AN OUTDOOR ACTIVITY TO LEARN OPERATIONS WITH INTEGERS

Maria Antonietta Lepellere, Dario Gasparo

University of Udine, Secondary school first grade “G. Caprin” Trieste

This article describes an example of research-informed teaching to help students to understand the operations with integers. The approach used is that of Inquiry and Embodied cognition in outdoor context, with the help of sagittal axis. The activity involved 15, eight grades (aged 13/14), students from a middle school in Trieste, Italy. The results were tested by proposing to the same students' different types of exercises and problems. 73% have obtained positive results with 72% of which very good. Finally, we investigated through Mentimeter the students' appreciation of the outdoor activity. 100% of students found the activity fun and helpful.

INTRODUCTION AND THEORETICAL BACKGROUND

Learning outside the classroom essentially can be defined as use of resources out of the classroom to achieve the goals and objectives of learning (Knapp, 2010; Smith & Walkington, 2020). Recently there has been an increased interest in the development of outdoor and adventure education programmes (Fägerstam & Samuelsson, 2012). The constant focus on textbooks and formal mathematical practice might invoke a view among students that mathematics is abstract, distanced and only useful in a in classroom context. Existing research on outdoor learning in mathematics indicates positive affective outcomes and possible academic benefits from learning mathematics in an out-of-school context (Daher & Baya'a, 2012; Moffett, 2011). Moreover, outdoor environments, are real-life contexts enabling children to internalise, transfer and apply mathematical ideas and provides direct experience, the students need to be active in the learning process (Moffett, 2011). It lends itself to the Inquiry-based mathematics education, a student-centred form of teaching whose guiding principle is that the students are supposed to work in ways like how professional mathematicians work (Artigue & Blomhøj, 2013; Dorier & Maass, 2014): they must observe phenomena, ask questions, look for mathematical and scientific ways of answer these questions, interpret, and evaluate their solutions, and communicate and discuss their solutions effectively. Cooperative learning gives the opportunity to discuss and reason with others and justify one's mathematical thoughts on how to solve different mathematical problems. Cooperative outdoor learning in mathematics gives the possibility to observe that a task at hand can be solved in more than one way and that more than one “right” solution to the problem may exist. The sensorimotor experiences arising from the environment also play a paramount role in learning (Wilson, 2002).

Embodied cognition is described as a bodily sense of knowing, expressed through physical movement and sensory exploration with environments (Merleau-Ponty, 2002;

2022. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.). *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 123-130). PME.

Varela et al., 1991). There is complexity in the processes that may be involved in the development of embodied cognition as “knowledge depends on being in a world that is inseparable from our bodies, our language, our social history” (Varela et al., 1991, p. 173). According to Glenberg (2010) perception and how memory works is affected by how people move their bodies. The role of gestures as semiotic tools, contributing to deeper understanding of mathematical concepts (Arzarello et al., 2009).

Fluidity with integer operations marks a transition from arithmetic to abstract algebra. They do not correspond to any of the pre-existing cognitive structures and destabilize the perceptions - established since elementary school - of students on numbers and operations. It is difficult for them to perceive that -27 is less than -1 or that addition can cause a reduction, while subtraction can cause an increase. Moreover, negative numbers are conceptually difficult because students spend much less time learning them. Not being able to attribute natural objects or quantities to them, they try to recall the rules that do not guarantee the validity of their results (Vlassis, 2002; Bofferding, 2014; Badarudin & Khalid, 2008). The key to a successful method is not to let them memorize a bunch of rules before they understand. Instead, students' understanding can be enhanced by using images or manipulating tools, to enable them to translate concepts into images. Additionally, giving students the opportunity to explore multiple representations of a particular mathematical concept can strengthen their conceptual understanding.

Numbers are closely related to space both in action and in thought. A now classic finding is the “spatial numerical association of response codes (SNARC) effect”: among literate individuals from cultures who read from left to right, smaller numbers induce dispositions to act in the left space and larger numbers in the right space. Negative integers also induce spatial arrangements, although the task requires influence whether they are “left” of zero, in line with their relative numerical magnitude, or mixed with positive integers based on their absolute value. Spatial arrangements can also play a role in more complex tasks: mental arithmetic, for example, induces systematic arrangements to respond spatially, with addition-bias responses to the right and subtraction bias responses to the left (Knops at al. 2009; Marghetis & Youngstrom, 2014). Anelli at al. (2014) found a significant “congruency effect” where subjects performed more correct addition operations when moving horizontally rightward (the inferred orientation for addition in cultures that read left to right). Citing earlier work on bodily movement and mathematical processes, these researchers offer more “evidence about the influence of active body movements on the calculation processes of additions and subtractions,” evidence which reveals, “…the direction of body motion can influence not only number magnitude in a number generation task, but also the more complex process of calculations that leads to a numerical magnitude” (2014, p. 4).

Typically, negative numbers are interpreted as a continuation of a horizontal number line, or number sequence, where numbers to the left of zero are negative and numbers to the right of zero are positive. Sometimes, they can mimic vertical number lines for example, a temperature gauge. Additionally, the meaning of the minus sign, the symbol most fundamental to integers, is ambiguous. Common meanings include the meaning of an operation (take away), a value (negative), and "the opposite of," and learners often apply multiple meanings during manipulation (Lamb et al., 2012).

Little attention has been given to the arrangements along the sagittal axis, which runs from behind the body forward. Things ahead can be seen, heard, touched; the things behind it are much more difficult to access. Furthermore, the sagittal axis is associated with another abstract domain: Time. Recent studies have shown that negative numbers are spontaneously associated with the space behind the body and positive numbers with the space in front. These spatial arrangements were evident only when the task involved both the positive and negative numbers. Whole reasoning, therefore, is not entirely abstract, but induces systematic dispositions to action (Marghetis & Youngstrom, 2014).

The purpose of this article is twofold: it is intended to show how an outdoor activity should be presented with a view to the Embodiment, the Inquiry and with the use of sagittal axis; to test, as first exploratory study, whether an hour and a half of outdoor activity was enough for the students to understand concepts and if the activity was appreciated by them. The study involved 15 students, eight grades (aged 13/14) 8 boys and 7 girls, from a middle school in Trieste, Italy.

THE METHODOLOGY

In the first part we describe those steps that led students to the discovery of properties regarding the addition and subtraction with integers. The approach used is that of Inquiry and Embodied cognition in an outdoor context. The activity takes place in the "Classroom under the sky" https://www.youtube.com/watch?v=lGJbz_d7OUs&t=80s (for another example see Lepellere & Gasparo 2021). The environment is already welcoming in itself: a small pond right on the edge of a laurel grove, an open lawn that converges to the maple tree in the centre of the space, under which a blackboard and seats for students are placed. The students can also make use of portable shelves, to support books and notebooks. The activity middle school in Trieste, Italy. The results were tested by proposing, to the same 15 students, just after an hour and half of outdoor activity 72 exercises on operations and five different problems. Finally, we investigated through Mentimeter the students' appreciation of the outdoor activity.

THE ACTIVITY

The straight line of numbers is represented by the stairs, the increasing direction to the right is not as intuitive as climbing the steps (positive numbers) or descending them (negative numbers). After having identified the zero point on the landing, we start by drawing positive and negative numbers on the wall next to the stairs. To further help visualization and memory, it is possible to paint negative numbers in red and positive numbers in blue, placing the relative sign in front of them. The students take their chosen position on the various stairs, after which we introduce some rules.

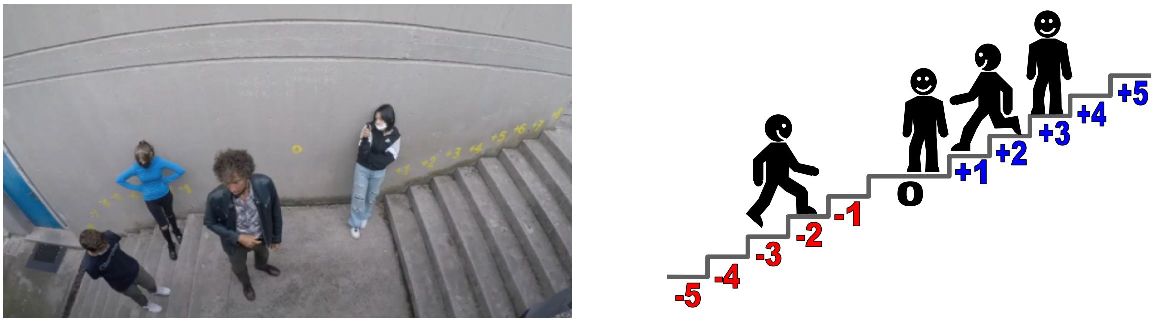

Figure 1: Stairs real and imaginary.

We invite students to discover the first oddity: how is it possible that by subtracting two numbers we get a larger number? I point out that subtracting means making a difference, so we invite students to calculate the difference between the +2 step and -4 step, that is, let's see how many steps we must do to go down from +2 to -4. We discover that there are 6! So, +2 - (-4) = + 6. Now we establish another rule that we have to do with the starting position that a student must take. Upon departure, a student stands in a neutral position, towards the teacher or fellow who gives the commands. Then he or she behaves like this: to add something they must turn upwards (positive numbers) and to subtract they must turn downwards. In Figure 1 on the right, we see the first student who passes from -3 to -2: he is turned upwards in the addition operation: -3 + (+1) = -2. The second stays in neutral position on 0, the third drops from +2 to +1, i.e., it is turned down in the subtraction position +2 - (+1) = +1. The fourth is neutral on +3 and wait instructions. It is time for the second rule: in front of a number there are two signs, one for the operation and one that indicates whether the number is positive or negative. If the number following the first is positive, we move forward and if the number is negative, we move backwards. We invite the students to move like fleas, amplifying the gesture with a jump. During the first calculations we often see some pupils simulating these jumps with their fingers in the notebook.

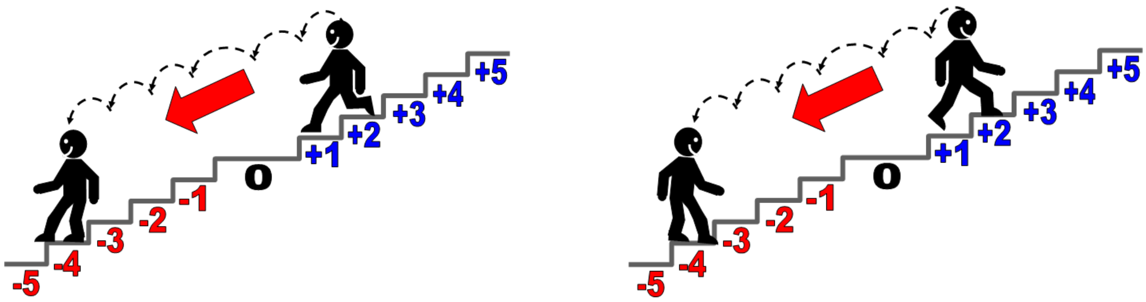

Figure 2: From step +2, we can get to step -4 with two methods.

Together we discover that starting from step +2, we can get to step -4 with two methods: turn left (-) and go forward (+) doing +2 - (+6) = -4 (Figure 2 on the left) or turn right (+) and go back (-) doing +2 + (-6) = -4 (Figure 2 on the right). At this point a series of games began: a team assigns an operation to the opponents who must solve problems such as -2 - (- 3). Then we ask how, starting from -1, we can get to +4. They start with simpler procedure; they turn left and go forward by 5. Then the students also discover the other system: they turn right, down, and go up to shrimp back by -5. The variations to the game are many: it is left to the students to find some. The whole outdoor activity lasted an hour and a half.

THE TEST

The next day and without any prior notice, students were offered a challenging test consisting of 72 operations to perform and 5 problems to set and solve. The operations were of type: $-20 + 15 = \boxdot$; $12 + (-16) = \boxdot$; $-4 + \boxdot = 10$; $-7 + \boxdot = -2$; $\boxdot + (-3) = 7$; $\boxdot + 13 = -5$ and so on in such a way as to cover all possibilities. Moreover, we give them the following 5 problems involving a vertical schema as temperature and sea level, horizontal schema as movement and timeline and finally a neutral schema as loans. Problem 1. Temperature: Lara looks at the thermometer: the temperature is -2. In the afternoon, however, the temperature rises by 11 degrees. What temperature do we have now? Write down the operation you did. Problem 2. Movement: Cristian walks 20 meters ahead and then returns 14 meters back. Where is he in relation to the starting point? Write down the operation you did. Problem 3. Sea level: Sofia is in a submarine with her friend Matteo. They are 150 meters below sea level! If the submarine rises 100 meters, what level is it now? Write down the operation you did. Problem 4. Timeline: Luca was a prominent Roman emperor, before reincarnating as a student of the Caprin. He was born in 510 BC. In the twentieth reincarnation he became a swallow, which died in 220 AD. How many years has he lived in these 20 reincarnations? Write down the operation you did. Problem 5. Loans: Isabel and Giada go to buy a sweatshirt from Scarface. Giada has 15 euros with her, but the sweatshirt costs 23. How much money does Isabel have to lend her for the purchase? In other words, how much money does Giada owe? Do you have to put + or - in front of the number? Write down the operation you did.

In Table 1. we show the results obtained. The 15 students numbered from 1 to 15 are placed in the column. The scores obtained from the operations are represented in the column "Operation Scores". One point has been assigned to each operation. The scores on the problems are reported from the third to the eighth column: 1 if it was correct and 0.5 if it was formulated correctly but the operation was wrongly made. The "Difficulty" column also shows the difficulty perceived by the students of the test (0 easy to 5 very difficult) and in the subsequent ones the preference between the schemes used (0 dislike 3 like very much), horizontal, vertical, scaled or the use of the rules. The last 2 columns contain the total score and the grade (in tenths) achieved by each individual student. The last row of the table contains the arithmetic means of the various results.

	Operation	Problems					Difficulty (1-5)	Preference Schemas or Rules (1-3)					
	Scores	1	2	3	4	5		Orizzontal	Vertical	Stairs	Rules	TOTAL	SCORE
Total Score	**72**	**1**	**1**	**1**	**1**	**1**						**77**	
1	**33**	1	1	1	1	0	3	2	1	2	3	37	**5-**
2	**46**	1	1	1	1	1		2	1	3		51	**6/7**
3	**12**	1	0,5	0	0	0						13,5	**3**
4	**35**	1	0	0	0,5	0,5	3	3	2	3	1	37	**5-**
5	**71**	1	1	0,5	0	0	3	3	0	2	3	73,5	**9/10**
6	**60**	1	1	1	0,5	1	3	2	2	3	3	64,5	**8/9**
7	**61**	1	1	1	1	1		3	3	3	0	66	**8/9**
8	**66**	1	1	1	1	1	2,5	0	2	1	2	71	**9+**
9	**57**	1	1	1	0	0	2,5	1	1	3	0	63	**8**
10	**55**	1	1	1	0,5	0	2	1	0	3	0	58,5	**7/8**
11	**66**	1	1	1	1	1	1	3	3	3	3	71	**9+**
12	**72**	1	1	1	1	1	1	1	2	3		77	**10**
13	**72**	1	1	1	1	1	1	3	3	3	3	77	**10**
14	**66**	1	1	0,5	1	1	2	1	0	0	3	70,5	**9**
15	**41**	1	1	0,5	0	0,5		0	0	3	0	44	**6-**
Avarage	**54**	**1**	**0,9**	**0,8**	**0,7**	**0,6**	**2,2**	**1,8**	**1,5**	**2,5**	**1,9**	**54,1**	**7**

Table 1: Results.

Immediately it is noted that only 3 out of 15 students obtained an insufficient total grade of which only one very negative (6/7 stands for 6.75 and so on). The first temperature problem, which is also what students must do most in real life, was solved correctly by all students, following the movement and sea level problem. The timeline exercise had an additional difficulty related to intrinsic knowledge of the subject and therefore less skill was expected. More unexpected is the result on the problem about loan and would need further study. The test was perceived as not very difficult even if 4 students did not answer to the question about it. As for the use of the schemes, the scale scheme (10 students gave preference grade 3), was appreciated more than the scheme in the horizontal (5 students gave preference grade 3) and vertical (3 students they gave preference grade 3). Finally, the use of the rules received some appreciation (6 students gave preference grade 3). Cases of non-response were not taken into consideration in the calculation of the mean.

Mentimeter was used to test student appreciation of the activity. First, they were asked to write the first 5 words that came to mind when thinking about the outdoor activity. Figure 3a shows the results. The words funny combined with fun, nice, beautiful are the most used. But understanding-related words such as interesting, simple, focus, ease, easy, and intelligence were also highly rated. We find the words numbers, scales, errors, comparison too. When asked to indicate on a scale from 0 to 5 (0 not at all and 5 very much) how much they liked the activity 6 students gave score 5, 5 students score 4 and 1 score 3, scores 0, 1 and 2 are not been voted on (Figure 3b.).

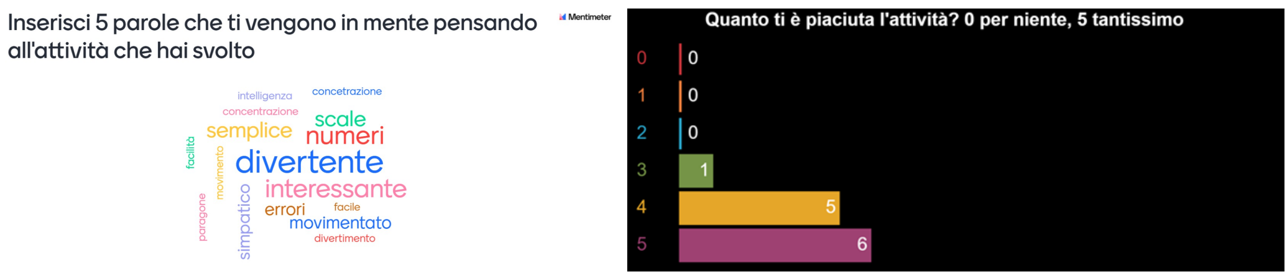

Figure 2: Mentimeter results.

It was also asked to indicate from 0 (not at all) to 5 (a lot) how much the activity on the stairs was of help compared to the study in class on the blackboard and the notebook. Here, too, 11 out of 13 young people who replied said that the activity was very helpful. Finally, it was asked whether by carrying out this activity the student was able to discover some rules on his own. 6 students voted yes and 7 no.

CONCLUSIONS

The lack of cognitive prerequisites based on personal experience, sensory deprivation, the lack of direct experiences are elements of risk that we detect in today's young people and that lead them to have difficulties even when they need to analyse, deduce, abstract. The term “educate” derives from the Latin ex ducere, “to lead out”, in the sense of trying to get the best out of each student but it can also be interpreted as “to lead out” from the classroom. Here, movement can represent a stimulus to learning if practiced in serenity and even more in an open environment (Moss, 2009). Covid-19 launches a challenge to schools today in a strong crisis and that of outdoor schools is a real way that connects students with reality, nature, dexterity, art and a new responsibility towards creation, others, themselves. The reduction of opportunities for socialization has led to various psychological disorders in adolescents: panic attacks and anxiety. It should therefore come as no surprise that fun and socializing activities are of interest and approval. The survey carried out anonymously at the end of the outdoor lesson shows that 100% of students found the activity fun. Several used terms such as “beautiful, joy, happiness”. After only an hour and a half of outdoor lessons most of the students obtained a very high score in a demanding test consisting of 72 operations of different types and 5 problems. This is a first investigative intervention that will lay the foundations for future experimental work.

References

Anelli, F., Lugli, L., Baroni, G., Borghi, A. M., & Nicoletti, R. (2014). Walking boosts your performance in making additions and subtractions. *Frontiers in Psychology, 5*.

Artigue, M., & Blomhøj, M. (2013). Conceptualising inquiry-based education in mathematics. *ZDM—The International Journal on Mathematics Education, 45*(6).

Arzarello, F., Paola, D., Robutti, O., & Sabena, C. (2009). Gestures as semiotic resources in the mathematics classroom. *Educational Studies in Mathematics, 70*(2), 9

Badarudin, B. R. H., & Khalid, M. (2008). Using the Jar Model to Improve Students’ Understanding of Operations on Integers. *Proceedings of icme-11–topic study group 10* *research and development in the teaching and learning of number systems and arithmetic*, 85.

Bofferding, L. (2019). Understanding Negative Numbers. In A. Norton & M. Alibali (Eds.), *Constructing Number* (pp. 251-277). Springer.

Daher, W., & Baya'a, N. (2012). Characteristics of middle school students learning actions in outdoor mathematical activities with the cellular phone. *Teaching Mathematics and its Applications*, 31(3), 133-152.

Dorier, J.L., & Maass, K. (2014). Inquiry-Based Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp.300-304). Springer Netherlands.

Fägerstam, E., & Samuelsson, J. (2012). Learning arithmetic outdoors in junior high school-influence on performance and self-regulating skills. *Education*, 1-13.

Glenberg, A. M. (2010). Embodiment as a unifying perspective for psychology. *WIREs Cognitive Science, 1*, 586-596.

Knops, A, Thirion, B, Hubbard, E, Michel, V, & Dehaene, S (2009). Recruitment of an area involved in eye movements during mental arithmetic. *Science, 324*, 1583.

Lamb, L.L., Bishop, J.P., Philipp, R.A., Schappelle, B.P., Whitacre, I., & Lewis, M. (2012). Developing symbol sense for the minus sign. *Mathematics Teaching in the Middle School, 18*(1), 5-9.

Lepellere, M. A., & Gasparo, D. (2021). Outdoor activities to discover medians and centroid of triangles. In M. Kingston & P. Grimes (eds.), *Proceedings of the Eighth Conference on Research in Mathematics Education in Ireland (MEI 8)*, 260–267.

Marghetis, T., and Youngstrom, K. (2014). Pierced by the number line: Integers are associated with back-to-front sagittal space. in *Proceedings of the 36th Annual Meeting of the Cognitive Science Society*; July 23-26, 2014; Quebec City, Canada, 36.

Merleau-Ponty, M. (2012). *The Phenomenology of Perception*. Transl. ed. D. A. Landes. Routledge.

Moffett, P. V. (2011). Outdoor mathematics trails: an evaluation of one training partnership. *Education 3–13, 39*(3), 277-287.

Smith C., Walkington C. (2020). *Four principles for designing embodied mathematics activities*.

Varela, F. J., Thompson, E., & Rosch, ER. (1991). *The embodied mind: cognitive science and human experience*. MIT Press.

Vlassis, J. (2002). The balance model: Hindrance or support for the solving of linear equations with one unknown. *Educational Studies in Mathematics*, 49(3), 341–359.

Wilson, M. (2002). Six views of embodied cognition. *Psychon. Bull. Rev., 9*, 625–636.