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Gamma ray detection: building a didactic proposal

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Abstract

A simple and economic scintillator kit in combination with a computer-based oscilloscope was used to develop a learning experience, comprised of a set of activities to introduce students in an interactive way to the physics of gamma-ray detection. The proposal for secondary school and university students is built by means of frequent on-the-fly formative assessment to collect difficulties of students and ways to overcome them. Internal decays of Lutetium–yttrium oxyorthosilicate (LYSO) crystals already present in the scintillator kit are used as sources of gamma rays, so no external sources were needed. In addition, the use of LYSO crystals offers a reason to discuss coincidence measurements, because of the inherent beta background present in the internal decay, which can be at least partially removed with coincidence measurements. The goal of the learning experience is to experience how gamma-ray detection is actually done, discussing the physics involved, motivated by the current frontline research on the detection of gamma-ray bursts.

Keywords: gamma ray detection, teaching sequence, formative assessment, students' difficulties, spectra

(Some figures may appear in colour only in the online journal)

1. Introduction

1.1. Motivation

An extensive research literature highlights a lack of competence and motivation in science, technology, engineering and mathematics [1–3]. The literature strongly indicates that the teaching of physics structured as in textbooks, with answers to unasked questions presented in an organized form does not produce conceptual knowledge and motivation [4–8]. It is necessary to offer intellectual challenges and be aware that there are specific angles of attack on the topics identified by literature [9–18]. Technologies and techniques of analysis are a fertile angle of attack both in the physics of matter [19, 20] and in quantum mechanics [21]. Several instruments and techniques that are used in avant-garde research are based on concepts that students have learned in school, like measuring resistivity or current. The relation between avant-garde research and high-school physics can be motivational and offers a clear perspective on how the basic physical concepts are relevant even for advanced research.

Therefore, there are growing attempts to introduce newer topics into the curriculum, which are more relevant in current times, like light-emitting diodes and quantum computers (for examples, see [22, 23]).

Spectroscopy is one of the most widespread and important methodologies in physical investigation [24–30]. It is crucial in many investigations of the microscopic world both with electromagnetic waves and with particles.

Among such topics are also topics currently at the forefront of physics research, like gamma-ray detection in astronomy, that gives us new insight into the mechanics and composition of supernovae explosions [31–33], and could shed new light on gravitational waves [34, 35]. Gamma-ray detection is also used in other relevant applications [28, 36].

The operation of the scintillating gamma-ray detector and some possibilities for coincidence measurements have been described elsewhere [37, 38], but none of these suggest a teaching/learning path. Building a path focusing on the operation of the gamma-ray detector, rather than on the interpretation of the detected results, gives two major advantages for novice students. It presents an example of how physics research is done in contemporary physics and allows students to apply their already existing knowledge in a new and synthetic way to produce an enrichment of learning and a result that is technologically significant. Both factors are motivating for students [24, 39].

1.2. Operation of the gamma-ray detector

In our experiments, we used a gamma-ray detector formed by a cerium-doped lutetium–yttrium oxyorthosilicate (LYSO) scintillator coupled to a silicon photomultiplier (SiPM). In order to discuss the learning difficulties that are expected, let us first briefly describe the operation of a LYSO-SiPM gamma-ray detector. Normally, the gamma particle would enter the scintillator from an external source. However, in LYSO, there are internal beta decays that also produce gamma photons in the decay chain. A small percentage of lutetium is radioactive and beta decays into excited hafnium. This, in turn, gamma decays into de-excited hafnium

$$^{176}\mathrm{Lu} \rightarrow {}^{176}\mathrm{Hf}^* + \beta + \bar{\nu} \tag{1}$$

$$^{176}\mathrm{Hf}^* \rightarrow {}^{176}\mathrm{Hf} + n\gamma. \tag{2}$$

These beta and gammas are produced in such short succession that they are detected as a single event. However, in a small crystal, some photons often escape and so events that

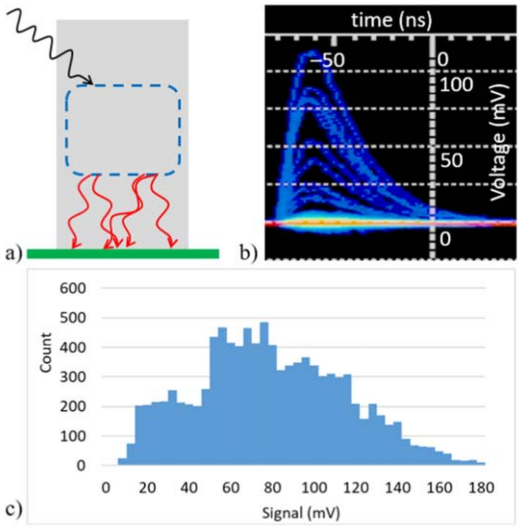


Figure 1. A schematic representation of the sequence of events when creating a spectrum. (a) A particle hits the crystal and creates an avalanche of visible photons. (b) The avalanche is registered as a pulse. Here various pulses are visible, each of a different height. (c) The number of occurrences of a range of heights is then plotted against the ranges of heights creating a spectrum.

correspond to different sums of photon energies are detected, together with the beta. This gives rise to the spectrum which is ultimately analysed in the activity.

The detection process of each particle begins with the particle releasing its energy in the scintillator crystal. This energy is transformed into a number of photons with energy in the visible spectrum. The number of produced photons is proportional to the energy of the original particle.

The visible spectrum photons fall on a SiPM, which is basically an array of photodiodes, operating as a single photon avalanche diode: a photon absorbed in one of them produces an electron-hole pair, which generates a cascade of charge carriers that saturates the signal of the photodiode. Summing up the signals from all the photodiodes one obtains a signal that is proportional to the number of incoming photons. The signal produced by the SiPM has a time distribution, the shape of which depends on the conversion process in the scintillator and the avalanche amplification in the SiPM: it results typically in a pulse of a few tens of nanoseconds. The area under the pulse is proportional to the number of visible light photons produced by the gamma photon. Moreover, the shape of the pulse is constant and its duration is constant, which means that the area under the pulse is proportional to the amplitude of the pulse. Thus the cause-effect link is established between the energy of the gamma photon and the amplitude of the pulse (see figure 1).

1.3. Educational path and research framework

We framed the research by combining the model of educational reconstruction [40] with design-based research (DBR) [41]. We designed an educational path by identifying the fundamental conceptual nodes of the topic and addressing them in a logical progression. In the logical progression we were guided by the learning cycle as described in the Investigative Science Learning Environment [42], a progression from observation through models to testing the models with testing experiments. The construction of the model is the main part of the course and is done mostly frontally with elements of peer instruction [43]. For each node we designed formative assessment in the form of clicker questions to assess how well the node has been appropriated by the students. For some nodes, we found known difficulties in literature, which helped us design the formative questions. When results showed unsatisfactory appropriation, peer discussion was used and a second round of the same clicker question was implemented as suggested by the peer instruction method. In each implementation, we identified elements of the course that we believed could be improved and modified them in the next implementation, as usually done in DBR.

A short overview of the educational path is presented in table 1. It runs through the steps of the detector as described in the previous section. Each step has its own learning goals and each step has possible learning difficulties that we discuss in this section. The results given by the detector are single energy values of single events. These need to be represented in the form of a spectrum (see figure 1). Spectroscopy has been shown to be a difficult topic for students [44–47]. Elements that are known to be challenging are distinguishing between energy levels and energy of emitted photons (differences between energy levels) [44–46] and creating a spectrum from single events. Students are known to misinterpret the vertical axis as energy rather than a number of events [46]. The spectrum in this activity presents an additional challenge: more than one particle is absorbed at the same time. This means that the energies of the various particles are summed in the spectrum. For example, one 100 keV particle and one 300 keV particle absorbed at the same time produce an event of 400 keV in the spectrum. Not two separate events of 100 and 300 keV. We have observed that this is a new concept even for physics education researchers and physics teachers that we talked to when developing the course. Another potentially difficult topic is the interpretation of electric current from the number of current carriers [48]. However, at this time, this was not an important part of the course and was not investigated.

For students to best gain an understanding of the involved phenomena, it is best if they are actively engaged in the course activities [49, 50]. Basing the course on experimental observations and gradually progressing through explanations allows students to formulate questions to which the course provides answers, building learning in an inquiry-based learning approach [51]. To build a research-based course, it is important to follow student reasoning at every possible step in accordance with a design-based research method. Therefore, on-the-fly quizzes were used to identify student reasoning, discover student difficulties and find ways to address them.

The main research questions that we posed were:

(a) How do students appropriate the main conceptual steps of the proposed path?
(b) How do students' difficulties emerge from the arguments proposed and how can they be overcome?
(c) Does on-the-fly formative assessment help identify these problems and address them?

Providing answers to these questions is relevant, because it shows how to build a learning path on a relatively difficult topic with active engagement of students, which topics among the crucial ones are those that, despite being taught, remain poorly understood, and what can be done to address them.

Table 1. An overview of the educational path.

Conceptual step	Learning goals	Activities	Conceptual knots
The pulse	Quantum particles produce cascades of photons in the visual spectrum. The photons produce current carriers in the SiPM. Bias voltage produces current (I). The area under the $I(t)$ curve is proportional to the energy of the original particle.	Identify that the area under the $I(t)$ curve is proportional to the energy of the original particle.	Students may think that the speed of the carriers affects the current.
The spectrum	Making a histogram (spectrum) from single events.	Make a spectrum from a sample of single pulses.	Students may not know how to make a spectrum from a set of data.
Energy spectrum of gamma particles	Nuclei have discrete energy levels like electrons in an atom. The emission of photons from nuclei is similar to the emission from electrons in an atom.	Predict the spectrum of the emitted gamma photons from the energy levels of the nucleus.	Students often think that the emitted photons have the same energies as the energy levels, instead of the differences between the levels.
Simultaneous absorption	When two or more particles are absorbed in a time smaller than the temporal resolution of the detector, their energies are summed. This affects the spectrum by changing the values on the horizontal axis.	Predict the spectrum when more than one particle is absorbed.	Students may sum the spectra as if they represented individual events – by summing the values of the vertical axis.
Escape events	When particles can escape from the crystal, the single energy peak of the sum of all energies splits to peaks representing partial sums.	Predict the energy spectrum taking into account possible escape events.	Students are expected not to have experience with this kind of task. All kinds of difficulties might arise.
Final spectrum	The final spectrum is a combination of all discussed phenomena.	Predict the shape of the final spectrum given a set of hypothetical energy levels of the nucleus.	Students are expected not to have experience with this kind of task. All kinds of difficulties might arise.

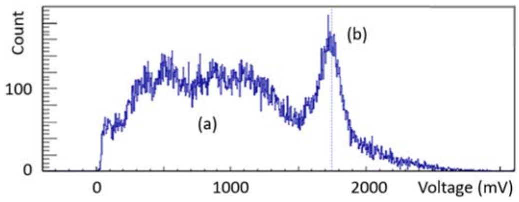


Figure 2. A measurement of an external ^{137}Cs source using the LYSO-SiPM detector. In range (a) the largest contribution is from LYSO's internal spectrum. The count in this range was very similar with and without the ^{137}Cs source. However, the characteristic peak of ^{137}Cs at 662 keV (b) is clearly visible and can be used for calibration.

2. Equipment and data collection

2.1. Equipment

While some equipment is commercially available for this sort of school experiments, this equipment is often very expensive. We wanted to develop the experience around a setup that students could use in groups by themselves, and should therefore be as easily available and as cost-effective as possible. We used a low-cost scintillation detector, developed by Ian Bearden [52] and a low-cost oscilloscope (Digilent Analog Discovery 2 with a dedicated Digilent BNC adapter) used in conjunction with a computer and the Digilent WaveForms software.

An analysis of the performance of the detector was done by means of a measurement campaign in which the detector was used in conjunction with a high performance acquisition system. A series of various setups were used. Among others, an external ^{137}Cs source was used for the calibration of the detector to relate voltage to energy. The result of this calibration is in figure 2. However, each experimental session requires its own calibration because the gain of the detector changes. So it is more convenient in a class activity to work with voltage or channels directly.

2.2. The research-based implementation of the designed proposal and data collection

The course has been piloted in four different settings: (1) a class of high-school students in Treviso (Cohort 'HST', $N = 17$, grade 13), (2) an extracurricular activity called *Experimental modules* for high-school students in Udine (Cohort 'EMU', $N = 21$, grade 13), (3) a summer school for motivated high-school students in Udine (Cohort 'SSU', $N = 32$, grade 12), and (4) a summer school for motivated high-school students in Ljubljana (Cohort 'SSL', $N = 17$, grades 10–13).

The activities lasted for 3 h, including an introductory part shortly explained in the next section. We collected data on clicker responses and graphing tasks, which will be explained in the description of the course. The findings will be presented along with the activities because the results of the activities are an integral part of the course. They inform further decisions and are crucial in the motivation for the activities that follow. We believe that discussing the activities without their results would make it difficult to follow the line of thought.

3. The activities and the results

3.1. Introduction to the problem

The way in which the sensor works is a preliminary step and can be explained to students at different levels.

Basic (high-school): the gamma produces a photoelectric-type effect and transforms photon energy into electrical signal.

Intermediate (motivated high-school, introductory university): the gamma enters the scintillator, which converts the high-energy gamma into a cascade of photons in the visible spectrum. These photons strike the photomultiplier and via a series of photoelectric-type effects results in an electric signal, the time distribution of which is shaped as a pulse. The signal pulse, converted to a voltage signal, is then measured and recorded by means of an oscilloscope.

Expert (university): to the description above are added the details of photon conversion in the scintillator and details of the SiPM operations.

In the first activity, students observe the voltage signal produced by the sensor on an oscilloscope. They compare the amplitude, shape and duration of different signals. The learning goal of this activity is to learn how to extract information about the energy of the particle from the spectrum. The task states:

The pulse of current, coming from the detector, is identical to the pulse you are observing, except for a scaling factor. The graph on the display can therefore be considered a graph of current versus time. What in this graph represents the total number of electrons incident on the photomultiplier?

Students usually respond with either amplitude or area. The correct answer is area, of course. Students are then told that given the same shape and duration of the pulses, the area is proportional to the peak value. The peak value is easier to address 'by hand'. However, when using a computer, the area can be calculated.

3.2. Creating a spectrum

Students are taught how to create a spectrum. We discuss that a spectrum is a graph of a number of events versus observed quantity. To verify how well students understand the concept, in the activity 'creating a spectrum' in table 2, they are asked to create a spectrum from a series of signals.

This activity has been introduced for the SSU cohort after observations from HST and EMU cohorts showed that this is a difficult conceptual node that requires additional consideration.

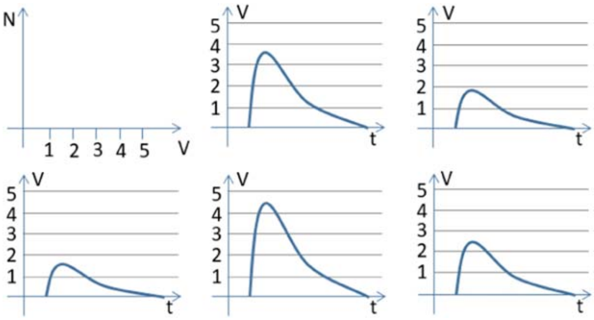
The results are reported in table 2. In the SSU cohort, 72% of students were able to produce the spectrum. However, observations from other cohorts gave us the impression that this number would be considerably lower in those cohorts. The SSU cohort significantly outperformed the other cohorts on almost all tasks, so might not be representative of all the cohorts.

3.3. The emission of gamma photons

First, we briefly discuss the beta decay and the spectrum of beta particles, since they are the first step in the decay process. The gamma decay is approached with the same tools used for the beta decay. Energy diagrams are used to discuss the energies of the gamma photons. Various energy levels are presented between the excited and the de-excited Hafnium.

Table 2. The task of creating a spectrum from single pulses, and students' results. The actual worksheet contains 17 pulses instead of 5 as in this sample.

Task: creating a spectrum.



From the pulses construct an appropriate spectrum.

Results

	SSU
Correct spectrum	0.72
Partially correct spectrum	0.06
Incorrect spectrum	0.22

Table 3. The task of predicting the emission spectrum from known energy levels of an atom, and students' results.

Task: Spectrum from energy levels.

A hypothetical atom has three energy states: 1 eV, 3 eV *and* 7 eV. *What are the possible energies of a photon emitted by this atom? (You can choose multiple answers.) Choices:* 1 eV; 2 eV; 3 eV; 4 eV; 5 eV; 6 eV; 7 eV.

Results (the correct answer is shaded)

	HST	EMU	SSU	SSL
2 eV, 4 eV, 6 eV[a]	0.32	0.09	0.74	0.07
1 eV, 3 eV, 7 eV	0.05	0.43	0.00	0.50
Other	0.42	0.48	0.20	0.43
Unanswered	0.21	0.00	0.06	0.00

[a] Correct answer.

To test how well students understand energy emission in an atom we use the clicker activity 'spectrum from energy levels' shown in table 3. Students are presented with a hypothetical system with three energy states with values 1 eV, 3 eV and 7 eV. Their task is to predict the possible energies of the emitted photons.

The results of the activity 'spectrum from energy levels' are shown in table 3. The answers are grouped into four groups: the '2 eV, 4 eV, 6 eV' ('correct') answer, the '1 eV, 3 eV, 7 eV' ('naive') answer, any other answer and unanswered. The results show that the identification of emitted photon energies is difficult for all the students, except the SSU cohort. This indicates that the best-prepared students even in secondary school are able to understand sufficiently well this important conceptual node. In all the other cohorts, there were approximately 50% of students whose answers were classified as other, and in no cohort, the answers were evenly distributed between the correct and the naive answers; instead, either one or the other dominated.

3.4. The absorption and spectrum of the gamma photons

After discussing the photon emission process, we are ready to look into the emission of gamma particles from our LYSO crystal. At this stage, we have no indication of how many energy levels there are between the excited and de-excited hafnium. So we proceed hypothetically with the simplest nontrivial possibility of three levels including the initial and final states. We suppose a difference in energy levels of 100 and 300 keV. This means three possible photon energies. But, a single decay can produce either (a) two photons of 100 and 300 keV or (b) a single photon of 400 keV.

The next step is to discuss how these photons are absorbed in the crystal. There are several possibilities:

(i) '*Separate detection*'. Each photon is absorbed as a separate event.
(ii) '*Simultaneous detection*'. The single 400 keV photon of case (b) is absorbed as a separate event, but the two 100 and 300 keV photons are absorbed together as a single event.
(iii) '*Escaping photons*'. Same as (ii) only with the possibility that some photons may escape the crystal undetected.

We list all the possibilities here for completeness, but in the course they are introduced one after the other.

Alternative 'separate detection' appears trivial and we have tested how well the students can predict the resulting spectrum already in the activity 'spectrum from energy levels' shown in table 3. The activity 'summing two gammas' in table 4 tests how well students can predict the spectrum in case of 'Simultaneous detection'. The answers are chosen to indicate particular mathematical operations to arrive at the result: (A) 'summing E and N' (sum E and N), where E stands for the energy of an event and N for the number of events; (B) 'summing E' (sum E); (C) 'summing N' (sum N) or 'tracing the outer line' (env.—for envelope) of both spectra.

The results of the activity 'summing two gamma' in table 4 indicate that many of the motivated students used correct reasoning. When informally asked, why they chose the correct option B, some students responded that option A violated energy conservation. This indicates that at least some students used fundamental physical laws to reason about a task, which does not explicitly invoke these laws. Interestingly, simply summing the spectra ('summing N') was the least favourite option in all cohorts.

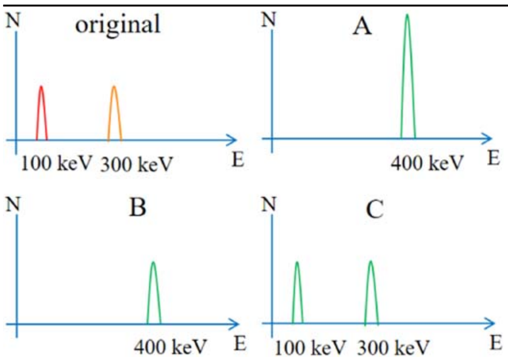
The purpose of this activity is for students to realize that a sum of energies will be reflected in the spectrum as a sum of horizontal values E, not vertical values N.

3.5. Simultaneous detection of a beta and a gamma

With the activities so far the students should have built an understanding of what energy photons are emitted and what measurement results we get, if multiple photons get absorbed in the crystal as a single event. The activity 'summing a gamma and a beta' in table 5 tests how well the students understand these phenomena and ask them to predict the spectrum of a simultaneously detected gamma and beta. The answers are selected to indicate similar reasoning as in the previous activity: (A) 'tracing the outer line', (B) 'summing E' and (C) 'summing N'. 'Summing E and N' is not present among these choices.

Table 4. The task of predicting the spectrum of two simultaneously emitted photons knowing the spectra of each photon, and students' results.

Task: Summing two gammas.



There are two photons in the crystal. Their energy distribution is indicated on the left. The photomultiplier can detect only the total energy absorbed in the crystal. Which of the spectra A, B or C represents the spectrum that will be the result of such a measurement?

Results

	HST	EMU	SSU	SSL
A (sum E and N)	0.71	0.40	0.20	0.33
B[a] (sum E)	0.29	0.40	0.65	0.67
C (sum N/env.)	0.00	0.20	0.15	0.00

[a] Correct answer.

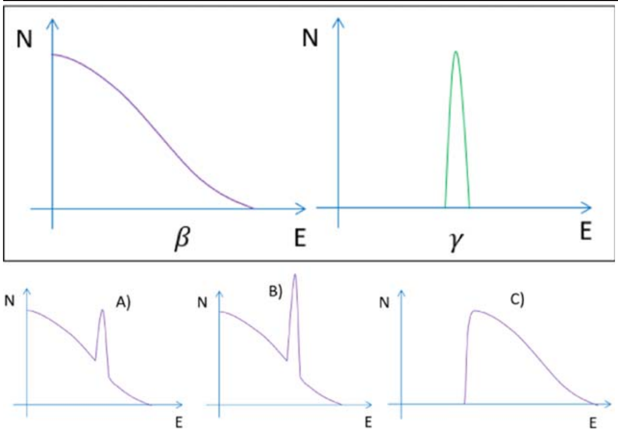
The results of the activity 'summing a gamma and a beta' in table 5 indicate that this topic is one of the crucial conceptual nodes to which it is necessary to pay significant attention. The success of students in the activity 'summing two gammas' (table 4), even in the SSU and SSL cohorts should not be taken to mean that the students have understood the concept. The results in table 5 show that the topic requires more attention. Changing context helps students to view the underlying principles in a different way. Peer-instruction-like discussion significantly helped students to arrive at the correct answer.

Another thing that emerged from the results is that when we compare the choices in the activities 'summing two gammas' in table 4 and 'summing a gamma and a beta' in table 5, we see inconsistent reasoning. Cohorts HST and SSL collectively chose A in table 5, indicating the reasoning 'tracing the outer line' that should have made them choose C in table 4. Instead, the majority chose A ('summing E and N'). Similarly, cohort SSU chose B in table 5, which would indicate the reasoning 'summing N' that should have made them also choose C in table 4. Instead they chose the correct answer B ('summing E').

In reasoning about spectra, it might help to refer to the statistical nature of the spectrum and ask: 'what happens in one event and where on the spectrum does it fall?'. Instead, the students might be considering the spectrum an entity in itself, and not comprised of smaller entities (events). Similar mental pictures have been observed for waves, where students consider a wave pulse an entity in and of itself instead of being comprised of smaller entities (displacements of point masses of the medium) [53–56].

Table 5. The task of predicting the spectrum of a simultaneously emitted beta and gamma knowing the spectra of each particle, and students' results.

Task: summing a gamma and a beta.



A beta and a gamma are absorbed simultaneously in the crystal. The separate spectra of the beta and the gamma are in the top row. What will be the spectrum of the simultaneous detection?

Results

	HST	SSU	SSL
A (env.)	0.82	0.10 (b) 0.04 (a)	0.58
B (sum N)	0.17	0.60 (b) 0.32 (a)	0.33
C[a] (sum E)	0.00	0.30 (b) 0.64 (a)	0.08

[a] Correct answer.
[b] (b) – before discussion; (a) – after discussion.

During our observation of student difficulties with deciding when to sum N and when to sum E, we found that it helps to address the events as *simultaneous* or *independent*. Simultaneous events are summed by E, while independent events are summed by N.

3.6. Escaping photons

The next activity addresses alternative (iii) in section 3.4: the possibility that some gamma photons escape the crystal and do not release their energy inside of it. In this case, the gamma spectrum, which should have been one single line, splits again into several lines. Only, instead of being lines of the photon energies, they are lines of the sum of all photon energies minus the escaped photons.

An example with only three energy levels E_0, E_1, and E_2, is simple. The corresponding photon energies are $E_{01} = E_1 - E_0$, $E_{02} = E_2 - E_0$, and $E_{12} = E_2 - E_1$. The energies present in the spectrum will thus be E_{01}, E_{02}, and E_{12}, instead of just E_{02}. The peak at energy E_{02} could be the result of either the detection of a single E_{02} photon or the simultaneous detection of E_{01}, and E_{12} photons. In the case of three photons, the spectrum is different from the

Table 6. The set of all possibly detected energies in case the photons are emitted simultaneously, but allowed to escape the scintillator crystal.

Possible decays	
Decay 1	E_{01}, E_{12}, E_{23}
Decay 2	E_{01}, E_{13}
Decay 3	E_{02}, E_{23}
Decay 4	E_{03}
Possible detected energies	
E_{01}	From decay 1 and decay 2
E_{02}	From decay 1 and decay 3
E_{03}	From decay 1, decay 2, decay 3 and decay 4
E_{12}	From decay 1
E_{13}	From decay 1 and decay 2
E_{23}	From decay 1 and decay 3
$E_{01} + E_{23}$	From decay 1

spectrum of single photons only in the amplitude of the peaks, but not in their position, which is not the case in general.

To differentiate between the single photon spectrum and the escaped-photon spectrum, in the course, we consider the next simplest case of four energy levels E_0, E_1, E_2, and E_3. These give 6 possible photon energies labelled $E_{nm} = E_m - E_n$, but only 4 possible decays as shown in table 6. Each decay can result in all the photons being absorbed or some photons escaping the crystal. This gives 7 possible lines in the spectrum also shown in table 6. There is only one line that cannot be found in the single photon spectrum.

Further discussion leads to the fact that the amplitudes of each of these peaks in the spectrum depends on the relative probability for that event to happen. For example, the peaks of the single photons are expected to be low, because the probability of two photons escaping the crystal is low. But, it gets higher when the size of the crystal gets smaller, as discussed very well in [57].

3.7. Predicting the final spectrum

Before giving students the final spectrum for analysis, we use another formative assessment to see how well students learned the topic. The activity is to predict the shape of the expected spectrum, knowing that the beta is always absorbed, while some of the gamma might escape the crystal. Students are presented with a hypothetical spectrum of the gamma and beta particles. Their task in activity 'predict the spectrum' shown in table 7 is to predict the shape of the spectrum using the knowledge acquired so far. This task combines summing by E the gamma and beta events which are simultaneous and then summing by N the various gamma +beta events which are independent.

We analysed the data by coding student responses with five codes. Examples of the codes are in figure 3, where 'correct' stands for correctly combining E and N; 'mostly correct w/o N' stands for correctly adding beta to each gamma, but not summing the final spectra of independent gamma+beta events; 'mostly correct unclear' was used when it was not possible to determine which of the above codes would be more appropriate; 'incorrect' was used for incorrect answers and 'missing' was used for unanswered.

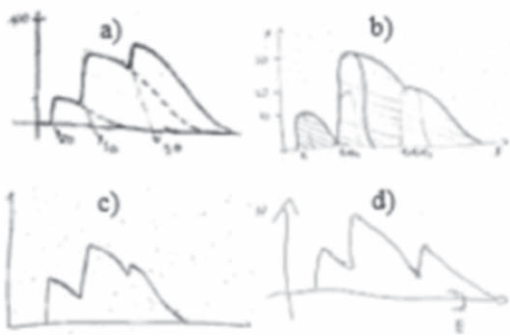
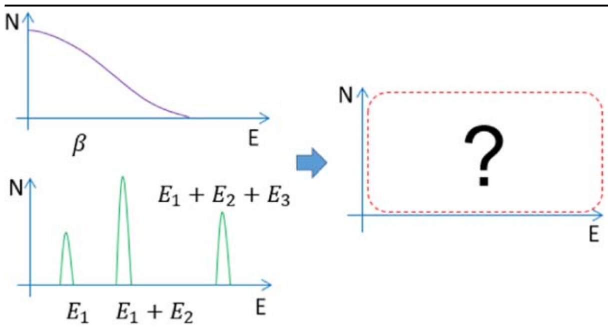


Figure 3. Examples of students' prediction of the spectrum. (a) 'correct'. The numbers associated with the lines clearly indicate superposition of the three spectra. (b) and (c) 'mostly correct w/o *N*'. The shape of the spectra clearly indicates an envelope-type tracing, not a superposition. (d) 'mostly correct unclear'. Due to the large spacing between the three gamma+beta spectra, it is unclear whether a superposition was used or an envelope-type tracing.

Table 7. The task of predicting the spectrum comprised of simultaneously detected beta and gamma particles with the possibility of escaped photons, and students' results.

Task: predict the spectrum.



One complete decay event consists of almost simultaneous beta decay and one or more gamma decays. The spectra of both are in the figure. Predict the spectrum of the complete decay events.

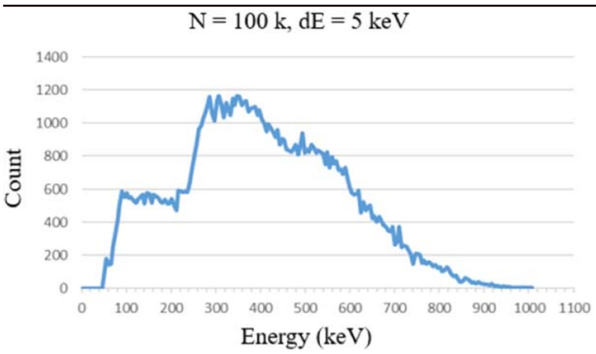
Results

	SSU
Correct (sum *E*)	0.12
Mostly correct w/o *N* (sum *E*)	0.28
Mostly correct unclear (sum *E*)	0.15
Incorrect	0.30
Missing	0.15

This activity was only performed in the SSU cohort. In the pilot implementations before SSU, the success rate for the activity in table 8 was very low. We hypothesized that if students were to first synthesize their acquired knowledge to create a fictional spectrum, the resources required to analyse a similar spectrum would be more easily triggered. Through DBR we

Table 8. The task of identifying single photon energies from a spectrum of internal LYSO decays given simultaneous detection and the possibility of escaped photons, and students' results.

Task: analyse the spectrum.



The spectrum of a LYSO crystal is in the figure. Identify the energies of the gamma photons emitted by LYSO.

Results

	HST		SSU	
	Corr.	All	Corr.	All
Identification	0.89	1.00	1.00	1.00
Energies	0.00	0.05	0.37	0.73
Probability	0.28	0.83	0.07	0.13

identified that the activity of predicting the spectrum was helpful for students to successfully analyse the final spectrum. It can be seen from the results in table 8 that in the HST cohort only a very small fraction of students attempted to identify the energy peaks in the spectrum. The added activity 'predicting the spectrum' in table 7 significantly increased the number of students in SSU cohort who attempted to identify the energy levels of the gamma photons, although their success rate was rather low.

On the other hand, the number of students attempting to determine the relevant probability for each event decreased in the SSU cohort. This is not a problem, because this is not a goal of the course. The physical models that students engage with do not address the probabilities. They are intentionally left out of the educational reconstruction. This is another indicator that the activity of predicting the spectrum helped activate the desired resources when analysing the spectrum, however, apparently, at the cost of not activating resources about the relative probabilities.

3.8. Analysing the final spectrum

All activities so far have been aimed for students to learn what a LYSO spectrum is comprised of so that they would be able to analyse one. In the final activity, students are given prints of an actual measured LYSO spectrum and the spectrum measured over the course of the session can be viewed. Students are now asked to identify the different gamma photon energies from the actual spectrum.

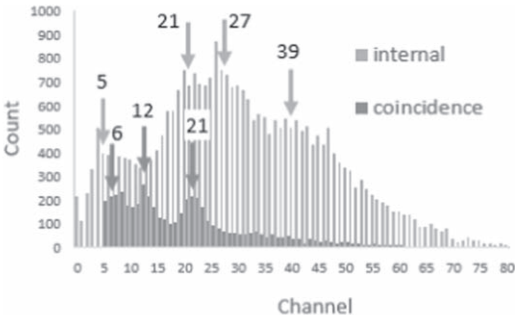


Figure 4. A comparison of the internal LYSO spectrum and the spectrum of coincidences.

This activity requires students to first identify the gamma lines from the combined gamma +beta spectrum of all possible gamma lines. Then they need to identify the gamma energies as possible sums of actual gamma photon energies. And then through trial and error identify the possible single photon energies that can sum up (as presented in table 6) to produce the observed gamma lines.

The data that we have is shown in table 8. Along with the fraction of successful students ('corr.' in table 8) we also report the fraction of students attempting a task ('All' in table 8).

We can see from table 8 that students are very successful in identifying the position of the gamma lines ('identification' in table 8), pointing at them with arrows or lines. Students were not particularly successful in identifying the actual energies of the gamma photons. The 36% of students in cohort SSU who incorrectly identified the lines, identified them as single photon energies instead of the possible sums of photon energies from which the possible individual photon energies are still to be determined.

The data in table 8 shows that students also attempt to determine the relative probability of each event (the amplitudes of the gamma lines).

3.9. Coincidence measurement

There is an additional experiment that can be performed as a testing experiment after the identification of the possible single gamma photon energies. This is to actually measure the single photon spectrum. The way to do it is to configure two LYSO gamma-ray detectors in a coincidence measurement. The two detectors are positioned facing each other. The coincidence measurement can be most easily done with the WaveForms software. The procedure is the following. (1) Connect each sensor to their own channel. (2) Multiply both signals and set the trigger to when the combined signal passes an arbitrary threshold. (3) At the trigger, log the value detected by one of the sensors. The rationale behind this setup is the following. When a beta decay happens in sensor 1 triggering an internal event, and one of the gamma photons escapes the crystal, it might end inside sensor 2 triggering an external event. Thus, when the two sensors are triggered simultaneously, one of them is almost certainly triggered by an external event provoked by the other sensor. This enables us to log the energies of the escaped gamma photons, which are not burdened by the beta spectrum, since the beta is entirely absorbed in the crystal where it originated. However, one sensor logs its own internal events when they trigger the other sensor and the external events coming from the other sensor. So, on average, the spectrum should be a superposition of single gamma lines and the internal LYSO spectrum with escape peaks only (only events where at least one photon escapes), as well as random coincidences, which are assumed to be very rare.

The spectra obtained by the above procedure are in figure 4. For the activity, the spectra should be prepared in advance, because they need to be calibrated. However, actual spectra can be collected before and during the course and shown to participants (the spectrum of coincidences in figure 4 took 8 h to collect). In the following paragraph, we will compare peaks from four spectra. To facilitate differentiation, we will label the peaks in the following way. Peaks from the measured internal spectrum with 'mi', those from the measured coincidence spectrum with 'mc', the single gamma peaks inferred from the internal spectrum, which should correspond with the measured peaks in the coincidence spectrum with 'hc' (for hypothesized coincidence), and the hypothesized internal peaks inferred from the measured coincidence peaks with 'hi'.

The cumulative gamma+beta peaks from the internal spectrum can be identified at channels 5(mi), 21(mi), 27(mi) and 39(mi). Students have identified them in the activity in table 8. To identify the possible single gamma energies (in units of channel), students have to consider the possible differences between the gamma+beta peaks. This leads to the possibilities 5(hc), 6(hc), 16(hc), 18(hc) and 22(hc). To test these results one uses the coincidence spectrum which shows single photon energies of 6(mc), 12(mc) and 21(mc). It appears that 6(mc) could account for the 5(hc) and 6(hc) possibilities and 21(mc) could account for the 22(hc) possibility. Could the 12(mc) account for both 16(hc) and 18(hc)?

The answer potentially lies in reversing the reasoning. The actual LYSO spectrum is comprised of peaks E_1, $E_1 + E_2$, $E_1 + E_3$ and $E_1 + E_2 + E_3$. Starting from the measured coincidence spectrum with peaks 6(mc), 12(mc) and 21(mc) this gives hypothesized internal peaks at 6(hi), 18(hi), 27(hi) and 39(hi), respectively. Comparing these with the gamma+beta lines of the measured internal spectrum we see that except for the 18(hi) versus 21(mi), all the other lines match very well. We currently do not have a convincing explanation for why the 18(hi) peak could appear as the 21(mi) peak.

4. Discussion

In this study, we have investigated the difficulties encountered by students in an active-learning course about a LYSO-SiPM gamma-ray detector. We show that the same course (largely) was very differently followed by the different cohorts. We show that formative assessment was helpful in identifying learning difficulties on-the-fly and enabled the correction of course as necessary.

We observed several difficulties that we discuss in the following paragraphs.

One set of difficulties was with simple spectra.

Students had some difficulty creating the spectrum from individual events. This led to the addition of an activity constructing spectra at the beginning of the course.

Students also had difficulties identifying photon energies from atomic energy levels. The most common incorrect answer was to simply take the energy levels to be the photon energies. However, about half of the students in most cohorts gave incorrect answers that cannot be explained in this way. Therefore, it would be worth investigating the reasoning resources that students use when selecting their answers. Such research could give valuable insight into students' reasoning about photon emission. Students' reasoning about energy is extremely important since energy considerations play a fundamental role in almost all physics involving microscopic phenomena. It would also be worth investigating whether spending more time on energy diagrams would help students arrive at more correct results.

A second set of difficulties was with complex spectra representing multiple simultaneous events.

Three tasks in the course address this concept. First summing two gammas in table 4, then summing a gamma and a beta in table 5, and then summing a beta and the escape spectrum of the gammas in table 7. The success rate of each cohort dropped significantly between summing two gammas and summing a gamma and a beta. In the task of summing two gammas, energy arguments were used by many students, but the same reasoning does not appear to have been used in the task of summing a gamma and a beta. Changing context apparently changed students' reasoning.

However, in the SSU cohort, where peer instruction was used, the success rate raised after peer instruction to the same level of summing two gammas, indicating that peer instruction was helpful in making students reconsider their arguments.

The activity of summing a beta and an escape spectrum of gammas was only done with the SSU cohort. The sum of mostly correct and entirely correct answers amounts to 55%. Given that the task of summing a gamma and a beta was successfully performed in this cohort by only 37% of students before peer discussion, and the latter task is more complex, a success rate of 55% at the first attempt indicates an improvement in students' learning. We believe that it might be helpful for students to address the building of the spectrum one event at a time, but this suggestion requires further investigation.

The finding that conceptually similar activities proved differently difficult for different cohorts emphasizes the need for formative assessment in lessons, as it clearly shows that simply explaining the correct procedure does not in general help students use the procedure in different situations and that different cohorts might encounter different problems that have to be addressed for that specific cohort. The results also show that peer discussion improved the answers when it was used.

The final activity of analysing an actual spectrum from the LYSO detector was performed significantly better in the later SSU cohort than in the initial HST cohort. We believe that the reason is the addition of the activity of predicting the spectrum before analysing it. The act of predicting requires students to activate the appropriate resources and identify the role of each one of them in the final shape of the spectrum. We believe this helped them activate the same resources when analysing an already existing real spectrum.

The success rate of 37% in the final activity of identifying energies in cohort SSU appears relatively low and certainly indicates a necessity for further improvement. However, the success of the SSU cohort (37%) with respect to the HST cohort (0%) indicates that the changes to the course had a positive effect on students' understanding. The success rate of 37% is also comparable to the success rate of this same cohort in summing a gamma and a beta before discussion. Thus, a more complicated task was successfully accomplished at the end of the course at the same rate as an easier task in the middle of the course. This indicates learning progress.

5. Conclusions

We developed a course about gamma-ray detection for complete novices in the field of particle physics. The topic offers multiple opportunities for active engagement and for engaging students in culturally relevant contemporary physics. The educational reconstruction presented in this article builds from simpler to more complex tasks, involving students at every possible step.

We asked three research questions and we answer them thusly:

(a) Which parts of the educational path can be done interactively with students?
Students could in more than 50% of cases identify that the area or peak of a signal represents the energy of a particle, they could build the spectrum from a given sample of data, and they could identify the effect that simultaneous detection has on the spectrum. They could predict the shape of the final spectrum and identify the spectral lines in the final spectrum. They were less than 50% successful in predicting the spectrum of simultaneously detected gamma and beta and identifying single gamma photon energies from the final spectrum. We, therefore, find that the simultaneous detection of particles with different spectra is a crucial node in gamma-ray detection and must be strengthened.

(b) Which problems do students encounter in their reasoning, or more specifically, which steps in the educational path cause most problems to students?
Most problems were observed in identifying spectral lines from energy levels, summing simultaneous events in the spectrum by energy, and determining single photon energies from the final spectrum. In the course, it was helpful to use frequent formative assessments and peer discussions to improve students' reasoning. It was also necessary to expose students to the phenomena several times in different contexts.

(c) Does on-the-fly formative assessment help identify these problems and correct them?
We argue that not only does it help identify them, but on-the-fly formative assessment is crucial to address them before arriving at the final activity of the course. We have shown that many aspects of spectrum formation have been poorly understood after they were introduced and had to be rectified. Sometimes multiple times in different contexts. This and the differences between the cohorts observed in our course lead us to believe that it might be indispensable to include formative assessment to tailor the course to the specific cohort of students and their needs, instead of aiming at a universal course without a formative assessment that would be equally efficient for all students.

- Answer to question (a) shows that a course on particle detection can be done in a largely active-engagement manner even with novice students. The type of active engagement was mostly formative assessment and peer instruction.
- Answer to question (b) shows that more emphasis needs to be given to spectra.

The results also show significant differences between different cohorts. Motivated students in summer schools tend to perform better than less motivated students in regular high-school classes or students enrolled in extracurricular experimental activities.

However, in all cohorts some inconsistent reasoning was observed. For example, students in some cohorts (SSU and SSL) correctly identified the spectrum of two gamma particles by summing their energy (more than 60%), but when discussing the spectrum of a simultaneous gamma and beta event switched to summing the number of events (SSU) or choosing the envelope of the two spectra (SSL). Peer discussion was used in the SSU cohort and was helpful in raising the success rate above 60%. This is entirely expected for a topic where knowledge is still being built, and it reinforces the previously identified problem of students not having a coherent reasoning scheme developed, but instead jumping between reasoning resources depending on the context [58].

In the end, we believe that we have shown that an active-engagement course for novices in such a complex topic as gamma particle detection can be successfully done. We have further shown that formative assessment during the course is crucial for the course's success because it allows us to catch and correct reasoning errors that might otherwise be missed and might lead to incorrect reasoning down the road. The cooperative learning coming from the discussion of the problems encountered are very important, too, for the overcoming of conceptual knots, as emerged where peer discussion was used (table 5). We have also shown that the activities can be expanded to include coincidence measurements. Further research could be done into students' reasoning resources about spectra and how simultaneous events affect a spectrum.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

Ethical statement

Written consent to participate in all the activities described in the article was obtained from all participants. In the consent form, the participants were informed that their answers to questionnaires would be anonymously collected and potentially used for research purposes, and that their collection and storage is in compliance with the General Data Protection Regulation (GDPR, EU 2016/679, 27/04/16) and the University of Udine's Quality Assurance System.

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