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From light polarization to quantum physics: Supporting lower secondary school students' transition from gestalt to functional thinking

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Abstract
In this paper, we present a new minimal mathematical conceptual approach to quantum mechanics using light polarization for lower secondary school students with the aim of bringing students closer to the so-called quantum mechanical way of thinking. We investigated how students think about some of the basic concepts and fundamental laws and we found that certain concepts are quite well-understandable in younger grades too. We studied the introduction of the so-called state circle, which can faithfully represent quantum mechanical formalism without involving students in abstract algebraic calculations. We then categorized and analyzed students' thoughts on the superposition principle and the lack of trajectory, finding that the concept of measurement and the lack of trajectory were problematic. We explored that younger students tend to hold gestalt-like mental models of quantum concepts, while at the same time being able to use visualizations correctly for reasoning in the quantum realm. Overall, this paper provides evidence in favor of introducing basic features of quantum mechanics as early as in lower secondary school.

Keywords: quantum physics, polarization, conceptual approach, photon, mental models

INTRODUCTION

The introduction of the basic ideas of quantum mechanics (QM) via two-state systems is receiving a lot of attention (Faletič et al., 2024; Greinert & Müller, 2023; Michelini et al., 2021, 2022a, 2022b; Nobel Prize Outreach AB, 2023; Quantum Technology Education Project, 2020; Tóth & Tél, 2023). One possible way of doing so is to use photon polarization, known as *Dirac approach*, named after Paul Dirac, who first applied light polarization to introduce QM (Dirac, 1958). One of the first developments of *Dirac approach* in secondary school material was carried out by researchers from Udine (Ghirardi et al., 1996; Michelini et al., 2000). They introduced the elements of quantum concepts and simplified mathematics through easy-to-implement student experiments and single photons thought experiments (Cobal et al., 2002a, 2002b; Michelini, 2008; Michelini & Stefanel, 2006, 2014, 2023). In addition, many educational pathways making use of light polarization have been developed to introduce university students (French & Taylor, 1978; Singh, 2008; Tóth, 2023, 2024) and secondary school learners to QM (Migdał et al., 2022; Müller & Wiesner, 2002; Schlummer et al., 2022) and to quantum computing (Berbhardt, 2019; Bondani et al., 2022; Migdał et al., 2022; Walsh et al., 2022).

Since the subject of physics ends at the age of 16 for several citizens, QM does not reach the wider society. In fact, material for lower grade is not only relevant for its cultural value (Stadermann & Goedhart, 2021) and for contributing to the statistical thinking that is so often lacking. However, to the best of our knowledge, no such implementation has been empirically researched yet. In this paper we present a new teaching-learning sequence, which comprises five lessons (45 minutes each) that requires almost no mathematics and focuses on the basic concepts for lower secondary school students. The presented learning path relies on the previous materials of Udine (Cobal et al., 2002a, 2022b; Ghirardi et al., 1996;

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Contribution to the literature

- We present a new conceptual approach to QM with minimal mathematics using light polarization for lower secondary school students based on previous research from Udine.
- We provide evidence that it is worth to introduce quantum formalism for some younger students too because a portion of them reached the functional thinking; in contrast several students have difficulties to understand the lack of trajectory and the concept of quantum measurement.
- We show that presenting hypotheses related to quantum probabilities and representing states on the state circle can enhance younger students' understanding.

Michelini, 2008; Michelini & Stefanel, 2021, 2023; Michelini et al., 2000, 2011).

As our goal is to present some conceptual foundations of QM for its cultural value and to try to guide students towards a *quantum mechanical way of thinking*, the mathematical formalism is beyond the scope, we focus on the students' initial thinking patterns. We investigate students' perceptions of some fundamental laws (Krijtenburg-Lewerissa et al., 2018): the probability law, the lack of trajectory, quantum measurement and the superposition principle. To this end, we conducted a field study with n=51 16-year-old students at Czuczor Gergely Benedictine Secondary School in Győr, Hungary, in the spring of 2023.

RESEARCH BACKGROUND

Teaching & Learning Quantum Physics

Students often interpret quantum phenomena in classical terms (Fischler & Lichtfeldt, 1992; Greca & Freire, 2003; Johnston et al., 1998; Kalkanis et al., 2003) and they tend to use previous classical knowledge in quantum situations (Bouchée et al., 2022), for instance using a temporal and spatial continuous description of quantum phenomena, whereas the quantum formalism does not allow for what happens between the preparation of state and the measurement: this is *the classical way of thinking*. In addition, researchers from Udine have described another reasoning pattern, *the hidden variable way of thinking*. That is, students often think that microscopic systems preserve some properties of classical macroscopic systems in theory, even if they are not knowable/detectable in real experiments, e.g., students believe in the temporal and spatial continuous description, but they also add that this behavior is not accessible due to experimental limitations; and the statistical behavior is often due to the uncontrollable disturbance caused by the (even ideal) measurement itself (Michelini & Stefanel, 2021; Michelini et al., 2011). Michelini et al. (2011) proposed that students' arguments about the lack of trajectory illustrates their way of thinking in QM.

Developing the *quantum mechanical way of thinking* is difficult in general (Stefani & Tsaparlis, 2009), even for younger students, who are more in need of tangible, visual models, and representations. However, the abstract and unintuitive nature of QM is exacerbated by the fact that quantum systems are very difficult to visualize and to imagine (Stefani & Tsaparlis, 2009). As highlighted in Ubben and Bitzenbauer (2022), the quantum models presented are often misunderstood by learners, who often believe that the models are an upscaled or downscaled exact representations of the real phenomena. German researchers identified two cognitive dimensions suitable to explain a substantial amount of the variance in learners' perceptions of quantum concepts. The dimension *fidelity of gestalt* (FG)

- describes that the one can understand their way of thinking *"as exact visual representations of phenomena or exact depictions of how things look"* (Ubben & Bitzenbauer, 2022, p. 1356) and
- *"how much the mental models' gestalt is perceived as accurate"* (Ubben & Bitzenbauer, 2022, p. 1360).

For example, gestalt thinking students imagine the photons as small balls with certain trajectories. On the other hand, the dimension *functional fidelity* (FF)

- … describes that the one can understand their mental models as *"appropriate descriptions of how phenomena work"* (Ubben & Bitzenbauer, 2022, p. 1356) and
- *"how much the mental models' underlying abstract functionality […] is perceived as accurate"* (Ubben & Bitzenbauer, 2022, p. 1360).

It is found that functional and gestalt thinking correlate with a high and low conceptual understanding of QM, respectively (Ubben & Bitzenbauer, 2023). Depending on the manifestation of FG and FF in the learners' mental models of the atomic shell, these archetypes were designated as *non-developed* type (low FG and low FF), *architectural* type (high FG and low FF), *dual* type (high FG and high FF) and *functional* type (low FG and high FF). In accordance with Ireson (2000), the Udine categories (Michelini & Stefanel, 2021; Michelini et al., 2011) can be associated with the German ones. The architectural, dual, and functional types correspond to classical, hidden variable and quantum mechanical thinking, respectively.

Teaching QM at the secondary school level should aim at fostering students' conceptual transition from a rather gestalt-oriented to a functional thinking about quantum concepts. In the following we therefore overview the students' difficulties in different approaches, in particular in *Dirac approach*. Then we focus on *Dirac approach* considering the gestalt-functional transition.

Introducing Learners to Quantum Physics via Light Polarization: A Brief Overview

Prior research has provided evidence that a polarization approach may be conducive to student learning about QM. However, learning obstacles hindering a transition to a fully functional understanding of quantum concepts have been documented in prior research. For example, a review reported that several university students mistakenly think that polarization state could not be described by two basic states because polarization can take an infinite direction, which requires an infinite basis (Singh & Marshman, 2015). There is also a problem in distinguishing the abstract Hilbert space from the real space (Singh & Marshman, 2015). This is like secondary school education, where students often confuse the concepts of property (which corresponds to the measured values), physical quantities and quantum states in *Dirac approach* (Chiofalo et al., 2022; Michelini & Stefanel, 2021; Pospiech et al., 2021).

Some students often refer to superposition states in a mathematical way or as a sum of possible properties or often explain the quantum state as a vector (Michelini & Stefanel, 2021; Michelini et al., 2022b), which is only a mathematical concept showing that students have difficulty using conceptual arguments in quantum situations. In Michelini et al. (2022b), it is argued that students often do not understand physical state and measurement even in classical physics, which makes them clueless in QM.

As shown in Passante et al. (2015) and Styer (1996), a common misconception is that the reason for the probabilistic measurement outcome is the unknown state or the mixed state indicating that students do not even understand why probability is generally used in QM. Several students believed that the effect of the measurement can be neglected as in classical physics (Montagnani et al., 2023). We note that polarization itself is a classical property of light and that this indeed is a problem of polarization approach, while students often think polarization is a quantum concept itself.

All approaches to QM agree on the difficulty of accepting that quantum objects cannot be described via the concept of trajectory. It is widely believed by students that quantum objects have well-defined position and momentum simultaneously, but this is inaccessible because the measurement itself disturbs the systems (Montagnani et al., 2023) (hidden variable thinking). Previous research in all educational approaches has found that students have difficulties in accepting the fact that the trajectory of the single quantum objects does not exist; and they try to develop strategies to preserve the classical concept of particle motion (which can be observed in *Dirac approach* (Michelini & Stefanel, 2011) and also in traditional approaches (Ambrose et al., 1999; Krijtenburg-Lewerissa et al., 2017; Thacker, 2003; Vokos et al., 2002). The above problems relate to the most central quantum concepts (Fischler & Lichtfeldt, 1992) highlighting the fact that conceptualization of QM and comparison with classical physics can be a crucial part of secondary school education.

DEVELOPMENT OF A NEW TEACHING-LEARNING SEQUENCE ON QUANTUM PHYSICS AT LOWER SECONDARY SCHOOL LEVEL

Definition of Key Idea & Goal of Teaching-Learning Sequence

Our aim is to present, through the phenomenon of light polarization, a QM learning path that is accessible to a wide range of people, with minimal mathematical apparatus, and that introduces some of the basic features of the subject at a conceptual level, while promoting a statistical mindset. The structure of the teaching-learning sequence is shown in **Table 1**.

Description of Design Process, Time Requirements, & Learning Prerequisites

The teaching-learning sequence is based on the following ideas.

- It is developed according to the model of educational reconstruction by Duit et al. (2012).
- It is embedded in a design-based research process of development, evaluation, and refinement (Anderson & Shattzcjm 2012), where this paper reports on one evaluation cycle withing a larger research project (cf. Michelini et al., 2022b).
- Students had to learn about polarization using polarizers and birefringent calcite crystals, including the measurement of the Malus law before participating in this teaching-learning sequence. We highlight that it is not required to learn about the wave model of light, since the presented material is based on the phenomenological approach to polarization as can be seen in (Michelini & Stefanel, 2006, 2014; Tóth et al., 2024a).
- To be appropriate for the use under real school conditions, in addition to the phenomenology of polarization, only a few lessons are sufficient.

Table 1. Summary of learning path (each lesson is 45 minutes long)

Lessons	Concepts & laws	Structure of learning path	Used materials
Before quantum studies	• Light intensity & polarization • Ideal polarizers • Malus law	(1) Light intensity & polarization of light (2) Ideal polarizers & birefringent calcite crystals: Measuring Malus law	Michelini and Stefanel (2006, 2014) & Tóth et al. (2024a)
#1	• Probability law • Relative frequency & probability • Law of large numbers & correspondence principle	(1) Existence of photons (2) Photonic interpretation of Malus law (3) Single photon interpretation of Malus & probability laws (4) Statistical predictions for many photons: Comparison of experimental result with theoretical prediction	JQM (Cobal et al., 2002a, 2022b; Tóth et al., 2024b) & Worksheet #1 (Tóth et al., 2024b)
#2	• Mutually exclusive, compatible, & incompatible properties • Uncertainty principle. • Superposition principle • Quantum measurement	(1) Polarization property & iconography (*, ◊, & Δ is horizontal, 45° & vertical polarization property, respectively) (2) Mutually exclusive properties in classical physics & quantum mechanics (polarization) (3) Compatible & incompatible properties comparing QM with macroscopic physics (4) Uncertainty principle: Existence of incompatible properties (5) Superposition principle: Existence of state in which measurement outcome is uncertain	Worksheet #2 (Tóth et al., 2024b)
#3	• Superposition principle	(1) A comparison of causes of probabilities in everyday life & QM (2) Comparing effect of measurement in classical & quantum physics (3) Lack of trajectory	JQM (Tóth et al., 2024b) & Worksheet #3 (Tóth et al., 2024b)
#4	• Probability law	(1) Two interpretative hypotheses are considered corresponding to classical interpretation of probabilities (2) Students see that quantum probabilities are due to internal nature of microscopic world (3) Superposition exists: Physics Nobel Prize 2022	Hypotheses (Tóth et al., 2024b)
#5	• Quantum states • Eigenstates • State circle • Quantum measurement	(1) Unit vectors assigned to polarization directions: Quantum state & Definition of eigenstates: Permitted states after a measurement (2) State circle: All states are represented in a circle (3) Effect of quantum measurement on state circle	Worksheet #4 (Tóth et al., 2024b)

Design Principles of Teaching-Learning Sequence

Two frames of interpretation: Classical & quantum physics

Although being a fully classical concept, a single photon can be assigned polarization states in the quantum realm. For example, students may perform experiments with high light intensity (Tóth et al., 2024a) and then they reinterpret the experiments in terms of single photons using a computer simulation Java quantum mechanics–JQM (Cobal et al., 2002a, 2022b; Tóth et al., 2024b).

The link between the two interpretive frameworks is established by the students accepting the idea that light consists of indistinguishable photons and that the intensity of monochromatic light is proportional to the number of photons, so that the experiments can be interpreted in terms of the number of photons.

Throughout the learning process, we constantly build on the possibility that certain incorrect answers given in QM are incompatible with classical physics.

Conceptual framework & reduced mathematics

The mathematical apparatus is reduced to a minimum as has already been proposed in earlier research (Hennig et al., 2024). The trigonometric functions are left: the students measure the Malus law and draw up a table showing the change in intensity for angles (in particular 0°, 30°, 45°, 60°, and 90°). This table is used as a learning instrument. Only the most basic concepts of probability theory are used, such as relative frequency, probability, law of large numbers observed in simulations, expected value, stochastic deviation; but these are all repeated in the quantum context. Statistical calculations are fully replaced by JQM simulations, the students perform simulations and analyze the stochastic result.

Comparison of classical physics with idealized quantum mechanics

The laws of QM are constantly compared with the laws of macroscopic world, which helps to review the concepts of classical physics and highlight the differences.

In the thought experiments, only ideal polarizers and calcites are considered: when the loss of light intensity is due only to the change in polarization. Therefore, the measurement uncertainties caused by the imperfect measuring devices are ignored, which helps the comparison with classical physics.

Geometric representation of states

As described in before, students tend to believe that visual models of phenomena are an exact representation of the phenomenon (gestalt thinking). Therefore, we try to avoid the narrow validity models: States are represented in the state circle (Pospiech et al., 2021; Tóth & Tél, 2023), where the whole measurement process is represented in a physically correct way. The only exception is JQM simulations; but students discuss why it cannot faithfully represent quantum phenomena.

Research Questions

To get an insight into students' thinking about the basic concepts, we address our first research question (RQ).

1. **RQ1.** How students think about the addressed concepts (the probability law, lack of trajectory, quantum measurement, and superposition state) via the presented conceptual approach?

As we are interested not only in the perceptions of the concepts, but also in the thinking style of each student, we address our second research question.

2. **RQ2.** To what extent does the teaching-learning sequence foster the transition towards a functional thinking of quantum concepts in secondary school students?

Description of the Teaching-Learning Sequence

Lesson 1: Probability law

As Michelini and Stefanel (2021) and Michelini et al. (2011, 2022b) propose and based on the design principle 1, at the beginning, the students are faced with the task *"If we replace the light source to a single photon source, what can the fraction I/I_0=N/N_0 in the Malus law mean in the case of a single photons?"* The answer is: it is the probability in a sequence of many repeated identical measurements of transmission because photons are undividable (*the probability law*). Since photons are indistinguishable, measuring many photons together is equivalent to a repeated measurement on only a single photon.

Referring to the *design principle 2*, the statistical analysis is done via JQM: a typical task is when students compare the theoretical prediction with the experimental result, linking the concept of *relative frequency* with *probability*, a value fluctuates around the relative frequency (*design principle 2*). Students then explore that (*design principle 2*) for many photons, the expected number of transmitted photons approaches the mean of the measured number in JQM. Since the Malus law gives this expected number, the classical theory of light polarization appears in a limit.

Lesson 2: Uncertainty principle

We again rely on the proposals of as Michelini and Stefanel (2021) and Michelini et al. (2011, 2022b), so we start the lesson begins with a thought experiment in which a single photon is emitted onto a system of polarizers with the same permitted directions. Photons will certainly pass through all if they have passed through the first one. They have acquired a property, called *polarization property*, which remains the same after passing through the others. This property implies that photons can certainly pass through a corresponding polarizer. Experiments have shown that (*design principle 1*) a horizontally polarized light would certainly be absorbed by a polarizer with vertical permitted direction: horizontally polarized photons do not possess the vertical polarization property. The vertical and horizontal polarization properties are *mutually exclusive* properties corresponding to mutually exclusive events. Measurements always define mutually exclusive observable properties; the sum of the probabilities of measuring them must be one.

The students then investigate a thought experiment when a single photon with diagonal polarization is emitted onto a polarizer with horizontal permitted direction. No one can say that a single photon possesses vertical/horizontal polarization property, because it is impossible to predict the transmission/absorption for a single measurement: these observable properties are *uncertain*. The diagonal polarization is *incompatible* with the horizontal/vertical ones. Two properties are incompatible if the possession of one makes the possession of the other *uncertain*. This is, where the *uncertainty principle* appears (for deeper explanation, see Tóth, 2023, 2024; Tóth & Tél, 2023): incompatible properties exist in QM. Two incompatible properties cannot certainly be associated with a system simultaneously, one of them will always remain uncertain; but the uncertain one can also be measured with a given probability. If a measurement is probabilistic, the photons are initially in a *superposition*: all indistinguishable photons are in the same state (this term is used in its ordinary sense) in which the measurement outcome is uncertain. Students then make comparisons (design principle 3): even ideal measurements in QM have effect.

Lesson 3: Superposition principle

The lesson begins with a game in which students explore the reason for the probabilities behind various cases (*design principle 3*): predicting the suit of the top card of a shuffled deck of French cards, whether tossed coins will come up heads or tails, or the polarization measurement of a single photon. In the previous everyday examples, the reason for the probability was the unknown properties of the cards and the complexity of dropped coins. In contrast in QM, the reason is the superposition. Students will also learn that ideal quantum measurements can change the polarization of photons (superposition collapse) in contrast with classical physics. At the end of the lesson, we take the idea from Michelini and Stefanel (2021) and Michelini et al. (2011, 2022b) articles that if a single photon is emitted onto a birefringent calcite crystal, then the spatial location of a single photon and its polarization are entangled. They explore that *a single photon has no certain trajectory*. Reconsidering the experiment of a single photon is emitted onto a polarizer, the photons have also no certain trajectory, they are "between" absorption (death) and transmission (life) between the polarizer and the detector (the analogy of the Schrödinger's cat).

Lesson 4: Interpretative hypotheses

Previously, students considered indistinguishable photons, but now, they focus on individual photons giving up the indistinguishability, the individual photons are endowed with properties; and students are introduced to two hypotheses (Michelini & Stefanel, 2021; Michelini et al., 2011, 2022b) based on our macroscopic view: the reason for probabilities will be the lack of knowledge. The first is the *statistical mixture theory*, which is analogous to a well-shuffled deck of cards (*design principle 3*). Students apply the shuffled deck theory (the top card can be in one of two colors, but we have no chance to know what it is) to photons. It is assumed that the photons cannot be in a superposition, when photons are emitted on a polarizer with horizontal permitted direction, they are previously polarized in two ways, corresponding to certain transmission and absorption (just like the cards are red or black). But we do not know the polarization of the individual photons: the fate of each photon is determined, but not knowing the properties of each photon forces us to use probability. Students can disprove this hypothesis by an experiment (*design principle 1*): if there are only two polarizations (vertical and horizontal), it would be impossible for light with diagonal polarization to pass completely through a corresponding ideal polarizer with diagonal permitted direction.

The second hypothesis, called *the co-existing properties hypothesis*, is an upgraded version of the previous one. Instead of one, two polarization properties are assigned to the individual photons simultaneously: All photons possess the property of certain transmission through the polarizer with diagonal permitted direction and, in addition, half of the photons possesses the property of certain transmission through a polarizer with horizontal permitted direction and the other half of them possess the property of certain absorption. Thus, a refutation as in the previous hypothesis cannot be applied, the property gives a result that is consistent with reality. Since we do not know which photon has which properties, this forces us to use probabilities. The hypothesis can nevertheless be disproved. If the polarizers with horizontal and diagonal permitted directions are placed one after the other, the real experiments show that the result varies depending on the order of the polarizers, but the hypothesis suggests that half of the photons are always expected to pass through the system. Students see that probabilities from everyday life leads to a contradiction in QM, hence the probabilistic behavior and the indistinguishability of photons are physical laws[1].

Lesson 5: State circle

In this lesson the state circle (Pospiech et al., 2021; Tóth & Tél, 2023) is introduced for representing quantum measurement (*design principle 4*). Like the teaching material of Udine (Michelini & Stefanel, 2021; Michelini et al., 2011, 2022b) we assign unit vectors to the polarization directions of photons, and these represent quantum states. The permitted states after a measurement are called eigenstates of the measurement. We illustrate all states in a state circle, as shown in **Table 2**. The red vector is the state $\boldsymbol{u}$ in which a measurement is performed, the dashed lines show the permitted states after the measurement ($\boldsymbol{H}$, $\boldsymbol{V}$). The eigenstates correspond to mutually exclusive properties and are *perpendicular* to each other, the eigenstate $\boldsymbol{H}$ is associated with certain transmission and the eigenstate $\boldsymbol{V}$ with certain absorption. The state $\boldsymbol{u}$ is a superposition state because it is not equal to any of the states that can be measured. This is the appearance of the superposition principle for states. When a measurement is performed, this state of a single photon "collapses" into one of the eigenstates.

METHODS

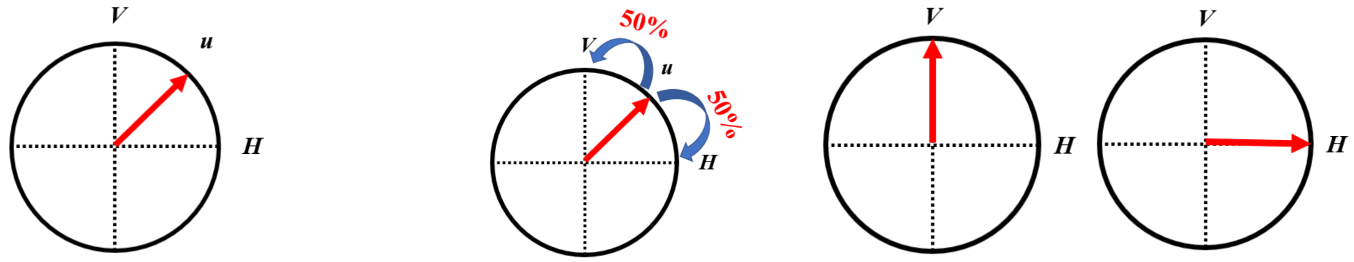
Study Design & Sample

This study is an evaluation study with a pre- and post-test. The sample comprises n=51 (25 females and 26 males) Hungarian secondary school (Czuczor Gergely Benedictine Secondary School, Győr, Hungary) students (grade 10 with age of 16) who participated in the teaching-learning sequence in spring of 2023 during their regular physics lessons. Since we examine the

[1] In two hypotheses, we have distinguished photons & associated them with properties. These are two examples of so-called hidden variable problem. Note that these examples are not suitable to disprove existence of hidden variables. Slightly more complicated hidden variable ideas would not be ruled out in such a simple situation.

Table 2. Pictorial representation of measurement process on state circle (Tóth & Tél, 2023)

Before measurement (prepared state)	Measurement process (superposition collapse)	After measurement (outcome state)
State (***u***) of single photons is assigned with a red vector. Eigenstates (***H*** & ***V***) of measurement are assigned with dashed lines.	Measurement process is indicated as a probabilistic change of state into one of eigenstates.	Measurements have changed state of single photons into one of eigenstates: ***H*** or ***V***.



impact of the last two lessons (hypotheses and state circle), the students completed the pre-test after *lesson 3* and the post-test after *lesson 5*. As they had no previous knowledge about QM, they did not write the test before the intervention.

Instrument

The instrument used in this study comprised a total of four tasks that were adopted from Michelini et al. (2022). *Tasks 1, 2,* and *3* are single-choice questions and *task 4* is an open-ended question. The questionnaire is shown in **Table 3**.

The answers to each single-choice question contained two gestalt thinking answers, one corresponding to a classical and one to a hidden variable way of thinking.

Task 1 is about the probability law in QM. QM way of thinking is option 'a' indicates that students understand that probabilities are unavoidable in QM. Option 'b' and 'c' are related to the classical and the hidden variable way of thinking.

Task 2 is about the effect of quantum measurement.

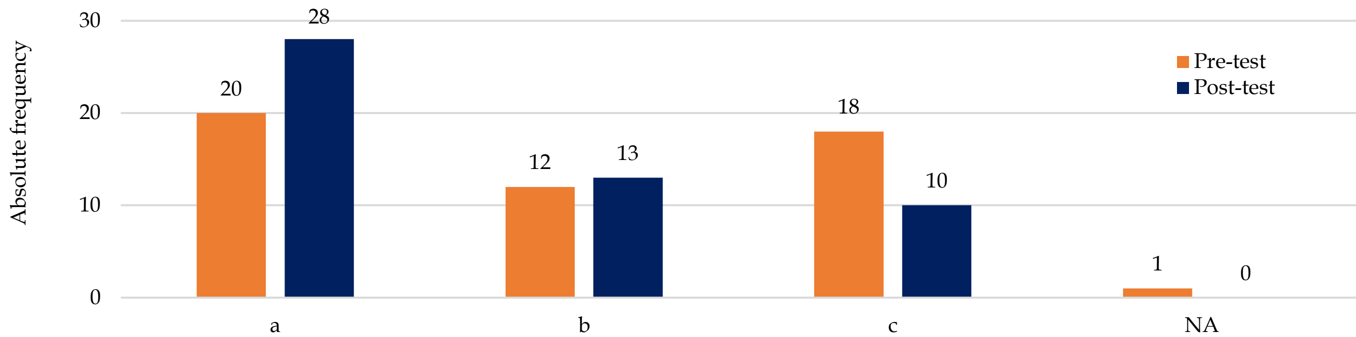
Task 3 investigates how students react to the lack of trajectory.

Option 'a' and option 'b' correspond to the classical (photons have well-defined trajectory) and hidden

Table 3. Questionnaire used in pre- & post-test (QM thinking responses are shown in blue, the gestalt thinking in red, and the mixed thinking remained black)

Task 1. Consider two probabilistic examples!
(1) There is a 50.0% probability that top card of a shuffled deck of French suit cards is red. Probabilities have to be because of lack of knowledge about cards.
(2) There is a 50.0% probability that a 45° polarized single photon will pass through an ideal polarizer with horizontal permitted direction.
What is reason for using probabilities in (2)?

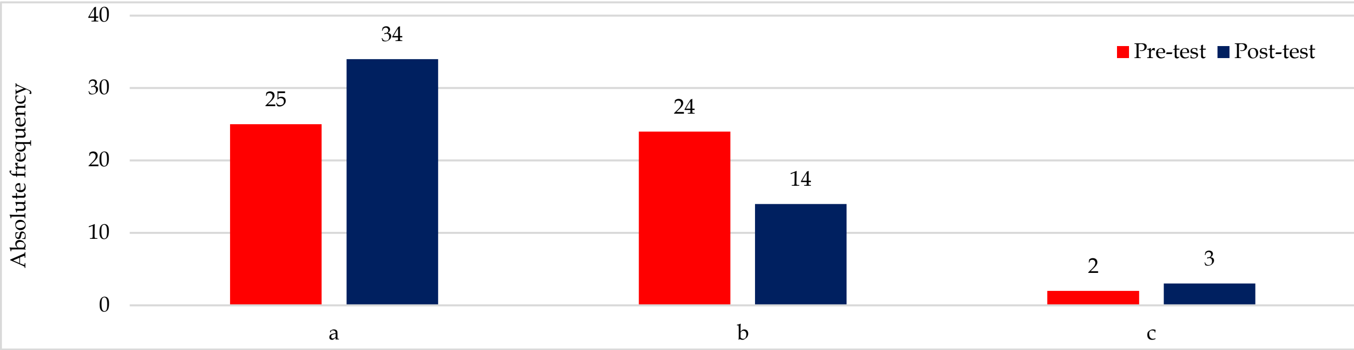
(a) In (2), probabilities are used even if we have enough knowledge about photons because uncertainty is an internal nature of microworld.
(b) In (2), probabilities are used because of lack of knowledge about photons.
(c) In (2), measurement process is unknown, little knowledge about photon-polarizer interaction forces us to use probabilities.



Task 2. Suppose we have prepared 45° polarized photons. These are emitted onto an ideal polarizer with horizontal permitted direction. Suppose that some of these photons have passed through it. What can we say if these transmitted photons encounter with a second ideal polarizer with horizontal permitted direction?

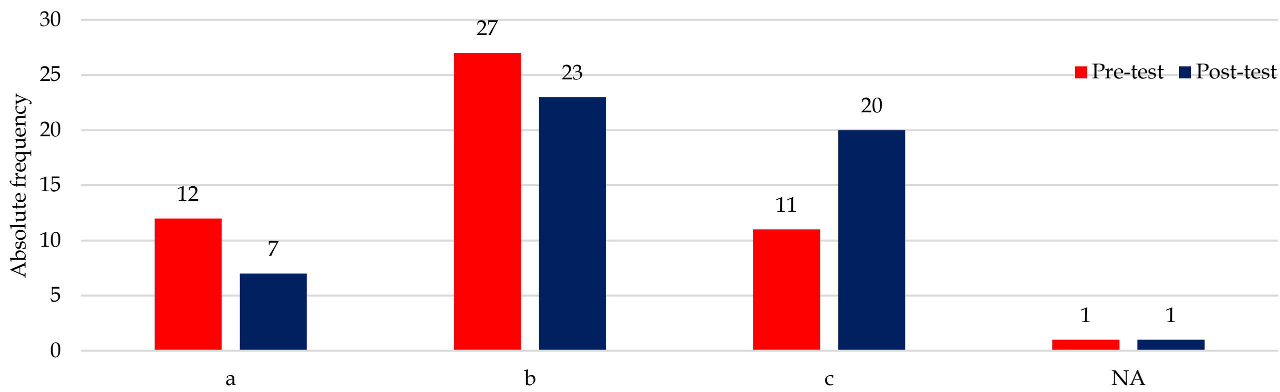
(a) Photons will go through it with certainty.
(b) Photons have a 50% probability of passing through.
(c) Photons will be absorbed with certainty.

Table 3 (Continued). Questionnaire used in pre- & post-test (QM thinking responses are shown in blue, the gestalt thinking in red, and the mixed thinking remained black)



Task 3. In classical physics a moving object always has a well-defined path, e.g., a kicked ball follows a parabolic path. What can we say about a single photon emitted on a birefringent calcite crystal? Explain your answer!

(a) As in example of ball, a certain path could also be associated with each individual photon, but in general we do not know enough about photons. If we knew enough, we would always know path of them.
(b) Contrary to example of ball, a certain path can only be theoretically assigned to each individual photons, but it is impossible to observe this experimentally, because measuring device disturbs path of photons, so that we get a result different from real one.
(c) In contrast to ball, it is usually impossible to assign a certain path to a single photon, we do not even have theoretical possibility of doing so.



Task 4. In your own words, how would you explain the superposition principle?

variable way of thinking about motion (photons have well-defined trajectory but it is inaccessible), respectively. In this task, students were also asked to justify their answer because the way of thinking about the motion is a good indicator of the students' perception. We inductively categorized all justifications to get a better insight into the students' ideas.

Task 4 asks students about superposition principles. Students' responses were analyzed, and different categories were established.

Data Analysis

We answer research question RQ1 by investigating the frequency of chosen answer options that are indicators for QM thinking apparent among the participants. We then examine the students' explanations for *task 3* (lack of trajectory) and *task 4* (superposition) and categorize them inductively.

We tend to answer research question RQ2 by categorizing each student's work into one of the thinking types: gestalt, mixed and functional thinking. Mixed thinking expresses a well-developed way of thinking about quantum phenomena, but also uses elements of gestalt thinking in argumentation. We define a student as having mixed thinking if:

- Two single-choice questions correspond to QM thinking and the argumentation on trajectory and superposition contains only a small gestalt element in addition to the functional elements.
- The trajectory or superposition argument is appropriate, but at least two single-choice answers do not correspond to QM thinking.

If somebody performs worse or better than the mixed thinking, we say that the student holds the gestalt or functional thinking type.

FINDINGS

Findings From Single-Choice Items

The number of responses to option 'c' in *task 1* (the belief that the probability is used due to the measurement process being unknown) is reduced in the post-test from 18 to 10, while the number of QM thinking responses (probabilities cannot be avoided) is improved from 20 to 28. It seems that the studied hypotheses were able to decrease the number of responses on option 'c'.

As an effect of *lesson 4-lesson 5*, there is a strong upward trend from 25 to 34 QM thinking answers in *task 2*. This shows that students basically remember that quantum measurement changes the state of photons, and the number of students answering correctly is further improved by the introduction of a state circle.

Task 3 was the most difficult question, in the pre-test only 11 and in the post-test 20 students answered correctly. There are six students who answered 'a' (classical motion) in the pre-test and then 'b' (hidden variable motion) in the post-test, while no students did the opposite. Ten students answered 'b' and one answered 'a' in the pre-test and then 'c' (quantum thinking) in the post-test. We can observe an evolution of thinking about trajectory, some students who chose the classical way of motion in the pre-test switched to a hidden variable way of motion in the post-test and only students who chose the hidden variable way of motion were able to give QM solution.

Findings From Open-Ended Questions

Students' justifications in *task 3* (33 responses in total: 18 students did not give any argument) are summarized in **Table 4**.

Only four students in the pre- and seven students in the post-test argued that single photons do not have a certain trajectory. Many students used the concept of 'measurement' (12 in the pre- and 18 in the post-test) and 'superposition' (nine and 18, respectively) as an argument. A common misconception reported in the literature, the 'measurement disturbance' (when students believe that the effect of the measurement is due to the disruptive effect of the non-ideal instruments), appeared remarkably few times, only once and twice in the pre- and post-test, respectively.

Table 4. Categories of students' justifications for task 3 (lack of trajectory) (frequency of each category is also shown in aggregate & broken down into response options & categories are not mutually exclusive)

Categories	Description	Anchor example	Total	
			Pre	Post
There is no path	This category is assigned to an answer, which suggests that it is impossible to assign a certain path to photons because of laws of QM.	*"Because of superposition, <u>even the photon itself does not 'know' where it is</u> until it is measured, so its path can only be determined by probabilities."*	4	7
A single photon has certain path	This category is assigned to an answer, which suggests that photons have certain well-defined trajectories in theory.	*"Photon is in a superposition until it reaches detector, then superposition collapses & <u>you find out which orbit photon was in</u>."*	7	4
Path of a single photon is not known by us	This category is assigned to an answer, which suggests that we do not know trajectories of photons. It is important that subject of statement is us/experimenter.	*"<u>We</u> cannot predict from a single photon, <u>which path it will follow</u>, because we do not know which of possible paths <u>it will eventually follow</u>."*	7	7
Single photons have two paths simultaneously	This category is assigned to an answer when students claim that photons have more trajectories simultaneously.	*"Because of principle of superposition until measurement result is known its orbit is uncertain <u>& it is present in both orbits at same time</u>. A photon can only be assigned a path after measurement result."*	6	1
Probabilistic description	This category is assigned to an answer when student refers to probabilistic description.	*"Photon is in superposition until moment of measurement, so there is no fixed position. It is only a <u>probability</u>, where it could be & since orbit is a sequence of successive positions, if it has no fixed position, it cannot have an orbit."*	3	5
Lack of knowledge	This category is assigned to an answer when it particularly highlights our lack of knowledge as a reason.	*"During measurement, although device indicates that photon has arrived, <u>we may get false information</u> because measuring device is disturbing photon. When detector signals, superposition collapses, & <u>we never know</u> what path it took."*	1	2
Measurement	We assign this category to an answer when a student involves measurement process into argument.	*"We know little about photons, we do not know what influences which path they take. Before photon has even passed through path into <u>detector</u>, we do not know which path it will choose, so from this point of view it can NOT be associated with a path. All we know is that, for example, it will certainly choose one of two paths, but it may get lost halfway through."*	12	18

Table 4 (Continued). Categories of students' justifications for task 3 (lack of trajectory) (frequency of each category is also shown in aggregate & broken down into response options & categories are not mutually exclusive)

Categories	Description	Anchor example	Total Pre	Total Post
It is impossible to observe a single photon	This is a subcategory of previous one when students highlights that it is impossible to observe path of photons even if it exists.	*"In many cases measurements will be different, we will not be able to observe exact results."*	6	3
Superposition	When students' argument contains term superposition.	*"Photon is in superposition until it reaches detector, at which point superposition collapses & we find out which orbit photon was in."*	9	18
Disturbance	When students' attribute to a disturbance effect to measurement's instruments.	*"Photon is attracted to measuring device, so it is deflected from its own orbit, & photons can travel in more than two orbits."*	1	2
Effect of measurement	When students' use effect of quantum measurements as an argument.	*"You cannot assign a path to a photon because it is in superposition until you measure it. As soon as we measure it, it collapses & falls, either through or not."*	0	2

In the written tests some students wrote a special argumentation in *task 3* and *task 4*, an example is: *"the photon goes into superposition after passing through a polarizer"* showing that students often believe that superposition only occurs when photons have passed through a polarizer or calcite. In this type of argument, the superposition in the polarization is not considered by the students to be a superposition state. Other two students wrote that *"the measurement has no effect on the photon, all we know by measurement is whether the photon has passed through"*, which indicates that some students identify the measurement with the signal of the detector, and it is not with the polarizer, which suggests a classical way of thinking about the motion.

For *task 4* (explain the superposition principle) we received one answer that fits into the previously mentioned pattern:

> "Due to the measurements [authors' comment: students identified measurements with the effect of the polarizer or calcite], the photons go into a superposition in which state we cannot say yes or no that they possess a property, we can only say maybe, e.g., if we turn over a card from a French-suited deck of cards, it will be red or black with a 50-50% probability, but we do not know which one will be, but the card 'knows about itself'."

This student claimed that the photons are not initially in a superposition state, but after passing through the polarizer/calcite the superposition appears. This means that in this students' mind the superposition only exists because of the measurement.

In **Table 5** we summarize our categorization of all the reasonable answers (27 in the pre-, and 34 in the post-test). Almost all students (22 then 19) gave an example, where the superposition was in the trajectory/position and very few students gave an example, where the superposition was in the polarization (one then six). The post-test shows an improvement in the category *"in general a certain property cannot be assigned to a single photon"* (from 16 to 22 responses) and many students used the term 'measurement' in their argument (13 then 19). The 'measurement device disturbs the system' misconception appeared only once by the same student who used this misconception in the argumentation of *Task 3*. It can also be seen that students who used the category *"the photons can possess more mutually exclusive properties simultaneously"* often also stated *"in general a certain property cannot be assigned to a single photon"* in relation to the superposition of trajectories/positions. The supplementary (in particular the state circle)

Table 5. Categories of students' answers for task 4 (superposition) (categories are not mutually exclusive)

Categories	Description: Category is used if…	Anchor example	Total Pre	Total Post
No answer/not understandable			24	17
In general, a certain property cannot be assigned to a single photon	students' answers highlight that it is impossible to say for sure that a single photon possess certain properties.	*"When given quantum is not in path, but a probability can be assigned to each path, which shows chance that given quantum is there."*	16	22
In general, a certain property cannot be known by us	students' answers state that we do not know properties of photon.	*"Superposition is state in which photon is until moment of measurement. We do not know if photon will be transmitted or absorbed until we measure it."*	4	5
Photons can possess more mutually exclusive properties simultaneously	students' answers state that photons possess all observable properties simultaneously.	*"When photon passes through calcite crystal, it is here & there at same time."*	10	9

Table 5 (Continued). Categories of students' answers for task 4 (superposition) (categories are not mutually exclusive)

Categories	Description: Category is used if...	Anchor example	Total	
			Pre	Post
Measurement disturbs system	students' answers state that measurement uncontrollably disturbs photons as detectors are macroscopic. Photons have certain properties, but this disturbance makes it impossible to observe them.	*"Photon refracted on calcite crystal is attracted by measuring device & deviates from its path."*	2	1
Photons are in between observable properties	students' answers describe superposition as a mixture of observable properties.	*"Situation is that we do not know anything about photon. We have launched xy-polarized photon & until our instrument signs, our photon floats somewhere between life & death."*	0	4
Photons live in another world	students' answers state that superposition is another world, where photons live before measurement.	*"Photon moves to a position that cannot be detected experimentally, only at end of process, detector reveals its existence."*	2	0
Position/path	students use position or path in their example to explain superposition.		22	19
Photon-polarizer interaction	students use example of photon polarizer interaction to explain superposition.		1	6
General property	students do not use certain examples, but state generally via arbitrary properties.		2	5
State	students use word "state" or "status" to describe superposition.		3	14
Measurement changes something	students highlight effect of measurements.		2	5
Measurement as an example	students include measurement process into explanation of superposition.		13	19
Strong highlighting of lack of knowledge	students highlight that superposition expresses our lack of knowledge showing a hidden variable way of thinking.		1	3
Probabilities	students state that probabilities are used when photons are in a superposition state.		4	3

material also created a new type of argument, *"the photons are in between the observable properties"*.

Analysis of Students' Conceptual Development: Gestalt-Functional Transitions

We have categorized each student into one of the cognitive mental models. We provide an archetype for each mental model. For instance, a student is identified as a gestalt thinker because she/he answered 'b', 'a' and 'a' in *tasks 1, 2,* and *3*, respectively. This means that the student believed that probability is used due to the lack of knowledge about the photons, and we do not know their trajectories because of the same external reason. As it is written by a student in *task 3*, *"We cannot predict the path of a single photon, because we do not know which of the possible trajectories it is following"*. Or in *task 4*, *"The photon is in such a situation that it can continue on any path, we can only know its position when it has been found by a detector"*.

As we can see in **Table 6**, the student argues that each photon follows a certain path, but which is unknown to us. One of the students is put in the mixed thinking mental model because the options 'b', 'a', and 'c' were answered in *tasks 1, 2,* and *3*, respectively. The student made a misconception in *task 1*, where the lack of knowledge (hidden variable way of thinking) about the photons was given as the reason for the probabilities.

Table 6. Number of students in each mental model after pre- & post-test

Gestalt thinking		Mixed thinking		Functional thinking	
Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test
43	34	0	9	8	8

However, the argument for the lack of trajectory is quite interesting:

> "Because of the superposition principle, until we know the measurement result, its path is uncertain, and it is present in both trajectories simultaneously. A photon can only be assigned a path after the measurement result."

This student explained the superposition principle as *"the photons have the property that they can be in a state between measurements"*. This answer is from the post-test, and the state circle appeared as a possible argument for superposition: the state of the photon can be between the measured states. It seems that this visual representation of mathematics is used by some students to explain superposition without using linear algebra.

A typical student with functional understanding correctly chose 'a', 'a', and 'c' in *tasks 1, 2,* and *3*, respectively. In *Task 3*, the argument is,

> "The photon is in superposition up to the moment of measurement, so there is no fixed position, only a probability of where it might be, and since the orbit is a sequence of successive positions, since it has no fixed position, it has no path either."

The superposition principle in *task 4* is explained as

> "Quanta are not at any point in space, but there is a probability associated with each path, which is the probability of finding the quantum there".

DISCUSSION

The presented teaching-learning sequence seems to be appropriate for younger students. To answer RQ1, we conclude that students' ideas about the probability law (*task 1)*, the measurement effect and a qualitative interpretation of the superposition principle (*task 4*) are quite well developed, while we experienced difficulties in understanding the lack of trajectory (*task 3*) and the meaning of quantum measurement. The data from *task 3* shows that students are very attached to the classical way of thinking about motion and that a high number of students follow the hidden variable pattern of thinking. The difficulties related to the lack of a trajectory are not surprising, as they have already been pointed out in a huge number of articles in older age groups (see research background section), which is new, but it is not surprising that younger students have similar problems. It was interesting to see that some of the students switched to the hidden variable way of thinking before reaching the quantum mechanical way of thinking. Something similar has been seen in Italian results (Michelini & Stefanel, 2021; Michelini et al., 2011), but an important addition may be that the emergence of a hidden variable way of thinking seems to be determined. A similar developmental path has been pointed out by German researchers in which the so-called "dual type of mental model" emerged as a transitional mental model (between gestalt and functional) (Ubben & Bitzenbauer, 2023). A new finding is that several students think that photons move like classical particles, and now when the photons reach the polarizer, the probability law is realized via an instantaneous decision by the photon. This belief corresponds to the gestalt mental model (Ubben & Bitzenbauer, 2022), the idea is very similar to the presentation of visual models like JQM. This way of thinking has two very important consequences that we explored and, to our best of our knowledge, cannot be found in the literature:

First, some students believe that the measurement has no effect. It is because if quantum measurement is just a signal from the detector, it has no effect, even though we know that the polarization of the photons has changed, but this change is due to the polarizer itself, which is by some students is not considered as part of the measurement.

Second, some students who identified the measurement with the effect of the polarizer believed that a measurement creates a superposition. This is because, as we observed in *task 4*, students forget that the superposition in polarization is "the" superposition; and they only focus on the superposition of trajectories or other everyday concepts. This might be because polarization is not an everyday concept.

We also observed (through *task 3* and *task 4*) that a common misconception 'measurement disturbs the system' appeared very few times, showing a fundamentally different pattern of thinking from what physics education researchers have found in traditional approaches in so much research (Ambrose et al., 1999; Krijtenburg-Lewerissa et al., 2017; Thacker, 2003; Vokos et al., 2002).

Regarding RQ2, we observe that a substantial proportion of students remain in gestalt thinking. For example, when students described the motion of photons in QM perfectly according to JQM visualization, i.e., they identified a narrow model as a faithful representation of the real world, although we discussed this in the lessons. Obviously, this is due to the lower level of abstraction, as the participants were lower secondary school students, and we also used a conceptual approach that reduced the mathematical formalism to a minimum. However, we stress that there are some students who have already achieved a functional understanding. Furthermore, the last two lessons (hypothesis and state circle) helped a certain group to switch to mixed thinking. In our opinion, it might be fruitful to analyze as to how a transition to a reduced mathematical formalism might help these learners take the step towards a more functional understanding of the respective quantum concepts.

Regarding RQ3, we found that the last two lessons (hypotheses and state circle) had a positive effect on half of the students' thinking. In the pre-test, 20 students understood that the use of probabilities is an inevitable approach, which increased to 28. A positive outcome is that a very low proportion of students, 12 in the pre-test, used the classical concept of motion as an argument, which is greatly reduced to seven in the post-test. The hypotheses and the state circle promoted students' thinking with limitations because almost half of the students stuck to the hidden variable way of thinking after the course. We can also see that the state circle helps to understand the measurement effect, the amount of QM thinking answers grew from 25 to 34 (*task 2*). The state circle represents the state changes in a physically correct way, by means of which we can correctly represent the superposition and the quantum measurement. In addition, much more sophisticated categories appeared in *task 3* and *task 4*.

The application of the presented teaching-learning material can be threefold:

(1) For half of the students, it may be sufficient to do only the first three lessons.

(2) Since the last two lessons brought improvement to half of the group the entire five-hour course could be appropriate for engaged students.

(3) It may be advisable for the mixed and functional thinking students to continue the course with the introduction of a simplified quantum formalism (Michelini et al., 2002, 2022b; Tóth, 2023, 2024).

There are limits to research, however. On the one hand, the sample size is small, and on the other hand, a deeper analysis could have been done with respect to the gestalt–functional analysis, as not too many questions were addressed to the students. However, we would point out that this is a pilot experiment on a well-constructed material that has yielded several new results and, to our knowledge, the educational potential of QM for this age group has not yet been explored. An exciting emerging research question is what other learning strategies can be used to move more students from classical thinking or the gestalt mental model to mixed or quantum thinking or the functional mental model.

CONCLUSIONS

In this paper we presented a new minimal mathematical conceptual approach to QM using light polarization for lower secondary school students based on previous research from Udine. We found that several students can comprehend the probability law, the effect of quantum measurement, and the superposition principle, while there are difficulties in understanding the lack of trajectory and the concept of quantum measurement. We inductively categorized students' thinking on the lack of trajectory and superposition. We explored that younger students tend to use gestalt thinking and use the visualization of JQM as an accurate representation of the phenomenology. However, there is a proportion of students who were able to achieve the mixed thinking or functional understanding.

An important message from our research is that we need to pay close attention to what students understand by superposition and quantum measurement. Students often think in a fundamentally sensible and well-constructed way that may not be obvious to the teacher at first: for example, students believe that the measurement creates a superposition basically due to the misconception that superposition in polarization is not recognized as a superposition, which should be taken with care in a next teaching experience. In addition, the effect of the polarizer is often not recognized by students because they attribute the measurement itself to the detector signal, this another point can also be developed. We believe, however, that it is possible and even worthwhile to teach QM to younger students if we are careful enough to accept that certain concepts, such as the fact that quantum objects do not obey the temporal and spatial description, will cause problems and that many students will remain in a semi-classical model.

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