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Abstract

We have proved in both human-based and computer-based tests that natural concepts generally 'entangle' when they combine to form complex sentences, violating the rules of classical compositional semantics. In this article, we present the results of an innovative video-based cognitive test on a specific conceptual combination, which significantly violates the Clauser–Horne–Shimony–Holt version of Bell's inequalities ('CHSH inequality'). We also show that collected data can be faithfully modelled within a quantum-theoretic framework elaborated by ourselves and a 'strong form of entanglement' occurs between the component concepts. While the video-based test confirms previous empirical results on entanglement in human cognition, our ground-breaking empirical approach surpasses language barriers and eliminates the need for prior knowledge, enabling universal accessibility. Finally, this transformative methodology allows one to unravel the underlying connections that drive our perception of reality. As a matter of fact, we provide a novel explanation for the appearance of entanglement in both physics and cognitive realms.

Keywords: human cognition, concept combinations, Bell's inequalities, quantum modelling, entanglement

1. Introduction

Completely Automated Public Turing test to tell Computers and Humans Apart (CAPTCHA[1]) have been widely used by websites to safeguard their services from bot-driven activities. Relatively simple tasks for people, but challenging for computers, are presented in CAPTCHA, such as clicking in a designated area, recognizing letters or numbers that are stretched, selecting objects in an image. Their simple yet effective mechanisms have contributed to maintain poll integrity, control registration, prevent ticket inflation, and counter false comments, ultimately ensuring a more secure and trustworthy online experience for users. Image-based CAPTCHAs require both image recognition and semantic classification, making them harder for bots to understand than text-based one. In addition, by collecting data from CAPTCHAs, one could train machine learning models.

It is commonly accepted that human perception and thought are essentially synthetic processes. We immediately form a 'Gestalt', or a general idea, of the object. Gestaltic patterns seem to be the primary basis of rational activity as well. Gestalt-thinking is not well captured by the analytical and compositional framework of classical logical semantics, which essentially deduces the meaning of a composite expression from the meanings of its component parts ('principle of compositionality', see, e.g., [1]). For an accurate study of natural languages and creative contexts, where holistic and ambiguous characteristics seem to be significant, classical semantics is no longer very useful. One example in this regard is the last line of Giacomo Leopardi's poem L'Infinito, "E 'l naufragar m'è dolce in questo mare" (And drowning in this sea is sweet to me, i.e. I think it's delightful to dedicate myself to the contemplation and meditation of the infinite). The meanings of the component terms "naufragar" (drowning), "dolce" (sweet), and "mare" (sea) do not correlate to the meanings they have in this context, which appears to be the fundamental cause of the poetic outcome in this instance. By the way, the poem refers to the settlement of Recanati, which is not near a sea. Nonetheless, our words' common meanings continue to be used and are loosely connected to the metaphorical meanings that the poem as a whole evokes, a semantic conundrum that frequently occurs in poetry and musical compositions, where meanings are inherently holistic, contextual and ambiguous.

Similar issues occur whenever people combine individual concepts to form

[1]The term "CAPTCHA" was coined in 2000 by Luis von Ahn, Manuel Blum, Nicholas Hopper and John Langford of Carnegie Mellon University, see, e.g., the webpage `http://www.captcha.net`.

conceptual combinations or more complex linguistic expressions as sentences and texts. A large empirical literature indeed reveals that the principle of compositionality is systematically violated and, more generally, concepts exhibit aspects of 'inherent vagueness', 'contextuality' and 'emergence' which prevent them to be modelled within classical Boolean logical and classical Kolmogorovian probabilistic structures (see, e.g., [2, 3, 4, 5, 6, 7, 8, 9, 10] and references therein).

Recently, various approaches have been put forward to theoretically cope with this structural inability of classical compositional semantics to handle the dynamics of human concepts. Remarkably, some of these approaches use the logico-mathematical formalism of quantum theory, detached from its physical interpretation, as a modelling tool to represent conceptual meaning. This research fits a growing research programme that applies quantum structures in the mathematical modelling of cognitive processes, with relevant extensions to information retrieval processes (see, e.g., [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25] and references therein).

In particular, in the 'holistic quantum computational semantics', meanings are typically represented by quantum superpositions of meanings and are treated as fundamentally dynamical objects. Any global meaning determines partial meanings, which are frequently vaguer than the global one. The majority of research on quantum computational logics has focused on sentential logics, whose alphabet is made up of atomic sentences and logical connectives. A first-order epistemic quantum computational logic with a semantic characterization that can express sentences like "the animal acts," "the animal eats the food", etc. was presented in [26]. An alternative but similar approach considers a concept as an entity whose meaning is incorporated into a given state, whose state can change under the influence of a context, and represents conceptual entities in the formalism of quantum theory in Hilbert space [27].

Within the two quantum approaches above, empirical and theoretical studies have been carried out with the aim of identifying 'quantum entanglement' in the combination of natural concepts [28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39]. In physics, entanglement is a statistical property of a composite system, or 'entity', made up, in the simplest case, of two individual entities. Entanglement is typically detected through the violation of suitable inequalities, called 'Bell's inequalities', the 'Clauser–Horne–Shimony–Holt (CHSH) inequality' [40, 41] being one of these inequalities. More precisely, we have recently performed various tests, including text-based cognitive tests on human participants, information retrieval tests on corpora of documents, and image retrieval tests on web search engines, which confirm that the CHSH inequality is systematically violated when two concepts combine [30, 31, 33, 34, 35, 36, 37, 38, 39].

In this article, we deepen and widely extend the research above and present the results of a video-based cognitive test that we have recently performed on the conceptual combination *The Animal Acts*, considered as a combination of the concepts *Animal* and *Acts*,[2] where the term "acts" refers to one of the sounds that can be made by an animal. The test used the videos produced by CAPTCHA through artificial intelligence (AI) and showed to the participants, who had to judge among the videos the best examples of *The Animal Acts*. As such, the test is more realistic and effective than the previous text-based tests, as it surpasses language barriers and eliminates the need for prior knowledge, thus enabling universal accessibility. We show that the CHSH inequality is significantly violated in the test, which reveals the presence of entanglement between the component concepts. We also work out a quantum theoretical model in Hilbert space for the statistical data ('judgement probabilities'), which reveals that entanglement occurs at both state and measurement levels, hence it is even stronger than the entanglement that is typically detected in quantum physics tests.

The presence of entanglement can be naturally explained if one observes that both concepts *Animal* and *Acts* carry meaning, but also the combination *The Animal Acts* carries its own meaning, and this meaning is not simply related to the separate meanings of *Animal* and *Acts* as prescribed by a classical compositional semantics. It also contains emergent meaning almost completely caused by its interaction with the wide overall context. We have called the complex mechanism by which meaning is attributed to the combined concept 'contextual updating', and it occurs at the level of entanglement creation [42].

For the sake of completeness, we summarize the content of this article in the following.

In Section 2, we illustrate the general setting of a Bell-type test for the empirical detection of entanglement in both physical and conceptual domains. The violation of the CHSH inequality is generally considered as conclusive towards the presence of entanglement in the given situation. We however observe that situations exist in which entanglement cannot only be attributed to the state of a composite entity, but also the measurements need to be considered to be entangled. This typically occurs in conceptual domains.

In Section 3, we describe the empirical setting of the video-based cognitive test

[2]In this article, we write concepts using capital letters and italics, e.g., *Animal*, *Fruit*, *Vegetables*, etc. This is frequently the way of referring to concepts in cognitive psychology.

we have performed, stressing the advantages of a cognitive test based on videos with respect to more traditional text-based tests. We also present the empirical results, which violate the CHSH inequality in agreement with those of previous tests, thus indicating that entanglement is a natural candidate to theoretically model the empirical situation.

In Section 4, we work out a quantum mathematical representation in Hilbert space for the data collected in Section 3 and show that the violation of the CHSH inequality can be explained in terms of a strong form of entanglement involving both states and measurements.

In Section 5, we finally provide a theoretical analysis of the results obtained in the test and the ensuing modelling, and establish some deep connections between the non-classical mechanism of meaning attribution and the appearance of entanglement in conceptual domains.

2. Identifying entanglement in physics and cognition

We present in this section the typical way in which entanglement is empirically detected in physical and cognitive domains.

In physics, entanglement is a statistical property of a composite entity, which can be identified by means of the empirical violation of Bell's inequalities [40, 41, 43]. One of Bell's inequalities that is more suited for empirical control is the CHSH inequality [41]. The violation of Bell's inequalities is generally interpreted by saying that, due to entanglement, micro-physical entities exhibit genuinely non-classical aspects, as 'nonseparability' and 'contextuality', where the latter also occurs when the component entities are far apart in space, an aspect known as 'nonlocality'. In addition, entanglement produces statistical correlations between the component entities that cannot be reproduced by any classical model of probability satisfying the axioms of Kolmogorov [44, 45]. Because of its deep physical implications, entanglement has been considered as the distinctive trait of quantum theory since its early days [46].

The empirical setting for the identification of entanglement refers to a 'Bell-type test', since the seminal papers by Einstein–Podolsky–Rosen (EPR), David Bohm and John Bell [40, 41, 43, 47, 48]. Let us consider a composite physical entity S_{12}, prepared in an initial state p, and such that the individual entities S_1 and S_2 can be recognized as component parts of S_{12}. Next, let us perform the coincidence measurements AB, AB', $A'B$ and $A'B'$ on S_{12}, where the coincidence measurement XY consists in performing the measurement X on S_1, with possible outcomes X_1 and X_2, and the measurement Y on S_2, with possible outcomes Y_1 and

Y_2, with $X = A, A'$, $Y = B, B'$. The component entities S_1 and S_2 have interacted in the past, but are spatially separated when the coincidence measurements are performed. If X_1, X_2, Y_1, Y_2 can only be ± 1, $X = A, A'$, $Y = B, B'$, the expected values of AB, AB', $A'B$ and $A'B'$ are the correlation functions $E(A,B)$, $E(A,B')$, $E(A',B)$ and $E(A',B')$, respectively. One can then prove that, under the reasonable, in classical physics, assumption of 'local separability', or 'local realism' [47], the CHSH inequality, namely,

$$-2 \leq E(A',B') + E(A',B) + E(A,B') - E(A,B) \leq 2 \tag{1}$$

should be satisfied [41]. We call 'CHSH factor' the quantity

$$\Delta_{CHSH} = E(A',B') + E(A',B) + E(A,B') - E(A,B) \tag{2}$$

and observe that Δ_{CHSH} is mathematically bound by -4 and $+4$.

According to modern manuals of quantum theory, the individual entities S_1 and S_2 are associated with the complex Hilbert spaces $\mathscr{H}_1$ and $\mathscr{H}_2$, respectively. In this case, both $\mathscr{H}_1$ and $\mathscr{H}_2$ are isomorphic to the complex Hilbert space $\mathbb{C}^2$ of all ordered couples of complex numbers. The composite entity S_{12} is instead associated with the tensor product Hilbert space $\mathscr{H}_1 \otimes \mathscr{H}_2$. In this case, $\mathscr{H}_1 \otimes \mathscr{H}_2$ is isomorphic to the complex Hilbert space $\mathbb{C}^2 \otimes \mathbb{C}^2$. The possible (pure) states of S_1 and S_2 are represented by unit vectors of $\mathscr{H}_1$ and $\mathscr{H}_2$, respectively, and the measurements that can be performed on S_1 and S_2 are represented by self-adjoint operators on $\mathscr{H}_1$ and $\mathscr{H}_2$, respectively. However, $\mathscr{H}_1 \otimes \mathscr{H}_2$ also contains vectors that cannot be written as the tensor product of a unit vector of $\mathscr{H}_1$ and a unit vector of $\mathscr{H}_2$. These non-product vectors of $\mathscr{H}_1 \otimes \mathscr{H}_2$ are said to represent 'non-product', or 'entangled', states of S_{12}. Analogously, the self-adjoint operators of $\mathscr{H}_1 \otimes \mathscr{H}_2$ are not limited to operators that are the tensor product of a self-adjoint operator of $\mathscr{H}_1$ and a self-adjoint operator of $\mathscr{H}_2$. In these cases, at least one eigenvector of these non-product self-adjoint operators represents an entangled state. These non-product self-adjoint operators are said to represent 'non-product', or 'entangled', measurements of S_{12} [43].

The CHSH inequality in Equation (1) is manifestly violated in quantum theory and, when a violation occurs, it is due to the presence of entanglement between the component entities S_1 and S_2 that can be recognized as component parts of S_{12}. Equivalently, in the absence of entanglement, the inequality in Equation (1) would not be violated. The typical situation in which the inequality in Equation (1) is violated in quantum theory consists in the state p of the composite entity S_{12} being the 'singlet spin state', which is an example of a 'maximally entangled state' [43], and the coincidence measurements XY being product measurements, $X=A,A'$, $B'B'$. This situation entails a CHSH factor equal to $\Delta_{QMC}=2\sqrt{2}\approx 2.83$, known as 'Cirel'son's bound' [49, 50]. The situation, however, also requires a special 'symmetry requirement on joint probabilities', namely, the 'marginal law of Kolmogorovian probability being satisfied', i.e. for every $i,j=1,2$,

$$\sum_{j=1,2}\mu(X_iY_j) = \sum_{j=1,2}\mu(X_iY'_j) \quad (3)$$

$$\sum_{i=1,2}\mu(X_iY_j) = \sum_{i=1,2}\mu(X'_iY_j) \quad (4)$$

In Equation (3), $\mu(X_iY_j)$ ($\mu(X_iY'_j)$) is the joint probability of obtaining the outcomes X_i in a measurement of X on S_1 and Y_j (Y'_j) in a measurement of Y (Y') on S_2, $X=A,A'$, $Y,Y'=B,B'$, $Y'\neq Y$. In Equation (4), $\mu(X_iY_j)$ ($\mu(X'_iY_j)$) is the joint probability of obtaining the outcomes X_i (X'_i) in a measurement of X (X') on S_1 and Y_j in a measurement of Y (Y') on S_2, $X,X'=A,A'$ and $Y=B,B'$, $X'\neq X$.

The needs of quantum computation and quantum information have intensified the theoretical research on Bell's inequalities (see, e.g., [43, 51]) as well as the research on the empirical consequences of entanglement (see, e.g., [51, 53, 54]), confirming the predictions of quantum theory.

In the meanwhile, it has become evident that the genuinely quantum aspects of contextuality, entanglement, indistinguishability, interference, and superposition, are not peculiar of micro-physical entities, as they also occur in the quantum mathematical modelling of complex cognitive processes, e.g., the processes involving judgement, decision, perception and language (see Section 1). In particular, both experimental and theoretical research has been dedicated to the identification of entanglement in the combination of natural concepts (see again Section 1), and our research team has provided substantial contributions along both lines of this investigation, as follows.

At an experimental level, we performed cognitive tests on human participants [30, 31, 38, 39], document retrieval tests on structured corpuses of documents [35, 37] and image retrieval tests on web search engines [33, 36, 39] using various combinations of two concepts. We mainly investigated the conceptual combination *The Animal Acts*, which we considered as a composite conceptual entity made up of the individual conceptual entities *Animal* and *Acts*. The tests had the form of the Bell-type test above, and we found (i) a systematic violation of the CHSH inequality in Equation (1). However, we also found (ii) a systematic violation of the marginal law (Equations (3) and (4)) and, in some cases, (iii) the CHSH factor in Equation (2) exceeded Cirel'son's bound.

While empirical finding (i) substantially agreed with the predictions of quantum theory and indicated the presence of 'conceptual entanglement', empirical findings (ii) and (iii) were unexpected, as they are not believed to occur in quantum physics. This led us to initiate a theoretical investigation on a problem that is usually overlooked in physics, the 'identification problem', that is, the problem of recognising individual entities of a composite entity by performing on the latter the typical coincidence measurements of Bell-type tests [31]. We thus elaborated a general theoretical framework to model any Bell-type situation, independently of the nature, physical or conceptual, of the entities involved, within the mathematical formalism of quantum theory in Hilbert space [31, 34]. In this theoretical framework, one applies the standard prescription of quantum theory according to which the composite entity S_{12} needs to be associated with a complex Hilbert space whose dimension is determined by the number of distinct outcomes of the performed measurements. Since in principle, each of the coincidence measurements AB, AB', $A'B$ and $A'B'$ have four distinct outcomes, S_{12} should be represented in the Hilbert space $\mathbb{C}^4$ of all ordered 4-tuples of complex numbers. Only in the attempt of 'recognizing' individual entities S_1 and S_2 within the composite entity, one considers possible isomorphisms with the tensor product Hilbert space $\mathbb{C}^2 \otimes \mathbb{C}^2$, where each copy of $\mathbb{C}^2$ takes into account the fact that measurements with two distinct outcomes can be performed on S_1 and S_2 in a Bell-type setting. And it is only at this stage, i.e. when individual entities are recognized from measurements performed on the composite entity, that entanglement can be identified. We proved in [31], that no unique isomorphism exists between $\mathbb{C}^4$ and $\mathbb{C}^2 \otimes \mathbb{C}^2$, and this is the reason why, from a mathematical point of view, different ways exist to account for entanglement being present within the composite entity S_{12} with respect to the individual entities S_1 and S_2 that are recognized as parts of S_{12}.

Essentially, entanglement manifests itself when the probabilities of a coincidence measurement on S_{12} cannot be written as products of probabilities of measurements on the component entities S_1 and S_2. Hence, entanglement is a property of the relation between the coincidence measurements and measurements on the component entities. Only when the additional symmetry mentioned above, i.e. the marginal law being satisfied, is present in all coincidence measurements, the entanglement of these different coincidence measurements can be captured in a state of the composite entity. If this is true for all coincidence measurements, one can prove that there is only one isomorphism connecting $\mathbb{C}^4$ with $\mathbb{C}^2 \otimes \mathbb{C}^2$, and $\mathbb{C}^4$ can be directly replaced by $\mathbb{C}^2 \otimes \mathbb{C}^2$ in that case. This means that the situation usually reported in modern manuals of quantum theory is exceptional and not the general one [31]. In this general situation, where the marginal law is empirically violated, no unique isomorphism exists between $\mathbb{C}^4$ and $\mathbb{C}^2 \otimes \mathbb{C}^2$.

The quantum theoretical framework above enables modelling of empirical data collected in Bell-type tests where the marginal law does not hold and the violation of the CHSH inequality in Equation (1) exceeds Cirel'son's bound, in those tests where this occurs, by introducing and representing entangled measurements. In particular, we have proved that, whenever the concepts *Animal* and *Acts* combine to form the combination *The Animal Acts*, a strong form of entanglement is created between *Animal* and *Acts*, which is such that, not only the state of the composite entity *The Animal Acts* is entangled, but also the coincidence measurements are entangled [31, 36, 37, 38].

We will see in Sections 4 and 5 that the entanglement above is due to the peculiar way in which the meaning of *The Animal Acts* relates to the meanings of *Animal* and *Acts*, which violates the classical semantic rules of composition. Before doing this, however, we need to present the details of the novel cognitive test based on videos which we have recently performed on this conceptual combination. This will be the aim of Section 3.

3. A novel video-based cognitive test

We present in this section the details of the video-based cognitive test on human participants for the detection of entanglement in the conceptual combination *The Animal Acts*.

As anticipated in Sections 1 and 2, we consider the concept *The Animal Acts* as a combination of the individual concepts *Animal* and *Acts*, where by "acts" we mean the action of producing a recognizable sound by the animal. Next, we consider two pairs of items of *Animal*, namely, (*Horse*, *Bear*) and (*Tiger*, *Cat*), and two pairs of items of *Acts*, namely, (*Growls*, *Whinnies*) and (*Snorts*, *Meows*). We are now ready to illustrate the test.

A sample of 221 individuals were presented in a 'within subjects design' a HTML5 questionnaire which contained four coincidence experiments AB, AB', $A'B$ and $A'B'$ whose setting was similar to the typical setting of a Bell-type test presented in Section 2. More specifically, participants were preliminarily asked to read an 'introductory text' where an explanation of the type of judgement test they had to complete and a description of the tasks involved in the judgement test were provided. More specifically, the participants had to preliminary complete a simple introductory test on the concept *Fruit*, judging the item that they considered as a good example of the concept *Fruit*. The items of *Fruit*, indicated in Figure 1, were generated by image-based generative intelligence, and we proposed the item

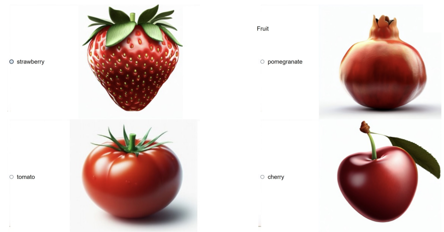

Figure 1: Introductory test based on different items of the concept *Fruit*.

Strawberry, which is an enlarged portion of an inflorescence, and *Tomato*, which is a fruit, to draw maximum attention to the choice. *Cherry* and *Pomegranate* were the remaining items of *Fruit* to judge upon.

Then, in each coincidence measurement, participants were asked to choose which item in a list of four items they judged as a good example of the conceptual combination *The Animal Acts*. In each coincidence measurement, each of the four possible items of *The Animal Act* was a video showing an animal and the corresponding action. More specifically, the videos, indicated in Figure 2, were created as follows:

(i) using a video search engine to find a video of the animal producing the sound;

(ii) extracting a cropped portion of the video where the animal produces the sound;

(iii) in cases where the animal and sound combination is unlikely, e.g., a cat that whinnies, using reverse image search to find a similar video of the animal instead of using AI services, as Sora AI, or morphing techniques to create a video of the animal producing the sound.

All of the operations above can be automated by an AI service. Future AI advancements will allow for the creation of 3D animated models that are virtually indistinguishable from real 3D video which can be activated by gaze and selected with pinch gestures, as in Vision Pro and Quest Pro, to create more engaging CAPTCHAs.

Among the 221 volunteers who participated in the test, 15 participants took the English version and the remainder took the Italian version. Video-based tests offers several advantages with respect to both text-based and image-based tests performed in previous empirical studies on the identification of entanglement in conceptual combinations. In particular, a video-based test reduces language dependence, captures actions more effectively than static images, and leverages au-

sub-test 1 sub-test 2 sub-test 3 sub-test 4

Figure 2: Video-based animal acts.

dio information to enhance the assessment.

Before analysing the results of the test, it is worth to make some remarks on the choice of its setting. The detection of entanglement in Bell-type cognitive tests, indeed, relies on a precise preparation of both the initial state and the coincidence measurements. Insufficient initial conditions, as lacking a brief introduction and prior knowledge of the animals and sounds involved, can hinder the ability to identify entanglement. Birds could offer an interesting case study because some species possess unique vocalizations distinct from other individuals. This individuality makes them well-suited for entanglement experiments involving experts. One could, e.g., consider, two pairs of items of *Animal*, namely, (*Cuckoo*, *Nightingale*) and (*Thrush*, *Goldfinch*), and two pairs of items of *Acts*, (*Cuckoos*, *Trills*) and (*Whistles*, *Snails*), without determining a violation of the CHSH inequality in Equation (1). In other words, cognitive tests on unskilled participants may fail to detect entanglement or yield lower value of the CHSH factor Δ_{CHSH} in Equation (2). A similar situation is likely to occur if the actions are not typical of the animals. Consider, e.g., two pairs of items of *Animal* (*Horse*, *Bear*) and (*Dog*, *Camel*), and two pairs of items of *Sport* (*Racing*, *Fishing*) and (*Sledding*, *Show Jumping*) for the conceptual combination *The Animal does Sport*. Also in this case, we expect the CHSH inequality in Equation (1) not to be violated or to show a little violation. This is why the identification of the items to use in the cognitive test followed strict prescriptions, as follows:

(1) Instance analysis. Identify all possible items of the two concepts under consideration. Search within corpora for pairs of items that appear together, evaluating the meaningfulness of such combinations.

(2) Pair generation. Consider all possible pairs of animals and sounds, verifying that at least one combination is present in the pairs identified in (1).

(3) CAPTCHA creation. If actions or sounds facilitate the selection, use generative AI to create video-based CAPTCHAs such as Sora AI. Otherwise, use

images and text.

(4) Correlation assessment. Administer a significant number of tests to participants. Calculate the maximum value of Δ_{CHSH} to establish the correlation between the two concepts.

We are now ready to illustrate the possible judgements of the individuals who participated in the video-based cognitive test.

In the coincidence measurement AB, participants had to choose the best example of *The Animal Acts* within the four items:

(A_1B_1) *The Horse Growls*

(A_2B_2) *The Bear Whinnies*

(A_1B_2) *The Horse Whinnies*

(A_2B_1) *The Bear Growls*

(sub-test 1 in Figure 1). If the response was A_1B_1 or A_2B_2, then the measurement AB was attributed the outcome $+1$; if the response was A_1B_2 or (A_2B_1), then the measurement AB was attributed the outcome -1.

In the coincidence measurement AB', participants had to choose the best example of *The Animal Acts* within the four items:

($A_1B'_1$) *The Horse Snorts*

($A_1B'_2$) *The Horse Meows*

($A_2B'_1$) *The Bear Snorts*

($A_2B'_2$) *The Bear Meows*

(sub-test 2 in Figure 1). If the response was $A_1B'_1$ or $A_2B'_1$, then the measurement AB' was attributed the outcome $+1$; if the response was $A_1B'_2$ or $A_2B'_1$, then the measurement AB' was attributed the outcome -1.

In the coincidence measurement $A'B$, participants had to choose the best example of *The Animal Acts* within the four items:

(A'_1B_1) *The Tiger Growls*

(A'_1B_2) *The Tiger Whinnies*

(A'_2B_1) *The Cat Growls*

(A'_2B_2) *The Cat Whinnies*

(sub-test 3 in Figure 1). If the response was A'_1B_1 or A'_2B_2, then the measurement $A'B$ was attributed the outcome $+1$; if the response was A'_1B_2 or A'_2B_1, then the measurement $A'B$ was attributed the outcome -1.

Finally, in the coincidence experiment $A'B'$, participants had to choose the best example of *The Animal Acts* within the four items:

($A'_1B'_1$) *The Tiger Snorts*

($A'_1B'_2$) *The Tiger Meows*

($A'_2B'_1$) *The Cat Snorts*

($A_2'B_2'$) *The Cat Meows*

(sub-test 4 in Figure 1). If the response was $A_1'B_1'$ or $A_2'B_2'$, then the measurement $A'B'$ was attributed the outcome $+1$; if the response was $A_1'B_2'$ or $A_2'B_1'$, then the measurement $A'B'$ was attributed the outcome -1.

For each coincidence measurement XY, we collected the relative frequencies of the obtained responses which we considered, in the large number limit, as the probability $\mu(X_iY_j)$ that the outcome $X_iY_j = \pm 1$ is obtained in the corresponding measurement, $X = A, A'$, $Y = B, B'$. Table 1 reports the judgement probabilities computed in this way. Referring to these probabilities, we can then calculate the expectation values, or correlation functions, of the coincidence measurements AB, AB', $A'B$ and $A'B'$, as follows:

$$E(A,B) = \mu(A_1B_1) - \mu(A_1B_2) - \mu(A_2B_1) + \mu(A_2B_2) = -0.8552 \quad (5)$$
$$E(A,B') = \mu(A_1B_1') - \mu(A_1B_2') - \mu(A_2B_1') + \mu(A_2B_2') = 0.5204 \quad (6)$$
$$E(A',B) = \mu(A_1'B_1) - \mu(A_1'B_2) - \mu(A_2'B_1) + \mu(A_2'B_2) = 0.7014 \quad (7)$$
$$E(A',B') = \mu(A_1'B_1') - \mu(A_1'B_2') - \mu(A_2'B_1') + \mu(A_2'B_2') = 0.9005 \quad (8)$$

Inserting Equations (5)–(8) into Equation (1), we get

$$\Delta_{CHSH} = E(A',B') + E(A',B) + E(A,B') - E(A,B) = 2.9774 \quad (9)$$

The numerical value 2.9774 exceeds the classical limit imposed by the CHSH inequality in Equation (1) and is also above Cirel'son's bound $2\sqrt{2} \approx 2.8284$. A simple check in Table 1 reveals that the marginal law in Equations (3) and (4) is also systematically violated here. This empirical pattern confirms and strengthens the results obtained in previous text-based cognitive tests on the conceptual combination *The Animal Acts*, namely [30, 38, 39]. Table 2 reports the judgement probabilities and the CHSH factor in these three text-based tests. By comparing the CHSH factors in Tables 1 and 2, we can see that a systematic violation of the CHSH inequality occurs in all tests. We also notice that the value of the CHSH factor in the test in [30] is far from Cirel'son's bound, the test in [38] is close to that bound, and the test in [39], together with the present test violate Cirel'son's bound. That the test in [30] has a relatively lower deviation from classicality than the other tests might be due to the fact that the introductory text in [30] encouraged the participants to also take into account emotions and imagination in their judgements. This led them to make choices that are less natural, as *The Bear Meows*, *The Cat Growls*, or *The Tiger Meows*, thus determining a relatively lower violation of the CHSH inequality in Equation (1).

Participants	221
Experiment AB	
Horse Growls	0.0452
Horse Whinnies	0.8824
Bear Growls	0.0452
Bear Whinnies	0.0271
Experiment AB'	
Horse Snorts	0.6833
Horse Meows	0.0226
Bear Snorts	0.2172
Bear Meows	0.0770
Experiment $A'B$	
Tiger Growls	0.7919
Tiger Whinnies	0.0362
Cat Growls	0.1131
Cat Whinnies	0.0588
Experiment $A'B'$	
Tiger Snorts	0.0633
Tiger Meows	0.0452
Cat Snorts	0.0045
Cat Meows	0.8869
Δ_{CHSH}	2.9774

Table 1: We report the statistical data collected in the video-based cognitive test presented in Section 3. The judgement probabilities are in substantial agreement with the results obtained in other text-based cognitive tests, namely, [30], [38] and [39], presented in Table 2. Also in this case, we get a significant violation of the CHSH inequalitiy in Equation (1), with a CHSH factor (see Equation (2)) exceeding Cirel'son's bound.

Test	Aerts & Sozzo (2011)	Aerts et al. (2023)	Bertini et al. (2023)
Participants	81	81	100
Experiment AB			
Horse Growls	0.0494	0.0494	0.03
Horse Whinnies	0.6296	0.1235	0.91
Bear Growls	0.0617	0.7778	0.04
Bear Whinnies	0.2593	0.0494	0.02
Experiment AB'			
Horse Snorts	0.5926	0.7160	0.83
Horse Meows	0.0247	0.0494	0.01
Bear Snorts	0.2963	0.2222	0.15
Bear Meows	0.0864	0.0123	0.01
Experiment $A'B$			
Tiger Growls	0.7778	0.7778	0.86
Tiger Whinnies	0.0864	0.0864	0
Cat Growls	0.0864	0.0617	0.14
Cat Whinnies	0.0494	0.0741	0
Experiment $A'B'$			
Tiger Snorts	0.1481	0.0864	0.01
Tiger Meows	0.0864	0.0617	0.02
Cat Snorts	0.0988	0.0247	0.02
Cat Meows	0.6667	0.8272	0.95
Δ_{CHSH}	2.4197	2.7901	3.22

Table 2: We report in comparison the statistical data of three text-based cognitive tests, namely, the Aerts & Sozzo (2011) test [30] in the first column, the Aerts et al. (2023) test [38] in the second column, and the Bertini et al. (2023) test [39] in the third column. The corresponding judgement probabilities are in substantial agreement across the tests and also with the video-based cognitive test in Table 1. All tests exhibit a significant violation of the CHSH inequality.

As noticed in Sections 1 and 2, the empirical results in the video-based cognitive test seem to indicate the presence of entanglement in the combination of *Animal* and *Acts*. We intend to show in Section 4 that this entanglement is present at both the state and the measurement level.

4. Data modelling in Hilbert space

In this section, we provide a quantum representation in Hilbert space of the data collected in the video-based cognitive test in Section 3. We apply the general quantum theoretical framework that we have elaborated for the modelling of any Bell-type test, as we have already done in various papers [31, 34, 37, 38].

The quantum theoretical framework for Bell-type situations consists in the implementation of three main steps, as follows.

(i) One identifies in the situation under study, the composite entity, together with the individual entities composing it.

(ii) One recognises in the composite conceptual entity the states, measurements and outcome probabilities that are relevant to the situation under study.

(iii) One represents entities, states, measurements and outcome probabilities using the Hilbert space representation of entities, states, measurements and outcome probabilities of quantum theory.

(i) The conceptual combination *The Animal Acts* is considered as a composite conceptual entity made up of the individual entities *Animal* and *Acts*.

(ii) Whenever an individual participating in the test reads the introductory text which explains the details of the test and nature of the concepts involved, this set of instructions prepares the composite entity *The Animal Acts* in an initial state p which describes the general situation of an animal that produces a recognizable sound. Each participant is then confronted with this uniquely prepared state p. More precisely, in the coincidence measurement XY, $X = A, A'$, $Y = B, B'$, each participant interacts with the entity *The Animal Acts* in the state p and operates as a measurement context for the entity. This interaction generally changes, in an intrinsically indeterministic way, p into a new state depending on the choice that is made, as a consequence of this 'contextual interaction'. E.g., if the participant chooses in *AB The Horse Whinnies*, which corresponds to the outcome A_1B_2 (see Section 3), the interaction between the entity *The Animal Acts* in the state p and the (mind of the) participant determines an indeterministic change of state of the entity from p to the state $p_{A_1B_2}$ which describes the more concrete situation of a horse that whinnies. More generally, for every $X = A, A'$, $Y = B, B'$, the coincidence measurement XY has four possible outcomes X_iY_j, where we choose $X_iY_j = +1$

if $i=j$ and $X_iY_j=-1$ if $i\neq j$, and four outcome states, or eigenstates, $p_{X_iY_j}$, describing the state of *The Animal Acts* after the outcome X_iY_j occurs in XY, $i,j=1,2$. When all responses are collected, a statistics of the outcomes X_iY_j arises which is interpreted, in the large number limit, as the probability $P_p(X_iY_j)$ that the outcome X_iY_j is obtained when the coincidence measurement XY is performed on the composite entity *The Animal Acts* in the initial state p.

(iii) We have identified the entities, initial state, coincidence measurements, outcome probabilities and eigenstates that are relevant to the *The Animal Acts* situation. Then, we have to work out a quantum representation in Hilbert space to model the data collected in this situation, that is, the entity *The Animal Acts* is associated with a complex Hilbert space and the initial state p is represented by a unit vector of this Hilbert space. Next, for every $X=A,A'$, $Y=B,B'$, the coincidence measurement XY is represented by a self-adjoint operator or, equivalently, by a spectral family, on the Hilbert space whose eigenvectors represent the eigenstates of XY, while the outcome probabilities are obtained from Born's rule of quantum probability.

Regarding the above Hilbert space representation, we preliminarily observe that all coincidence measurements XY, $X=A,A'$, $Y=B,B'$, have four outcomes X_iY_j, $i,j=1,2$, which entails that the composite entity *The Animal Acts* is associated, as an overall entity, with the complex Hilbert space $\mathbb{C}^4$ of all ordered 4-tuples of complex numbers. Moreover, each state p of *The Animal Acts* is represented by a unit vector of $\mathbb{C}^4$ and each coincidence measurement on *The Animal Acts* is represented by a self-adjoint operator or, equivalently, by a spectral family, on $\mathbb{C}^4$. On the other hand, for every $i,j=1,2$, each outcome X_iY_j is obtained by juxtaposing the outcomes X_i and Y_j, e.g., *The Tiger Growls*, is obtained by syntactically juxtaposing the words "tiger" and "growls". This operation defines a 2-outcome measurement X, $X=A,A'$, on the individual entity *Animal* and a 2-outcome measurement Y, $Y=B,B'$, on the individual entity *Acts*. Hence, each of these individual entities is associated with the complex Hilbert space $\mathbb{C}^2$ of all ordered couples of complex numbers. Should we had performed separate measurements on *Animal* and *Acts*, the Hilbert space formalism would have prescribed that the composite entity *The Animal Acts* would have been associated with the tensor product $\mathbb{C}^2\otimes\mathbb{C}^2$. But, we remind that we are studying here the identification problem (see Section 2), that is, the problem of how the composite entity *The Animal Acts* can be decomposed into the individual entities *Animal* and *Acts* in such a way that these individual entities can be recognised from measurements performed on the composite entity. As such, we are doing an operation that is the inverse of what one typically does in Bell-type situations in quantum physics, where one constructs or, better, composes, the measurements on the composite entity from measurements performed on individual entities.

From a mathematical point of view, the vector spaces $\mathbb{C}^4$ and $\mathbb{C}^2 \otimes \mathbb{C}^2$ are isomorphic, where each isomorphism is defined by the relationship between the corresponding orthonormal (ON) bases. The states of *The Animal Acts* are represented by unit vectors of $\mathbb{C}^4$, hence of $\mathbb{C}^2 \otimes \mathbb{C}^2$, which contains both vectors representing product states and vectors representing entangled states. Moreover, the vector space $L(\mathbb{C}^4)$ of all linear operators on $\mathbb{C}^4$ is isomorphic to the tensor product $L(\mathbb{C}^2) \otimes L(\mathbb{C}^2)$, where $L(\mathbb{C}^2)$ of all linear operators on $\mathbb{C}^2$. Analogously, the tensor product $L(\mathbb{C}^2) \otimes L(\mathbb{C}^2)$ contains both self-adjoint operators representing product measurements and self-adjoint operators representing entangled measurements (see Section 2).

Now, let $I : \mathbb{C}^4 \longrightarrow \mathbb{C}^2 \otimes \mathbb{C}^2$ be an isomorphism mapping a given ON basis of $\mathbb{C}^4$ onto a given ON basis of $\mathbb{C}^2 \otimes \mathbb{C}^2$. We say that a state p represented by the unit vector $|p\rangle \in \mathbb{C}^4$ is a 'product state with respect to I', if two states p_A and p_B, represented by the unit vectors $|p_A\rangle \in \mathbb{C}^2$ and $|p_B\rangle \in \mathbb{C}^2$, respectively, exist such that $I|p\rangle = |p_A\rangle \otimes |p_B\rangle$. Otherwise, p is an 'entangled state with respect to I'. Then, we say that a measurement e represented by the self-adjoint operator $\mathscr{E}$ on $\mathbb{C}^4$ is a 'product measurement with respect to I', if two measurements e_X and e_Y, represented by the self-adjoint operators $\mathscr{E}_X$ and $\mathscr{E}_Y$, respectively, on $\mathbb{C}^2$ exist such that $I\mathscr{E}I^{-1} = \mathscr{E}_X \otimes \mathscr{E}_Y$. Otherwise, e is an 'entangled measurement with respect to I'. Hence, the notion of entanglement crucially depends on the 'isomorphism that is used to identify individual entities within a given composite entity'.

With reference to a Bell-type setting, one can now prove that, if the coincidence measurements XY and XY', $X = A, A'$, $Y, Y' = B, B'$, $Y \neq Y'$, are product measurements with respect to the isomorphism I, then, for every state p of the composed entity, the marginal law of Kolmogorovian probability expressed by Equation (3) is satisfied. Analogously, if the coincidence measurements XY and $X'Y$, $X, X' = A, A'$, $Y = B, B'$, $X \neq X'$, are product measurements with respect to the isomorphism I, then, for every state p of the composed entity, the marginal law expressed by Equation (4) is satisfied. One also proves that, if the marginal law is satisfied in all coincidence measurements, then a unique isomorphism exists, which can be chosen to be the identity operator [31].

It follows from the above that, if the marginal law is violated, then one cannot find a unique isomorphism between $\mathbb{C}^4$ and $\mathbb{C}^2 \otimes \mathbb{C}^2$ such that all measurements are product measurements with respect to this isomorphism. In this case, one cannot explain the violation of the CHSH inequality as due to the usual situation in quantum physics where all measurements are product measurements and only the initial pre-measurement state is entangled

[3]. Furthermore, if the marginal law is systematically violated, as it occurs in our test (see Table 1), then four distinct isomorphisms I_{XY}, exist such that the measurement XY is a product measurement with respect to I_{XY}, $X = A, A'$, $Y = B, B'$. As a consequence, there is no unique isomorphism allowing to identify individual entities of a given composite entity. Finally, if we consider a given isomorphism between $\mathbb{C}^4$ and $\mathbb{C}^2 \otimes \mathbb{C}^2$ with respect to which identifying individual entities of a composite entity in a given test, then it may happen that both the initial pre-measurement state and all measurements are entangled [31].

Now, the theoretical considerations above allow one to formulate the following hypotheses.

Firstly, the non-classical correlations that violate the CHSH inequality in Equation (1) in the *The Animal Acts* situation can be reasonably attributed to the fact that 'the component concepts carry meaning and further meaning is created in the combination process'. Since the violation of the CHSH inequality indicates the presence of entanglement between the individual conceptual entities, then it is reasonable that 'it is the quantum structure of entanglement that theoretically capture the meaning that is non-classically created in this case'. This suggests that the initial state p of the composite entity *The Animal Acts* should be an entangled state.

Secondly, in the *The Animal Acts* situation, since all coincidence measurements XY, $X = A, A'$, $Y = B, B'$, violate the marginal law of Kolmogorovian probability, also these measurements should be entangled measurements. In addition, in each measurement XY, all outcomes X_iY_j, correspond to combined concepts, though less abstract than *The Animal Acts*, e.g., in *The Cat Meows*, meaning is created with respect to *Cat* and *Meows* taken separately, which suggests that all eigenstates $p_{X_iY_j}$, $i, j = 1, 2$, should also be entangled states.

We are now ready to work out the required quantum representation in Hilbert space of the data in Table 1.

The composite conceptual entity *The Animal Acts* is associated with the complex Hilbert space $\mathbb{C}^4$. Let $(1,0,0,0)$, $(0,1,0,0)$, $(0,0,1,0)$ and $(0,0,0,1)\}$ be the unit vectors of the canonical ON basis of $\mathbb{C}^4$, and let us consider the isomorphism $I : \mathbb{C}^4 \longrightarrow \mathbb{C}^2 \otimes \mathbb{C}^2$, where the canonical ON basis of $\mathbb{C}^4$ coincides with the ON

[3]We add that there are reasons to believe that the marginal law is also violated in typical Bell-type tests on quantum physical entities, which indicates that entangled measurements are involved also in physical domain. However, the experimental violation of the marginal law in these tests is not large, hence has hardly been reflected about from a theoretical point of view []

basis of the tensor product Hilbert space $\mathbb{C}^2 \otimes \mathbb{C}^2$ made up of the unit vectors $(1,0) \otimes (1,0)$, $(1,0) \otimes (0,1)$, $(0,1) \otimes (1,0)$ and $(0,1) \otimes (0,1)$.

In the ON bases above, a given state q of the composite entity is represented by the unit vector $|q\rangle = (ae^{i\alpha}, be^{i\beta}, ce^{i\gamma}, de^{i\delta})$, where $a, b, c, d \geq 0$, $a^2 + b^2 + c^2 + d^2 = 1$, α, β, γ, $\delta \in \Re$ and $\Re$ is the real line. One easily proves that $|q\rangle$ represents a product state if and only if

$$ade^{i(\alpha+\delta)} - bce^{i(\beta+\gamma)} = 0 \tag{10}$$

Otherwise, $|q\rangle$ represents an entangled state.

Let us now come to the representation of the initial, or preparation, state p of the composite entity *The Animal Acts*. In previous articles, we represented the state p of the conceptual entity *The Animal Acts* by the unit vector

$$|p\rangle = \frac{1}{\sqrt{2}}(0, 1, -1, 0) \tag{11}$$

which represents the maximally entangled state corresponding to the singlet spin state, as typically done in Bell-type tests in quantum physics (see Section 2). However, before making this choice here too, it is worth to reflect about the general modelling scheme we adopted to represent the states of conceptual entities (see, e.g., [25]). In [52] we used the unit vector

$$|p_{AB}\rangle = \sum_{i,j=1,2} \sqrt{\mu(A_i B_j)} |p_{A_i B_j}\rangle \tag{12}$$

where $\mu(A_i B_j)$ are the judgement probabilities of the coincidence measurement AB and the unit vectors $|p_{A_i B_j}\rangle$ form an ON basis of eigenvectors representing the eigenstates of AB, $i, j = 1, 2$, in the 4-dimensional Hilbert space describing the AB-measurement situation. This representation is in agreement with the general modelling scheme in [25], because the unit vector in Equation (12) represents the initial state of *The Animal Acts* in a measurement having *The Horse Growls*, *The Horse Whinnies*, *The Bear Growls*, and *The Bear Whinnies* as possible outcomes. Analogously, one could use, with obvious changes of symbols, the unit vectors

$$|p_{AB'}\rangle = \sum_{i,j=1,2} \sqrt{\mu(A_i B'_j)} |p_{A_i B'_j}\rangle \tag{13}$$

to represent the initial state of *The Animal Acts* when the coincidence measurement AB' is performed, the unit vector

$$|p_{A'B}\rangle = \sum_{i,j=1,2} \sqrt{\mu(A'_i B_j)} |p_{A'_i B_j}\rangle \tag{14}$$

to represent the initial state of *The Animal Acts* when the coincidence measurement $A'B$ is performed, and the unit vector

$$|p_{A'B'}\rangle = \sum_{i,j=1,2} \sqrt{\mu(A'_iB'_j)}|p_{A'_iB'_j}\rangle \tag{15}$$

to represent the initial state of *The Animal Acts* when the coincidence measurement $A'B'$ is performed. Indeed, also the unit vectors in Equations (13), (14) and (15) reproduce the correct judgement probabilities in the corresponding coincidence measurements.

How will we then represent the initial state of *The Animal Acts* with respect to the overall experiment that tests the CHSH version of Bell's inequalities according to the general modelling scheme in [25]? The straightforward answer is to take the normalized superposition state represented by the linear combination

$$|p_{CHSH}\rangle = \frac{|p_{AB}\rangle + |p_{AB'}\rangle + |p_{A'B}\rangle + |p_{A'B'}\rangle}{\||p_{AB}\rangle + |p_{AB'}\rangle + |p_{A'B}\rangle + |p_{A'B'}\rangle\|} \tag{16}$$

Let us recall that the considered basis vectors for each of the coincidence measurements are not necessarily the same, their relation depending on how these coincidence measurements relate experimentally to the CHSH form of the Bell's test experiment, and only experiments to test these relations can give us this exact information. This however does not avoid the linear combination in Equation (16) to be well defined and representing the initial state of *The Animal Acts* with respect to the CHSH form of the Bell's test.

Let us also recall that the shift from, e.g., an initial, or preparation, state represented by the unit vector $|p_{AB}\rangle$ to an initial, or preparation, state represented by the unit vector $|p_{CHSH}\rangle$ corresponds to (i) an application of a projection operator in Hilbert space which projects onto the one-dimensional subspace generated by the unit vector $|p_{AB}\rangle$, and (ii) a normalization of the resulting projected vector. The opposite shift of the initial, or preparation, state corresponds instead, to a superposition in Hilbert space, as we have explained above.

The considerations above indicate that the unit vector in Equation (16) is the most appropriate candidate to represent the initial state corresponding to a preparation of the entity *The Animal Acts* according to the general modelling scheme in [25]. However, for the experiments where the data are obtained by calculating relative frequencies of occurrence of different choices, using separately the fours vectors in Equations (12), (13), (14) and (15) comes to the same concerning the predictions made by the model, because relative frequencies of occurrence of outcomes are always determined separately for each of the four coincidence cases. Or, more concretely, for these experiments we have, e.g.,

$$\mu(X_iY_j) = |\langle p_{X_iY_j}|p_{XY}\rangle|^2 = |\langle p_{X_iY_j}|p_{CHSH}\rangle|^2 \tag{17}$$

for every $X = A, A'$, $Y = B, B'$, $i, j = 1, 2$.

Since the outcomes in cognitive tests with human participants are to a large extent also determined by estimating relative frequencies of occurrence, one can expect that in this case the same, this time almost, equalities. Indeed, since some human participants in the tests will have their choices determined in a totally different way, these equalities do not hold in this case, although even those participants will have the tendency not to let their choice about one of the coincidence pairs depend on that they gave answers before or after for the other coincidence pairs, which again makes the effect on outcomes for the two states rather little different, and hence we can speak of 'almost equalities'. Hence, given the complexity of the human mind, it is not excluded that the answers given for one of the coincidence pairs are indeed partly determined by the presence in the same test of the other coincidence pairs, which means that for psychology experiments we should in principle consider both states giving rise to different predictions for the to be rested probabilities.

Now, what about assuming the maximally entangled singlet spin state represented by the unit vector $|p\rangle$ in Equation (11) to be the initial state corresponding to the preparation of *The Animal Acts*?

The above is a justifiable choice if we think of a preparation that is very minimal and leads to the test person considering a kind of bare *The Animal Acts*. For example, before the instruction sheet is read by the test person, it could have been communicated that it involves measurements involving the sentence "the animal acts". For this minimal initial preparation, that maximally entangled state could serve in the quantum theoretical model we are building also according to the general modelling scheme in [25].

We wish to digress briefly in connection with this problem of precisely determining the prepared state, firstly and foremost with the intention of fully clarifying it, and secondly because an element emerges here where our approach shows its strength in connection with a possible use for AI based on quantum structures. Even if we wish to determine the prepared state of an experiment where only the sentence "the animal acts" is presented to the test subjects, the singlet spin state is not the best choice to serve as the prepared state. A superposition state in which all animals and all actions that can be performed figure would be the real correct mathematical representation. This would lead however to having to consider a Hilbert space of giant dimension as tensor product of all these possibilities. The human mind is probably not capable of handling this condition, but does make an attempt in that direction. Studies on how concepts are represented in the human mind preferentially point to working with, on the one hand, a precedence for the representation of some leading exemplars (see, e.g., [55] on exemplar theories of concepts) and, on the other hand, with the representation of a prototype of the concept in question (see, e.g., [56] on prototype theories of concepts). In any case, if these exemplar and prototype theories are correct, they also point to a shortcoming of the human mind in connection with such a representation, and it can be expected that a powerful AI would indeed be able to represent such a giant superposition in every detail. Indeed, in the end, the collection of all animals and all possible actions that each is capable of and that can be conceptually described is still a finite collection.

Coming back to our quantum representation in Hilbert space of the data presented in Section 3, there are however also good independent reasons for choosing the unit vector in Equation (11) to represent the initial state of the entity *The Animal Acts*, as we did in previous articles on cognitive entanglement. Indeed, let us recall that our aim is to incorporate as much as possible the entanglement of *The Animal Acts* situation into the state preparation. Moreover, the singlet spin state has specific symmetry properties, namely, it is always represented by a unit vector of the form in Equation (11) independently of the ON basis in which the unit vector is expressed. This would intuitively correspond to the fact that *The Animal Acts* expresses a more abstract concept than the corresponding outcomes. Finally, this choice allows one to more easily capture the theoretical connections between entanglement and meaning, as we will see in the rest of this section.

Then, coming to measurements, let us represent the coincidence measurements XY, $X = A, A'$, $Y = B, B'$. As we have seen above, each measurement has four outcomes $X_iY_j = \pm 1$ and four eigenstates $p_{X_iY_j}$, $i, j = 1, 2$. For every $X = A, A'$, $Y = B, B'$, we represent XY by the spectral family defined by the ON basis of the four eigenvectors $|p_{X_iY_j}\rangle$, where we set, for every $i, j = 1, 2$,

$$|p_{X_iY_j}\rangle = (a_{X_iY_j} e^{i\alpha_{X_iY_j}}, b_{X_iY_j} e^{i\beta_{X_iY_j}}, c_{X_iY_j} e^{i\gamma_{X_iY_j}}, d_{X_iY_j} e^{i\delta_{X_iY_j}}) \tag{18}$$

In Equation (18), the coefficients are such that $a_{X_iY_j}, b_{X_iY_j}, c_{X_iY_j}, d_{X_iY_j} \geq 0$ and $\alpha_{X_iY_j}, \beta_{X_iY_j}, \gamma_{X_iY_j}, \delta_{X_iY_j} \in \Re$. One easily verifies that, for every $X = A, A'$, $Y = B, B'$, XY is a product measurement if and only if all $|p_{X_iY_j}\rangle$s are product vectors. Otherwise, XY is an entangled measurement.

Next, for every $X = A, A'$, $Y = B, B'$, $i, j = 1, 2$, the probability $P_p(X_iY_j)$ of obtaining the outcome X_iY_j in a measurement of XY on the composite entity in the state p is given by Born's rule of quantum probability, that is,

$$P_p(X_iY_j) = |\langle p_{X_iY_j}|p\rangle|^2 \tag{19}$$

$i, j = 1, 2$.

To find a quantum mathematical representation of the data in Table 1, for every measurement e_{XY}, the four unit vectors in Equation (18) have to satisfy the following three sets of conditions.

(i) Normalization. The eigenvectors in Equation (18) are unit vectors, that is, for every $X = A, A'$, $Y = B, B'$, $i, j = 1, 2$,

$$a^2_{X_iY_j} + b^2_{X_iY_j} + c^2_{X_iY_j} + d^2_{X_iY_j} = 1 \tag{20}$$

This corresponds to four conditions for each coincidence measurement XY.

(ii) Orthogonality. The eigenvectors in Equation (18) are mutually orthogonal, that is, for every $X = A, A'$, $Y = B, B'$, $i, i', j, j' = 1, 2$, $i \neq i'$, $j \neq j'$,

$$\langle p_{X_iY_j}|p_{X_{i'}Y_{j'}}\rangle = 0 \tag{21}$$
$$\langle p_{X_iY_j}|p_{X_iY_{j'}}\rangle = 0 \tag{22}$$
$$\langle p_{X_iY_j}|p_{X_iY_{j'}}\rangle = 0 \tag{23}$$

This corresponds to six additional conditions for each coincidence measurement XY.

(iii) Probabilities. For every $X = A, A'$, $Y = B, B'$, $i, j = 1, 2$, the probability $P_p(X_iY_j)$ coincides with the empirical probability $\mu(X_iY_j)$ in Table 1, that is,

$$P_p(X_iY_j) = |\langle p_{X_iY_j}|p\rangle|^2 = \mu(X_iY_j) \tag{24}$$

where we have used Born's rule in Equation (19). This corresponds to four additional conditions for each coincidence measurement XY.

Finally, let us set, for every $X = A, A'$, $Y = B, B'$, $i, j = 1, 2$, $\alpha_{X_iY_j} = \beta_{X_iY_j} = \gamma_{X_iY_j} = \delta_{X_iY_j} = \theta_{X_iY_j}$, where $\theta_{X_iY_j} \in \Re$, for the sake of simplicity.

The empirical data in Table 1 can be represented in Hilbert space, as follows.

The eigenstates of the measurement AB are represented by the unit vectors

$$|p_{A_1B_1}\rangle = e^{i311.20^\circ}(0.75, -0.59, -0.29, 0) \tag{25}$$
$$|p_{A_1B_2}\rangle = e^{i64.38^\circ}(0.01, 0.44, -0.88, 0.15) \tag{26}$$
$$|p_{A_2B_1}\rangle = e^{i224.23^\circ}(0.24, 0.30, 0, -0.92) \tag{27}$$
$$|p_{A_2B_2}\rangle = e^{i0.53^\circ}(0.61, 0.60, 0.37, 0.36) \tag{28}$$

By applying the entanglement condition in Equation (10), we can verify that all eigenstates are entangled, hence AB is an entangled measurement. However, one observes that the condition in Equation (10) shows a larger deviation from zero in the unit vector $|p_{A_1B_2}\rangle$. We can then say that the eigenstate $p_{A_2B_1}$, corresponding to *The Horse Whinnies*, is a 'relatively more entangled state', which is reasonably intuitive, as *The Horse Whinnies* carries higher meaning with respect to *Horse* and *Whinnies*. Analogously, the eigenstate $p_{A_2B_2}$, corresponding to *The Bear Whinnies*, is a 'relatively less entangled state', which is again reasonably intuitive, as *The Bear Whinnies* carries lower meaning with respect to *Bear* and *Whinnies*.

The eigenstates of the measurement AB' are represented by the unit vectors

$$|p_{A_1B'_1}\rangle = e^{i0.07^\circ}(0.41, 0.31, -0.85, 0.06) \quad (29)$$

$$|p_{A_1B'_2}\rangle = e^{i16.59^\circ}(-0.01, 0.21, 0, -0.98) \quad (30)$$

$$|p_{A_2B'_1}\rangle = e^{i131.66^\circ}(-0.19, 0.92, 0.26, 0.20) \quad (31)$$

$$|p_{A_2B'_2}\rangle = e^{i180.20^\circ}(0.89, 0.05, 0.45, 0) \quad (32)$$

Also in this case, all eigenstates are entangled, hence AB' is an entangled measurement. However, the condition in Equation (10) shows a larger deviation from zero in the unit vector $|p_{A_1B'_1}\rangle$. We can then say that the eigenstate $p_{A_1B'_1}$, corresponding to *The Horse Snorts*, is a 'relatively more entangled state', which is reasonably intuitive, as *The Horse Snorts* carries higher meaning with respect to *Horse* and *Snorts*. Analogously, the eigenstates $p_{A_2B'_1}$ and $p_{A_2B'_2}$, corresponding to *The Horse Meows* and *The Bear Meows*, are 'relatively less entangled states', which is again reasonably intuitive, as *The Horse Meows* and *The Bear Meows* carry lower meaning with respect to *Horse* and *Meows* and *Bear* and *Meows*, respectively.

The eigenstates of the measurement $A'B$ are represented by the unit vectors

$$|p_{A'_1B_1}\rangle = e^{i32.52^\circ}(0.20, 0.35, -0.90, 0.14) \quad (33)$$

$$|p_{A'_1B_2}\rangle = e^{i174.92^\circ}(0.98, -0.09, 0.18, 0) \quad (34)$$

$$|p_{A'_2B_1}\rangle = e^{i0.39^\circ}(0.01, 0.86, 0.39, 0.32) \quad (35)$$

$$|p_{A'_2B_2}\rangle = e^{i205.95^\circ}(0.03, 0.35, 0, -0.94) \quad (36)$$

All eigenstates are entangled, hence $A'B$ is an entangled measurement. However, the condition in Equation (10) shows a larger deviation from zero in the unit vector $|p_{A'_1B_1}\rangle$. We can then say that the eigenstate $p_{A'_1B_1}$, corresponding to *The Tiger*

Snorts, is a 'relatively more entangled state', which is reasonably intuitive, as *The Tiger Snorts* carries higher meaning with respect to *Tiger* and *Snorts*. Analogously, the eigenstates $p_{A'_1B_2}$ and $p_{A'_2B_2}$, corresponding to *The Tiger Whinnies* and *The Cat Whinnies*, are 'relatively less entangled states', which is again reasonably intuitive, as *The Tiger Whinnies* and *The Cat Whinnies* carry lower meaning with respect to *Tiger* and *Whinnies* and *Cat* and *Whinnies*, respectively.

Finally, the eigenstates of the measurement $A'B'$ are represented by the unit vectors

$$|p_{A'_1B'_1}\rangle = e^{i99.21^\circ}(0.73,-0.63,-0.27,0) \quad (37)$$

$$|p_{A'_1B'_2}\rangle = e^{i20.16^\circ}(0.27,0.31,0.01,-0.91) \quad (38)$$

$$|p_{A'_2B'_1}\rangle = e^{i0.69^\circ}(0.62,0.53,0.44,0.38) \quad (39)$$

$$|p_{A'_2B'_2}\rangle = e^{i353.58^\circ}(0.09,0.47,-0.86,0.18) \quad (40)$$

Also in this case, all eigenstates are entangled, hence $A'B'$ is an entangled measurement. However, the condition in Equation (10) shows a larger deviation from zero in the unit vector $|p_{A'_1B'_1}\rangle$. We can then say that the eigenstate $p_{A'_2B'_2}$, corresponding to *The Cat Meows*, is a 'relatively more entangled state', which is reasonably intuitive, as *The Cat Meows* carries higher meaning with respect to *Cat* and *Meows*. Analogously, the eigenstate $p_{A'_2B'_1}$, corresponding to *The Cat Snorts*, is a 'relatively less entangled state', which is again reasonably intuitive, as *The Cat Snorts* carries lower meaning with respect to *Cat* and *Snorts*.

We have thus completed the quantum mathematical representation of the data on the video-based cognitive test in Section 3. This representation, however, also suggests relevant considerations. This will be the content of Section 5.

5. Entanglement as a mechanism of contextual updating

The quantum theoretical modelling elaborated in Section 4 allows one to draw some interesting conclusions regarding the appearance of entanglement in the combination of natural concepts and, more important, on the nature of this entanglement. We stress, however, that these conclusions are independent of the domain, conceptual or physical, where entanglement is applied. This means that the results obtained in the present article may also shed new light on the nature of physics entanglement.

Firstly, we have explicitly worked out a quantum mathematical model in Hilbert space which explains the violation of the CHSH inequality in the video-based cognitive test on *The Animal Acts* as a demonstration of the presence of quantum entanglement. More explicitly, the individual conceptual entities *Animal* and *Acts* entangle when they combine to form the combined, or composite, conceptual entity *The Animal Acts*. The reason of this entanglement is that both concepts *Animal* and *Acts* carry meaning. But, also the combination *The Animal Acts* carries its own meaning. And, the meaning of *The Animal Acts* is not attributed by separately attributing meaning to *Animal* and *Acts*, as would be prescribed by a classical compositional semantics.

Secondly, we have seen that, not only the initial state of the composite entity is entangled, but also all coincidence measurements are entangled, in the quantum representation of *The Animal Acts* situation. This result is due to the violation of the marginal law of Kolmogorovian probability in the cognitive test which forbids concentrating all the entanglement of the state-measurement situation into the state of the composite entity, as we have seen in Section 2.

Thirdly, in each coincidence measurement, all eigenstates are entangled. This result is due to the fact that, in each coincidence measurement, all possible outcomes correspond to combinations of concepts, e.g., *The Bear Snorts* is itself a combination of the concepts *Bear* and *Snorts*, hence it is reasonable to expect that a non-classical mechanism of meaning attribution, similar to *The Animal Acts*, occurs.

Fourthly, in each coincidence measurement, some eigenstates, which are the final states at the end of the coincidence measurement when a defined outcome is obtained, exhibit a relatively higher degree of entanglement than others, which can exactly be explained by the fact that entanglement captures meaning attribution, hence higher degrees of entanglement correspond to higher meaning attribution, thus higher judgement probabilities. For example, the eigenstates corresponding to *The Horse Whinnies*, *The Horse Snorts*, *The Tiger Growls* and *The Cat Meows* are the states that exhibit the highest degree of entanglement in the corresponding coincidence measurement. As observed in Section 4, this can be explained as due to the fact that higher meaning is attributed in the combination process for these items. As a matter of fact, these items score the highest probability of being judged as a good example of the conceptual combination *The Animal Acts*. By contrast, the eigenstates corresponding to *The Bear Whinnies*, *The Horse Meows*, *The Tiger Whinnies* and *The Cat Snorts* are the states that exhibit the lowest degree of entanglement, close to product states, in the corresponding coincidence measurement. Again this can be explained as due to the fact that lower meaning is attributed in the combination process for these items. As a matter of fact, these items score the lowest probability of being judged as a good example of the conceptual combination *The Animal Acts*.

Fifthly, by looking at the CHSH factor in Table 1, we notice that the video-based cognitive test violates the CHSH inequality by an amount that exceeds the value $2\sqrt{2} \approx 2.8284$, i.e. Cirel'son's bound, which is usually believed to be the theoretical limit to represent in Hilbert space the statistical correlations that are observed in Bell-type tests by pushing all entanglement into the state of the composite entity and considering only product measurements. Again, this is due to the fact that the coincidence measurements are actually entangled, rather than product, measurements. As we have argued in [38], independently of the physical or conceptual domain of reference, if one allows entangled measurements, then a Hilbert space representation is possible also for Bell-type tests which violate Cirel'son's bound.

We would like to conclude the present article by deepening the mechanism of meaning attribution to concepts and its relationship with quantum entanglement. As we have seen above, the appearance of entanglement in *The Animal Acts* is due to the fact that people attribute meaning to the combination *The Animal Acts* as a whole entity, without firstly attributing meaning to *Animal* and *Acts* and then combining these separate meanings into a meaning for *The Animal Acts*. One way to characterize this process of meaning assignment is the following. A concept carries a meaning, and a second concept carries a meaning, however, the combination of these two concepts also carries its proper meaning, and this is not the simple combination of the two meanings of the component concepts as prescribed by a classical compositional semantics. On the contrary, 'the new emergent meaning of the combined concept arises in a complex contextual way', in which the whole of the context relevant to the combination plays a role.

The above is even more evident if one considers an entire text produced by human language and its meaning relationship with the words (concepts) composing it. Each time a word (concept) is added to a text, one can speak of an 'updating of contextuality', and this updating continues to occur until the end of the text that contains all the words. As we have argued in [38, 42], this mechanism of 'contextual updating' to attribute meaning has to be carried by an entangled state, because this is exactly how entangled states are formed in the tensor product of Hilbert spaces. In other words, it is these entangled states that accomplish the contextual updating in the mathematical formalism of quantum theory. The deep structural similarities between physical and conceptual domains, suggest that the mechanism of contextual updating could also explain, better than the 'spooky action at a distance' mechanism, how entanglement is produced in physics [42].

Coming back to *The Animal Acts* situation, the concept *Animal* is an abstraction of all possible animals and the concept *Acts* is an abstraction of all possible sounds produced by animals. But, people do not construct the meaning of *The Animal Acts* by separately considering abstractions of animals and abstractions of acts and then combining these abstractions. On the contrary, they take directly abstractions of animals making a sound, and this occurs in a coherent way that is represented by a superposed, more precisely, entangled, state. This is exactly what we have defined above as the mechanism of contextual updating.

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