Journal of Economics & Management Strategy

WILEY

ORIGINAL ARTICLE OPEN ACCESS

Do Firms Gain From Managerial Overconfidence? Managerial Bargaining Power and the Role of Severance Pay

Clara Graziano[1] | Annalisa Luporini[2]

[1]Department of Economics and Statistics, University of Udine and CESifo, Udine, Italy | [2]Department of Economics and Management, University of Florence and CESifo, Firenze, Italy

Correspondence: Clara Graziano (clara.graziano@uniud.it)

Received: 7 January 2023 | **Revised:** 26 July 2024 | **Accepted:** 19 August 2024

Keywords: entrenchment | managerial compensation | optimism | overconfidence | severance pay

ABSTRACT

We analyze the effects of optimism and overconfidence when the manager has bargaining power and the compensation package includes severance pay. Optimism implies that the manager overestimates the probability of success, while overconfidence induces the manager to overestimate the increase in the probability of success due to her investment. If the manager can renegotiate the initial contract, the advantage of using severance pay to induce the manager to invest, commonly found in the literature, is reduced by the presence of the biases. Optimism increases severance pay and managerial entrenchement with a negative effect on expected profit. Overconfidence reduces incentive pay, as shown by the previous literature, but its effect on severance pay depends on the intensity of the bias. A moderate overconfidence reduces severance pay and increases expected profit. Conversely, extreme overconfidence increases severance pay and this may offset the beneficial effect on incentive pay. Thus, the attempt to exploit managerial overconfidence to reduce incentive pay may backfire if the manager is replaced. Our model suggests that the large severance payments documented by the empirical literature represent a form of efficient contracting when the optimistic and overconfident manager has bargaining power.

JEL Classification: J33, D86, D90, L21

1 | Introduction

Severance agreements specify the payments the executives receive in case of departure. They are used by a vast majority of firms and both the number of firms signing such contracts and the average amount of the severance payments have increased in the last decades (Cadman et al., 2016; Callahan, 2024).[1] Despite their widespread use, according to Callahan (2024, p. 159), "severance pay is perhaps the most controversial yet least understood form of executive compensation." Indeed, theoretical research investigating severance agreements has provided mixed results so far. Severance payments have been criticized because they occur when the board of directors dismisses the incumbent manager after a period of poor performance. Thus, these payments have been considered as a "reward for failure" that violates the pay-for-performance principle of agency theory (Bebchuk and Fried 2004). Criticisms are particularly severe when the dismissed manager receives a payment in excess of the severance amount specified by the contract, a documented and common practice (Goldman and Huang 2015). Other scholars however, suggest that severance payments are part of an efficient contract because they provide CEOs with insurance for their human capital and, by offsetting the manager's risk aversion, they offer incentives for investments in valuable risky projects. Furthermore, the separation pay in addition to the contractual amount may be part of an implicit contract.

The present paper investigates the efficiency of severance pay by studying how managerial biases on the probability of success influence the optimal design of the contract when the manager has bargaining power and can renegotiate the initial agreement. Following de la Rosa (2011) we distinguish between *optimism* and *overconfidence*. Optimism occurs when the manager has a subjective belief on the probability of success higher than the "true" probability, while overconfidence distorts the manager's assessment of the increase in the probability of success due to her effort. In our context, effort takes the form of a firm-specific investment that is observable but unverifiable. We investigate how optimism and overconfidence affect the amount of severance pay necessary to induce the manager to leave when separation from the firm is profit enhancing but the manager can oppose replacement.

Similarly to what happens in the previous literature on the optimality of severance agreements, in our model severance pay mitigates the incentive to dismiss the manager when the replacement is only marginally better. This, in turn, helps inducing the manager to undertake the level of investment desired by the board. We assume that the incumbent manager has some bargaining power and can credibly threaten to resist being replaced. The idea underlying this assumption is that the manager can oppose replacement by making it a costly and contentious process so that valuable opportunities are missed and firm value decreases. To allow a smooth replacement, the board is willing to renegotiate the separation agreement and consent to a payment high enough to avoid costly opposition. The severance pay is renegotiated when the board knows whether the investment has been undertaken and can be tailored to its level as in Almazan and Suarez (2003) who show that this may be cheaper than just motivating the manager through an incentive pay that has to satisfy an ex-ante incentive compatibility constraint (ICC). However, overconfidence and optimism create a wedge between board's and manager's beliefs on expected profit, increasing the amount of severance pay asked by the manager. This, in turn, leads to managerial entrenchment as the board retains the incumbent manager even when the expected probability of success is significantly lower than that of the replacement. Then, a new trade-off emerges between the saving on (ex ante) incentive compensation resulting from the relaxed ICC and the higher cost for the (ex post) severance payment.

The main findings of the paper can be summarized as follows. First, if the manager has bargaining power and can oppose replacement, the advantage of using severance pay to provide incentives is reduced by managerial optimism and overconfidence. These biases, that are usually beneficial for the firm when only the incentive pay is considered (de la Rosa 2011; Santos-Pinto 2008), may turn out to be detrimental when turnover and severance pay are taken into account. Second, both the degree and the kind of managerial bias matter because overconfidence and optimism have different impact both on managerial compensation and on expected profit. In our model, optimism does not affect incentive pay but raises contractual severance pay and thus reduces profit. Overconfidence, on the other hand, may be either advantageous or detrimental for the firm depending on the degree of the bias. Moderate overconfidence reduces both incentive pay and severance pay without affecting the investment choice, thus increasing expected profit. Extreme overconfidence, on the contrary, may have a negative impact on profit. In fact it reduces the incentive pay, but such positive effect may be offset by the distortion in the investment choice resulting in a higher renegotiated severance pay. Thus, expected profit is non-monotonic in the degree of overconfidence. In other terms, the attempt to exploit executive overconfidence through a heavy use of incentive pay, documented for example by Humphery-Jenner et al. (2016), may backfire when the investment choice and the opportunity of replacing a manager who holds some bargaining power are considered.

The role played by managerial bargaining power in our model is then discussed by analyzing the consequences of different assumptions as to such power. By doing so, we show that the optimality of severance payments is strictly related to managerial bargaining power. In particular, when the manager cannot oppose replacement, zero severance pay is optimal. Without severance pay, the payment to the manager decreases in both optimism and overconfidence. Thus, in this case, both biases are beneficial for the firm as in previous literature.

Finally, our model helps explaining the common practice of granting a separation pay largely exceeding its contractual level, as reported by empirical studies and anecdotal evidence. We rationalize such discretionary payment in forced turnover as a result of the bargaining between the board and the manager and we show that paying the fired manager a sum in excess of the contractual severance pay may be optimal for shareholders. When instead the manager has no bargaining power as it is usually the case in voluntary turnover, zero severance payment is optimal. Our results are in line with the empirical evidence of Goldman and Huang (2015) who show that discretionary separation pay in forced turnovers is motivated by the desire to facilitate a smooth transfer of power from a poorly performing CEO to a potentially better replacement, while in voluntary turnovers the separation pay signals weak internal corporate governance.

This paper contributes to two streams of literature. First, it is related to the literature on the role of the severance pay from an optimal contracting point of view. In particular, in addition to Almazan and Suarez (2003), several other papers suggest that severance agreements, by inducing a more lenient replacement policy, mitigate the moral hazard problem on the part of the board and induce the manager to make a risky but profitable investment. Wu and Weng (2018) consider a setting where the contract designed by the board depends on the informativeness of an observed signal on the manager ability. In such a setting, both entrenchment and anti-entrenchment may emerge in the optimal contract. Laux (2015) shows that the optimal pay mix consists of restricted stock, to combat excessive risk-taking, and severance pay, to combat excessive conservatism.[2] In a similar vein, a few papers demonstrate that severance pay, by protecting the manager from the cost of dismissal, can alleviate information revelation problems. For instance, Inderst and Mueller (2010) find that offering a combination of severance pay and steep incentive pay may be the cheapest way to induce the manager to disclose information that may lead to her dismissal. Green and Taylor (2016) show that severance payment may be necessary to induce truthtelling when the manager has an informational advantage over the principal and the latter has to decide whether to terminate a multistage project on the basis of such information. Similarly, Vladimirov (2021) focuses on the interplay between severance pay and contract length and indicates that severance pay may discourage managers from using window dressing or information concealment to avoid replacement.[3] Our paper contributes to this literature by showing that the quasi-rents necessary to induce the manager to leave are likely to be larger when the manager is optimistic and highly overconfident than in the absence of such biases, thus limiting the optimality of severance payments.

Second, we contribute to the research on managerial overconfidence and optimism in a principal/agent relationship (see, e.g., Santos-Pinto 2008; de la Rosa 2011; Otto 2014) by showing that models that do not account for the possibility of managerial turnover and severance payment, may overstate the positive contribution of overconfidence and optimism. In a standard agency model with moral hazard and risk averse agent, the optimal contract trades off risk insurance and incentive provision. Managerial overconfidence, and the resulting divergence of beliefs between principal and agent, makes it easier to satisfy the participation and the ICCs, thus reducing the cost of eliciting effort (Santos-Pinto 2008; de la Rosa 2011; Gervais, Heaton, and Odean 2011; Otto 2014; Köszegi 2014). Firms can take advantage of this effect either by inducing the same level of effort required to an unbiased manager at a lower cost, or by offering a compensation structure with a particularly heavy incentive pay (the so-called exploitation hypothesis).[4]

We depart from this literature by assuming risk neutrality and bargaining power on the manager's side. In a standard principal/agent setting, managerial bargaining power would generally result in a different division of the gains from trade. In our framework, managerial bargaining power implies that the manager can oppose firing so that it may be necessary to renegotiate the severance payment to induce her to leave. The outcome of such renegotiation positively depends on the probability of success perceived by the manager, so that a trade-off between the ex ante saving on incentive pay and the ex post higher payment for severance arises. Consequently, the impact of the managerial biases is different from that occurring in a standard principal/agent model. Optimism does not affect incentive pay but increases renegotiated severance pay. This has a negative effect on expected profit, contrary to what happens in the principal/agent literature where optimism makes high-powered incentives more profitable for the firm. As to overconfidence, the size of the bias matters. We find a positive effect of moderate overconfidence on expected profit in line with the results obtained in the agency literature, but a negative impact in the case of extreme overconfidence.

Our paper complements the previous literature where the bias on beliefs leads to systematic differences in "payoffs, effort levels and incentives between over- and under-confident agents" (Sautmann 2013), by showing that the bias impacts also the amount of severance pay and the entrenchment. Moreover, our results suggest that different degrees of managerial biases may be beneficial for the firm according to the bargaining position of the manager: moderate overconfidence benefits the firm when the manager has a strong bargaining position while extreme overconfidence is beneficial when the manager has no bargaining power.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3 we discuss the difference between contractual and renegotiated severance pay in a simplified setting. Section 4 studies the replacement decision and the renegotiation stage. Section 5 analyzes the optimal compensation package and the investment choice. Given the optimal severance and incentive pay, Section 6 investigates the separate effects of optimism and overconfidence on the optimal contract and on firm expected profit. Section 7 presents an extension of the model where we discuss the role played by managerial bargaining power. Finally, Section 8 concludes.

2 | The Model

Consider a board that perfectly represents the shareholders and maximizes firm's value. The board hires a risk neutral manager to implement a project. The cash flow generated by the project can take either value 0 or value $R > 0$. The probability of success of the project, denoted by p_k, depends on a firm-specific investment $I_k, k = L, M, H$, made by the manager after joining the firm. In the absence of investment ($I = I_L = 0$), the probability of success is $p_L > 0$. If the manager makes investment I_M, the probability of success increases to $p_M > p_L$, while it becomes $p_H > p_M$ when the larger investment $I_H > I_M$ is chosen. The cost of the investment $c(I_k) = I_k$ is borne by the manager. The investment is unverifiable, though it is observable by the board that, consequently, comes to know the manager's probability of success before the results of the project become publicly known.

Only after the manager has undertaken the investment, a new manager materializes. We denote the probability of success of the new manager by $q \in [0, 1]$ and, since we have no reason to consider any particular value of q more/less likely to occur than other values, we assume that q is uniformly distributed.[5] Both the board and the incumbent manager observe the realization of q which may be higher than the probability of success of the incumbent even when the latter makes the investment desired by the board. The new manager does not need to make any investment when joining the firm. A high value of q may result, for example, from a better match between the new manager's ability and the skills required by the firm (possibly the replacement has made elsewhere an investment in human capital that is valuable also in this firm). In other words, the firm-specific investment of the incumbent may not be sufficient to avoid being less productive than the replacement. In such a case, the board may prefer to fire the incumbent and hire the new manager. Note that this implies that we are focusing on forced, not voluntary, turnover. Consistently with the evidence that CEOs have some power over the board, we model a situation where the manager can oppose dismissal. In other words, the incumbent has the ability to prolong the firing process and make it difficult for the company to move on in a different direction with a new manager. Thus, replacement can occur only with mutual agreement between board and incumbent. Specifically, we assume that the latter has a high enough bargaining power to oppose replacement if contractual severance pay is smaller than what the manager believes she would receive by staying with the firm, an amount that can be considered her "outside option" in the bargaining process. This is meant to capture the fact that actual severance payments are indeed related to the annual managerial compensation.[6] In Section 7, we introduce uncertainty on the manager having bargaining power and we also consider the case of no managerial bargaining power at all.

The incumbent manager and the board hold heterogeneous beliefs regarding the probability of success and are aware of such divergence that affects both the original contract and subsequent renegotiation, if any. Following the previous literature (de la Rosa 2011), we decompose the managerial bias into two components: optimism, , that is independent of the manager's action and is always at work, and overconfidence Δ_i, $i = M, H$, that captures the manager's distorted belief on the productivity of her investment. The effect of such biases on beliefs is made explicit by the following assumption.

Assumption 1. The manager's beliefs about the probability of success are:

1. $p_L + \theta$, if no investment is made, $I = I_L = 0$;
2. $p_M + \theta + \Delta_M$, if the manager makes investment I_M;
3. $p_H + \theta + \Delta_H$, if the manager makes investment I_H, where $\Delta_H = \Delta_M(1 + z)$ denotes the high overconfidence resulting from the high investment, with $z > \frac{I_H - I_M}{I_M} > 0$ and $1 - p_H - \theta \geqslant \Delta_H > 0$.

Assumption 1 specifies that the manager's beliefs differ from the "true" probabilities used by the board in two dimensions: level (optimism) and differences (overconfidence). Optimism has a uniform effect on the manager's beliefs, while overconfidence increases in the investment level: it is higher in the case of I_H than in the case of I_M and it is null in the case of I_L. In other words, overconfidence and investment are complements.[7] We consider the rate z of increase in overconfidence when moving from I_M to I_H as a parameter and we refer to an increase in Δ_M as an increase in overconfidence. Given $z > 0$, the slope of the manager's beliefs of success (considered as a function of Δ_M) is everywhere steeper in case of I_H than in case of I_M.[8] Then, a rise in overconfidence results in a higher increase in the manager's beliefs of success if the latter chooses I_H than if she chooses I_M. The assumption that $z > \frac{I_H - I_M}{I_M}$ implies that the increase in the manager's subjective belief of success is higher than the relative increase in the cost of the investment, making the high investment particularly attractive to the overconfident manager. In the following, we show that, given this assumption, a high value of Δ_M can induce a distortion in the choice of the investment. We define such high values of Δ_M as extreme overconfidence. We also consider the cases where the manager is not optimistic ($\theta = 0$) or not overconfident ($\Delta_M = 0$) or both ($\theta = \Delta_M = 0$). A manager who is neither optimistic nor overconfident is called a rational manager.

The following assumption completes the framework.

Assumption 2. Investment I_M is efficient: $(p_M - p_L)R > I_M$, while investment I_H is not: $(p_H - p_M)R < I_H - I_M$, though $(p_H - p_L)R > I_H$.

Note that Assumption 2 implies $\frac{I_H - I_M}{p_H - p_M} \geqslant R \geqslant \frac{I_M}{p_M - p_L}$ which can be satisfied if and only if

$$\frac{I_M}{I_H} \leq \frac{p_M - p_L}{p_H - p_L}.$$

Assumption 2 ensures that I_M is more efficient than both I_L and I_H. Indeed, I_M is more efficient than $I_L = 0$ because its cost is lower than the increase in the expected return. The high investment I_H instead, is better than no investment but it is inefficient when compared to I_M because the additional cost of choosing I_H rather than I_M is larger than the increment in the expected return.

If the incumbent remains in office, she enjoys benefit of control $B \geqslant 0$.[9]

The contract offered by the board maximizes the expected final cash flow of the project net of managerial compensation. Recalling that the new manager is not necessarily more productive than an incumbent manager who has made a positive investment, we focus on cases where the profit-maximizing board wants to provide the incumbent with the incentive to invest even if replacement may occur with positive probability. To this end, we consider a simple incentive contract with base salary, incentive pay w contingent on the high return R, and severance pay s. We normalize the reservation level of utility to 0, so that the base salary of the incumbent takes value 0, as well as the compensation of the new manager when replacement occurs. Our results would not change if we assumed a positive level of reservation utility as long as such level is smaller than the expected compensation in case of no investment (see discussion in footnote 11) and they would remain qualitatively the same even in the case of a larger reservation value.

The timing of the model can be summarized as follows:

$t = 0$: The board observes whether the manager is overconfident, optimistic or both and, accordingly, offers a compensation contract aimed at inducing investment I_i, $i = M, H$. The contract is enforceable. We denote by $\{w(\theta, \Delta_i; I_i), s(\theta, \Delta_i; I_i)\}$ the contract offered to an optimistic and overconfident manager who is required to choose I_i. In general, the first or (and) the second argument of the two functions can take value zero if the manager is either not optimistic or not overconfident (or both). The manager decides whether to accept the offer.

$t = 1$: If the contract is accepted, the manager decides how much to invest.

$t = 2$: The board observes the investment level and deduces the probability of success.

$t = 3$: A rival manager appears. Board and incumbent manager observe the rival's ability. The board evaluates whether it is profitable to replace the incumbent. If this is the case, and contractual $s(\theta, \Delta_i; I_i)$ is too low for the incumbent to accept replacement, renegotiation occurs and a new level of severance pay s' is agreed upon.

$t = 4$: The cash flow realizes. The manager is paid the compensation/severance pay agreed upon.

Note that, at $t = 3$, the board knows whether the level of investment is $I_j = I_i$ as desired or whether a different level of investment $I_j \neq I_i$ has been chosen. Thus, the renegotiated severance payment can be contigent on the observed investment level. Then, $s'(\theta, \Delta_j | I_j \neq I_i)$ indicates that the initial contract was meant to induce investment I_i while the manager has chosen a level of investment, $I_j \neq I_i$, resulting in a level of overconfidence Δ_j. In the case in which the manager has indeed undertaken the investment required by the board we simplify the notation and write $s'(\theta, \Delta_i | I_i)$ rather than $s'(\theta, \Delta_i | I_i = I_i)$.[10]

The model is solved by working backwardly. We first discuss renegotiation under an arbitrary initial contract. Then, we discuss the board's replacement decision. Given the replacement decision, we derive the incentive compatible contract $\{w(\theta, \Delta_i; I_i), s(\theta, \Delta_i; I_i)\}$ that maximizes the firm final cash flow and we determine the investment level chosen by the manager. Before developing our model, however, we discuss the role of contractual and renegotiated severance pay in the incentive contract.

3 | Contractual Versus Renegotiated Severance Pay: An Example

Let us discuss the different characteristics of contractual and renegotiated severance pays and their impact on incentive pay with a simple example. Our aim is to underline the implications of the difference in the information available to the board when the contractual and the renegotiated severance pays are agreed upon. The board can fire the incumbent manager if a new and more productive manager materializes. In such a case, the incumbent manager will be paid either the contractual severance pay or a renegotiated amount according to her bargaining power and to contract provisions.

Consider a simplified framework with no private benefits ($B = 0$) and two possible investment levels: $I = I_L = 0$ or $I = I_M$, with the cash flow generated by the project specified above, that is, either 0 or $R > 0$. The associated probabilities of success are $p_L > 0$ if $I = 0$ and $p_M > p_L$ if $I = I_M$. Assume further that the manager is rational, that is, that she has the same beliefs as the board regarding the probability of success. This assumption does not affect the qualitative difference between contractual and renegotiated severance pay that is our focus here.

Let us also simplify the analysis by assuming that the board uses an exogenously given value as a threshold for dismissal. Specifically, we assume that it dismisses the incumbent if the new manager has a probability of success q higher than p_M when investment $I = I_M$ is undertaken, and higher than p_L if $I = I_L = 0$.

Given that the investment is unverifiable, the board cannot condition the contract on the investment level but has to offer a combination of incentive and severance pay such that the manager is induced in making the desired investment, I_M. Consider first the case with a positive contractual severance pay ($s > 0$) and no renegotiation ($s' = 0$). Since s is decided before the investment stage, it cannot be contingent on I. This implies that, in case of dismissal, the manager will receive the same payment irrespective of the level of I she has previously chosen. As in standard agency problems, the optimal compensation must induce the manager to accept the contract (the participation constraint [PC]) and to choose investment I_M rather than $I = I_L = 0$ (the ICC). Under the zero reservation utility assumption and the assumption that q is uniformly distributed, the PC can be written as

$$\int_0^{p_M} p_M w f(q) dq + \int_{p_M}^1 s f(q) dq - I_M$$
$$= (p_M w) p_M + s(1 - p_M) - I_M \geqslant 0,$$

where the first integral represents the expected compensation in case of retention ($q \leq p_M$), and the second integral represents the expected payment in case of dismissal ($q > p_M$) when the manager chooses I_M.

The ICC is

$$\int_0^{p_M} p_M w f(q) dq + \int_{p_M}^1 s f(q) dq - I_M$$
$$= (p_M w) p_M + s(1 - p_M) - I_M$$
$$\geqslant (p_L w) p_L + s(1 - p_L)$$
$$= \int_0^{p_L} p_L w f(q) dq + \int_{p_L}^1 s f(q) dq,$$

where the first line is the expected compensation when the manager chooses I_M and the second line is the expected compensation when she chooses . It is immediate to see that the assumption of zero reservation utility together with the assumption that $w, s \geqslant 0$ imply that PC is not binding. Consequently, we can focus our attention on ICC.

The lowest incentive pay that satisfies ICC is

$$w(s > 0, s' = 0) = \frac{I_M}{p_M^2 - p_L^2} + \frac{s}{p_M + p_L}. \tag{1}$$

The severance pay s makes the incentive pay w increase, ($\frac{\partial w}{\partial s} > 0$) because it reduces the penalty provided by the higher probability of dismissal when $I = 0$ is chosen. Thus, in case of a positive contractual severance payment, the contract offered to incentivize I_M is more expensive.

Consider now the situation where the original contract does not include any severance pay ($s = 0$) but the manager has bargaining power so that she can oppose the board decision and ask for a positive payment $s' > 0$ to leave the firm. Note that from an incentive point of view there is no reason to pay the manager once the investment has already been made. Then, a renegotiated severance pay can only arise if the manager has bargaining power and can oppose replacement. At the renegotiation stage the board knows the level of the investment, consequently the renegotiated severance payment, contrary to the contractual one, can be contingent on I. In particular, let assume that the bargaining power of the manager positively depends on the investment level, so that a higher amount is paid when I_M is chosen than when no investment is undertaken: $s'_M > s'_L = 0$. Then, the ICC becomes $(p_M w)p_M + s'_M(1 - p_M) - I_M \geqslant (p_L w)p_L$, so that the lowest incentive-compatible pay is

$$w\left(s = s'_L = 0, s'_M > 0\right) = \frac{I_M}{p_M^2 - p_L^2} - \frac{(1 - p_M)s'_M}{p_M^2 - p_L^2}. \quad (2)$$

The term that is subtracted from the RHS is the expected value of the severance pay. By comparing (2) to (1), it is immediate to see that it is now easier to satisfy the ICC and that the incentive pay to induce $I = I_M$ is reduced. Thus, there is a negative relation between incentive pay and renegotiated severance pay ($\frac{\partial w}{\partial s'} < 0$) because the higher renegotiated severance pay when $I = I_M$ is chosen (which is anticipated at the contracting stage) contributes to incentivize the manager to choose this investment level.

4 | Renegotiation and Replacement Decision

Let us now use the general framework introduced in Section 2, to analyze the renegotiation of a contract designed to induce a positive level of investment I_i, with i equal either to M or to H. We first determine the condition for the manager to accept dismissal. Given that the manager can oppose such decision, the severance pay must compensate her for the expected loss she will suffer when fired, corresponding to her belief of what she can obtain by remaining with the firm, $(p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B$. In the present section, we consider the case where the contractual severance pay does not meet this requirement. Consequently, a board willing to fire the incumbent, must renegotiate the contract by making a take-it-or-leave-it offer,

$$s'(\theta, \Delta_i|I_i) \geqslant (p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B, \quad (3)$$

that is acceptable by the manager. The optimal level of renegotiated severance pay $s'(\theta, \Delta_i|I_i)$ is clearly equal to the RHS of the above inequality. Given that the investment decision has already been made, there is in fact no reason for the board to increase the renegotiated payment above the minimum level necessary to overcome the incumbent's opposition to replacement. Such payment is correctly anticipated by both the manager and the board at the time when the contract is signed and thus contributes to the expected compensation or expected cost calculated by the manager and the board, respectively.

The board wants to replace the incumbent whenever the expected profit is higher under the new manager, that is, when the gain from replacement, computed by using the "right" probability of success, is higher than the cost. Anticipating that severance pay will be renegotiated, the board will then fire the manager when

$$qR - s'(\theta, \Delta_i|I_i) \geqslant p_i(R - w(\theta, \Delta_i; I_i)), \quad i = M, H, \quad (4)$$

where the LHS represents the expected profit under the new manager and the RHS the expected profit under the incumbent. Note that, since the new manager does not make any investment, she does not receive any incentive pay. The above inequality implies that the manager is fired when

$$q \geqslant p_i - \left(\frac{p_i w(\theta, \Delta_i; I_i) - s'(\theta, \Delta_i|I_i)}{R}\right), \quad (5)$$

meaning that the firing decision is based on the difference between the probability of success of the replacement with respect to that of the incumbent, "adjusted" for the difference in the payment to the incumbent manager in case of retention and dismissal. When the firing cost represented by the severance pay exceeds the sum saved by replacing the manager, that is, when $p_i w(\theta, \Delta_i; I_i) < s'_i(\theta, \Delta_i|I_i)$, the board will replace the manager for higher values of q than in the opposite case.

Note that if no severance pay (either renegotiated or contractual) is paid, the manager is fired even when the replacement has a lower probability of success than her own. In other words, if the contract does not contemplate any severance payment the manager is dismissed too frequently.

For replacement to occur, both condition (4) and condition (3), the latter in the form of an equality, must be simultaneously satisfied

$$(q - p_i)R + p_i w(\theta, \Delta_i; I_i) \geqslant s'(\theta, \Delta_i|I_i)$$
$$= (p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B.$$

In the LHS of this inequality, the payment to the manager in case of retention is what the board expects to pay and is therefore computed using the "right" probability of success, while in the RHS the payment required by the manager in case of dismissal is computed using her subjective beliefs of success.

This condition, taken in the form of an equality, determines the cutoff value of q, above which the board will replace the incumbent

$$\hat{q}(\theta, \Delta_i|I_i) = p_i + \frac{B + (\theta + \Delta_i)w(\theta, \Delta_i; I_i)}{R}. \quad (6)$$

Since the severance pay required by the manager to leave is the monetary equivalent of her expected utility in case she stays with the firm, the above cutoff value is increasing in $w(\theta, \Delta_i; I_i)$ and in B. Note that the term $\frac{B + (\theta + \Delta_i)w(\theta, \Delta_i; I_i)}{R}$ represents the difference between the two expected payment in case of dismissal and in case of retention.

To fully characterize the contract offered to the manager, we have to determine the incentive pay $w(\theta, \Delta_i; I_i)$ and the contractual severance pay $s(\theta, \Delta_i; I_i)$ (which in turn determine the outcome of the renegotiation process, $s'(\theta, \Delta_i|I_i)$, and the cutoff value $\hat{q}(\theta, \Delta_i|I_i)$). This is done in the following section. Note, however, that in the above discussion we have taken it for granted that the incumbent makes investment

$I_i, i = M, H$, when the contract has an incentive component equal to $w(\theta, \Delta_i; I_i)$. To determine the optimal contract, we also need to consider the hypothetical case where the incumbent makes investment I_j when the contract requires her to choose I_i, so that the renegotiated severance pay becomes $s'(\theta, \Delta_j | I_j \neq I_i)$ and the cutoff value above which the manager is dismissed becomes $\hat{q}(\theta, \Delta_j | I_j \neq I_i)$. This never occurs in equilibrium but, precisely to provide the appropriate incentives to discourage such behavior, we need to take into account what payment the incumbent would obtain in such a case.

The next lemma summarizes the above discussion, and indicates the optimal payments that induce the incumbent to leave both when she has made investment I_i and when she has made investment I_j, despite the incentive pay offered at $t = 0$, $w(\theta, \Delta_i; I_i)$ was meant to induce I_i. Note that the latter case may occur if either the manager has not invested ($I_j = I_L = 0$) or if she has chosen a positive level of investment different from the one required by the board (I_H when I_M is required or viceversa).

Lemma 1. *If renegotiation between the board and the incumbent manager takes place under a contract with incentive pay equal to $w(\theta, \Delta_i; I_i)$, the optimal renegotiated severance payment is*

i. $s'(\theta, \Delta_i | I_i) = (p_i + \theta + \Delta_i) w(\theta, \Delta_i; I_i) + B$ *if the incumbent has made investment $I_i, i = M, H$;*
ii. $s'(\theta, \Delta_j | I_j \neq I_i) = (p_j + \theta + \Delta_j) w(\theta, \Delta_i; I_i) + B$ *if the incumbent has made investment $I_j \neq I_i$ $i = M, H, j = L, M, H$.*

4.1 | Renegotiation in the Case of a Rational Manager

To establish a benchmark for our analysis, we consider a rational manager whose subjective beliefs are equal to the "true" probabilities, that is we consider the case where $\theta = \Delta_M = 0$. The cutoff value of q above which the manager is dismissed then is

$$\hat{q}(0, 0 | I_i) = p_i + \frac{B}{R}.$$

Again, the replacement decision is determined by the difference between the probability of success of the replacement and that of the incumbent, "adjusted" for the difference in what should be paid to the incumbent manager in case of retention, $p_i w(0, 0; I_i)$, and in case of dismissal, $s'(0, 0 | I_i)$. Now such difference reduces to $\frac{B}{R}$.

Note that the cutoff $\hat{q}(0, 0 | I_i)$ is lower than the cutoff determined in the case of a biased manager, $\hat{q}(\theta, \Delta_i | I_i)$, and does not depend on the incentive pay. In fact, optimism and overconfidence give rise to an entrenchment effect by distorting the replacement decision because of the higher severance pay (depending in turn on the incentive pay) required by the overconfident and optimistic manager to accept replacement. Since it is more costly to replace a biased manager, the board will replace her only when the probability of success of the replacement is higher.

As far as the renegotiated severance pay is concerned, by setting $\theta = \Delta_i = 0$ in Lemma 1, we have that

$$s'_i(0, 0 | I_i) = p_i w(0, 0; I_i) + B, \quad i = M, H.$$

As in the case of an overconfident manager, the renegotiated severance pay corresponds to the amount that the manager expects to obtain by staying with the firm, because any payment lower than that would be rejected.

5 | Manager's Investment and Optimal Compensation

Having established the optimal renegotiated severance pay, we can now determine the contractual severance pay and the incentive pay necessary to induce a positive level of investment. Then, we will be able to determine the optimal investment level.

5.1 | Contractual Severance Pay

The following proposition establishes that, for any given positive level of investment $I_i, i = M, H$, there is an entire range of optimal contractual severance payments $s(\theta, \Delta_i; I_i)$, all lower than (or equal to) the minimum payment the manager can get at the renegotiation stage $s'(\theta, 0 | 0 \neq I_i)$, where such minimum payment corresponds to the case in which the manager makes no investment ($I = I_L = 0$) even if the contract prescribes $I_i > 0$. Consequently, contractual severance pay will always be renegotiated.

Proposition 1. *Any level of contractual severance pay $s(\theta, \Delta_i; I_i)$ such that $0 \leq s(\theta, \Delta_i; I_i) \leq s'(\theta, 0 | 0 \neq I_i)$ is optimal. Given that $s'(\theta, 0 | 0 \neq I_i) < s'(\theta, \Delta_i | I_i)$, renegotiation occurs at $t = 3$ whenever the manager is replaced after a positive investment.*

Proof. See Appendix 1. □

The intuition for this result comes from the fact that the severance pay established at the contracting stage cannot be contingent on the investment. As shown in the example of Section 3, this reduces the expected penalty from not investing and implies a higher incentive pay to induce the manager to invest. Moreover, since the contract is enforceable, the board must pay any amount specified in the contract even if the (unverifiable) investment, is not the desired one. Conversely, at the renegotiation stage, the board knows the investment chosen by the manager, so that the severance payment can depend on such investment. There follows that, at the contracting stage, the board doesn't want to commit to any level of severance pay higher than the minimum the manager can receive at the renegotiation stage. This minimum amount is what the manager obtains in the case where the contract requires a positive investment but the manager does not comply and chooses $I_L = 0$.

To better understand why even a contractual severance pay equal to zero can be optimal, consider that the manager anticipates that in case of a low $s(\theta, \Delta_i; I_i)$, an additional payment will be agreed upon at the renegotiation stage so as to make replacement acceptable. This clearly provides the incentive to invest even when contractual severance pay is zero. Hence, what really matters for the investment is the severance pay that the manager can bargain in case of dismissal, not the amount specified in the contract.

Finally, note that this result may seem in contrast with the generous severance payments we observe in managerial contracts. While any level between zero and $s'(\theta, 0; 0 \neq I_i)$ is equally optimal in our model, in the real world a zero contractual severance payment is the exception rather than the rule. A possible reason to set the contractual severance pay at a higher level, included the highest optimal level, $s'(\theta, 0; 0 \neq I_i)$, is that shareholders generally oppose a large renegotiated payment in excess of the contractual agreements. Shareholders usually want to limit such discretionary payment that allows the departing manager to obtain more than its contractual entitlement with no economic justification from their point of view. However, as explained above, this can give rise to costly ligation. Then, the board may want to keep the additional component at the minimum by setting the contractual component at the highest among the optimal levels.

5.1.1 | Contractual Severance Pay for a Rational and for a Biased Manager

All what we said on contractual severance pay holds both for an optimistic ($\theta > 0$) and/or overconfident ($\Delta_i > 0$) manager as well as for a rational one ($\theta = \Delta_i = 0$). The only difference is obviously in the values taken by the maximum level of contractual severance pay. In the case of a biased manager this is $s'(\theta, 0; 0 \neq I_i) = p_L w(\theta, \Delta_i; I_i) + B$ while in the case of a rational manager it is $s'(0, 0|0 \neq I_i) = p_L w(0, 0; I_i) + B$

5.2 | Investment Levels

To determine the optimal level of investment, we need to refer to the PC and the ICCs. Recall that we have normalized the reservation utility to zero and consider that the firm wants to induce the manager to choose investment I_i, where i can alternatively be M or H. As the parties anticipate that the severance pay will be renegotiated (see Proposition 1), the PC is

$$\int_0^{\hat{q}(\theta,\Delta_i|I_i)} \Big[(p_i + \theta + \Delta_i) w(\theta, \Delta_i; I_i) + B\Big] f(q) dq + \int_{\hat{q}(\theta,\Delta_i|I_i)}^1 s'(\theta, \Delta_i|I_i) f(q) dq - I_i \geqslant 0, \qquad \text{(PC)}$$

where the first integral represents the expected compensation in case of retention ($q \leq \hat{q}(\theta, \Delta_i|I_i)$), and the second integral represents the expected payment in case of dismissal ($q > \hat{q}(\theta, \Delta_i|I_i)$), provided that the manager has undertaken the investment desired by the board.

Moreover, two incentive constraints must now be satisfied. The first one, ICC 1, guarantees that the manager prefers I_i to not investing

$$\int_0^{\hat{q}(\theta,\Delta_i|I_i)} \Big[(p_i + \theta + \Delta_i) w(\theta, \Delta_i; I_i) + B\Big] f(q) dq + \int_{\hat{q}(\theta,\Delta_i|I_i)}^1 s'(\theta, \Delta_i|I_i) f(q) dq - I_i \geqslant \qquad \text{(ICC 1)}$$
$$\int_0^{\hat{q}(\theta,0|0\neq I_i)} \Big[(p_L + \theta) w(\theta, \Delta_i; I_i) + B\Big] f(q) dq + \int_{\hat{q}(\theta,0|0\neq I_i)}^1 s'(\theta, 0|0 \neq I_i) f(q) dq,$$

where according to the notation introduced in Section 2, $s'(\theta, \Delta_i|I_i)$ and $\hat{q}(\theta, \Delta_i|I_i)$ are, respectively, the renegotiated severance pay and the cutoff when the manager complies with the contract aimed to induce investment I_i, while $s'(\theta, 0|0 \neq I_i)$ and $\hat{q}(\theta, 0|0 \neq I_i)$ would be the values of these variables if the manager were to undertake no investment. In the latter case, the manager would not be overconfident, that is, $\Delta_i = 0$. However, this would not affect the value of incentive pay $w(\theta, \Delta_i; I_i)$ as such value was established ex ante at the contractual stage, on the ground of the required level of investment I_i.

The second incentive constraint, ICC 2 requires that the manager prefers I_i to I_j when the contract prescribes investment I_i

$$\int_0^{\hat{q}(\theta,\Delta_i|I_i)} \Big[(p_i + \theta + \Delta_i) w(\theta, \Delta_i; I_i) + B\Big] f(q) dq + \int_{\hat{q}(\theta,\Delta_i|I_i)}^1 s'(\theta, \Delta_i|I_i) f(q) dq - I_i \geqslant \qquad \text{(ICC 2)}$$
$$\int_0^{\hat{q}(\theta,\Delta_j|I_j\neq I_i)} \Big[(p_j + \theta + \Delta_j) w(\theta, \Delta_i; I_i) + B\Big] f(q) dq + \int_{\hat{q}(\theta,\Delta_j|I_j\neq I_i)}^1 s'(\theta, \Delta_j|I_j \neq I_i) f(q) dq - I_j.$$

Here $\hat{q}(\theta, \Delta_j|I_j \neq I_i)$ denotes the cutoff when the contract aims to induce I_i but the manager undertakes investment I_j. In the latter case, the manager's overconfidence is $\Delta_j \neq \Delta_i$ but again this, while affecting the renegotiated severance pay and the ex post condition for dismissal, does not affect the incentive pay that is determined *ex ante* when the contract is offered.

By comparing PC and ICC 1, we can immediately check that, consistently with the result in the example of Section 3, if ICC 1 is satisfied, the PC is satisfied as well. In fact, by substituting $s'(\theta, 0|0 \neq I_i)$ from Lemma 1, the RHS of ICC 1 becomes $(p_L + \theta) w(\theta, \Delta_i; I_i) + B > 0$ in the case of a biased manager and $p_L w(0, 0; I_i) + B > 0$ in the case of a rational one. Consequently, the PC holds in the form of an inequality. Given that the manager has enough bargaining power to obtain the same utility both in case of retention and in case of dismissal, the PC is never binding.[11] We then focus our attention on the ICCs.

5.3 | Incentive Pay and Investment Level I_M

In this section, we determine the incentive pay necessary to induce the level of investment requested by the board. Consider first the case where the board wants to induce the efficient investment level I_M. The compensation must satisfy incentive compatibility constraints ICC 1 and ICC 2. By substituting $s'(\theta, \Delta_M | I_M)$ and $s'(\theta, 0 | 0 \neq I_M)$ from Lemma 1, ICC 1 can be written as:

$$(p_M + \theta + \Delta_M) w(\theta, \Delta_M; I_M) - (p_L + \theta) w(\theta, \Delta_M; I_M) \geqslant I_M,$$

implying that the incentive pay must satisfy

$$w(\theta, \Delta_M; I_M) \geqslant \frac{I_M}{(p_M + \Delta_M - p_L)}. \tag{7}$$

Consider then ICC 2 with $i = M$ and $j = H$. Substituting for $s'(\theta, \Delta_M | I_M)$ and $s'(\theta, \Delta_H | I_H \neq I_M)$ from Lemma 1, ICC 2 becomes

$$(p_H + \Delta_H - p_M - \Delta_M) w(\theta, \Delta_M; I_M) \leq I_H - I_M.$$

Therefore, to guarantee that both ICC 1 and ICC 2 are satisfied and I_M is chosen, it must be the case that

$$\begin{aligned} \frac{I_M}{p_M + \Delta_M - p_L} &\leq w(\theta, \Delta_M; I_M) \\ &\leq \frac{I_H - I_M}{p_H + \Delta_H - p_M - \Delta_M}. \end{aligned} \tag{8}$$

Recalling that $\Delta_H = \Delta_M(1 + z)$ such inequality can be satisfied if and only if

$$\frac{I_M}{I_H} \leq \frac{p_M + \Delta_M - p_L}{p_H + \Delta_M(1 + z) - p_L}. \tag{9}$$

When the value of Δ_M is high, the value of the RHS of the above inequality is small so that it can be difficult to satisfy such condition and guarantee that the manager chooses investment I_M.[12] In other words, when the degree of overconfidence is large, the manager even if offered $w(\theta, \Delta_M; I_M)$ will choose I_H. This happens because a high level of overconfidence increases the subjective probability of success to such an extent that, by choosing I_H, the manager expects that the increase in her expected compensation more than compensates the additional cost of the investment. This implies that, for a high enough level of overconfidence, ICC 2 does not hold.

Consider the following definition.

Definition 1. The manager is moderately overconfident when $\Delta_M \leq \Delta_M^* = \frac{I_M(p_H - p_L) - I_H(p_M - p_L)}{I_H - I_M(1+z)}$ so that ICC 2 is satisfied. Conversely, the manager is extremely overconfident when $\Delta_M > \Delta_M^*$ and ICC 2 does not hold.

Note that $\Delta_M^* > 0$ follows from $z > \frac{I_H - I_M}{I_M}$.[13] When the manager is moderately overconfident, I_M is incentive compatible and a board willing to induce such a level of investment will offer the lowest possible level of $w(\theta, \Delta_M; I_M)$ satisfying the ICC, that is

$$w(\theta, \Delta_M; I_M) = \frac{I_M}{(p_M + \Delta_M - p_L)}. \tag{10}$$

When instead the manager is extremely overconfident, I_M cannot be implemented. This highlights the potential downside of overconfidence that may induce the manager to choose the inefficient investment.

5.4 | Incentive Pay and Investment Level I_H

Consider then the incentive pay necessary to implement the investment level I_H. The ICCs for the high level of investment, I_H, ICC 1 and ICC 2, respectively imply

$$w(\theta, \Delta_H; I_H) \geqslant \frac{I_H}{(p_H + \Delta_H - p_L)} \tag{11}$$

and

$$\begin{aligned} w(\theta, \Delta_H; I_H) &\geqslant \frac{I_H - I_M}{(p_H + \Delta_H - p_M - \Delta_M)} \\ &= \frac{I_H - I_M}{p_H - p_M + z\Delta_M}. \end{aligned} \tag{12}$$

The first inequality guarantees that the manager prefers I_H to zero investment, while the second one guarantees that I_H is preferred to I_M. The minimum incentive pay required to induce investment I_H depends on which constraint is binding.

Let us first consider the case in which the manager is extremely overconfident so that I_M is not implementable. It is immediate to verify that the binding constraint to implement I_H is the first one. In fact, Definition 1 says that extreme overconfidence, $\Delta_M > \Delta_M^*$, occurs when

$$\frac{I_M}{I_H} > \frac{p_M + \Delta_M - p_L}{p_H + \Delta_M(1 + z) - p_L}, \tag{13}$$

and it can be easily verified that this corresponds to the case where the binding constraint is (11). In this case, the increase in the perceived probability of success due to the choice of I_H is so high that the manager always prefers I_H to I_M. Consequently, the optimal incentive pay is given by the lowest value of w satisfying condition (11) $w(\theta, \Delta_H; I_H) = \frac{I_H}{p_H + \Delta_H - p_L}$.[14]

Let us then investigate whether investment I_H can be incentive compatible under moderate overconfidence. The next proposition proves that, even if this is possible, it is generally unprofitable. Thus, Proposition 2 allows us to restrict our attention to two mutually exclusive cases: extreme overconfidence with investment level I_H, and zero or moderate overconfidence with investment level I_M.[15]

Proposition 2. *The incentive pay offered by the board depends on the degree of managerial overconfidence while it is independent of optimism:*

- $w(\theta, \Delta_M; I_M) = w(0, \Delta_M; I_M) = \frac{I_M}{(p_M + \Delta_M - p_L)}$ *is generally offered when the manager is moderately overconfident* ($\Delta_M \leq \Delta_M^*$) *so that the manager chooses* I_M
- $w(\theta, \Delta_H; I_H) = w(0, \Delta_H; I_H) = \frac{I_H}{(p_H + \Delta_H - p_L)}$ *is offered when the manager is extremely overconfident* ($\Delta_M > \Delta_M^*$) *so that the manager chooses* I_H.

Proof. See Appendix 1. □

The effect of overconfidence follows immediately from the discussion above. Optimism has no impact on the bonus because it uniformly shifts upwards the probabilities of success with no distortion in the marginal probabilities. A similar "no distortion" result is found in de la Rosa (2011) where, however, optimism tends to make the incentive pay steeper by relaxing the PC. Note that, since in our model optimism increases the return of both investment levels without changing their relative profitability, it does not affect the choice of the investment.

5.4.1 | Incentive Pay When the Manager is Rational

When the manager is rational ($\theta = \Delta_M = 0$), the efficient level of investment I_M can always be implemented while I_H is not. Consider condition (8). This implies that the two ICCs for investment level I_M are now satisfied when

$$\frac{I_M}{p_M - p_L} \leq w_M(0, 0; I_M) \leq \frac{I_H - I_M}{p_H - p_M}. \tag{14}$$

For such condition to hold it must be

$$\frac{I_M}{I_H} \leq \frac{p_M - p_L}{p_H - p_L},$$

which, however, is always the case by Assumption 2. On the other hand, it can be easily proved that I_H cannot be incentive compatible, so that I_M will always be implemented in the case of a rational manager.

Corollary 1. *When the manager is rational, only* I_M *is incentive compatible and the board offers incentive pay* $w(0, 0|I_M) = \frac{I_M}{(p_M - p_L)}$.

Proof. Consider investment I_H. In this case ICC 2 implies

$$w(\theta, \Delta_H; I_H) \geqslant \frac{I_H - I_M}{(p_H - p_M)}$$

but we know that $\frac{I_H - I_M}{(p_H - p_M)} > R$ by Assumption 2, so that the board will never offer such incentive pay. Then I_M will be incentivized with the lowest level of w that satisfies (14). □

Proposition 2 and Corollary 1 highlight the role of overconfidence in the investment choice. When the manager is rational and holds correct beliefs about the probability of success, it is not possible to implement I_H because the manager is aware that the additional cost cannot be compensated by the increase in the expected compensation. In fact, Assumption 2 guarantees that the rise in the cost is higher than the gain in the expected return. However, this may not be enough to prevent an overconfident manager from choosing the inefficient investment because of her biased assessment of the probability of success. Thus, our model accounts for the possibility, documented by a large literature (see, among others, Malmendier and Tate 2005 and 2015), that an optimistic and overconfident manager may choose an investment level higher than the optimal one. The differential effects of optimism and overconfidence are analyzed in more detail in the following section.

6 | The Effects of Optimism and Overconfidence on the Optimal Contract and Firm Expected Profits

In this section, we analyze in detail how each single bias affects the contractual components (incentive pay and severance pay), the cutoff used by the board in the replacement decision and the firm's expected profit.

6.1 | Optimistic, But Not Overconfident, Manager

Consider first a manager who is only optimistic ($\theta > 0$, $\Delta_M = 0$). The following corollary clarifies the impact of a change in optimism on the contractual components.

Corollary 2. *Optimism has no effect on incentive pay,* $\frac{\partial w(\theta, 0; I_i)}{\partial \theta} = 0$, *while it increases the renegotiated severance pay and the cutoff value for the replacement decision:* $\frac{\partial s'(\theta, 0 \mid I_i)}{\partial \theta} > 0$ *and* $\frac{\partial \hat{q}(\theta, 0 \mid I_i)}{\partial \theta} > 0$, $i = M, H$.

Proof. See Appendix 1. □

Optimism shifts the perceived probability of success upward by the same amount for all investment levels and therefore does not affect the incentive pay. Hence, an optimistic manager has no reason to prefer the high investment I_H when I_M is required. In fact, given that incentive pay is not affected by optimism, the optimal level of the bonus for an optimistic (but not overconfident) manager is equal to that of a rational one

$$w(\theta, 0; I_M) = \frac{I_M}{(p_M - p_L)} = w(0, 0; I_M).$$

An increase in optimism (θ), however, raises the renegotiated severance pay because the higher perceived probability of success induces the manager to overvalue the expected incentive pay if confirmed[16]

$$s'(\theta, 0|I_M) = (p_M + \theta)w(\theta, 0; I_M) + B > p_M w(0, 0; I_M) + B = s'(0, 0|I_M).$$

The higher renegotiated severance pay increases the cutoff for the retention/dismissal decision

$$\hat{q}(\theta, 0|I_M) = p_M + \frac{B}{R} + \frac{\theta \cdot w(\theta, 0; I_M)}{R}$$
$$> p_M + \frac{B}{R} = \hat{q}(0, 0|I_M).$$

Thus, a positive entrenchment effect is at work even if the manager is only optimistic.

Consider now the expected profit of the firm

$$V(\theta, 0|I_M) = \int_0^{\hat{q}(\theta,0|I_M)} [p_M(R - w(\theta, 0; I_M))] f(q) dq + \int_{\hat{q}(\theta,0|I_M)}^1 (qR - (p_M + \theta) w(\theta, 0; I_M)) f(q) dq. \quad (15)$$

A natural question concerns the overall impact of optimism on expected profits resulting from the combined effects on expected incentive pay, retention policy and expected severance payment. This is established by the following corollary.

Corollary 3. *The expected profit of the firm is decreasing in optimism* θ : $\frac{dV(\theta,0\mid I_M)}{d\theta} < 0$.

Proof. See Appendix 1. □

The negative relation between optimism and expected profit arising in our model is in contrast with the positive effect found in de la Rosa (2011) and in the previous principal/agent literature where optimism, by relaxing the incentive/insurance trade-off due to the agent being risk-averse, makes high-powered incentives feasible and has thus a positive influence on expected profit. The difference is that, in our model, optimism increases the expected payment to be made to the manager through the severance pay. Being the manager risk neutral, there is no positive insurance effect.

Corollary 3 suggests that, if the board were to know the type of the manager, it would prefer a rational manager to an optimistic (but not overconfident) one. Indeed, considering that in both cases the investment level I_M will always be chosen, the above discussion shows that the expected profit is lower under an optimistic (but not overconfident) manager than under a rational one: $V(\theta, 0|I_i) < V(0, 0|I_i)$.

6.2 | Overconfident But Not Optimistic Manager

Consider now the opposite case where the manager is overconfident but not optimistic ($\Delta_M > 0, \theta = 0$). The following corollary shows how a change in overconfidence affects the contract offered by the board when $\theta = 0$ and $\Delta_i > 0$.

Corollary 4. *Overconfidence continuously reduces incentive pay, and increases the cutoff value for dismissal. For a given investment level* $I_i, i = M, H$, *it also reduces the renegotiated severance payment:* $\frac{\partial s'(0,\Delta_i \mid I_i)}{\partial \Delta_i}|_{I_i} < 0$. *Only at* Δ_M^* *where the shift from* I_M *to* I_H *occurs, overconfidence has an increasing impact on severance pay.*

Proof. See Appendix 1. □

Incentive pay is continuously decreasing in overconfidence because overconfidence induces the manager to overestimate the effect of her investment on the probability of success so that not only a lower bonus is needed to incentivize the same level of investment but the effect is also preserved when shifting to the higher (and more costly) level of investment I_H. This finding implies that the incentive pay of an overconfident manager is always lower than that of a rational one

$$w(0, \Delta_i; I_i) = \frac{I_i}{(p_i + \Delta_i - p_L)} < \frac{I_M}{(p_M - p_L)} = w(0, 0; I_M)$$

and is in line with previous theoretical literature as well as with empirical evidence (Otto 2014; Humphery-Jenner et al. 2016).

The positive relation between overconfidence and entrenchment is the result of two contrasting effect. On the one hand, overconfidence has a positive direct impact on $\hat{q}(0, \Delta_i|I_i)$ through the increase in the belief of success that multiplies the incentive pay (the term Δ_i). On the other hand, it has a negative indirect effect due to the fact that the incentive pay is decreasing in Δ_i. The former effect dominates, creating an upward distortion in the replacement decision. In particular, if we compare the optimal cutoff for an overconfident manager with that for a rational manager, we obtain

$$\hat{q}(0, \Delta_i|I_i) = p_i + \frac{B}{R} + \frac{\Delta_i \cdot w(0, \Delta_i; I_i)}{R} > p_M + \frac{B}{R}$$
$$= \hat{q}(0, 0|I_M).$$

Also the negative relation between overconfidence and renegotiated severance pay is the result of two contrasting effects. On the one hand, there is an increase in the manager's perceived probability of success that in turn increases the required severance pay. On the other hand, there is a reduction in the incentive pay that is the main component of the severance payment. For a moderately overconfident manager, this latter effect is dominant so that the reduction in incentive pay more than compensates the increase in the subjective belief of success. As a result, the renegotiated severance payment is lower than that of a rational one: $s'(0, \Delta_M|I_M) < s'(0, 0|I_M)$. However, we cannot say the same for an extremely overconfident manager because, at Δ_M^*, the shift from I_M to I_H induces a spike in the belief of success that leads also to an increase in s' such that $s'(0, \Delta_H^*|I_H) > s'(0, \Delta_M^*|I_M)$ where $\Delta_H^* = (1 + z)\Delta_M^*$. For $\Delta_M > \Delta_M^*$, the renegotiated severance pay $s'(0, \Delta_H|I_H)$ is again decreasing in Δ_M. Consequently, we cannot say whether the rise occurring at Δ_M^* will make $s'(0, \Delta_H|I_H)$ everywhere higher than $s'(0, \Delta_M^*|I_M)$. We cannot exclude that for very high values of Δ_M, $s'(0, \Delta_H|I_H)$ falls below $s'(0, \Delta_M^*|I_M)$.

To complete our analysis let us consider z. This parameter measures the increase in overconfidence when moving from I_M to I_H for any given level of the "basic" overconfidence parameter Δ_M (see Assumption 1). Therefore, z represents another route through which overconfidence affects incentive pay. We may wonder what is the effect of z on the contractual components, keeping the "basic" overconfidence Δ_M constant. Obviously, this is relevant only in the case in which the manager chooses I_H. In such a case the impact of a rise in z is similar to the impact of Δ_M just analyzed: when z rises, the incentive pay decreases pushing down the value of the cutoff. However, the entrenchment increases because of the higher belief of success on the part of the manager. Again the higher entrenchment follows from the dominance of the latter effect that makes firing more costly.

Let us now evaluate the total effect of an increase in overconfidence on expected profit.

Corollary 5. *For a given investment level $I_i, i = M, H$, the expected profit is increasing in overconfidence. When $\Delta_M = \Delta_M^*$ so that a further increase in Δ_M implies a shift from I_M to I_H, there is a discontinuity: the expected profit generally has an initial drop and then resumes an increasing trend.*

Proof. See Appendix 1. □

Profit is increasing in overconfidence as long as overconfidence is moderate and does not lead to a change in the investment level. This also implies that expected profit is higher with a moderately overconfident but not optimistic manager ($\theta = 0, 0 < \Delta_M \leq \Delta_M^*$) than with a rational manager. When a rise in overconfidence induces the shift from I_M to I_H, there is a discontinuity and expected profit suddenly drops because, due to the rise in the subjective belief of success, the manager requires a higher renegotiated severance pay. Observe that, the difference between renegotiated and contractual severance pay is at least equal to the value of the investment I_i. By substituting for $w(\theta, \Delta_i; I_i)$ it is immediate to verify that the renegotiated severance pay is $s'(\theta, \Delta_i | I_i) = s'(\theta, 0 | 0 \neq I_i) + I_i$, where $s'(\theta, 0 | 0 \neq I_i)$ is the maximum optimal level of contractual severance pay from Proposition 1. Then, the severance pay suddenly increases by $I_H - I_M$ when the manager switches from investment I_M to I_H.[17] Once the new level of investment, I_H, is chosen, the expected profit is again increasing in overconfidence. Note, however, that there is no guarantee that it will reach again the level corresponding to Δ_M^*, because the increase in Δ_M is bounded by the constraint that the belief of success cannot exceed one.

Summarizing, a moderate level of overconfidence that does not distort the investment choice, is beneficial for the firm but this may not hold true for extreme levels of overconfidence. This issue is analyzed by the following corollary.

Corollary 6. *Expected profit is higher with a moderately overconfident manager than with a rational manager, $V(0, \Delta_M | I_M) > V(0, 0 | I_M)$ for $\Delta_M \leq \Delta_M^*$, but the reverse may hold true with an extremely overconfident manager, that is, when $\Delta_M > \Delta_M^*$.*

Proof. See Appendix 1. □

Corollary 6 highlights the negative impact on expected profit resulting from the shift in the investment from I_M to I_H. The drop in expected profit occurring at Δ_M^* can be high enough to result in an expected profit higher with a rational manager than with an extremely overconfident one. In particular, the large bias of an extremely overconfident manager (z high) may induce her to overestimate the probability of success of investment I_H with respect to I_M to a large extent even if the difference in the objective probabilities of success is relatively small ($p_H - p_M$ small). Then, the choice of I_H leads to a high severance payment in case of replacement but induces a small gain in terms of probability of success. Notice that Corollary 6 is proved for $\theta = 0$, that is, without considering the negative impact of optimism. This leads to the conclusion that, even in the absence of optimism, the firm benefits from hiring a moderately overconfident manager while it may prefer a rational manager to an extremely overconfident one.

6.3 | The Combined Effects of Optimism and Overconfidence

Much of the above analysis also holds in the case of a manager who is both optimistic and overconfident. In particular, corollaries 2 and 3 are also valid in the case of a given positive value of Δ_M (see their proofs in Appendix 1) and corollaries 4 and 5 also hold in the case of a given positive value of θ (see again their proofs in Appendix 1). Summarizing, the two managerial biases affect the contract in very different ways: while overconfidence reduces the cost of the contract by lowering the incentive pay necessary to satisfy the ICCs and by reducing the severance pay except at $\Delta_M = \Delta_M^*$, optimism always increases the cost through its effect on severance pay. Then, if we consider a manager who is both optimistic and overconfident, the negative impact on profit found in Corollary 6 for an extremely overconfident manager can be generalized to a moderately overconfident and optimistic one. Indeed, the negative impact of a sufficiently large degree of optimism can overcome the positive effect of moderate overconfidence. As a result, the firm may be worse off by hiring a biased manager than by hiring a rational one.[18]

7 | The Role of Managerial Bargaining Power

Anecdotal evidence suggests that CEOs are powerful and hold a strong bargaining position vis-a-vis the board. We believe our assumption that the manager can oppose replacement captures many situations of CEO turnover. Observe that the manager's bargaining power derives from the possibility to resist a decision taken against her will. This implies that we are focusing on forced turnovers rather than voluntary turnovers where it is the manager the one who wants to leave.[19] Given that possible opposition to replacement and the resulting payment in case of dismissal is a key feature of our model, we here discuss the robustness of our findings to alternative assumptions on managerial bargaining power. Specifically, we discuss two settings. In the first one we consider the extreme case where the manager has no bargaining at all and the board can fire her at will. In the second one we briefly consider a more general framework where the manager has a variable bargaining power that depends on the "type" of the board. We present the main findings in the text and we relegate all the formal results to Appendix 2 where the optimal values for the incentive pay, the severance pay, and the cutoff are derived. We develop our analysis considering investment I_M and then we tackle the question whether such efficient level of investment is incentive compatible with respect to I_H. For the sake of simplicity, we assume in this section that the manager's private benefit is zero: $B = 0$.

7.1 | No Managerial Bargaining Power

Let us consider the case where the manager has no bargaining power. We want to derive the values for $w_{NB}(\theta, \Delta_M; I_M)$, $s_{NB}(\theta, \Delta_M; I_M)$, and $\hat{q}_{NB}(\theta, \Delta_M | I_M)$ where the subscript NB indicates no bargaining power. Given that the manager has no bargaining power and cannot oppose replacement, no renegotiation will occur. Then, only contractual severance pay, if any, will be paid in case of dismissal. Consequently, in this section we focus on contractual provisions and we show that the (contractual) severance pay will be optimally set to zero.

As discussed in Section 4, the board wants to hire the new manager whenever the latter yields higher expected profit than the incumbent:

$$qR - s_{NB}(\theta, \Delta_M; I_M) \geqslant p_M (R - w_{NB}(\theta, \Delta_M; I_M))$$

which results in a cutoff value equal to

$$\hat{q}_{NB}(\theta, \Delta_M | I_M) = p_M - \frac{p_M w_{NB}(\theta, \Delta_M; I_M) - s_{NB}(\theta, \Delta_M; I_M)}{R}. \quad (16)$$

This expression is equivalent to condition (5) with the difference that here we have the contractual severance pay rather than the renegotiated one. Again, the cutoff is negatively related to the incentive pay and positively related to the severance pay, but $s_{NB}(\theta, \Delta_M; I_M)$ is not exogenously determined by the manager's bargaining power as was $s'(\theta, \Delta_i | I_i)$ in expression (5).[20] Here, the optimal value of $s_{NB}(\theta, \Delta_M; I_M)$ must be determined together with the other components of the contract.

Let us consider the profit of the firm:

$$V_{NB}(\theta, \Delta_M; I_M) = \int_0^{\hat{q}_{NB}(\theta, \Delta_M | I_M)} [p_M (R - w_{NB}(\theta, \Delta_M; I_M))] f(q)dq + \int_{\hat{q}_{NB}(\theta, \Delta_M | I_M)}^{1} (qR - s_{NB}(\theta, \Delta_M; I_M)) f(q)dq.$$

This is the objective function to be maximized subject to participation and incentive-compatibility constraints analogous to PC, ICC 1 and ICC 2, with contractual severance pay $s_{NB}(\theta, \Delta_M; I_M)$ in the place of the renegotiated one. As in the baseline model, the PC is not binding and the optimal value of the incentive pay $w_{NB}(\theta, \Delta_M; I_M)$ is determined by the binding ICC 1. This implies that the optimal value of the bonus depends on the level of the severance pay.

To show that the optimal severance pay is equal to zero, let us consider the effect of $s_{NB}(\theta, \Delta_M; I_M)$ on the expected profit of the firm. Such effect is both direct and indirect as $s_{NB}(\theta, \Delta_M; I_M)$ also affects the cutoff value and the incentive pay (derived from ICC 1). Thus, the overall effect of the severance pay on the firm expected profit is given by

$$\frac{dV_{NB}(\theta, \Delta_M; I_M)}{ds_{NB}(\theta, \Delta_M; I_M)} = \underbrace{\frac{\partial V_{NB}}{\partial s_{NB}(\theta, \Delta_M; I_M)}}_{<0} + \underbrace{\frac{\partial V_{NB}}{\partial w_{NB}(\theta, \Delta_M; I_M)}}_{<0} \underbrace{\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial s_{NB}(\theta, \Delta_M; I_M)}}_{?} \underbrace{\frac{\partial V_{NB}}{\partial \hat{q}_{NB}(\theta, \Delta_M | I_M)}}_{=0} \left(\frac{\partial \hat{q}_{NB}(\theta, \Delta_M | I_M)}{\partial w_{NB}(\theta, \Delta_M; I_M)} \frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial s_{NB}(\theta, \Delta_M; I_M)} + \frac{\partial \hat{q}_{NB}(\theta, \Delta_M | I_M)}{\partial s_{NB}(\theta, \Delta_M; I_M)} \right). \quad (17)$$

The first term on the right hand side of (17) is the direct negative effect. The second term is the indirect effect through the incentive pay. The third term represents the indirect effect on the cutoff value but is equal to zero because the optimal cutoff automatically adjusts to offset any change in the level of $s_{NB}(\theta, \Delta_M; I_M)$.

If the total effect $\frac{dV(\cdot)}{ds_{NB}(\cdot)}$ is negative, the optimal contract will offer zero severance pay, while in the case of a positive $\frac{dV(\cdot)}{ds_{NB}(\cdot)}$, a positive severance payment should be included in the contract. It immediately derives from (17) that a positive impact of the severance pay on the incentive pay $\left(\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial s_{NB}(\theta, \Delta_M; I_M)} \geqslant 0 \right)$ is a sufficient condition for an overall negative sign of $\frac{dV(\cdot)}{ds_{NB}(\cdot)}$. In the example of Section 3, we saw that, in the absence of renegotiation, a positive contractual severance pay results in a higher incentive pay because it reduces the penalty suffered by a (rational) manager in case of no investment. Consequently, a higher incentive pay is needed to induce the manager to choose the desired level of investment. In Appendix 2 we prove that this is a general result, that is, that $\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial s_{NB}(\theta, \Delta_M; I_M)} > 0$ for any level of $s_{NB}(\theta, \Delta_M; I_M) \geqslant 0$.[21] We can then conclude that

Proposition 3. *The optimal contractual severance pay when the manager has no bargaining power and the board wants to induce investment I_M, is equal to zero, $s_{NB}(\theta, \Delta_M; I_M) = 0$.*

Proof. See Appendix 2. □

Note that a zero contractual level of the severance pay is optimal also when the manager has bargaining power (see Proposition 1). However, this occurs for very different reasons. When the manager can renegotiate the contractual agreement, both parties anticipate the outcome of the possible renegotiation so that a zero contractual severance pay becomes irrelevant: firm expected profit is the same for all the values of $s(\theta, \Delta_M; I_M)$ smaller than $s'(\theta, 0 | 0 \neq I_M)$. Conversely, in the present framework without renegotiation, a zero contractual severance pay is optimal because it makes it easier to satisfy the ICC. This occurs because, a positive contractual severance pay requires a higher incentive pay.

Interestingly, in the absence of managerial bargaining power, the optimal incentive pay is decreasing in both optimism and overconfidence: $\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial \theta} < 0$ and $\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial \Delta_M} < 0$. Thus, the effect of overconfidence is the same as in the baseline model: it makes it easier to satisfy the ICC by increasing the subjective beliefs of success. The negative impact of optimism on incentive pay, instead, is in contrast with the findings in the model with managerial bargaining power where optimism had no effect on this component of the contract. Such difference is due to the fact that, in the baseline model, optimism increases managerial beliefs by the same amount irrespective of the level of the investment so that the ICCs are not affected. Conversely, in the present case, optimism increases what the manager expects to receive when investing more than what she expects to receive with no investment (due to the effect on the cutoff values for dismissal).[22].

The negative relationships between incentive pay and managerial biases imply that the optimal incentive pay for an optimistic and overconfident manager is lower than the one for a rational manager: $w_{NB}(\theta, \Delta_M; I_M) < w_{NB}(0, 0; I_M)$. This finding is consistent with the results of the principal/agent literature where managerial bargaining power is not considered and there are no negative effects from hiring an optimistic and/or overconfident manager.

As in the baseline model, the question whether investment I_M is incentive compatible arises. In fact, high overconfidence may lead to overinvestment also in the present case with no managerial bargaining power. In Appendix 2, a condition analogous to expression (13) is derived, so that for values of Δ_M higher than the threshold derived from such condition, the manager chooses investment I_H. Despite the apparent similarity, there is however a crucial difference with respect to the analysis in Section 5. There, in the case with bargaining power, the manager is able to transfer the investment cost to the firm. Here, with no renegotiation and no severance pay, it is the manager who eventually bears the cost of the investment. Then, the manager's benefit from choosing investment I_H is reduced by such high cost, even if it is not canceled. Observe that now the firm benefits from the choice of I_H because it enjoys a higher probability of success for free. Furthermore, the switch from investment I_M to investment I_H does not lead to any increase in the severance payment, contrary to what happens in the model with bargaining power. In conclusion, with no managerial bargaining power, the expected profit is continuously increasing in overconfidence. A relevant implication is that different degrees of overconfidence may be optimal for the firm according to whether the manager has bargaining power: moderate overconfidence is optimal when the manager has bargaining power while extreme overconfidence is optimal when the board can fire the manager at will.

7.2 | Variable Bargaining Power

Let us now briefly consider a more general framework with variable managerial bargaining power. Specifically, with probability α the board is strong and can fire the manager at will, while with probability $1 - \alpha$ the board is weak and the manager can oppose replacement.[23] In this framework, the baseline model analyzed in Section 5 corresponds to $\alpha = 0$, while the case without bargaining power corresponds to $\alpha = 1$. We further assume that the type of the board is unknown at the contracting stage when only the value of α is common knowledge. Whether the board is weak or strong, however, becomes common knowledge after the investment is undertaken but before the new manager shows up.

Having established that zero contractual severance pay is optimal both when the board is weak and when it is strong, also in the present extension we can set the value of the contractual severance pay to zero. Then, we have to determine the incentive pay necessary to induce investment I_M. The amount of such incentive pay depends on α because the manager anticipates that the possibility to receive the severance payment depends on whether the board is weak or strong. The ICCs for this case and the optimal incentive pay $w_\alpha(\theta, \Delta_M; I_M)$ are presented in Appendix 2. When $\alpha = 1$, the value of $w_\alpha(\theta, \Delta_M; I_M)$ reduces to the expression derived in the case without managerial bargaining power. When instead $\alpha = 0$, the expression reduces to Equation (10). In general, a sufficient condition for $w_\alpha(\theta, \Delta_M; I_M)$ being monotonically increasing in α is $p_M + p_L + \theta < 1$. In other words, the lower is managerial bargaining power, that is, the higher the probability that the manager will not obtain a positive renegotiated severance pay, the larger is the incentive pay necessary to satisfy the binding ICC. The effects of overconfidence and optimism in this setting are in line with those found in the two benchmark cases. Overconfidence reduces the optimal amount of incentive pay, confirming the effect found both with and without bargaining power. The consequences of optimism, instead, depend on the value of alpha: optimism reduces the incentive pay when the board is likely to be strong (α high) while it increases such payment when the board is likely to be weak (α low). This follows from the different impacts of optimism in the two benchmark cases: (i) no effect on incentive pay and positive effect on renegotiated severance pay when the manager has bargaining power ($\alpha = 0$) and (ii) negative effect on incentive pay when the manager has no bargaining power ($\alpha = 1$).

Finally, in Appendix 2 we derive the condition that must be satisfied for I_M to be incentive compatible and we show that it similar to the condition found in the no bargaining case.

8 | Conclusion

The paper examines the interplay between managerial biases and remuneration when the manager has bargaining power so that she can oppose replacement. We consider a setting where the board can fire the manager if a better replacement appears after the manager has undertaken a firm-specific and unverifiable investment. The board offers a contract that comprises incentive pay and severance pay. Severance pay helps motivating the manager to invest despite the anticipated possibility of being replaced. It turns out that the best way to motivate the manager is to offer a low contractual severance pay and to renegotiate the payment ex post in case replacement becomes profitable. The manager anticipates that, by renegotiating the contractual severance agreements, she will be able to recover the cost of the investment. This provides the incentive to invest, despite the risk of being replaced. In other words, renegotiation allows the board to provide the necessary ex-ante incentive by reimbursing the manager for the investment only if this has been actually undertaken.

We show that the degree and the kind of managerial bias matter. Indeed, optimism and overconfidence have different effects on the components of the compensation package. Optimism does not affect incentive pay but raises severance pay (contractual and renegotiated), consequently the expected profit is lower than the one obtained when the manager is unbiased. Overconfidence, instead, decreases incentive pay with a positive effect on profits, while its impact on severance pay depends on the degree of the bias. A moderate level of overconfidence reduces severance pay with no distortion in the investment, but a sufficiently high level of overconfidence induces the manager to choose an inefficiently high investment, which in turn results in a very high renegotiated severance pay, lowering expected profits. Hence, there is a discontinuity with a drop in expected profits at the level of overconfidence that induces the switch from the efficient to the inefficient investment. In summary, the firm benefits from moderate overconfidence while extreme overconfidence and optimism are detrimental. Overall, our model indicates that when the manager has bargaining power there may be a trade-off between the lower cost of the incentive compensation and the higher cost of the severance payment. Hence, it is important to consider severance agreements when studying the effect of managerial optimism and overconfidence because their beneficial impact may be overstated otherwise.

Our findings explain the high payments observed in several turnover events as the result of optimal contract provisions in the presence of managerial biases coupled with some bargaining power originated by the possibility to oppose replacement. We show that some bargaining power is necessary to explain the severance agreements but the severance payments in forced turnover should not be considered a "reward for failure" explained only by the control of a powerful CEOs over weak board. Indeed, they facilitate unverifiable and risky investment by the manager and therefore they may be efficient. Our model suggests that it is optimal to have a minimum contractual severance pay and then renegotiate this amount when the firm has more information consistently with the empirical evidence provided by Goldman and Huang (2015).

We also extend the model to consider alternative assumptions on the manager's bargaining power. We show that when the board can fire the manager at will both contractual and renegotiated severance payments are zero. Furthermore, incentive pay is decreasing not only in overconfidence but also in optimism. Finally we consider a setting with variable bargaining power by assuming that the board can be either weak (corresponding to our baseline model with bargaining power) or strong (corresponding to the case with no managerial bargaining power) with positive probability and we find that the results are in line with those in the two extreme cases.

Acknowledgments

We are grateful to an anonymous referee for helpful comments and suggestions. We thank Laura Abrardi and Stefano Comino for valuable comments. We also benefited from the comments of participants at the ASSET 2021 Annual Conference and GRASS 2023 Workshop on earlier versions of the paper. Open access publishing facilitated by Universita degli Studi di Udine, as part of the Wiley - CRUI-CARE agreement.

Endnotes

[1] See also Cowen, King, and Marcel (2016).

[2] The positive role of severance pay in encouraging risk-taking investment is confirmed by several empirical studies (see among others Cadman, Campbell, and Johnson 2023).

[3] Empirical evidence suggesting that severance pay may curb the manager's incentive to misreport financial information reducing agency costs is provided by Brown (2015).

[4] Note, however, that several papers highlight also the downsides of optimism and overconfidence. De La Rosa (2011) finds that the principal's expected profit decreases in the agent's overconfidence when the agent is significantly optimistic and no effort is implemented. Overconfidence may also induce managers to take too much risk in project choice (Malmendier and Tate 2008; Gervais, Heaton, and Odean 2011) or it may reduce the incentive to gather information about a project leading to inefficient implementation, (Downs 2023). For a comprehensive survey of the large literature on managerial optimism and overconfidence see Malmendier and Tate (2015) and Santos-Pinto and de la Rosa (2020).

[5] This assumption is not necessary for most of our results, it is only used in the proof of Corollary 6 and in Section 7.

[6] "A typical severance contract would detail payments, usually equaling multiple times the CEO's base salary and bonus, as well as continuing/immediate vesting of existing executive stocks and options." (Goldman and Huang 2015; 1110). See also Brown (2015).

[7] The complementarity between overconfidence and effort (investment) is a common assumption in the theoretical literature. Chen and Schildberg-Hörisch (2019) provide experimental evidence that supports it.

[8] Similarly to de La Rosa (2011) and Santos-Pinto (2022), we are assuming that the marginal contribution of the investment to the probability of success is increasing. Note that also the assumption that optimism has a uniform effect on managerial belief is common to de La Rosa and Santos-Pinto.

[9] We allow for the presence of such a nonmonetary benefit for the sake of the comparison with the literature on severance pay. Contrary to what happens in the corporate governance literature, where the benefit usually incentivizes the manager to exert effort at the cost of a distorted replacement decision, in our setting no such trade-off occurs and the benefit plays no role in the results.

[10] The notation used for the contractual components differs from the one used for the renegotiated pay because the contractual payments (both incentive and severance pay) are agreed upon before the investment choice. Therefore the investment level specified in these payments (and the resulting overconfidence) is the one desidered by the board: $\{w(\theta, \Delta_i; I_i), s(\theta, \Delta_i; I_i)\}$. Conversely, the renegotiated severance pay takes into account the possible divergence between the investment undertaken and the one desidered by the board.

[11] A positive level of the reservation utility would not affect this result as long as such level is smaller than the expected compensation in case of no investment, $(p_L + \theta)w(\theta, \Delta_i, I_i) + B$. Note that if it were $(p_L + \theta)w(\theta, \Delta_i, I_i) + B < \bar{u}$, the reverse would hold because ICC 1 would always be satisfied and could be disregarded in the subsequent analysis. However, in the latter case, the PC would become binding which would make (i) the case of overinvestment more likely and (ii) incentive pay also depend on optimism.

[12] Note that $\frac{\partial}{\partial \Delta_M}\left(\frac{p_M + \Delta_M - p_L}{p_H + \Delta_M(1+z) - p_L}\right) = \frac{(p_H - p_L) - (1+z)(p_M - p_L)}{(p_H + \Delta_M(1+z) - p_L)^2} < 0$. It is in fact $1 + z > \frac{I_H}{I_M}$ from Assumption 1, and $\frac{I_H}{I_M} \geqslant \frac{p_H - p_L}{p_M - p_L}$ from Assumption 2.

[13]In fact, if the rate of increase in overconfidence when moving from I_M to I_H were low ($z < \frac{I_H - I_M}{I_M}$), (9) would always be satisfied. Here we want to analyze the interesting case where overconfidence results in the distortion in the investment level.

[14]Note that if $\frac{I_M}{I_H} > \frac{p_M + \Delta_M - p_L}{p_H + \Delta_M(1+z) - p_L}$, it follows that $w(\theta, \Delta_H; I_H) = \frac{I_H}{p_H + \Delta_H - p_L} < \frac{I_M}{p_M + \Delta_M - p_L} = w(\theta, \Delta_M; I_M)$.

[15]The terms moderate and extreme overconfidence in our model only refer to the bias in the belief of the manager's investment productivity (overconfidence in a strict sense). This is different from the distinction in de la Rosa who considers "slight" or "significant overconfidence overall" referring to the sum of the two biases.

[16]This has no effect on the ICCs because $s'(\theta, 0|0 \neq I_M)$ and $s'(\theta, 0|I_j \neq I_M)$ are raised by the same proportion.

[17]It is in fact $s'(\theta, \Delta_H|I_H) - s'(\theta, \Delta_M|I_M) = I_H - I_M$.

[18]This result is quite intuitive, a formal proof that there always exist a value $\tilde{\theta}$: $1 - p_H > \tilde{\theta} \geqslant 0$ such that $V(\theta > \tilde{\theta}, \Delta_M|I_M) < V(0, 0|I_M)$ can be provided upon request.

[19]Specifically, we are looking at forced turnover "without cause" where "for cause" usually refers to conditions such as willful misconduct or breach of fiduciary duties.

[20]This feature of the baseline model allowed us to substitute such exogenously given value of $s'(\theta, \Delta_i|I_i)$ to obtain Equation (6).

[21]Such a proof requires a mild condition on the parameters for the case of a biased manager. No condition is required when the manager is rational. The condition in the case of a biased manager is slightly more restrictive than the one that allows to determine the value for the bonus. Note that, in any case, $\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial s_{NB}(\theta, \Delta_M; I_M)} > 0$ is just a sufficient condition for the optimal severance pay to be equal to zero. The overall effect is negative even when it is $\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial s_{NB}(\theta, \Delta_M; I_M)} < 0$ but small in absolute value, $\left|\frac{\partial V_{NB}}{\partial s_{NB}(\theta, \Delta_i; I_i)}\right| > \frac{\partial V_{NB}}{\partial w_{NB}(\theta, \Delta_i; I_i)} \frac{\partial w_{NB}(\theta, \Delta_i; I_i)}{\partial s_{NB}(\theta, \Delta_i; I_i)}$.

[22]Formally, an increase in θ relaxes the incentive-compatibility constraints in the present context.

[23]We thank an anonimous referee for this suggestion.

References

Almazan, A., and J. Suarez. 2003. "Entrenchment and Severance Pay in Optimal Governance Structures." *Journal of Finance* 58, no. 2: 519–547.

Bebchuk, L. A., and J. Fried. 2004. *Pay Without Performance: The Unfulfilled Promise of Executive Compensation*. Cambridge: Harvard University Press.

Brown, K. E. 2015. "Ex Ante Severance Agreements and Earnings Management." *Contemporary Accounting Research* 32, no. 3: 897–940.

Cadman, B. D., J. L. Campbell, and S. Klasa. 2016. "Are Ex Ante CEO Severance Pay Contracts Consistent With Efficient Contracting?" *Journal of Financial Quantitative Analysis* 51, no. 3: 737–769.

Cadman, B. D., J. L. Campbell, and R. G. Johnson. 2023. "Time-Series Variation in the Efficacy of Executive Risk-Taking Incentives: The Role of Uncertainty." *Accounting Review* 99: 113–141.

Callahan, C. 2024. "Arranging Golden Goodbyes for Executive Exits: A Review and Agenda for Severance Pay." *Journal of Management* 50: 158–187.

Chen, S., and H. Schildberg-Hörisch. 2019. "Looking at the Bright Side: The Motivational Value of Confidence." *European Economic Review* 120: 103302.

Cowen, A. P., A. W. King, and J. J. Marcel. 2016. "CEO Severance Agreements: A Theoretical Examination and Research Agenda." *Academy of Management Review* 41, no. 1: 151–169.

de la Rosa, L. E. 2011. "Overconfidence and Moral Hazard." *Games and Economic Behavior* 73: 429–451.

Downs, J. 2023. "Information Gathering by Overconfident Agents." *Journal of Economics & Management Strategy* 30: 554–568.

Gervais, S., J. B. Heaton, and T. Odean. 2011. "Overconfidence, Compensation Contracts and Capital Budgeting." *Journal of Finance* 66, no. 5: 1735–1777.

Goldman, E. M., and P. P. Huang. 2015. "Contractual vs. Actual Separation Pay Following CEO Turnover." *Management Science* 61, no. 5: 1108–1120.

Green, B., and C. R. Taylor. 2016. "Breakthroughs, Deadlines, and Self-Reported Progress: Contracting for Multistage Projects." *American Economic Review* 106, no. 12: 3660–3699.

Humphery-Jenner, M., L. Lisic, V. Nanda, and S. D. Silveri. 2016. "Executive Overconfidence and Compensation Structure." *Journal of Financial Economics* 119: 533–558.

Inderst, R., and H. Mueller. 2010. "CEO Replacement Under Private Information." *Review of Financial Studies* 23: 2935–2969.

Köszegi, B. 2014. "Behavioral Contract Theory." *Journal of Economic Literature* 52, no. 4: 1075–1118.

Laux, V. 2015. "Executive Pay, Innovation, and Risk-Taking." *Journal of Economics & Management Strategy* 24, no. 2: 275–303.

Malmendier, U., and G. Tate. 2005. "CEO Overconfidence and Corporate Investment." *The Journal of Finance* 60: 2661–2700.

Malmendier, U., and G. Tate. 2008. "Who Makes Acquisitions? CEO Overconfidence and the Market's Reaction." *Journal of Financial Economics* 89, no. 1: 20–43.

Malmendier, U., and G. Tate. 2015. "Behavioral CEOs: The Role of Managerial Overconfidence." *Journal of Economic Perspectives* 29, no. 4: 37–60.

Otto, C. 2014. "CEO Optimism and Incentive Compensation." *Journal of Financial Economics* 114: 366–404.

Santos-Pinto, L. 2008. "Positive Self-Image and Incentives in Organizations." *The Economic Journal* 118: 1315–1332.

Santos-Pinto, L. 2022. "Are Overconfident Workers More Likely to Win Bonuses or to be Promoted?" Discussion Paper.

Santos-Pinto, L., and L. E. de la Rosa. 2020. "Overconfidence in Labor Markets." In *Handbook of Labor, Human Resources and Population Economics*, edited by K. Zimmerman, 1–42. Cham: Springer International Publishing. https://doi.org/10.1007/978-3-319-57365-6.

Sautmann, A. 2013. "Contracts for Agents With Biased Beliefs: Some Theory and an Experiment." *American Economic Journal: Microeconomics* 5, no. 3: 124–156.

Vladimirov, V. 2021. "Contract Length and Severance Pay." CEPR DP 16288.

Wu, Z., and X. Weng. 2018. "Managerial Turnover and Entrenchment." *Journal of Economics & Management Strategy* 27, no. 4: 742–771.

Appendix 1

Proof of Proposition 1. For any given positive level of investment $I_i, i = M, H$, the board maximizes its profit by keeping both the incentive and the severance pay that is actually paid (it can be either the contractual one or the renegotiated one) as low as possible. The incentive compatibility constraints (ICCs) and the participation constraint (PC) must be taken into account. We then want to prove that raising contractual $s(\theta, \Delta_i|I_i)$ above $(p_L + \theta)w(\theta, \Delta_i; I_i) + B \equiv s'(\theta, 0|0 \neq I_i)$ makes the ICCs more binding, thus raising both $w(\theta, \Delta_i; I_i)$ and the severance pay that is paid in case of replacement.

For $I_i, i = M, H$, the PC is

$$\int_0^{\hat{q}(\theta,\Delta_i|I_i)} \left[(p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B\right] f(q)dq + \int_{\hat{q}(\theta,\Delta_i|I_i)}^1 [\max s(\theta, \Delta_i; I_i), s'(\theta, \Delta_i|I_i)] f(q)dq - I_i \geqslant 0, \quad \text{(PC)}$$

where the first term represents the expected compensation in case of retention ($q \leq \hat{q}(\theta, \Delta_i|I_i)$), and the second term represents the expected payment in case of dismissal ($q > \hat{q}(\theta, \Delta_i|I_i)$), provided that the manager has undertaken investment I_i. With an abuse of notation we here denote $\hat{q}(\theta, \Delta_i|I_i) = p_i - \left(\frac{p_i w(\theta, \Delta_i; I_i) - [\max s(\theta, \Delta_i; I_i), s'(\theta, \Delta_i \mid I_i)]}{R}\right)$.

Moreover, two incentive constraints must be satisfied. The first one, ICC 1, guarantees that the manager prefers $I_i, i = M, H$, to not investing

$$\int_0^{\hat{q}(\theta,\Delta_i|I_i)} [(p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B] f(q)dq + \int_{\hat{q}(\theta,\Delta_i|I_i)}^1 [\max s(\theta, \Delta_i; I_i), s'(\theta, \Delta_i|I_i)] f(q)dq - I_i \geqslant \quad \text{(ICC 1)}$$
$$\int_0^{\hat{q}(\theta,0|0\neq I_i)} [(p_L + \theta)w(\theta, \Delta_i, I_i) + B] f(q)dq + \int_{\hat{q}(\theta,0|0\neq I_i)}^1 [\max s(\theta, \Delta_i; I_i), s'(\theta, 0|0 \neq I_i)] f(q)dq,$$

where $\hat{q}(\theta, 0|0 \neq I_i) = p_L - \left(\frac{p_L w(\theta, \Delta_i; I_i) - [\max s(\theta, \Delta_i; I_i), s'(\theta, 0 \mid 0 \neq I_i)]}{R}\right)$.

The second incentive constraint, ICC 2 requires that the manager prefers I_i to I_j $i, j = M, H, i \neq j$ when the contract prescribes investment I_i

$$\int_0^{\hat{q}(\theta,\Delta_i|I_i)} [(p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B] f(q)dq + \int_{\hat{q}(\theta,\Delta_i|I_i)}^1 [\max s(\theta, \Delta_i; I_i), s'(\theta, \Delta_i|I_i)] f(q)dq - I_i \geqslant \quad \text{(ICC 2)}$$
$$\int_0^{\hat{q}(\theta,\Delta_j|I_j\neq I_i)} [(p_j + \theta + \Delta_j)w(\theta, \Delta_i; I_i) + B] f(q)dq + \int_{\hat{q}(\theta,\Delta_j|I_j\neq I_i)}^1 [\max s(\theta, \Delta_i; I_i), s'(\theta, \Delta_j|I_j \neq I_i)] f(q)dq - I_j.$$

where $\hat{q}(\theta, \Delta_j|I_j \neq I_i) = p_j - \left(\frac{p_j w(\theta, \Delta_i; I_i) - [\max s(\theta, \Delta_i; I_i), s'(\theta, \Delta_j \mid I_j \neq I_i)]}{R}\right)$.

Note that the RHS of ICC 1 is positive and that the LHSs of PC and ICC 1 coincide, implying that (PC) is never binding if ICC 1 is satisfied. We then focus on the ICCs consider first ICC 1, dividing the analysis in different cases according to the values of the contractual severance pay $s(\theta, \Delta_i; I_i)$.

Case 1. $s(\theta, \Delta_i; I_i) < (p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B = s'(\theta, \Delta_i|I_i)$. The contract will then be renegotiated in case the manager is dismissed after undertaking I_i. Consequently, the severance pay in the LHS of ICC 1 becomes $s'(\theta, \Delta_i|I_i)$ by Lemma 1 while the cutoff becomes $\hat{q}(\theta, \Delta_i|I_i) \equiv p_i + \frac{(\theta + \Delta_i)w(\theta, \Delta_i; I_i)}{R}$. As to the RHS of ICC 1, two subcases are possible according to the level of the contractual severance pay

a. $s(\theta, \Delta_i; I_i) \leq (p_L + \theta)w(\theta, \Delta_i; I_i) + B = s'(\theta, 0|0)$. In this case renegotiation will occur even if the manager were not to comply with the contract and choose $I_L = 0$. Substituting the expressions for $s'(\theta, \Delta_i|I_i)$ and $s'(\theta, 0|0)$, ICC 1 becomes

$$(p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B - I_i \geqslant (p_L + \theta)w(\theta, \Delta_i; I_i) + B.$$

b. $s(\theta, \Delta_i; I_i) = (p_L + \theta)w(\theta, \Delta_i; I_i) + B + t \leq (p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B = s'(\theta, \Delta_i|I_i)$, with $t > 0$. This makes ICC 1 more binding with respect to the previous subcase, because the RHS is increased. In fact, by substituting the expressions for $s(\theta, \Delta_i; I_i)$ and $s'(\theta, \Delta_i|I_i)$, ICC 1 becomes

$$(p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B - I_i \geqslant (p_L + \theta)w(\theta, \Delta_i; I_i) + B + \int_{\hat{q}(\theta,0|0\neq I_i)}^1 tf(q)dq,$$

where $\hat{q}(\theta, 0|0 \neq I_i) = p_L + \frac{s(\theta, \Delta_i; I_i) - p_L w(\theta, \Delta_i; I_i)}{R}$. The RHS of the above expression is clearly increasing in t.

Case 2. $s(\theta, \Delta_i; I_i) = (p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B + t$ *with* $t > 0$. In this case, the contract will not be renegotiated. If dismissed, the manager will now be paid $s(\theta, \Delta_i; I_i)$ independently of whether she has chosen $I_i, i = M, H$, or $I_L = 0$. This results in cutoff values equal to $\hat{q}(\theta, \Delta_i|I_i) \equiv p_i + \frac{s(\theta, \Delta_i; I_i) - p_i w(\theta, \Delta_i; I_i)}{R}$ if the manager chooses I_i and equal to $\hat{q}(\theta, 0|0 \neq I_i) \equiv p_L + \frac{s(\theta, \Delta_i; I_i) - p_L w(\theta, \Delta_i; I_i)}{R}$ if she were to choose $I_L = 0$. Substituting these values in ICC 1 together with the value of $s(\theta, \Delta_i; I_i)$, the constraint becomes

$$[(p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B] + \int_{\hat{q}(\theta,\Delta_i|I_i)}^1 tf(q)dq - I_i$$
$$\geqslant [(p_L + \theta)w(\theta, \Delta_i; I_i) + B] + \int_{\hat{q}(\theta,0|0\neq I_i)}^1 [(p_i + \Delta_i - p_L)w(\theta, \Delta_i; I_i) + t] f(q)dq$$

which, considering that it is $\hat{q}(\theta, \Delta_i|I_i) > \hat{q}(\theta, 0|0 \neq I_i)$, can also be written as

$$[(p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B] - I_i$$
$$\geqslant [(p_L + \theta)w(\theta, \Delta_i; I_i) + B] + \int_{\hat{q}(\theta,0|0\neq I_i)}^1 [(p_i + \Delta_i - p_L)w(\theta, \Delta_i; I_i)] f(q)dq + \int_{\hat{q}(\theta,0|0\neq I_i)}^{\hat{q}(\theta,\Delta_i|I_i)} tf(q)dq$$

thus making it clear that such expression is more stringent than the one obtained in case (1a) for any given level of $t \geqslant 0$.

In conclusion, as far as ICC 1 is concerned, it would be optimal to set $s(\theta, \Delta_i; I_i) \leq s'(\theta, 0|0)$.

Let us then discuss ICC 2. Consider first the case where the board offers a contract aimed at incentivizing I_M. ICC 2 is

$$\begin{aligned}&\int_0^{\hat{q}(\theta,\Delta_M|I_M)}[(p_M+\theta+\Delta_i)w(\theta,\Delta_M;I_M)+B]f(q)dq\\&+\int_{\hat{q}(\theta,\Delta_M|I_M)}^1[\max s(\theta,\Delta_M;I_M),s'(\theta,\Delta_M|I_M)]f(q)dq-I_M\\\geqslant&\int_0^{\hat{q}(\theta,\Delta_H|I_H\neq I_M)}[(p_H+\theta+\Delta_H)w(\theta,\Delta_M;I_M)+B]f(q)dq\\&+\int_{\hat{q}(\theta,\Delta_H|I_H\neq I_M)}^1[\max s(\theta,\Delta_M;I_M),s'(\theta,\Delta_H|I_H\neq I_M)]\\&f(q)dq\end{aligned}$$

Now the three relevant cases are the following.

i. $s(\theta, \Delta_M; I_M) \leq (p_M + \theta + \Delta_M)w(\theta, \Delta_M; I_M) + B = s'(\theta, \Delta_M|I_M)$, where renegotiation occurs both if the manager chooses I_M and if she chooses I_H. In this case, contractual severance pay does not affect the constraint.

ii. $s(\theta, \Delta_M; I_M) = (p_M + \theta + \Delta_M)w(\theta, \Delta_M; I_M) + B + t < (p_H + \theta + \Delta_H)w(\theta, \Delta_M; I_M) + B$, with $t > 0$. If this is the case, there is no renegotiation if I_M is chosen while there would be renegotiation if I_H were chosen. Then, by substituting the expressions for $s(\theta, \Delta_M; I_M)$ and $s'(\theta, \Delta_H|I_H \neq I_M)$, ICC 2 becomes

$$\begin{aligned}&[(p_M+\theta+\Delta_M)w(\theta,\Delta_M;I_M)+B]-I_M\\&\quad+\int_{\hat{q}(\theta,\Delta_M|I_M)}^1 tf(q)dq\\&\geqslant[(p_H+\theta+\Delta_H)w(\theta,\Delta_M;I_M)+B]-I_H\end{aligned}$$

and is then relaxed by a positive value of t.

iii. $s(\theta, \Delta_M; I_M) = (p_H + \theta + \Delta_H)w(\theta, \Delta_M; I_M) + B + t$, with $t > 0$. In this case there is no renegotiation, either if I_M is chosen or if I_H is chosen instead. ICC 2 is equal to

$$\begin{aligned}&[(p_M+\theta+\Delta_M)w(\theta,\Delta_M;I_M)+B]-I_M+\\&\int_{\hat{q}(\theta,\Delta_M|I_M)}^1[(p_H+\Delta_H-p_M-\Delta_M)w(\theta,\Delta_M;I_M)+B+t]f\\&(q)dq\\\geqslant&[(p_H+\theta+\Delta_H)w(\theta,\Delta_M;I_M)+B]+\int_{\hat{q}(\theta,\Delta_H|I_H\neq I_M)}^1 tf(q)dq\\&-I_H,i,j=M,H,j\neq i,\end{aligned}$$

and is again relaxed by a positive value of t as $\hat{q}(\theta, \Delta_M|I_M) < \hat{q}(\theta, \Delta_H|I_H \neq I_M)$.

The fact that the ICC 2 constraint is relaxed by a high contractual severance pay in cases 2 and 3 raises the question whether such an increase could be beneficial for the firm. We can however verify that the negative effect on ICC 1 dominates the positive effect on ICC 2.

Consider case (ii) where $s(\theta, \Delta_M; I_M) = (p_M + \theta + \Delta_M) w(\theta, \Delta_M; I_M) + B + t < (p_H + \theta + \Delta_H)w(\theta, \Delta_M; I_M) + B$. From ICC 1 (case 2) we obtain that $w(\theta, \Delta_M; I_M)$ must satisfy

$$w(\theta,\Delta_M;I_M)\geqslant\frac{I_M+\int_{\hat{q}(\theta,0|0\neq I_M)}^{\hat{q}(\theta,\Delta_M|I_M)}tf(q)dq}{\int_0^{\hat{q}(\theta,0|0\neq I_M)}(p_M+\Delta_M-p_L)f(q)dq}$$

while from ICC 2 we have

$$\frac{I_H-I_M+\int_{\hat{q}(\theta,\Delta_M|I_M)}^1 tf(q)dq}{p_H+\Delta_H-p_M-\Delta_M}\geqslant w(\theta,\Delta_M;I_M).$$

For both constraints to be simultaneously satisfied it must then be the case that

$$\begin{aligned}&\frac{I_H-I_M+\int_{\hat{q}(\theta,\Delta_M|I_M)}^1 tf(q)dq}{p_H+\Delta_H-p_M-\Delta_M}\\&\geqslant\frac{I_M+\int_{\hat{q}(\theta,0|0\neq I_M)}^{\hat{q}(\theta,\Delta_M|I_M)}tf(q)dq}{\int_0^{\hat{q}(\theta,0|0\neq I_M)}(p_M+\Delta_M-p_L)f(q)dq}.\end{aligned}$$

This is to be compared with the case where $s(\theta, \Delta_M; I_M) \leq s'(\theta, 0|0)$, namely the optimal level of contractual severance pay as far as ICC 1 is concerned. In the latter case, the board renegotiates whenever it wants to dismiss the manager. The relevant inequality that can be derived from the corresponding forms of ICC 1 and ICC 2 is

$$\frac{I_H-I_M}{p_H+\Delta_H-p_M-\Delta_M}\geqslant\frac{I_M}{(p_M+\Delta_M-p_L)}$$

which is less stringent than the inequality above implying that, by choosing $s(\theta, \Delta_M; I_M) \leq s'(\theta, 0|0)$ the board can maximize over a larger set of values for $w(\theta, \Delta_M; I_M)$.

For case (iii) where $s(\theta, \Delta_M; I_M) = (p_H + \theta + \Delta_H)w(\theta, \Delta_M; I_M) + B + t$ we have that it is not possible to satisfy ICC 1 unless the denominator of the following inequality is positive (which depends on the values of the parameters). Clearly if ICC 1 is not satisfied such level of contractual severance pay is not feasible. If, on the contrary, the parameters are such that ICC 1 can be satisfied, $w(\theta, \Delta_M; I_M)$ must now simultaneously satisfy

$$w(\theta,\Delta_M;I_M)\geqslant\frac{I_M+\int_{\hat{q}(\theta,0|0\neq I_M)}^{\hat{q}(\theta,\Delta_M|I_M)}tf(q)dq}{\int_0^{\hat{q}(\theta,0|0\neq I_M)}(p_M+\Delta_M-p_L)f(q)dq-\int_{\hat{q}(\theta,0|0\neq I_M)}^{\hat{q}(\theta,\Delta_M|I_M)}(p_H+\Delta_H-p_M-\Delta_M)f(q)dq}$$

from ICC 1 and

$$\frac{I_H - I_M + \int_{\hat{q}(\theta,\Delta_M|I_M)}^{\hat{q}(\theta,\Delta_H|I_H\neq I_M)} tf(q)dq}{\int_0^{\hat{q}(\theta,\Delta_M|I_M)} (p_H + \Delta_H - p_M - \Delta_M)f(q)dq} \geqslant w(\theta, \Delta_M; I_M).$$

from ICC 2. We can then apply the same line of reasoning as above to show that, by choosing $s(\theta, \Delta_M; I_M) \leq s'(\theta, 0|0)$ the board can maximize over a larger set of values for $w(\theta, \Delta_M; I_M)$.

Consider finally ICC 2 for the case where the board offers a contract aimed at incentivizing I_H. As far as $s(\theta, \Delta_H; I_H)$ is concerned, the relevant cases are the following:

i. if $s(\theta, \Delta_H; I_H) < (p_M + \theta + \Delta_M)w(\theta, \Delta_H; I_H) + B]$, renegotiation occurs both if the manager chooses I_H and if she were to choose I_M so that the value of contractual severance pay is irrelevant.

ii. if $s(\theta, \Delta_H; I_H) = (p_M + \theta + \Delta_M)w(\theta, \Delta_H; I_H) + B + t < (p_H + \theta + \Delta_H)w(\theta, \Delta_H; I_H) + B$ with $t > 0$, we observe renegotiation if I_H is chosen but not if I_M is chosen. By substituting for $s(\theta, \Delta_H; I_H)$ and $s'(\theta, \Delta_M; I_M \neq I_H)$, ICC 2 can be written as

$$\begin{aligned}&[(p_H + \theta + \Delta_H)w(\theta, \Delta_H; I_H) + B] - I_H \\ \geqslant\ & [(p_M + \theta + \Delta_M)w(\theta, \Delta_H; I_H) + B] \\ &\quad + \int_{\hat{q}(\theta,\Delta_M|I_M\neq I_H)}^{1} tf(q)dq - I_M, \\ &\quad i, j = M, H, j \neq i.\end{aligned}$$

which obviously becomes more stringent with a positive t.

iii. if $s(\theta, \Delta_H; I_H) = (p_H + \theta + \Delta_H)w(\theta, \Delta_H; I_H) + B + t$, renegotiation does not occur and $s(\theta, \Delta_H; I_H)$ is paid both if the manager chooses I_H and if she were to choose I_M. By substituting for $s(\theta, \Delta_H; I_H)$, ICC 2 becomes

$$\begin{aligned}&[(p_H + \theta + \Delta_H)w(\theta, \Delta_H; I_H) + B] \\ &\quad + \int_{\hat{q}(\theta,\Delta_H|I_H)}^{1} tf(q)dq - I_H \\ \geqslant\ & [(p_M + \theta + \Delta_M)w(\theta, \Delta_H; I_H) + B] - I_M \\ &\quad + \int_{\hat{q}(\theta,\Delta_M|I_M\neq I_H)}^{1} [(p_H + \Delta_H - p_M - \Delta_M) \\ &\quad w(\theta, \Delta_H; I_H) + B + t]f(q)dq\end{aligned}$$

which is again made more stringent the larger is t as $\hat{q}(\theta, \Delta_M|I_M \neq I_H) < \hat{q}(\theta, \Delta_H|I_H)$.

□

Proof of Proposition 2. The part of the proposition concerning extreme overconfidence is proved in the text. To prove the part on moderate overconfidence, recall that in this case both I_M and I_H can be made incentive compatible. Incentive compatibility of I_M has been discussed in the text where we have shown that it implies offering $w(\theta, \Delta_M; I_M) = \frac{I_M}{(p_M + \Delta_M - p_L)}$. Consider now the ICCs for I_H and note that the condition for moderate overconfidence (9) can be written as $\frac{I_H}{(p_H + \Delta_H - p_L)} \leq \frac{I_H - I_M}{(p_H + \Delta_H - p_M - \Delta_M)}$ implying that, in case the board wants to implement I_H, (12) is binding, and the lowest value of incentive pay is

$$w(\theta, \Delta_H; I_H) = \frac{I_H - I_M}{(p_H + \Delta_H - p_M - \Delta_M)}.$$

Observe that, given (9), it is $w(\theta, \Delta_H; I_H) = \frac{I_H - I_M}{p_H + \Delta_H - (p_M + \Delta_M)} > \frac{I_M}{(p_M + \Delta_M - p_L)} = w(\theta, \Delta_M; I_M)$. This also implies that $\hat{q}(\theta, \Delta_M|I_M) = p_M + \frac{B}{R} + \frac{\theta + \Delta_M}{R} w(\theta, \Delta_M; I_M) < p_H + \frac{B}{R} + \frac{\theta + \Delta_H}{R} w(\theta, \Delta_H; I_H) = \hat{q}(\theta, \Delta_H|I_H)$. We must prove that these two effects make I_H generally unprofitable for the firm that will consequently offer the manager $w(\theta, \Delta_M; I_M)$ to induce I_M. In other words we must prove that the expected profit of the firm is higher under a contract based on $w(\theta, \Delta_M; I_M) = \frac{I_M}{(p_M + \Delta_M - p_L)}$ (to induce the choice of I_M) than under a contract based on $w(\theta, \Delta_H; I_H) = \frac{I_H - I_M}{p_H + \Delta_H - (p_M + \Delta_M)}$ (to induce the choice of I_H).

Taking into account that renegotiation occurs when the manager is replaced (Proposition 1) and that $s'(\theta, \Delta_i|I_i) = (p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B$, the expected profit of the firm $V(\theta, \Delta_i|I_i)$ $i = M, H$, can be written as

$$\begin{aligned}V(\theta, \Delta_i|I_i) = \int_0^{\hat{q}(\theta,\Delta_i|I_i)} & [p_i(R - w(\theta, \Delta_i; I_i))] \\ & f(q)dq + \int_{\hat{q}(\theta,\Delta_i|I_i)}^{1} \Big(qR - (p_i + \theta + \Delta_i) \\ & w(\theta, \Delta_i; I_i) - B\Big)f(q)dq.\end{aligned} \tag{A1}$$

The difference between the expected profit that can be obtained by offering $w(\theta, \Delta_H; I_H) = \frac{I_H - I_M}{p_H + \Delta_H - (p_M + \Delta_M)}$ and $w(\theta, \Delta_M; I_M) = \frac{I_M}{(p_M + \Delta_M - p_L)}$ can consequently be written as

$$\begin{aligned}&V(\theta, \Delta_H|I_H) - V(\theta, \Delta_M|I_M) \\ &= \int_0^{\hat{q}(\theta,\Delta_M|I_M)} (p_H - p_M)(R - w(\theta, \Delta_M; I_M))f(q)dq \\ &\quad - \int_0^{\hat{q}(\theta,\Delta_M|I_M)} p_H(w(\theta, \Delta_H; I_H) - w(\theta, \Delta_M; I_M))f(q)dq + \\ &\quad + \int_{\hat{q}(\theta,\Delta_M|I_M)}^{\hat{q}(\theta,\Delta_H|I_H)} \{p_H(R - (w(\theta, \Delta_M; I_M)) \\ &\quad - [qR - (p_M + \Delta_M + \theta)w(\theta, \Delta_M; I_M) - B]\}f(q)dq + \\ &\quad - \int_{\hat{q}(\theta,\Delta_M|I_M)}^{\hat{q}(\theta,\Delta_H|I_H)} p_H(w(\theta, \Delta_H; I_H) - w(\theta, \Delta_M; I_M))f(q)dq - F,\end{aligned} \tag{A2}$$

where $F \equiv \int_{\hat{q}(\theta,\Delta_H|I_H)}^{1} \{(p_H + \Delta_H - p_M - \Delta_M)w(\theta, \Delta_M; I_M) + (p_H + \Delta_H + \theta)[w(\theta, \Delta_H, I_H) - w(\theta, \Delta_M; I_M)]\}f(q)dq$.

By summing and subtracting $p_M R$ in the first integral of the second line, $V(\theta, \Delta_H|I_H) - V(\theta, \Delta_M|I_M)$ becomes

$$\int_0^{\hat{q}(\theta,\Delta_M|I_M)} (p_H - p_M)(R - w(\theta, \Delta_M; I_M))f(q)dq$$
$$- \int_0^{\hat{q}(\theta,\Delta_M|I_M)} p_H(w(\theta, \Delta_H; I_H) - w(\theta, \Delta_M; I_M))f(q)dq +$$
$$+ \int_{\hat{q}(\theta,\Delta_M|I_M)}^{\hat{q}(\theta,\Delta_H|I_H)} (p_H - p_M)Rf(q)dq - \int_{\hat{q}(\theta,\Delta_M|I_M)}^{\hat{q}(\theta,\Delta_H|I_H)} p_H$$
$$(w(\theta, \Delta_H; I_H) - w(\theta, \Delta_M; I_M))f(q)dq +$$
$$- \int_{\hat{q}(\theta,\Delta_M|I_M)}^{\hat{q}(\theta,\Delta_H|I_H)} \{p_H w(\theta, \Delta_M; I_M) - p_M R$$
$$- [qR - (p_M + \Delta_M + \theta)w(\theta, \Delta_M; I_M) - B]\}f(q)dq - F.$$

Summing $p_M(w(\theta, \Delta_H; I_H) - w(\theta, \Delta_M; I_M))$ to the second and fourth integral and summing up the integrals in the first two lines we can then write

$$V(\theta, \Delta_H|I_H) - V(\theta, \Delta_M|I_M) \leq \int_0^{\hat{q}(\theta,\Delta_H|I_H)} (p_H - p_M)(R - w(\theta, \Delta_M; I_M))f(q)dq +$$
$$- \int_0^{\hat{q}(\theta,\Delta_H|I_H)} (p_H - p_M)(w(\theta, \Delta_H; I_H) - w(\theta, \Delta_M; I_M))f(q)dq +$$
$$- \int_{\hat{q}(\theta,\Delta_M|I_M)}^{\hat{q}(\theta,\Delta_H|I_H)} \{p_H w(\theta, \Delta_M; I_M) - p_M R + [qR - (p_M + \Delta_M + \theta)w(\theta, \Delta_M; I_M) - B]\}f(q)dq - F.$$

Considering also that $I_H - I_M \geqslant (p_H - p_M)R$ we have

$$V(\theta, \Delta_H|I_H) - V(\theta, \Delta_M|I_M) \leq \int_0^{\hat{q}(\theta,\Delta_H|I_H)} [I_H - I_M - (p_H - p_M) w(\theta, \Delta_H; I_H)]f(q)dq +$$
$$- \int_{\hat{q}(\theta,\Delta_M|I_M)}^{\hat{q}(\theta,\Delta_H|I_H)} \{p_H w(\theta, \Delta_M; I_M) - p_M R + [qR - (p_M + \Delta_M + \theta) w(\theta, \Delta_M; I_M) - B]\}f(q)dq - F.$$

Substituting $I_H - I_M = [p_H + \Delta_H - (p_M + \Delta_M)]w(\theta, \Delta_H; I_H)$, recalling that $\Delta_H - \Delta_M = z\Delta_M$, and substituting back for F, the above inequality becomes

$$V(\theta, \Delta_H|I_H) - V(\theta, \Delta_M|I_M) \leq \int_0^{\hat{q}(\theta,\Delta_H|I_H)} z\Delta_M w(\theta, \Delta_H|I_H)f(q)dq$$
$$- \int_{\hat{q}(\theta,\Delta_M|I_M)}^{\hat{q}(\theta,\Delta_H|I_H)} \{p_H w(\theta, \Delta_M; I_M) - p_M R + [qR - (p_M + \Delta_M + \theta)w(\theta, \Delta_M; I_M) - B]\}f(q)dq +$$
$$- \int_{\hat{q}(\theta,\Delta_H|I_H)}^1 (p_H + \Delta_H - p_M - \Delta_M)w(\theta, \Delta_M; I_M)f(q)dq +$$
$$- \int_{\hat{q}(\theta,\Delta_H|I_H)}^1 (p_H + \Delta_H + \theta)[w(\theta, \Delta_H, I_H) - w(\theta, \Delta_M; I_M)]f(q)dq.$$

Considering that $z\Delta_M$ is very small and all the other terms are negative, $V(\theta, \Delta_H|I_H) - V(\theta, \Delta_M|I_M)$ will generally be negative. Recall that we are here considering the case of moderate overconfidence, that is values of $\Delta_M \leq \Delta_M^*$. □

Proof of Corollary 2. The following proof shows that the corollary holds for any value of $\Delta_i \geqslant 0$, including $\Delta_i = 0$. $\frac{\partial w(\theta,\Delta_i;I_i)}{\partial \theta} = 0$ immediately follows from the expression for $w(\theta, \Delta_i; I_i)$ which does not depend on θ. It is immediate that $\frac{\partial s'(\theta,\Delta_i|I_i)}{\partial \theta} = (p_i + \Delta_i)w(\theta, \Delta_i; I_i) > 0$. Finally, $\frac{\partial \hat{q}(\theta,\Delta_i|I_i)}{\partial \theta} > 0$ follows from substituting $w(\theta, \Delta_i; I_i) = \frac{I_i}{(p_i + \Delta_i - p_L)}$ in (6) thus obtaining $\hat{q}(\theta, \Delta_i|I_i) = p_i + \frac{B}{R} + \frac{(\theta + \Delta_i)I_i}{R(p_i + \Delta_i - p_L)}$ which is clearly increasing in θ. □

Proof of Corollary 3. As for the previous corollary, the proof considers any possible value of $\Delta_i \geqslant 0$ including $\Delta_i = 0$. Note that in this case it is $V(\theta, \Delta_i|I_i) = \int_0^{\hat{q}(\theta,\Delta_i|I_i)} [p_i(R - w(\theta, \Delta_i; I_i))]f(q)dq + \int_{\hat{q}(\theta,\Delta_i|I_i)}^1 (qR - (p_i + \theta)w(\theta, \Delta_i; I_i))f(q)dq, i = M, H$. The overall impact of optimism on expected profit, resulting from the combined effects on expected incentive pay, retention policy and expected severance pay is given by

$$\frac{dV(\theta,\Delta_i|I_i)}{d\theta} = \frac{\partial V}{\partial w(\theta,\Delta_i;I_i)}\frac{\partial w(\theta,\Delta_i;I_i)}{\partial \theta} + \frac{\partial V}{\partial s'(\theta,\Delta_i|I_i)}\frac{\partial s'(\theta,\Delta_i|I_i)}{\partial \theta} + \frac{\partial V}{\partial \hat{q}(\theta,\Delta_i|I_i)}\frac{\partial \hat{q}(\theta,\Delta_i|I_i)}{\partial \theta},$$

where the first term on RHS is zero (incentive pay is not affected by θ) as well as the last term because $\hat{q}$ is optimally determined by balancing what is gained from replacement and the payment necessary to have the incumbent leave. It is in fact $\frac{\partial V}{\partial \hat{q}(\theta,\Delta_i|I_i)} = \underbrace{[(\hat{q}(\theta, \Delta_i|I_i) - p_i)R - ((\theta + \Delta_i)w(\theta, \Delta_i|I_i) + B)]}_{=0} f(\hat{q}(\theta, \Delta_i|I_i))$. Then, the total effect on firm profit is negative because we are left with the only effect of severance pay which is indeed negative: $\frac{\partial V}{\partial s'(\theta,\Delta_i;I_i)}\frac{\partial s'(\theta,\Delta_i|I_i)}{\partial \theta} = -w(\theta, \Delta_i|I_i)\int_{\hat{q}(\theta,\Delta_i|I_i)}^1 f(q)dq < 0$. □

Proof of Corollary 4. Again, we provide the proof for values of $p_i - p_L > \theta$, including $\theta = 0$. That the incentive pay is continuously decreasing in Δ_M immediately follows from $w(\theta, \Delta_M; I_M) = \frac{I_M}{(p_M + \Delta_M - p_L)}$ and $w(\theta, \Delta_H; I_H) = \frac{I_H}{(p_H + \Delta_M(1+z) - p_L)}$, considering that at Δ_M^* it is $\frac{I_M}{(p_M + \Delta_M^* - p_L)} = \frac{I_H}{(p_H + \Delta_M^*(1+z) - p_L)}$.

That $s'(\theta, \Delta_i|I_i)$ is decreasing in overconfidence for a given level of investment I_i with $i = M, H$ follows from

$$\frac{\partial s'(\theta,\Delta_M|I_M)}{\partial \Delta_M} = -\left(\frac{I_M}{(p_M + \Delta_M - p_L)}\right)\left(\frac{(p_L + \theta)}{(p_M + \Delta_M - p_L)}\right) < 0.$$
$$\frac{\partial s'(\theta,\Delta_H|I_H)}{\partial \Delta_M} = -\left(\frac{(1+z)I_H}{(p_H + \Delta_M(1+z) - p_L)}\right)\left(\frac{(p_L + \theta)}{(p_H + \Delta_M(1+z) - p_L)}\right) < 0.$$

To verify that $s'(\theta, \Delta_i|I_i)$ is increasing in overconfidence at Δ_M^*, where the shift from I_M to I_H occurs, note that at Δ_M^* it is $w(\theta, \Delta_M|I_M) = w(\theta, \Delta_H|I_H) \equiv w$ implying $s'(\theta, \Delta_M|I_M) = (p_M + \theta + \Delta_M^*)w < (p_H + \theta + \Delta_M^*(1+z))w = s'(\theta, \Delta_H|I_H)$.

To evaluate the effect of overconfidence on the cutoff value for dismissal, substitute $w(\theta, \Delta_i|I_i) = \frac{I_i}{(p_i + \Delta_i - p_L)}$ in (6) thus obtaining $\hat{q}(\theta, \Delta_i|I_i) = p_i + \frac{B}{R} + \frac{(\theta + \Delta_i)I_i}{R(p_i + \Delta_i - p_L)}$. For a given level of investment I_i, $\hat{q}(\theta, \Delta_i|I_i)$ is increasing in the overconfidence parameter Δ_M as

$$\frac{\partial \hat{q}(\theta, \Delta_i|I_i)}{\partial \Delta_M} = \frac{I_i R(p_i + \Delta_i - p_L) - I_i R(\theta + \Delta_i)}{[R(p_i + \Delta_i - p_L)]^2} = \frac{I_i(p_i - p_L - \theta)}{R(p_i + \Delta_i - p_L)^2} > 0, i = M, H$$

which is satisfied for $p_i - p_L > \theta$.

To evaluate what happens at Δ_M^* where the shift from I_M to I_H occurs, note that at Δ_M^* it is $w(\theta, \Delta_i|I_M) = w(\theta, \Delta_i|I_H) \equiv w$ implying that $\hat{q}(\theta, \Delta_i|I_M) = p_M + \frac{B}{R} + \frac{\theta + \Delta_M^*}{R}w < p_H + \frac{B}{R} + \frac{\theta + \Delta_M^*(1+z)}{R}w = \hat{q}(\theta, \Delta_i|I_H)$. Then the cutoff value is increasing also in this point (even if there is a discontinuity). □

Proof of Corollary 5. Note that the proof considers any possible value of $\theta \geqslant 0$ including $\theta = 0$. To prove the corollary we must show that (a) when I_i is chosen, profit is increasing in Δ_i and (b) when (9) holds as an equality, profit is generally higher if I_M is chosen. Consider expression (A1) representing expected profit $V(\theta, \Delta_i|I_i)$, $i = M, H$. Then

(a) For a given $i = M, H$ it is

$$\frac{\partial V(\theta, \Delta_i|I_i)}{\partial \Delta_i} = -p_i \frac{\partial w(\theta, \Delta_i; I_i)}{\partial \Delta_i} - \int_{\hat{q}(\theta, \Delta_i|I_i)}^{1} \left[w(\theta, \Delta_i; I_i) + (\theta + \Delta_i)\frac{\partial w(\theta, \Delta_i; I_i)}{\partial \Delta_i} \right] f(q)dq + -\frac{\partial \hat{q}(\theta, \Delta_i|I_i)}{\partial \Delta_M} \underbrace{[(\hat{q}(\theta, \Delta_i|I_i) - p_i)R - ((\theta + \Delta_i)w(\theta, \Delta_i; I_i) + B)]f}_{=0} (\hat{q}(\theta, \Delta_i|I_i)).$$

In fact, by substituting $\hat{q}(\theta, \Delta_i|I_i) = p_i + \frac{B}{R} + \frac{(\theta + \Delta_i)w(\theta, \Delta_i; I_i)}{R}$, we can immediately verify that the square bracket in the last term of the RHS is equal to zero. Substituting $\frac{\partial w(\theta, \Delta_i; I_i)}{\partial \Delta_i} = -\frac{I_i}{(p_i + \Delta_\theta - p_L)^2} = -\frac{w(\theta, \Delta_i; I_i)}{(p_i + \Delta_i - p_L)} < 0$, we then obtain

$$\frac{\partial V(\theta, \Delta_i|I_i)}{\partial \Delta_i} = p_i \frac{w(\theta, \Delta_i; I_i)}{(p_i + \Delta_i - p_L)} - \int_{\hat{q}(\theta, \Delta_i|I_i)}^{1} \left[w(\theta, \Delta_i; I_i) - \frac{(\theta + \Delta_i)w(\theta, \Delta_i; I_i)}{(p_i + \Delta_i - p_L)} \right] f(q)dq = \frac{w(\theta, \Delta_i; I_i)}{p_i + \Delta_i - p_L} \left[p_i - \int_{\hat{q}(\theta, \Delta_i|I_i)}^{1} (p_i - \theta - p_L) f(q)dq \right] > 0, i = M, H.$$

b) When (9) holds as an equality, the manager is indifferent between I_M and I_H but profits are generally higher in the former case. Define $\Delta_H^* \equiv \Delta_M^*(1+z)$. Note that $w(\theta, \Delta_M^*; I_M) = w(\theta, \Delta_H^*; I_H) = w$ while $\hat{q}(\theta, \Delta_M^*|I_M) = p_M + \frac{B}{R} + \frac{\theta + \Delta_M^*}{R}w < p_H + \frac{B}{R} + \frac{\theta + \Delta_H^*}{R}w = \hat{q}(\theta, \Delta_H^*|I_H)$. Consider that the difference $V(\theta, \Delta_H^*|I_H) - V(\theta, \Delta_M^*|I_M)$ can be written as

$$V\left(\theta, \Delta_H^*|I_H\right) - V\left(\theta, \Delta_M^*|I_M\right) = \int_0^{\hat{q}(\theta, \Delta_M^*|I_M)} (p_H - p_M)(R - w)f(q)dq + \int_{\hat{q}(\theta, \Delta_M^*|I_M)}^{\hat{q}(\theta, \Delta_H^*|I_H)} p_H(R - w)f(q)dq - \int_{\hat{q}(\theta, \Delta_H^*|I_H)}^{1} \left(p_H + \Delta_H^* - p_M - \Delta_M^*\right) wf(q)dq - \int_{\hat{q}(\theta, \Delta_M^*|I_M)}^{\hat{q}(\theta, \Delta_H^*|I_H)} [qR - (p_M + \Delta_M + \theta)w - B] f(q)dq$$

which is equivalent to (A2) in the proof of Proposition 2, where we have substituted $w(\theta, \Delta_M^*; I_M) = w(\theta, \Delta_H^*; I_H) = w$. We can then apply the same line of reasoning to show that

$$V\left(\theta, \Delta_H^*|I_H\right) - V\left(\theta, \Delta_M^*|I_M\right) = \int_0^{\hat{q}\left(\theta, \Delta_H^*|I_H\right)} z\Delta_M wf(q)dq - \int_{\hat{q}\left(\theta, \Delta_M^*|I_M\right)}^{\hat{q}\left(\theta, \Delta_H^*|I_H\right)} \{[q - (p_M + \Delta_M + \theta)w - B] - p_M R + p_H w\} f(q)dq - \int_{\hat{q}\left(\theta, \Delta_H^*|I_H\right)}^{1} (p_H + \Delta_H - p_M - \Delta_M) wf(q)dq,$$

and, considering that $z\Delta_M$ is very small and both the second and the third term are negative, $V\left(\theta, \Delta_H^*|I_H\right) - V\left(\theta, \Delta_M^*|I_M\right)$ will generally be negative. □

Proof of Corollary 6. Consider that expected profits for $\theta = 0$ can be written as

$$V(0, \Delta_i|I_i) = p_i(R - w(0, \Delta_i; I_i)) + \int_{\hat{q}(0, \Delta_i|I_i)}^{1} [(q - p_i)R - \Delta_i w(0, \Delta_i; I_i) - B)] f(q)dq.$$

Then, using the assumption of uniform distribution of q, the difference between $V(0, \Delta_H|I_H)$ and $V(0, 0|I_M)$ is equal to

$$V(0, \Delta_H|I_H) - V(0, 0|I_M) = (p_H - p_M)R - p_H w(0, \Delta_H; I_H)) + p_M w(0, 0; I_M) + -\frac{\hat{q}(0, \Delta_H|I_H)^2 - \hat{q}(0, 0|I_M)^2}{2} - (p_H R + \Delta_H w(0, \Delta_H; I_H) + B)(1 - \hat{q}(0, \Delta_H|I_H)) + (p_M R + B)(1 - \hat{q}(0, 0|I_M)).$$

After some manipulation we can then write

$$
\begin{aligned}
& V(0,\Delta_H|I_H) - V(0,0|I_M) \\
= & \left[p_H\hat{q}(0,\Delta_H|I_H) - p_M\hat{q}(0,0|I_M)\right]R - \frac{\hat{q}(0,\Delta_H|I_H)^2 - \hat{q}(0,0|I_M)^2}{2} \\
& -\Delta_H w(0,\Delta_H;I_H)(1-\hat{q}(0,\Delta_H|I_H)) + [\hat{q}(0,\Delta_H|I_H) - \hat{q}(0,0|I_M)]B \\
& -p_H w(0,\Delta_H;I_H)) + p_M w(0,0;I_M)
\end{aligned}
$$

Recalling that $\hat{q}(0,\Delta_H|I_H) = p_H + \Delta_H\frac{w(0,\Delta_H;I_H)}{R} + \frac{B}{R}$ and $\hat{q}(0,0|I_M) = p_M + \frac{B}{R}$, such expression is equal to

$$
\begin{aligned}
V(0,\Delta_H|I_H) - V(0,0|I_M) = & -\frac{[\Delta_H w(0,\Delta_H;I_H)]^2}{2R} + \frac{(p_H)^2-(p_M)^2}{2}R - \Delta_H\frac{w(0,\Delta_H;I_H)}{R}B \\
& -\Delta_H w(0,\Delta_H;I_H)\left(1-p_H-\Delta_H\frac{w(0,\Delta_H;I_H)+B}{R}\right) + \left(p_H-p_M+\Delta_H\frac{w(0,\Delta_H;I_H)}{R}\right)B+ \\
& +p_M w(0,0;I_M) - p_H w(0,\Delta_H;I_H).
\end{aligned}
$$

To prove the corollary, consider the difference in profits at $\Delta_M^* + \varepsilon$ for $\varepsilon \to 0$. Recall that we have defined $\Delta_H^* \equiv \Delta_M^*(1+z)$ and that $w(0,\Delta_H^*;I_H) = w(0,\Delta_M^*;I_M) = \frac{I_M}{p_M+\Delta_M^*-p_L}$ while $w(0,0;I_M) = \frac{I_M}{p_M-p_L}$. Then, if we consider the case where $B \to 0$, it is

$$
\begin{aligned}
V\left(0,\Delta_H^*|I_H\right) - V(0,0|I_M) = & -\frac{\left[\Delta_H^* w\left(0,\Delta_H^*;I_H\right)\right]^2}{2R} + \\
& +\frac{(p_H)^2-(p_M)^2}{2}R - \Delta_H^* w\left(0,\Delta_H^*;I_H\right)\left(1-p_H-\Delta_H^*\frac{w\left(0,\Delta_H^*;I_H\right)}{R}\right) \\
& +\frac{p_M\Delta_M^* I_M}{\left(p_M+\Delta_M^*-p_L\right)(p_M-p_L)} - \frac{(p_H-p_M)I_M}{\left(p_M+\Delta_M^*-p_L\right)} \\
= & -\Delta_H w\left(0,\Delta_H^*;I_H\right)\left(1-p_H-\frac{\Delta_H^* w\left(0,\Delta_H^*;I_H\right)}{2}\right) + \frac{(p_H)^2-(p_M)^2}{2}R \\
& +\frac{p_M\Delta_M^* w\left(0,\Delta_H^*;I_H\right)}{(p_M-p_L)} - (p_H-p_M)w\left(0,\Delta_H^*;I_H\right),
\end{aligned}
$$

which can also be written as

$$
\begin{aligned}
& V\left(0,\Delta_H^*|I_H\right) - V(0,0|I_M) \\
= & \Delta_M^* w\left(0,\Delta_H^*;I_H\right)\left[\frac{p_M}{(p_M-p_L)} - 1 + p_H + \frac{\Delta_H w\left(0,\Delta_H^*;I_H\right)}{2R}\right] \\
& -z\Delta_M^* w\left(0,\Delta_H^*;I_H\right)\left[1-p_H-\frac{\Delta_H^* w(0,\Delta_H;I_H)}{2R}\right] \\
& +\frac{(p_H)^2-(p_M)^2}{2}R.
\end{aligned}
$$

This expression can be negative for $p_H - p_M$ small, possibly $p_H - p_M \to 0$, if

$$
z > \frac{\frac{p_L}{(p_M-p_L)} + p_H + \frac{\Delta_H^* w\left(0,\Delta_H^*;I_H\right)}{2R}}{1-p_H-\frac{\Delta_H^* w\left(0,\Delta_H^*;I_H\right)}{2R}}.
$$

□

Appendix 2

Proof of Proposition 3. To prove that the optimal value of the severance pay is equal to zero, we need to derive the values of $w_{NB}(\theta,\Delta_M;I_M)$ and to show that $\frac{\partial w_{NB}(\theta,\Delta_M;I_M)}{\partial s_{NB}(\theta,\Delta_M;I_M)} > 0$.

Assume for simplicity that $B = 0$ and consider investment I_M. Then ICC 1 becomes

$$
\begin{aligned}
& \int_0^{\hat{q}_{NB}(\theta,\Delta_M|I_M)} (p_M+\Delta_M+\theta)w_{NB}(\theta,\Delta_M;I_M)f(q)dq \\
& + \int_{\hat{q}_{NB}(\theta,\Delta_M|I_M)}^1 s_{NB}(\theta,\Delta_M;I_M)f(q)dq - I_M \\
\geqslant & \int_0^{\hat{q}_{NB}(\theta,0|0\neq I_M)} (p_L+\theta)w_{NB}(\theta,\Delta_M;I_M)f(q)dq \\
& + \int_{\hat{q}_{NB}(\theta,0|0\neq I_M)}^1 s_{NB}(\theta,\Delta_M;I_M)f(q)dq
\end{aligned}
$$

where

$$
\hat{q}_{NB}(\theta,\Delta_M|I_M) = p_M - \frac{(p_M w_{NB}(\theta,\Delta_M;I_M) - s_{NB}(\theta,\Delta_M;I_M))}{R}
$$

and

$$
\hat{q}_{NB}(\theta,0|0\neq I_M) = p_L - \frac{(p_L w_{NB}(\theta,\Delta_M;I_M) - s_{NB}(\theta,\Delta_M;I_M))}{R}.
$$

Let $x = (p_M + \theta + \Delta_M)$ and $y = (p_L + \theta)$. Using the assumption of uniform distribution we can write ICC 1 as

$$w_{NB}(\theta, \Delta_M; I_M)[\hat{q}_{NB}(\theta, \Delta_M|I_M)x - \hat{q}_{NB}(\theta, 0|I_L \neq I_M)y] - s(\hat{q}_{NB}(\theta, \Delta_M|I_M) - \hat{q}_{NB}(\theta, 0|I_L \neq I_M)) - I_M \geqslant 0. \quad (B1)$$

Substituting the expression for $\hat{q}_{NB}(\theta, \Delta_M|I_M)$ and $\hat{q}_{NB}(\theta, 0|0 \neq I_M)$ we obtain

$$-[w_{NB}(\theta, \Delta_M; I_M)]^2\left(\frac{xp_M - yp_L}{R}\right) + w_{NB}(\theta, \Delta_M; I_M)\left[(xp_M - yp_L) + \frac{s_{NB}(\theta, \Delta_M; I_M)}{R}[2(p_M - p_L) + \Delta_M]\right] - s_{NB}(\theta, \Delta_M; I_M)(p_M - p_L) - I_M \geqslant 0.$$

Suppose that ICC 2 is satisfied. Then the incentive bonus, which satisfies the above condition in the form of an equality, is given by

$$w_{NB}(\theta, \Delta_M; I_M) = \frac{R}{2} + \frac{s_{NB}(\theta, \Delta_M; I_M)[2(p_M - p_L) + \Delta_M]}{2(xp_M - yp_L)} + - \frac{R}{2(xp_M - yp_L)} * \left\{(xp_M - yp_L)^2 + \frac{[s_{NB}(\theta, \Delta_M; I_M)]^2[2(p_M - p_L) + \Delta_M]^2}{R^2} + 2\frac{s_{NB}(\theta, \Delta_M; I_M)}{R}(xp_M - yp_L)\Delta_M . - \frac{4(xp_M - yp_L)I_M}{R}\right\}^{\frac{1}{2}} \quad (B2)$$

It is immediate to verify that the value of the discriminant is increasing in s. Consequently the condition for the discriminant to be positive for any value of $s_{NB}(\theta, \Delta_M; I_M) \geqslant 0$ is $(p_M^2 - p_L^2 + \theta(p_M - p_L) + p_M\Delta_M) = (xp_M - yp_L) > \frac{4I_M}{R}$.

The derivative of the incentive pay with respect to is

$$\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial s_{NB}(\theta, \Delta_M; I_M)} = \frac{2(p_M - p_L) + \Delta_M}{2(xp_M - yp_L)} + -\frac{1}{2}\left\{(xp_M - yp_L)^2 + \frac{[s_{NB}(\theta, \Delta_M; I_M)]^2[2(p_M - p_L) + \Delta_M]^2}{R^2} + 2\frac{s_{NB}(\theta, \Delta_M; I_M)}{R}(xp_M - yp_L)\Delta_M - \frac{4(xp_M - yp_L)I_M}{R}\right\}^{-\frac{1}{2}} \left(\frac{s_{NB}(\theta, \Delta_M; I_M)[2(p_M - p_L) + \Delta_M]^2}{R(xp_M - yp_L)} + \Delta_M\right]\bigg).$$

Evaluated at $s_{NB}(\theta, \Delta_M; I_M) = 0$, the above expression becomes

$$\left.\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial s_{NB}(\theta, \Delta_M; I_M)}\right|_{s=0} = \frac{2(p_M - p_L) + \Delta_M}{2(xp_M - yp_L)} - \left((xp_M - yp_L)^2 - \frac{4(xp_M - yp_L)I_M}{R}\right)^{-\frac{1}{2}} \frac{\Delta_M}{2}.$$

Then, $\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial s_{NB}(\theta, \Delta_M; I_M)} > 0$ if and only if

$$(xp_M - yp_L)\left(1 - \frac{\Delta_M^2}{[2(p_M - p_L) + \Delta_M]^2}\right) > \frac{4I_M}{R} \quad (B3)$$

which is satisfied for I_M sufficiently small with respect to R. Such condition is slightly more restrictive than the condition for the discriminant to be positive. □

The effect of optimism and overconfidence on incentive pay

As to the effects of optimism and overconfidence on incentive pay, they can be immediately derived from (B2). Having established that it is optimal not to pay any severance pay, we evaluate these effects at $s^{NB}(\theta, \Delta_M; I_M) = 0$

$$\left.\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial \theta}\right|_{s^{NB}(\theta, \Delta_M; I_M)=0} = \frac{R(p_M - p_L)}{2(xp_M - yp_L)^2}\left\{(xp_M - yp_L)^2 - \frac{4(xp_M - yp_L)I_M}{R}\right\}^{\frac{1}{2}} + -\frac{R[(p_M - p_L)(xp_M - yp_L)^2 - 2(p_M - p_L)(xp_M - yp_L)I_M)/R]}{2(xp_M - yp_L)^2} \left\{(xp_M - yp_L)^2 - \frac{4(xp_M - yp_L)I_M}{R}\right\}^{-\frac{1}{2}}$$

which is negative as

$$sign\left.\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial \theta}\right|_{s^{NB}(\theta, \Delta_M; I_M)=0} = sig \; n - \frac{2(xp_M - yp_L)I_M}{R} < 0.$$

$$\left.\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial \Delta_M}\right|_{s^{NB}(\theta, \Delta_M; I_M)=0} = \frac{Rp_M}{2(xp_M - yp_L)^2}\left\{(xp_M - yp_L)^2 + - \frac{4(xp_M - yp_L)I_M}{R}\right\}^{\frac{1}{2}} - \frac{Rp_M[2(xp_M - yp_L)^2 - 4(xp_M - yp_L)I_M/R]}{2(xp_M - yp_L)^2}\left\{(xp_M - yp_L)^2 - \frac{4(xp_M - yp_L)I_M}{R}\right\}^{-\frac{1}{2}}$$

which is negative as

$$sign\frac{\partial w_{NB}(\theta,\Delta_M;I_M)}{\partial \Delta_M}\Bigg|_{s^{NB}(\theta,\Delta_M;I_M)=0} = sig$$
$$n - \frac{2(xp_M - yp_L)I_M}{R} < 0.$$

We can then conclude that $w_{NB}(\theta,\Delta_M;I_M)$ is decreasing in both θ and Δ_M implying that $w_{NB}(\theta,\Delta_M;I_M) < w_{NB}(0,0;I_M)$. In other words, also when the manager has no bargaining power, incentive pay for a biased manager is lower than for a rational one. The difference is that here also optimism reduces incentive pay.

Incentive compatibility with respect to I_H

Finally, consider the issue of whether the above solution with investment I_M is incentive compatible with respect to the choice of I_H. Setting $s_{NB}(\theta,\Delta_M;I_M)=0$, ICC 2 can now be written as

$$\int_0^{\hat{q}_{NB}(\theta,\Delta_M|I_M)}(p_M+\Delta_M+\theta)w_{NB}(\theta,\Delta_M;I_M)f(q)dq - I_M$$
$$\geqslant \int_0^{\hat{q}_{NB}(\theta,\Delta_H|I_H\neq I_M)}(p_H+\Delta_H+\theta)w_{NB}(\theta,\Delta_M;I_M)f(q)dq - I_H$$

which, considering the assumption of uniform distribution, becomes

$$w_{NB}(\theta,\Delta_M;I_M)[\hat{q}_{NB}(\theta,\Delta_M|I_M)x - \hat{q}_{NB}(\theta,\Delta_H|I_H\neq I_M)j] \geqslant I_M - I_H$$

where $j \equiv (p_H+\theta+\Delta_H)$.

For the solution found for $w_{NB}(\theta,\Delta_M;I_M)$ using ICC 1 to be incentive compatible, the following must then hold

$$\frac{I_M}{\hat{q}_{NB}(\theta,\Delta_M|I_M)x - \hat{q}_{NB}(\theta,0|I_L\neq I_M)y} = w_{NB}(\theta,\Delta_M;I_M) \leq \frac{I_H - I_M}{\hat{q}_{NB}(\theta,\Delta_H|I_H)j - \hat{q}_{NB}(\theta,\Delta_M|I_M)x}$$

which requires

$$\frac{I_M}{I_H} \leq \frac{xp_M - yp_L}{jp_H - yp_L}. \tag{B4}$$

The manager chooses I_M when the above condition is satisfied while she chooses I_H otherwise.

Variable bargaining power

We here derive the optimal value of the incentive bonus $w_\alpha(\theta,\Delta_M;I_M)$ in the case of variable bargaining power. Recall that with probability α the board is strong and the manager cannot oppose replacement while with probability $(1-\alpha)$ the board is weak and pays the renegotiated severance pay derived in the baseline model. Contractual severance pay $s_\alpha(\theta,\Delta_M;I_M)$ is equal to zero. Then, the ICC 1 for investment I_M is

$$(1-\alpha)(p_M+\Delta_M+\theta)w_\alpha(\theta,\Delta_M;I_M) + \int_0^{\hat{q}_{NB}(\theta,\Delta_M|I_M)}\alpha(p_M+\Delta_M+\theta)w_\alpha(\theta,\Delta_M;I_M)f(q)dq - I_M$$
$$\geqslant (1-\alpha)(p_L+\theta)w_\alpha(\theta,\Delta_M;I_M) + \int_0^{\hat{q}_{NB}(\theta,0|0\neq I_M)}\alpha(p_L+\theta)w_\alpha(\theta,\Delta_M;I_M)f(q)dq$$

which simplifies to

$$(1-\alpha)(p_M - p_L + \Delta_M)w_\alpha(\theta,\Delta_M;I_M) + \alpha\left\{\int_0^{\hat{q}_{NB}(\theta,\Delta_M|I_M)}(p_M+\Delta_M+\theta)w_\alpha(\theta,\Delta_M;I_M)f(q)dq - \int_0^{\hat{q}_{NB}(\theta,0|0\neq I_M)}(p_L+\theta)w_\alpha(\theta,\Delta_M;I_M)f(q)dq\right\} \geqslant I_M \tag{B5}$$

where the bracket is positive.

Solving the equality for $w_\alpha(\theta,\Delta_M;I_M)$ and recalling that $x \equiv (p_M+\theta+\Delta_M)$ and $y \equiv (p_L+\theta)$ we have the following second degree equation where we simplify notation by writing w_α rather than $w_\alpha(\theta,\Delta_M;I_M)$.

$$-w_\alpha^2\frac{\alpha}{R}(xp_M - yp_L) + w_\alpha[\alpha(xp_M - yp_L) + (1-\alpha)(p_M - p_L + \Delta_M)] - I_M = 0$$

The solution is:

$$w_\alpha = \frac{[\alpha(xp_M - yp_L) + (1-\alpha)(x-y)] - \sqrt[2]{[\alpha(xp_M - yp_L) + (1-\alpha)(x-y)]^2 - 4\frac{\alpha}{R}(xp_M - yp_L)I_M}}{2\frac{\alpha}{R}(xp_M - yp_L)}. \tag{B6}$$

Consider then the issue of whether the above solution w_α with investment I_M is incentive compatible with respect to the choice of I_H. Considering that $s_\alpha(\theta,\Delta_M;I_M)=0$, ICC 2 can now be written as

$$(1-\alpha)(p_M+\Delta_M+\theta)w_\alpha(\theta,\Delta_M;I_M) + \int_0^{\hat{q}_{NB}(\theta,\Delta_M|I_M)}\alpha(p_M+\Delta_M+\theta)w_\alpha(\theta,\Delta_M;I_M)f(q)dq - I_M$$
$$\geqslant (1-\alpha)(p_H+\Delta_H+\theta)w_\alpha(\theta,\Delta_M;I_M) + \int_0^{\hat{q}_{NB}(\theta,\Delta_H|I_H\neq I_M)}\alpha(p_H+\Delta_H+\theta)w_\alpha(\theta,\Delta_M;I_M)f(q)dq - I_H$$

which, using the assumption of uniform distribution, becomes

$$(1-\alpha)w_\alpha(\theta,\Delta_M;I_M)(x-j) + \alpha w_\alpha(\theta,\Delta_M;I_M)[\hat{q}_{NB}(\theta,\Delta_M|I_M)x - \hat{q}_{NB}(\theta,\Delta_H|I_H\neq I_M)j] \geqslant I_M - I_H$$

where $j \equiv (p_H+\theta+\Delta_H)$.

For the above solution to be incentive compatible, the following must then hold

$$w_\alpha(\theta,\Delta_M;I_M) \leq \frac{I_H - I_M}{(1-\alpha)(j-x) + \alpha[\hat{q}_{NB}(\theta,\Delta_H|I_H)j - \hat{q}_{NB}(\theta,\Delta_M|I_M)x]}.$$

The above inequality is satisfied if

$$\frac{I_M}{I_H} \leq \frac{(1-\alpha)(x-y) + \alpha[\hat{q}_{NB}(\theta,\Delta_M|I_M)x - \hat{q}_{NB}(\theta,0|I_L\neq I_M)y]}{(1-\alpha)(j-x) + \alpha[\hat{q}_{NB}(\theta,\Delta_H|I_H)j - \hat{q}_{NB}(\theta,0|I_L\neq I_M)y]}.$$

Finally, let us prove that $w_\alpha(\theta, \Delta_M; I_M)$ is increasing in α when $p_M + p_L + \theta < 1$.

Let us define $A \equiv \alpha(xp_M - yp_L) + (1-\alpha)(x-y)$, $C \equiv (xp_M - yp_L)$, $B \equiv (x-y)$ and rewrite expression (B6) in the following way

$$w_\alpha(\theta, \Delta_M; I_M) = \frac{R}{2}\left\{1 + \frac{(1-\alpha)B}{\alpha C} - \frac{\sqrt{A^2 - \frac{4}{R}\alpha C I_M}}{\alpha C}\right\}.$$

Observe that neither C nor B depend on α while $\frac{\partial A}{\partial \alpha} = C - B$. Then

$$\frac{\partial w_\alpha}{\partial \alpha} = \frac{R}{2}\left\{-\frac{(\alpha BC + (1-\alpha)BC)}{\alpha^2 C^2} - \frac{\left(2A(C-B) - \frac{4\alpha C I_M}{R}\right)}{2\sqrt{A^2 - \frac{4\alpha C I_M}{R}}}\frac{\alpha C}{\alpha^2 C^2} + \frac{C\sqrt{A^2 - \frac{4\alpha C I_M}{R}}}{\alpha^2 C^2}\right\},$$

and

$$sign\frac{\partial w_\alpha}{\partial \alpha} = sign\left\{-B\sqrt{A^2 - \frac{4\alpha C I_M}{R}} - A\left((C-B) - \frac{2CI_M}{R}\right)\alpha + \left(A^2 - \frac{4\alpha C I_M}{R}\right)\right\}$$

Note that $\left(A^2 - \frac{4\alpha C I_M}{R}\right) \geqslant B\sqrt{A^2 - \frac{4\alpha C I_M}{R}}$. Then, a sufficient condition for $sign\frac{\partial w_\alpha}{\partial \alpha} > 0$ is $C - B < 0$ or $(xp_M - yp_L) < (x-y)$ and this is always the case for $p_M + p_L + \theta < 1$.