}\mathbf{q} \quad (6)$$

To parameterize the joint-space robot motion ($\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \ddot{\mathbf{q}}$) over time through the prescribed waypoints, we consider spline curve that adopt fourth-degree polynomials for the starting and final segments, and cubic polynomial functions for the intermediate intervals, i.e., 434 spline curves [31]. The 434 parameterization

Table 4 Joint positions in the four waypoints in approach 2 (q_5^*, θ^*)

wp	q_1 (rad)	q_2 (rad)	q_3 (rad)	q_4 (rad)	q_5^* (rad)	q_6 (rad)	q_7 (rad)	θ^* (rad)
1	-0.776	-1.286	1.703	-2.468	1.185	1.881	1.593	2.850
2	-0.752	-1.207	0.998	-2.226	0.892	1.385	1.557	3.108
3	-0.857	-0.204	-0.060	-2.074	0.459	1.243	0.665	3.921
4	-0.953	0.272	0.057	-2.326	0.531	1.744	0.345	4.094

Table 5 Joint positions in the four waypoints in approach 3 (q_5^*, ϕ^*, θ^*)

wp	q_1 (rad)	q_2 (rad)	q_3 (rad)	q_4 (rad)	q_5^* (rad)	q_6 (rad)	q_7 (rad)	ϕ^* (rad)	θ^* (rad)
1	-0.807	-0.969	1.752	-2.541	1.403	2.182	2.001	-2.118	3.207
2	-0.435	-0.939	0.921	-2.379	0.944	1.830	2.162	-2.055	3.435
3	0.526	-0.938	-1.139	-2.224	0.094	1.665	1.528	-1.903	4.058
4	0.853	-1.008	-1.847	-2.411	-0.128	2.027	1.131	-1.826	4.189

Table 6 Joint positions in the four waypoints in approach 4 ($q_5^*, \phi^*, \theta^*, \psi^*$)

wp	q_1 (rad)	q_2 (rad)	q_3 (rad)	q_4 (rad)	q_5^* (rad)	q_6 (rad)	q_7 (rad)	ϕ^* (rad)	θ^* (rad)	ψ^* (rad)
1	0.589	0.129	0.221	-2.618	0.454	2.675	0.396	2.494	2.913	5.637
2	0.738	-0.462	0.012	-2.400	0.297	2.079	0.617	1.768	2.843	4.905
3	0.756	-1.340	-1.142	-2.186	-0.241	1.563	-0.103	-4.967	2.240	4.360
4	0.769	-1.364	-1.790	-2.378	-0.476	1.896	-0.550	4.487	4.189	-5.096

Table 7 Numerical results of the maximum end-effector jerk and integral of squared jerk in the compared approaches

	$\ddot{x}_{\max}$ (m/s ³)	$\ddot{y}_{\max}$ (m/s ³)	$\ddot{z}_{\max}$ (m/s ³)	$\sum_{n=1}^6 \int_0^T \dot{X}_n^2(q, \dot{q}, \ddot{q}, \ddot{q}) dt$
Reference case	11.427	6.532	8.951	952.938
Approach 1 (q_5^*)	0.870	5.640	7.635	182.783
Approach 2 (q_5^*, θ^*)	4.056	4.157	7.925	58.524
Approach 3 (q_5^*, ϕ^*, θ^*)	2.294	4.195	7.340	56.659
Approach 4 ($q_5^*, \phi^*, \theta^*, \psi^*$)	4.311	4.129	7.904	55.138

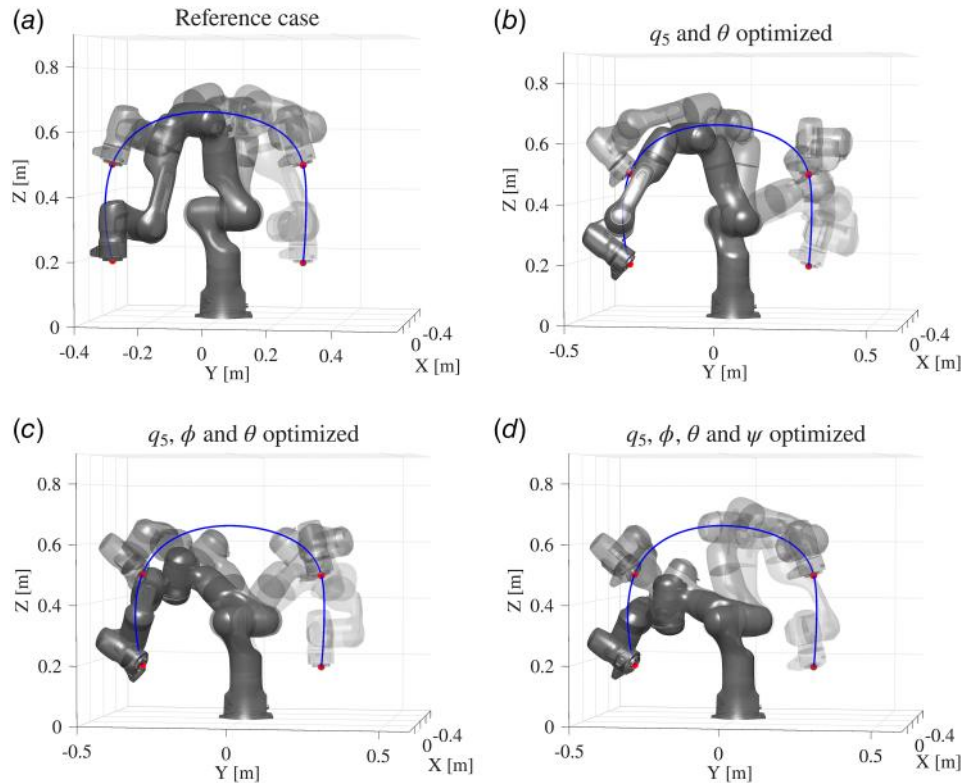


Fig. 4 3D path and robot configuration at each of the waypoints in the compared approaches: (a) reference case, (b) approach 2, (c) approach 3, and (d) approach 4

ensures that the generated joint-space trajectory interpolates the prescribed joint positions at all waypoints. In addition, it provides continuous velocity and acceleration profiles throughout the motion, with user-defined boundary conditions for the initial and final velocities and accelerations, which are set equal to zero. However, alternative motion planning primitives can also be integrated within the proposed approaches, as long as they conform to an analogous time parameterization. In this work, the robotic manipulator is modeled as a serial chain of rigid links and ideal joints. The nominal kinematic parameters of the robot together with its kinematic limits are assumed to be known. Furthermore, parameter uncertainties are neglected.

A flowchart of the proposed minimum-jerk trajectory planning strategy based on kinematic redundancy for robotic manipulators is reported in Fig. 2. From the flowchart, the optimization procedure can be summarized as follows. First, the optimization algorithm generates a candidate solution consisting of the optimization variables, namely the selected redundant joint positions and, depending on the considered approach, one or more end-effector orientation angles at each waypoint. Then, the complete robot configuration at each waypoint is determined by solving the inverse kinematics problem. The resulting joint

configurations are then interpolated to obtain a continuous joint-space trajectory over time. Forward kinematics is applied to compute the corresponding end-effector motion, from which the jerk profile is evaluated. Finally, the cost function is computed from the end-effector jerk and returned to the optimization algorithm, which iteratively updates the candidate solutions until the termination criterion is satisfied.

3 Case Study

This work considers a 7-DOF Panda arm by Franka Emika GmbH executing a pick-and-place task, as illustrated in Fig. 2. The task is specified by four Cartesian waypoints defining the pick-and-place operation, whereas the corresponding robot motion is parameterized in the joint space using a 434 spline interpolation, following an approach similar to that adopted in Ref. [11]. The proposed approach, however, is readily extensible to tasks involving a larger number of waypoints and to more complex scenarios. The desired end-effector poses $[x, y, z, \phi, \theta, \psi]$ at each waypoint in the reference case are listed in Table 1. The end-effector positions reported in Table 1 are fixed for all

compared incremental approaches, whereas one or more Euler angles are considered as optimization variables in approaches 2, 3, and 4. The allocation of time intervals between each couple of successive waypoints is fixed, with time intervals of 0.9 s assigned to the transitions between waypoints 1–2 and 3–4, and an interval of 1.2 s for the intermediate segment between waypoints 2 and 3. The total execution time is therefore equal to $T = 3$ s.

In approach 1, the task is modeled as six-dimensional, with the robot exhibiting a redundancy degree of $N - R = 1$. Therefore, the position q^* of a single joint must be optimized at each of the four waypoints defining the operation, resulting in a total of four optimization variables. All seven joints are individually evaluated through numerical simulations to identify which joint yields the best improvement in trajectory smoothness and jerk reduction compared to a reference scenario. Table 2 reports the joint positions in the four waypoints in the reference case. In approach 2, the pick-and-place task is modeled as five-dimensional by allowing one Euler angle at each waypoint to vary freely ($R = 5$). This results in a redundancy degree of $N - R = 2$. Accordingly, the optimization problem involves both the position q^* of a selected robot joint and one Euler angle defining the end-effector orientation at

each waypoint, yielding a total of eight optimization variables. Approach 2 is therefore evaluated through 21 simulations carried out for each combination of joint and Euler angle. In a similar way, the number of operational space variables describing the task is equal to $R = 4$ in approach 3 (i.e., three optimization variables at each waypoint) and to $R = 3$ in approach 4 (i.e., four optimization variables at each waypoint). The optimization problem is solved 21 times also for approach 3 to cover the whole combinations of joint and couple of free angles, whereas seven tests are performed to evaluate the performance of approach 4, considering all seven robot joints.

The numerical tests are performed on a computer running Windows 10 Pro with an Intel Core i9-10900 CPU at 2.80 GHz and 32.0 GB of RAM. The optimization problems in (1), (3), (4), and (5), all subject to the constraints in (2), are implemented in Matlab 2024b using the genetic algorithm *ga*. The size of the population is set equal to 100 for approach 1, 150 for approach 2, 200 for approach 3, and 300 for approach 4. Furthermore, the maximum number of iterations is set equal to 1000 for approach 1, 2000 for approaches 2 and 3, and 3000 for approach 4. A random initial population within specified lower and upper bounds of joint positions

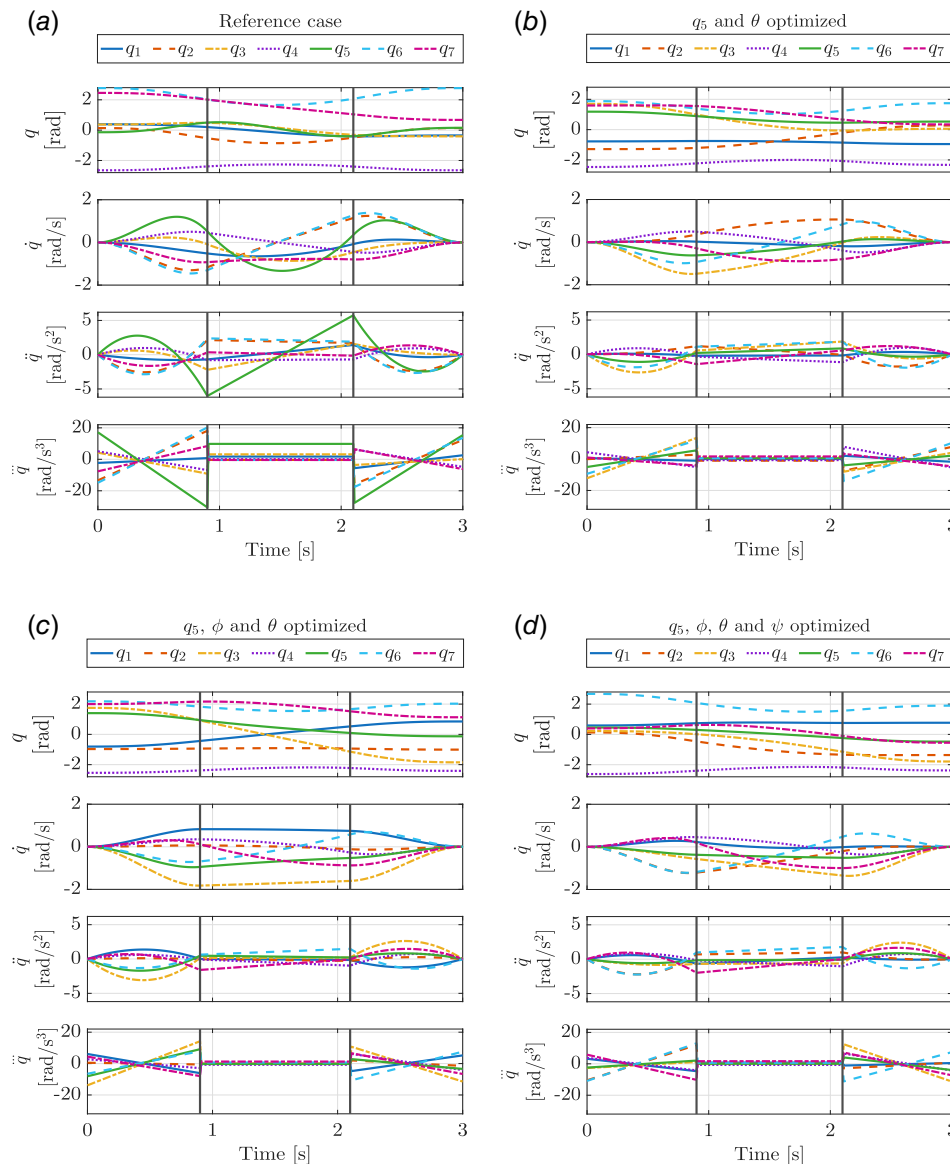


Fig. 5 Joint positions, velocities, accelerations, and jerks in the compared approaches: (a) reference case, (b) approach 2, (c) approach 3, and (d) approach 4

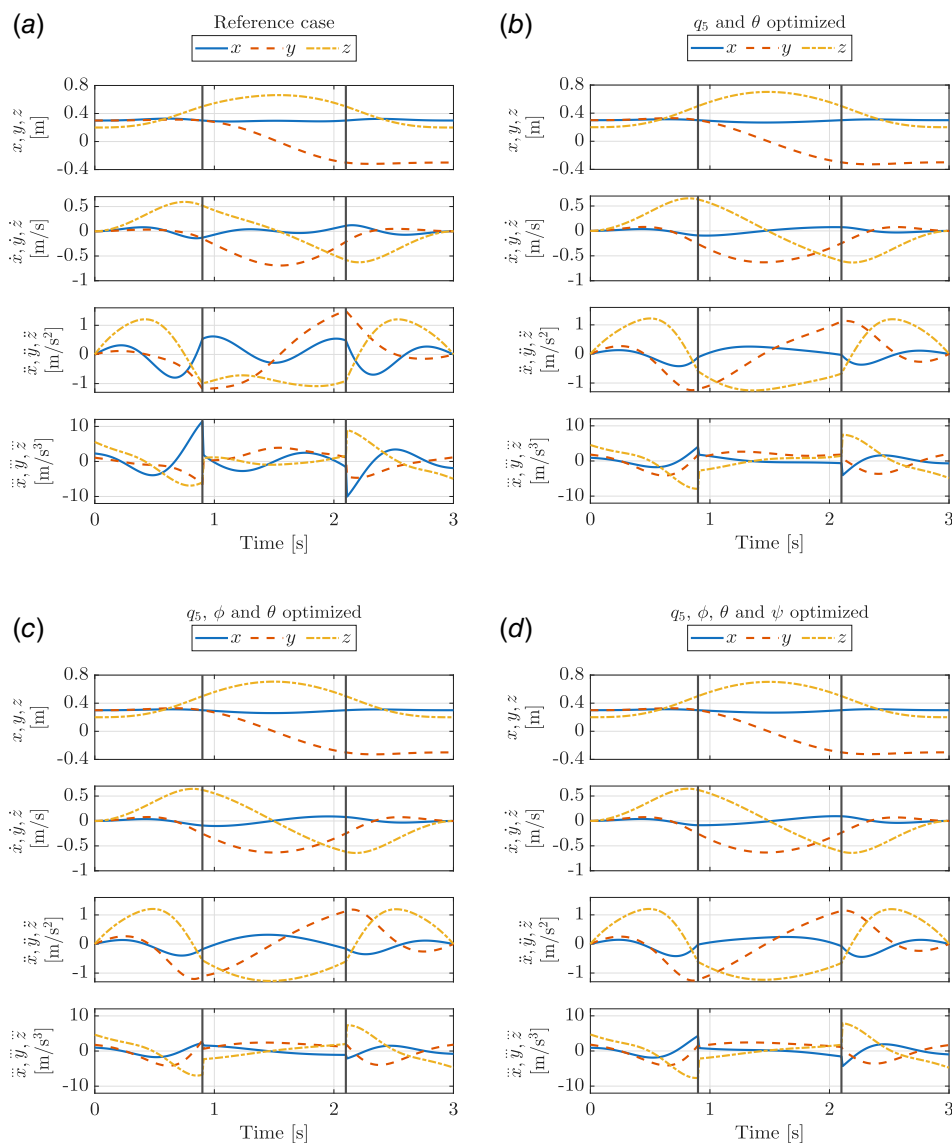


Fig. 6 End-effector positions, velocities, accelerations, and jerks in the compared approaches: (a) reference case, (b) approach 2, (c) approach 3, and (d) approach 4



Fig. 7 The Franka Emika Panda robot used in the experimental tests

and orientation angles is chosen. The genetic algorithm is employed to enable a broader and more effective exploration of the solution space by evolving a population of candidate solutions, compared to gradient-based optimization approaches, which may be prone to convergence to local minima.

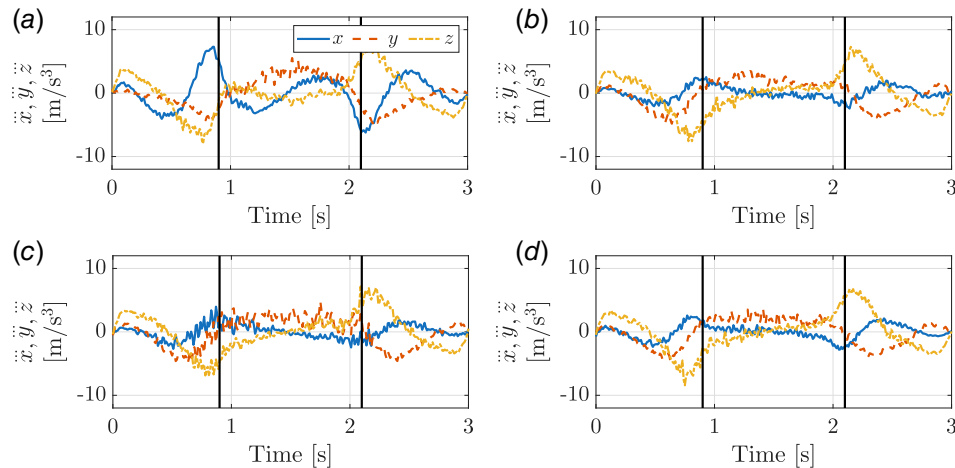
4 Numerical Results

The numerical results are first presented in Fig. 3 as percentage jerk reduction in the compared approaches with respect to the reference case. In the figure, each box of the board represents the result of one optimization problem considering a different joint and, in addition to that, a different orientation angle (or set of angles) for approaches 2, 3, and 4, as optimization variables. The percentages of jerk reduction are computed with respect to the trajectory of the reference case (Tables 1 and 2).

Considering approach 1, the best results are obtained when joint q_2^* or joint q_5^* is considered as optimization variable: in both cases, the percentage of jerk reduction is equal to 80.8%. However, the

Table 8 Experimental results of the root-mean-square values of the end-effector jerk and integral of squared jerk in the compared approaches

	$\ddot{x}_{\text{meas,rms}}$ (m/s ³)	$\ddot{y}_{\text{meas,rms}}$ (m/s ³)	$\ddot{z}_{\text{meas,rms}}$ (m/s ³)	$\sum_{n=1}^6 \int_0^T \ddot{x}_{\text{meas},n}^2(q, \dot{q}, \ddot{q}, \ddot{q})dt$
Reference case	2.717	2.290	3.050	829.624
Approach 1 (q_5^*)	0.612	2.684	3.141	168.527
Approach 2 (q_5^*, θ^*)	1.013	2.146	2.933	64.084
Approach 3 (q_5^*, ϕ^*, θ^*)	1.097	2.226	2.857	73.373
Approach 4 ($q_5^*, \phi^*, \theta^*, \psi^*$)	1.143	2.126	2.914	61.401

**Fig. 8** Experimental results: jerk of the robot end-effector in the compared approaches: (a) reference case, (b) approach 2, (c) approach 3, and (d) approach 4

worst result (59.9% when q_4^* is optimized) still represents a large improvement with respect to the reference case. When moving to approach 2, the best result in terms of jerk reduction is obtained when the joint q_5^* is considered together with angle θ^* (93.9%). However, from Fig. 3, it can be seen that all the results obtained with approach 2 are greater than or equal to 69.6%, indicating that considering the functional redundancy of the task in addition to the structural redundancy of the manipulator leads to improvements in the optimization of minimum-jerk trajectories.

These results are further enhanced when moving to approach 3, which considers three optimization variables. In that case, the best result is the reduction of 94.3% (considering q_7^*, ϕ^* , and θ^*), followed by the reduction of 94.1% obtained in both cases considering q_5^*, ϕ^* , and θ^* , as well as q_7^*, θ^* , and ψ^* . Finally, in approach 4, the combined optimization of one joint position and all three Euler angles yields percentage reduction results always greater than 92.4% with respect to the reference case. The best jerk reduction is obtained when q_7^* is considered in the optimization together with all three orientation angles (94.7%), followed by the cases with q_5^* and q_3^* (94.2%).

For the sake of simplicity, in the following, we only consider the best cases for all approaches when joint q_5^* is considered, since the fifth joint of the Panda arm can be considered one of the structurally redundant joints with respect to traditional anthropomorphic industrial manipulators with 6 DOFs. The optimal joint positions in the four waypoints in approach 1 (q_5^*), approach 2 (q_5^*, θ^*), approach 3 (q_5^*, ϕ^*, θ^*), and approach 4 ($q_5^*, \phi^*, \theta^*, \psi^*$) are reported in Tables 3–6, respectively. Furthermore, the maximum end-effector jerk and integral of squared jerk in the compared approaches are reported in Table 7. From the table, it can be seen that the integral of squared jerk is more and more reduced moving from the reference case to the proposed approaches as additional sources of redundancy are exploited in the optimization problem. Moreover, the maximum values of jerk of the

end-effector in all compared approaches are always lower than the reference case. The proposed formulation does not simply increase the number of optimization variables. Instead, it leverages an additional source of redundancy arising from the task specification, i.e., the unconstrained end-effector orientation, while preserving the prescribed task and execution time.

Figure 4 shows the 3D path and robot configuration at each of the waypoints in the compared approaches (only the reference case, approaches 2, 3, and 4 are reported due to space constraints). Figure 5 illustrates the joint positions, velocities, accelerations, and jerks in the compared approaches (reference case, approach 2 with q_5^* and θ^* , approach 3 with q_5^*, ϕ^* , and θ^* , and approach 4 with q_5^*, ϕ^*, θ^* , and ψ^*). In the plots, the vertical solid lines indicate the desired waypoints. Figure 5 shows that the velocities, accelerations, and jerks of the robot in the proposed approaches are significantly reduced compared to the reference case (the vertical limits of the graphs are deliberately kept the same for all plots for comparison purposes). Furthermore, a reduction in the translational velocities, accelerations, and jerks of the robot end-effector can be appreciated from Fig. 6, which reports the Cartesian-space trajectory of the robot in the reference case, approach 2 with q_5^* and θ^* , approach 3 with q_5^*, ϕ^* , and θ^* , and approach 4 with q_5^*, ϕ^*, θ^* , and ψ^* .

5 Experimental Results

The optimal trajectories obtained with the proposed approaches are tested on a real Franka Emika robot with seven DOFs (Fig. 7). The robot has a reach of 855 mm and a maximum payload of 3 kg. The trajectories are sent to the robot controller via the robot operating system Melodic Morenia and PYTHON 3.6 using a workstation with an Intel i5-10600k processor and 32 GB of RAM running Ubuntu 18.04 LTS Bionic Beaver. The desired trajectory is

provided to the robot controller in terms of joint positions, velocities, and accelerations over time. The robot controller executes the received trajectory by tracking the reference values contained in the trajectory file. Any communication delay or latency is common to all experiments and therefore does not influence the comparison between the reference and optimized trajectories. The measured outputs consist of the actual joint positions, joint velocities, and joint torques. The accelerations and jerks of the measured trajectory are then obtained via numerical differentiation starting from the joint velocities.

Table 8 reports the results of the experimental tests in terms of root-mean-square (rms) values of the end-effector jerk and integral of squared jerk in the compared approaches. Moreover, Fig. 8 reports the jerks of the robot end-effector along the x , y , and z coordinates. From Table 8 and Fig. 8, it can be seen that all compared approaches perform better than the reference case in terms of both maximum jerk and integral of squared jerk. Furthermore, approaches 2, 3, and 4, which leverage both the structural redundancy of the robot and the functional redundancy of the task, show superior performance with respect to the state-of-the-art techniques of approach 1, which considers only a selected redundant joint in the optimization routine. These results demonstrate the feasibility of the proposed approach and the capability of the optimization strategy to reduce the jerk of the robot end-effector in an experimental scenario. Furthermore, comparing Table 7 and Table 8, it can be seen that a good agreement between numerical and experimental tests is found.

A critical aspect of the proposed methodology concerns the choice of the joint and angle whose positions are included in the optimization. However, defining a criterion for selecting the most suitable joint and angle, and investigating their relationship with the robot kinematics, task geometry, and optimization outcomes, lies beyond the scope of this work. Moreover, although the proposed approach does not guarantee convergence to a global optimum, it achieves a substantial reduction in jerk of the robot end-effector with respect to the reference case and to the state-of-the-art technique in Ref. [27], as confirmed by the extensive numerical and experimental results.

6 Conclusions

This article presented a novel redundancy-aware minimum-jerk trajectory planning approach for redundant manipulators that jointly exploits the structural redundancy of the robot and the functional redundancy of the task to achieve highly smooth motions and significantly reduce end-effector vibrations. The performance of the approach has been validated on a robot with seven DOFs through extensive numerical tests, and compared with the state-of-the-art strategy in Ref. [27]. Furthermore, bespoke experiments have proven the effectiveness of the proposed approach in reducing the jerk of the end-effector in a real scenario. The results demonstrated that, by jointly optimizing both the position of a redundant joint and the angles that define the orientation of the end-effector for each of the task waypoints, jerk can be reduced by more than 90% with respect to a reference case.

Future work will include the extension of the proposed strategy to robotic systems with more than seven degrees-of-freedom, the comparison with additional alternative strategies from the literature, as for instance Ref. [10], and the application of the proposed approaches to more complex scenarios, also including dynamics constraints and obstacle avoidance, as in Ref. [32]. Furthermore, artificial intelligence will also be considered for optimizing the solution of the inverse kinematic problem in redundant robots.

Acknowledgment

This research was developed within the Laboratory for Big Data, IoT, Cyber Security (LABIC) funded by Friuli Venezia Giulia region (Italy) and the Laboratory for Artificial Intelligence for Human-Robot Collaboration (AI4HRC) funded by Fondazione

Friuli (Italy). This study was carried out within the Interconnected Nord-Est Innovation Ecosystem (iNEST) and received funding from the European Union Next-GenerationEU (National Recovery and Resilience Plan, PNRR, Mission 4, Component 2, Investment 1.5, D.D. 1058 23/06/2022, ECS00000043). This article reflects only the authors' views and opinions, neither the European Union nor the European Commission can be considered responsible for them.

Conflict of Interest

There are no conflicts of interest.

Data Availability Statement

The datasets generated and supporting the findings of this article are obtainable from the corresponding author upon reasonable request.

References

- [1] Piazzzi, A., and Visioli, A., 2000, "Global Minimum-Jerk Trajectory Planning of Robot Manipulators," *IEEE Trans. Ind. Electron.*, **47**(1), pp. 140–149.
- [2] Biagiotti, L., and Melchiorri, C., 2008, *Trajectory Planning for Automatic Machines and Robots*, Springer Science & Business Media, Berlin, Heidelberg.
- [3] Trigatti, G., Boscaroli, P., Scalera, L., Pillan, D., and Gasparetto, A., 2018, "A Look-Ahead Trajectory Planning Algorithm for Spray Painting Robots With Non-spherical Wrists," Gasparetto, A., and Ceccarelli, M., eds., *Mechanism Design for Robotics*, MEDER 2018, Mechanisms and Machine Science, Udine, Italy, Sept. 11–13, Vol. 66, Springer, Cham, pp. 235–242.
- [4] Brunello, A., Fabris, G., Gasparetto, A., Montanari, A., Saccomanno, N., and Scalera, L., 2025, "A Survey on Recent Trends in Robotics and Artificial Intelligence in the Furniture Industry," *Rob. Comput. Integr. Manuf.*, **93**, p. 102920.
- [5] Xiao, Z., Sun, D., Zhu, D., Wang, Y., Yan, Y., and Wu, H., 2026, "Time-Torque Coordinated Optimization for Trajectory Planning of Industrial Robots," *Rob. Comput. Integr. Manuf.*, **99**, p. 103199.
- [6] Gasparetto, A., and Zanotto, V., 2007, "A New Method for Smooth Trajectory Planning of Robot Manipulators," *Mech. Mach. Theory*, **42**(4), pp. 455–471.
- [7] Bendali, N., Mohammed, O., Minh-Tuan, N., and Abderrezak, S., 2022, "Optimal Trajectory Generation Method to Find a Smooth Robot Joint Trajectory Based on Multiquadric Radial Basis Functions," *Int. J. Adv. Manuf. Technol.*, **120**(1), pp. 297–312.
- [8] Bendali, N., Scalera, L., Sundaram, A., Said, A., and Gasparetto, A., 2025, "An Approach Based on Multiquadric Radial Basis Functions for Smooth Trajectory Planning of Robotic Manipulators With Kinematic Constraints," *Nonlinear Dyn.*, **113**, pp. 21511–21529.
- [9] Liu, Z., Li, J., Li, Y., Yao, S., and Wang, G., 2026, "Design of Jerk Model and Optimal Time Trajectory Planning Method Based on Smooth Continuous Oscillatory Type Curves," *Mech. Mach. Theory*, **219**, p. 106342.
- [10] Berscheid, L., and Kroger, T., 2021, "Jerk-Limited Real-Time Trajectory Generation With Arbitrary Target States," *Robotics: Science and Systems 2021*, Virtual, July 12–16.
- [11] Lozer, F., Scalera, L., Boscaroli, P., and Gasparetto, A., 2023, "An Experimental Setup to Test Time-Jerk Optimal Trajectories for Robotic Manipulators," Petrič, T., Ude, A., and Žlajpah, L., eds., *Advances in Service and Industrial Robotics*, RAAD 2023, Mechanisms and Machine Science, Bled, Slovenia, June 14–16, Vol. 135, Springer, Cham, pp. 309–316.
- [12] Fang, Y., Qi, J., Hu, J., Wang, W., and Peng, Y., 2020, "An Approach for Jerk-Continuous Trajectory Generation of Robotic Manipulators With Kinematic Constraints," *Mech. Mach. Theory*, **153**, p. 103957.
- [13] Lu, S., Zhao, J., Jiang, L., and Liu, H., 2017, "Solving the Time-Jerk Optimal Trajectory Planning Problem of a Robot Using Augmented Lagrange Constrained Particle Swarm Optimization," *Math. Probl. Eng.*, **2017**(1), p. 1921479.
- [14] Siciliano, B., Sciavicco, L., Villani, L., and Oriolo, G., 2009, "Robotics: Modelling, Planning and Control," *Advanced Textbooks in Control and Signal Processing*, Springer London.
- [15] Haug, E. J., 2024, "Redundant Serial Manipulator Inverse Position Kinematics and Dynamics," *ASME J. Mech. Rob.*, **16**(8), p. 081008.
- [16] Fabregat-Jaén, M., Peidro, A., Colombo, M., Rocco, P., and Reinoso, Ó., 2025, "Topological and Spatial Analysis of Self-motion Manifolds for Global Redundancy Resolution in Kinematically Redundant Robots," *Mech. Mach. Theory*, **210**, p. 106020.
- [17] Monari, E., Chen, Y., and Verthey, R., 2023, "On Locally Optimal Redundancy Resolution Using the Basis of the Null Space," 2023 IEEE International Conference on Robotics and Automation (ICRA), London, UK, May 29–June 2, IEEE, pp. 9665–9671.
- [18] Hong, T., Li, W., and Huang, K., 2024, "A Reinforcement Learning Enhanced Pseudo-inverse Approach to Self-collision Avoidance of Redundant Robots," *Front. Neurobot.*, **18**, p. 1375309.

- [19] Azizkhani, M., Ha, J., Gunderman, A. L., and Chen, Y., 2025, "Soft Robot Kinematic Control Via Manipulability-Aware Redundancy Resolution," *J. Mech. Rob.*, **17**(7), p. 071013.
- [20] Feller, D., 2023, "Dexterity, Workspace and Performance Analysis of the Conceptual Design of a Novel Three-Legged, Redundant, Lightweight, Compliant, Serial-Parallel Robot," *J. Intel. Rob. Syst.*, **109**(1), p. 6.
- [21] Doan, N. C. N., Tao, P. Y., and Lin, W., 2016, "Optimal Redundancy Resolution for Robotic Arc Welding Using Modified Particle Swarm Optimization," 2016 IEEE International Conference on Advanced Intelligent Mechatronics (AIM), Stockholm, Sweden, May 16–21, IEEE, pp. 554–559.
- [22] Ruiz, A. G., Santos, J. C., Croes, J., Desmet, W., and da Silva, M. M., 2018, "On Redundancy Resolution and Energy Consumption of Kinetically Redundant Planar Parallel Manipulators," *Robotica*, **36**(6), pp. 809–821.
- [23] Li, X., Liu, H., and Dong, M., 2021, "A General Framework of Motion Planning for Redundant Robot Manipulator Based on Deep Reinforcement Learning," *IEEE Trans. Ind. Inform.*, **18**(8), pp. 5253–5263.
- [24] Chen, D., Li, S., Li, W., and Wu, Q., 2019, "A Multi-Level Simultaneous Minimization Scheme Applied to Jerk-Bounded Redundant Robot Manipulators," *IEEE Trans. Autom. Sci. Eng.*, **17**(1), pp. 463–474.
- [25] Dai, C., Lefebvre, S., Yu, K.-M., Geraedts, J. M., and Wang, C. C., 2020, "Planning Jerk-Optimized Trajectory With Discrete Time Constraints for Redundant Robots," *IEEE Trans. Autom. Sci. Eng.*, **17**(4), pp. 1711–1724.
- [26] Lu, Y.-A., Tang, K., and Wang, C.-Y., 2021, "Collision-Free and Smooth Joint Motion Planning for Six-Axis Industrial Robots by Redundancy Optimization," *Rob. Comput. Integr. Manuf.*, **68**, p. 102091.
- [27] Lozer, F., Scalera, L., Boscariol, P., and Gasparetto, A., 2025, "Planning Optimal Minimum-Jerk Trajectories for Redundant Robots," *Rob. Auton. Syst.*, **192**, p. 105049.
- [28] Lagomarsino, M., Lorenzini, M., De Momi, E., and Ajoudani, A., 2022, "Robot Trajectory Adaptation to Optimise the Trade-Off Between Human Cognitive Ergonomics and Workplace Productivity in Collaborative Tasks," 2022 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Kyoto, Japan, Oct. 23–27, IEEE, pp. 663–669.
- [29] Yang, Q., Qu, W., Wang, Y., Song, X., Guo, Y., and Ke, Y., 2025, "Smooth Joint Motion Planning for Redundant Fiber Placement Manipulator Based on Improved RRT," *Rob. Comput. Integr. Manuf.*, **91**, p. 102851.
- [30] Barre, P.-J., Bearee, R., Borne, P., and Dumetz, E., 2005, "Influence of a Jerk Controlled Movement Law on the Vibratory Behaviour of High-Dynamics Systems," *J. Intel. Rob. Syst.*, **42**, pp. 275–293.
- [31] Cook, C., and Ho, C., 1984, "The Application of Spline Functions to Trajectory Generation for Computer-Controlled Manipulators," *Computing Techniques for Robots*, Springer, Boston, MA, pp. 101–110.
- [32] Duong, T., Ramsey, C. W., Kingston, Z., Thomason, W., and Kavraki, L. E., 2026, "Ultrafast Sampling-Based Kinodynamic Planning Via Differential Flatness," preprint [arXiv:2603.16059](https://arxiv.org/abs/2603.16059).