A dynamic version of the Massey's rating system with an application in basketball

Paolo Vidoni and Enrico Bozzo

Abstract This paper proposes a flexible, dynamic extension of the popular Massey's method for rating players and teams involved in sports competitions. The original Massey's approach is static since the computation of a team rating is based on the strength of the opponent teams evaluated at the current time. The proposed dynamic extension updates the team rating considering the strength of the opponent teams evaluated at the time when the matches were played. An application of the new rating procedure to the Euroleague Basketball 2018-2019 is presented.

Key words: basketball, match prediction, ranking, rating, sport analytics

1 A brief introduction to the Massey's method

The main idea underlying the method proposed by Massey [4] in his honors thesis, and shared by most models and rating methods, is that the match outcome between two teams depends on the team strengths only through their difference. Let us consider a tournament involving $I \geq 2$ teams and $N \geq 1$ games. If y_n, $n = 1, \ldots, N$, is an outcome of match n played by teams i_n, j_n, then

$$y_n \approx g(r_{i_n} - r_{j_n}),$$

where r_{i_n} and r_{j_n} are the ratings of teams i_n and j_n. Although there are various possibilities, here and in what follows the outcome y_n is the final score difference

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in the game from the perspective of team i_n and, usually, $g(x) = x$. In this case, a simple approach for computing the team ratings after N matches is to minimize the quadratic objective function

$$\sum_{n=1}^{N} \{y_n - (r_{i_n} - r_{j_n})\}^2. \tag{1}$$

It is immediate to see that (1) can be viewed as the sum of the squared residual for a linear regression model with a suitable design matrix, however the associated linear system is overdetermined due to linear dependence. A standard approach to address this problem is to impose a linear constraint on the team ratings; following [4], we assume the sum constraint so that $\sum_{i=1}^{I} r_i = 0$.

It is interesting to note that Massey's ratings are interdependent and that the rating of the team i after N matches can be written as the sum of the mean rating of the teams matched by i and the mean of the score differences from the perspective of i; that is,

$$r_i = \frac{1}{m_i} \sum_{j=1}^{I} A_{ij} r_j + \frac{p_i}{m_i}, \tag{2}$$

where m_i is number of games played by i, A_{ij} is the number of matches between i and j and p_i is the sum of score differences from the perspective of i. In this case, the rating of the opponent teams is evaluated at the end of the sequence of matches.

An effective description of Massey's method, with the introduction of an appropriate statistical model for score differences, can be found in [2].

2 A temporized extension of the Massey's method

A first proposal for obtaining time-dependent ratings, i.e. to consider the strength of the opponent teams at the time of the matches, can be found in [1] and it is based on a suitable generalization of formula (2). In this case, the rating of team i, after the match played at time $t \in \{1, 2, \dots, T\}$, is defined as

$$r_i(t) = \frac{1}{m_i(t)} \sum_{k=1}^{m_i(t)} \{r_{j_k}(t_{k-1}) + y_i(t_k)\}, \tag{3}$$

where $m_i(t)$ is the number of games played by i until time t, j_k, $k = 1, \dots, m_i(t)$ are all the teams matched by i until time t, at the timestamps $t_1, \dots, t_{m_i(t)} = t$, and $y_i(t_k)$ is the score difference from the perspective of i in the match of time t_k. The initial ratings $r_i(t_0) = r_i(0)$, $i = 1, \dots, I$, are usually set as equal to zero; alternatively, they can be estimated or specified as the team ratings computed at the end of the previous season. With simple algebra, it is quite easy to obtain the iterative formula

$$r_i(t) = r_i(t^-) + \frac{1}{m_i(t)} [y_i(t) - \{r_i(t^-) - r_j(t^-)\}],$$

where j is the team faced by i at time t and $r_i(t^-)$ and $r_j(t^-)$ are the ratings of i and j before the match. Then, the rating of i at time t can be iteratively defined as an update of the rating before time t, which involves the difference between the realized score difference $y_i(t)$ and the expected score difference as described by the ratings difference $r_i(t^-) - r_j(t^-)$. Notice that this time-aware version of the Massey method has the advantage of considering the rating of the opponent teams at the time of the matches. However, it suffers from an *excess of stability*, since as the number of games $m_i(t)$ increases the rating becomes more and more stable, and a new result can only slightly modify the current reputation of the team.

A more general dynamic extension of Massey's method can be specified by considering a suitable weighted mean, instead of the simple mean, in equation (3). Therefore,

$$r_i(t) = \sum_{k=1}^{m_i(t)} \omega_{m_i(t),k}\{r_{j_k}(t_{k-1}) + y_i(t_k)\}, \quad t \in \{1,2,\ldots,T\}, \tag{4}$$

with $\omega_{m_i(t),k}$, $k = 1,\ldots,m_i(t)$, a system of non-negative (normalized) weights. With an appropriate choice of the weights, it is possible to calibrate the influence that the most recent results have in defining the present team rating. Under this respect, an interesting option is to set $\omega_{1,1} = 1$ and, for $m > 1$, $\omega_{m,1} = (1-\gamma)^{m-1}$, $\omega_{m,k} = (1-\gamma)^{m-k}\gamma$, $k = 2,\ldots,m-1$, $\omega_{m,m} = \gamma$, with $\gamma \in (0,1)$. In this case, the average assigns weights that decrease exponentially as the matches come from further in the past and it can be interpreted as a simple exponential smoothing procedure, giving the following iterative relation

$$r_i(t) = r_i(t^-) + \gamma[y_i(t) - \{r_i(t^-) - r_j(t^-)\}],$$

with smoothing parameter γ.

To account for the uncertainty associated with the rating values, and then assess the significance of the differences in teams' abilities, and to specify a procedure to predict match results, we introduce a suitable statistical model for the match outcomes y_n, $n = 1,\ldots,N$. This model can be viewed as the natural stochastic data-generating process that includes the new dynamic rating methodology just defined. More precisely, we consider the innovations state space model defined by the following measurement and state equations

$$\begin{aligned} y_i(t) &= r_i(t^-) - r_j(t^-) + \varepsilon_i(t) \\ r_i(t) &= r_i(t^-) + \gamma\varepsilon_i(t), \end{aligned}$$

where $\varepsilon_i(t) \sim N(\delta,\sigma^2)$, $t = 1,\ldots,T$, $i = 1,\ldots,I$, is a sequence of uncorrelated Gaussian error terms; δ describes the potential home field advantage of team i (see, e.g., [2]). An accurate description of this type of model can be found in [3]. To improve the flexibility of the model, we may consider a convenient transformation of the ratings difference, in particular $g\{r_i(t^-) - r_j(t^-);\alpha,\beta\} = \alpha + \beta\{r_i(t^-) - r_j(t^-)\}$. In this framework, the estimates of the unknown model parameters, with the associated estimated standard errors, can be obtained quite easily using the maximum likelihood approach, since the likelihood function has a simple explicit expression.

Finally, we emphasize that this model can be generalized by considering a specification for the match outcomes y_n, $n = 1,\ldots,N$, different from the score difference and by defining offensive and defensive rating instead of a single general rating for each team. A further, useful improvement requires the inclusion of suitable team- and game-specific covariate information, such as injuries, coaching changes, weather conditions, time of the day, participation in other competitions, etc.

3 An application to the Euroleague Basketball 2018-2019

As a simple application, we consider the match results of the 2018/2019 regular season of the Turkish Airlines Euroleague Championship. The regular season of the Euroleague Championship is a round-robin tournament where sixteen teams play each other home and away and the top eight teams advance to the playoffs. Then, the total number of matches is 240, since each team competes twice against other teams in the league.

The original Massey method (M) and the two time-aware versions, that one based on the simple mean (3) (Msimple) and that one based on the weighted mean (4) with simple exponential smoothing weights (Mses), are applied to the results of the tournament. The estimated ratings, with the associated ranking, are reported in Table 1. The estimates for the unknown model parameters, namely the home field advantage

Table 1 Teams ranked according to the official score at the end of the season, with the games won and the point difference, and the ratings (with the associated ranks in parentheses) obtained using the original Massey method (M) and the two temporalized methods (Msimple and Mses).

Team	Won	Difference	M	Msimple	Mses
Fenerbahce Istanbul	25	267	8.03 (1)	85.73 (1)	86.03 (1)
CSKA Moscow	24	193	6.09 (3)	65.92 (3)	63.34 (3)
Real Madrid	22	236	7.37 (2)	70.25 (2)	73.28 (2)
Anadolu Efes Istanbul	20	156	4.87 (4)	62.26 (4)	62.18 (4)
FC Barcelona	18	76	2.47 (5)	28.901 (9)	26.06 (9)
Panathinaikos Athens	16	37	1.03 (8)	47.25 (5)	47.61 (5)
Baskonia Vitoria-Gasteiz	15	71	2.09 (6)	35.72 (7)	32.93 (8)
Zalgiris Kaunas	15	37	1.12 (7)	40.11 (6)	43.03 (6)
Olympiacos Piraeus	15	25	0.91 (10)	-14.39 (11)	-15.45 (11)
Maccabi Tel Aviv	14	30	0.94 (9)	33.99 (8)	33.27 (7)
FC Bayern Munich	14	-56	-1.62 (12)	-32.35 (12)	-29.40 (12)
Olimpia Milan	14	1	0.09 (11)	-2.89 (10)	-2.53 (10)
Khimki Moscow Region	9	-116	-3.44 (13)	-61.51 (13)	-62.58 (13)
Gran Canaria	8	-299	-9.41 (14)	-117.60 (15)	-116.35 (15)
Buducnost Podgorica	6	-320	-10.00 (15)	-101.69 (14)	-101.13 (14)
Darussafaka Istanbul	5	-338	-10.56 (16)	-139.72 (16)	-140.28 (16)

δ, the variance of the error terms σ^2, and (for Msimple and Mses) α and β, are obtained using the maximum likelihood approach. Note that the three methods produce the same ranking for the four best teams, and this corresponds to the order that we have considering the difference in points. The two time-dependent procedures give similar results and they are both affected by any variations in the quality of the teams' performances that may occur, in particular, at the end of the season. In this respect, the Massey method is more rigid and stable, which can be a disadvantage in some cases. The following Figure 1 describes the pattern of the ratings of Fenerbahce Istanbul, CSKA Moscow, FC Barcelona, Maccabi Tel Aviv and Baskonia Vitoria-Gasteiz from day 10 to the end of the regular season, by considering only the time-dependent approaches.

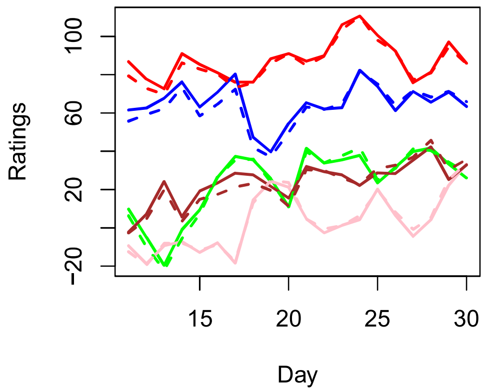

Fig. 1 Pattern of the ratings of Fenerbahce Istanbul (red), CSKA Moscow (blue), FC Barcelona (green), Maccabi Tel Aviv (pink), Baskonia Vitoria-Gasteiz (brown) obtained using Msimple (dashed line) and Mses (solid line).

Finally, we discuss the predictive ability of the statistical models based on the three rating methods. In particular, we evaluate both the accuracy (understood as the percentage of correct predictions for the win, both in general and for the home and the away team) and the mean squared prediction error, concerning the prediction of score differences. We consider a sequential procedure, so that at each day of the season, the model parameters are estimated using the results of the matches played up to that time. The findings are reported in Table 2 and show that, regarding the predictive accuracy, the behaviour of all three methods is quite good, with a slight prevalence for Massey's method. On the other hand, the temporalized method Mses has the best performance regarding the prediction of point differences. It must be said that the present analysis refers to a specific sports tournament, which does not present interesting dynamics, such as upsets, significant variations in performance, etc.

Table 2 Percentage of correct predictions for the win, in general (AccW) and for the home (AccWH) and the away team (AccWA), and mean squared prediction error (MSPE) for the point difference, using models based on the Massey (M) and on the two temporalized (Msimple and Mses) methods.

Method	AccW	AccWH	AccWA	MSPE
M	**0.750**	0.855	**0.520**	199.77
Msimple	0.744	**0.864**	0.480	189.45
Mses	0.744	**0.864**	0.480	**189.05**

4 Final remarks

In this paper, we introduced a new temporalized rating method. This approach adequately takes into account the fact that teams' capabilities change over time. In addition, the associated statistical model is flexible and easily extensible to improve its predictive ability. Sports organizations or analysts could effectively use this dynamic method to achieve a less rigid, more realistic, and up-to-date assessment of the current strength of teams engaged in sports competitions. In addition, the associated statistical model, even in its simplest version, has demonstrated good predictive accuracy and thus can be used in the context of sports event betting.

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