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RESEARCH

The Role of Absolute Space in Newton's Physics

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Abstract
In this paper, we examine Newton's concept of absolute space. We aim at clarifying: (1) the reasons that led Newton to introduce this notion; (2) the arguments he used to justify its existence; (3) the relationship between absolute space and the principle of inertia; and (4) the role of absolute space within Newtonian physics. We then analyse the criticisms raised by Leibniz and Mach, focusing on their specific arguments rather than their broader philosophical frameworks, due to the space limitations of this article. We conclude by discussing Einstein's views, particularly how his theory of relativity ultimately ruled out the existence of absolute space and time. Nevertheless, we argue that in the context of Newtonian physics, the concept of absolute space was not at all absurd, even if it presented certain conceptual difficulties. The paper ends with an appendix highlighting noteworthy annotations from the editors of the Geneva Edition to Proposition XIX of the third book of Newton's *Principia*.

Keywords Absolute space · Dynamics · Force · Inertia · Newton · Mach · Einstein · Geneva edition

1 Introduction

Newton supported the existence of absolute time and space –this is well known. In particular, he emphasized the role of absolute motions and absolute space through the famous bucket experiment and through other arguments found in the *Principia* themselves as well as in the early work *De gravitatione et equipondio fluidorum* and in the letters to Richard

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Bentley[1] (1662–1742) written between 1692 and 1693. Accordingly, this paper focuses on the concept of absolute space, rather than that of absolute time.

The criticisms of Newton's absolute entities are also well known – for instance, those formulated by Leibniz (1646–1716) and Berkeley (1685–1753) and, more recently, the views of Ernst Mach (1838–1916) and Henri Poincaré (1854–1912)[2].

The view that space is a neutral container for phenomena—independent of those phenomena themselves—rather than an order of coexistences defined by the relative positions and motions of objects, appears philosophically naïve. Why, then, did a scientist of Newton's caliber insist so strongly on the necessity of absolute space? The answer is twofold: as the General Scholium to the *Principia*, the letters to Bentely and the queries 28 and 31 to *Optiks* show, the concept of absolute space is tied to Newton's theological and metaphysical convictions[3]. On the other hand, Newton introduced this notion due to internal necessities within his physics since absolute space is deeply connected with the main law of Newtonian physics, the principle of inertia, or, at least, this was Newton's view. We will deal only with the physical aspect that induced Newton to introduce absolute space and absolute motions.

In Sect. 2, we shall analyse the famous bucket experiment, through which Newton believed he had demonstrated the existence of absolute space and absolute motions. In Sect. 3 the role of absolute space in Newtonian physics will be clarified. In Sect. 4 the connection between movement in absolute space and form of celestial bodies is analysed. In Sect. 5 Leibniz's and Mach's criticisms to the notion of absolute space are examined. In the Conclusion some opinions of Albert Einstein (1879–1955) on absolute space will be explained. They are useful to understand how the man who constructed the theory that eliminated forever absolute space from physics, expressed less severe criticisms towards this concept than those raised by other physicists and philosophers. Finally, in the Appendix we will refer to the notes of the GE relative to Proposition XIX of *Principia*'s third book.

2 Existence of Absolute Space: The Bucket Experiment

Newton did not take the existence of absolute space for granted; rather he sought to prove it through an *experimentum crucis*. He remained faithful to the experimental philosophy which he regarded as the only methodologically valid approach in scientific inquiry. If absolute space is to be considered a physical concept – albeit of a particular kind – then, as Newton saw it, there must be at least one experiment capable of demonstrating its reality. In this regard, it is sufficient to recall Newton's famous statement in the General Scolium at the end of the second and third editions of the *Principia* concerning the origin of the force of gravity: Newton refused to hazard any hypothesis about its origin, because such hypotheses would have been experimentally unverifiable, "For whatever is not deduced from the

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[1] The four letters written by Newton to Bentley between 1692 and 1693 deal to a large extent with the problem of the relationship between God and the physical world. They also include interesting considerations concerning the possible finiteness or infinity of the universe (see Newton 1692-93).

[2] We will deal with Leibniz's and Mach's criticisms to Newton's concept of absolute space. As to the observations by Berkeley and Poincaré see resp.: Berkeley, 1721 (English translation 2008), Poincaré, 1908, Second part, Fifth Chapter. English translation 2007.

[3] On the connection between Newton's concept of absolute space and his theological as well as metaphysical convictions, see, e,g. Bussotti & Lotti, 2022, pp. 239–246, Iliffe, 2017; Lotti, 2006; McGuire, 1978a, b, 1995; Snobelen, 2005.

phenomena must be called a hypothesis; and hypotheses, whether metaphysical or physical, or based on occult qualities, or mechanical, have no place in experimental philosophy. In this experimental philosophy, propositions are deduced from the phenomena, and are made general by induction"[4].

In defining absolute space and its associated notion of absolute motion, Newton proceeded with logical precision: he provided definitions, but carefully avoided treating them as sufficient grounds for asserting existence via pure reasoning alone. In the long Scolium that closes the *Definitions*, he writes:

> Absolute space, of its own nature without reference to anything external, always remains homogeneous and immovable. Relative space is any movable measure or dimension of this absolute space; such a measure or dimension is determined by our senses from the situation of the space with respect to bodies and is popularly used for immovable space, as in the case of space under the earth or in the air or in the heavens, where the dimension is determined from the situation of the space with respect to the earth. Absolute and relative space are the same in species and in magnitude, but they do not always remain the same numerically. For example, if the earth moves, the space of our air, which in a relative sense and with respect to the earth always remains the same, will now be one part of the absolute space into which the air passes, now another part of it, and thus will be changing continually in an absolute sense. (Newton, 1999, pp. 408–409).

Newton's definition is clear:

1) Absolute space exists independently of physical phenomena.
2) It differs from relative space "by number"—that is, by position with respect to various frames of reference. For example, if the Earth is taken as the relative reference frame, the space containing the air appears stationary; yet, with respect to absolute space, it is in motion.

Absolute motions are then the motions considered with respect to absolute space.

Having defined the concept, Newton then sought to justify the existence of absolute motion via his bucket experiment:

> The effects distinguishing absolute motion from relative motion are the forces of receding from the axis of circular motion. For in purely relative circular motion these forces are null, while in true and absolute circular motion they are larger or smaller in proportion to the quantity of motion. If a bucket is hanging from a very long cord and is continually turned around until the cord becomes twisted tight, and if the bucket is thereupon filled with water and is at rest along with the water and then, by some sudden force, is made to turn around in the opposite direction and, as the cord unwinds, perseveres for a while in this motion; then the surface of the water will at first be level, just as it was before the vessel began to move. But after the vessel, by the force

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[4] Newton, 1999, p. 943. It should be emphasised that hypothesis, in Newton, is a polysemic term. Here we refer to hypotheses about the possible existence of physical objects under conditions in which no experiment to determine such existence is conceivable.

> gradually impressed upon the water, has caused the water also to begin revolving perceptibly, the water will gradually recede from the middle and rise up the sides of the vessel, assuming a concave shape (as experience has shown me), and, with an ever faster motion, will rise further and further until, when it completes its revolutions in the same times as the vessel, it is relatively at rest in the vessel. The rise of the water reveals its endeavor to recede from the axis of motion, and from such an endeavor one can find out and measure the true and absolute circular motion of the water, which here is the direct opposite of its relative motion. In the beginning, when the relative motion of the water in the vessel was greatest, that motion was not giving rise to any endeavor to recede from the axis; the water did not seek the circumference by rising up the sides of the vessel but remained level, and therefore its true circular motion had not yet begun. But afterward, when the relative motion of the water decreased, its rise up the sides of the vessel revealed its endeavor to recede from the axis, and this endeavor showed the true circular motion of the water to be continually increasing and finally becoming greatest when the water was relatively at rest in the vessel. Therefore, that endeavor does not depend on the change of position of the water with respect to surrounding bodies, and thus true circular motion cannot be determined by means of such changes of position (*Ivi*, pp. 412–413).

The phases of the experiment can be expounded as follows:

1) The rope suspending the bucket is twisted
2) The bucket is filled with water and left stationary
3) The rope is released, causing the bucket to spin
4) Initially, the water remains still unaffected by the bucket's motion
5) After a while, the bucket's rotation is transmitted to the water, which begins to rotate and climb the walls of the vessel. This behaviour is due entirely to the principle of inertia: if unconstrained by the bucket's walls, each particle of water would move tangentially with the velocity it acquired from the rotation, effectively flying outward. This, as Newton writes, can be seen as an endeavor to move away from the axis of motion. From the point of view of the individual water particle, a centrifugal force is felt because the particle cannot move along the tangent, but must remain in the bucket. Therefore, the particles rise up the walls of the vessel forming a concave shape. Such a shape depends, hence, on the inertia principle.
6) The circular motion of the water increases as the motion relative to the bucket decreases
7) This means that to the same kinetic state of the bucket (its rotation around the rope) two different kinetic states of the water corresponds: at the beginning when the relative motion water-bucket is maximal because the bucket rotates, while this is not the case for water, no centrifugal force exists; in the second phase, when the relative motion water-bucket is null (for both rotate), inertial forces are present. Since the same kinetic state of the bucket corresponds to two kinetic states of the water, the rotation and the ascent of the water on the bucket's walls cannot be referred to the bucket, and, in fact, to none of the neighboring bodies

The experiment may be extended, in keeping with Newton's reasoning: after some time, the bucket ceases to rotate, yet the water continues to spin for a while. Eventually, the water comes to rest and its surface flattens once again (Fig. 1).

The four phases can be expressed in the following table (Table 1):

This extended version of the experiment provides further confirmation that the rotational motion of the water is not determined by its relative motion with respect to the bucket. For, when the relative motion is null (phases 2 and 4) the water rotates in the phase 2, but does not rotate in the phase 4; when the relative motion is maximal (phases 1 and 3), the water does not rotate in the phase 1, but rotates in the phase 3. Newton's conclusion is thus legitimate: the rotation and ascent of the water cannot be attributed to its motion relative to surrounding bodies—as Descartes had proposed. Since such movements indicate water's inertia, inertial state itself cannot be referred to the bodies around the water.

From a geometrical and kinematic standpoint, the problem can be set as follows: since rotational motion and ascent indicate water's inertia with respect to what entity is such inertia to be defined? Since inertia implies uniform rectilinear motion, with respect to what a uniform rectilinear motion must be determined? It is worth noting that even from a purely geometrical point of view, this issue is complex: how does one establish whether a line

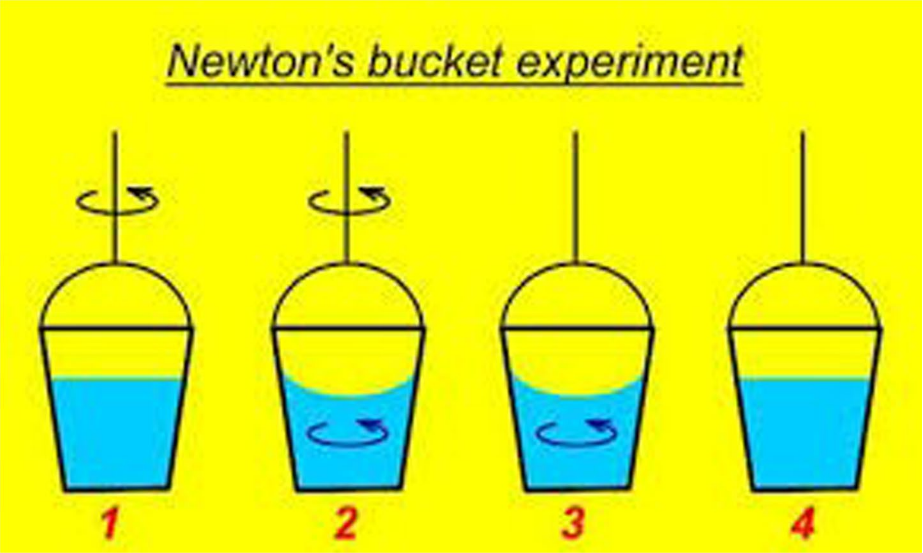

Fig. 1 The four phase of the bucket experiment

Table 1 Lack of connection between bucket's and water's movement

	Relative motion water/bucket	Surface of the water	Rotation of the water
1	Maximal	Flat	No
2	Absent	Curved	Yes
3	Maximal	Curved	Yes
4	Absent	Flat	No

is truly straight, and with respect to what is that determined? Newton's answer is: (1) the rotation and ascent of the water along the bucket's walls reveals absolute motion; (2) such a motion must be referred to the previously defined entity: absolute space. We will clarify these topics in the next Section where, in connection, to the bucket experiment and to the criticism to Descartes' physics the idea that Newton thought to have proved the existence of absolute space is argued.

3 Absolute Space and Newton's Physics

As mathematical reasoning became central to physics, and as increasingly complex problems—such as those Newton addressed—were explored, the ability to establish a reference system for each phenomenon became crucial.

Given a particular problem, one is, in principle, free to choose and adjust a reference frame as needed. We can then compare the results that are obtained, for the same problem, in different reference systems and find the transformations that allow us to pass from one system to another. While this approach is given from granted today, it was far from obvious when Newton began developing mechanics.

For the association of points in space with coordinate triplets to define a physical meaningful reference system, it is essential that, once the co-ordinates of a point in system F are fixed - using, for example, Cartesian coordinates for the point $P=(x, y, z)$ - that point remains stationary with respect to F. The point P must be identified unambiguously through its coordinates, which may vary across systems, but must remain fixed within any given system.

On this topic, Newton's *De Gravitatione et Aequipondio Fluidorum*[5] presents several insightful observations. Most of this text is dedicated to a sharp criticism of the Cartesian conception of vortices, of the idea that the motions of bodies can only be determined in relation to neighboring bodies, and of the belief that space is identified with extension, i.e. that there can be no space separate from bodies. Newton formulated a series of critiques against the Cartesian conception of space and motion, several of which are examined below. The main one can be summarised as follows:

If one accepts that a body's motion can only be determined relative to surrounding bodies, that space is identical to extension, and that space cannot exist independently of bodies and their relations, then it becomes impossible to establish a reference system with the necessary characteristics described above. There would be no guarantee that the points of a reference system would be fixed once and forever, even with respect to the system itself. In other words, the concept of reference system would not even exist, and it would not be possible to do physics beyond the most elementary results[6].

Newton's argument proceeds as follows: according to Descartes, the position of a body B can only be determined in relation to the position occupied by close bodies. But when

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[5] The date of composition of *De Gravitatione et Aequipondio Fluidorum* (Newton, 1666) is generally fixed at 1666. the pamphlet was not published during Newton's lifetime. Today some scholars doubt that it was written in 1666. We do not enter into this issue here. See, e.g., Henry, 2011, Hill, 2003, Kochiras, 2012, Slowik, 2002. See, specifically, the Section, 1.2. entitled *Newton's Arguments Against Cartesian Relationalism*, pp. 12–17, and the entire Chap. 2 on the structure of space in Newton. For the translation we refer to Newton 1962, pp. 121–156.

[6] On this topic see Bussotti & Pisano, 2013, Bussotti, 2016, Bussotti, 2017, Bussotti & Lotti, 2022, pp. 209–212.

motion takes place, the position of close bodies changes, often precisely because it is influenced by the motion of B itself (think of possible collisions). Then, if space cannot be separated from the bodies, and if the mutual positions of the bodies change, influencing each other, how is it possible to fix even a single reference system with the characteristics seen before? The points of the reference system itself would move in time not only with respect to other reference systems, but with respect to the one initially chosen. It can be specified, following the basis of Newton's reasoning, that a series of observer-based reference systems could not even be fixed, because for this to be possible it is necessary that the observer be considered at rest and that the positions of the points of the system can be fixed for the observer who represents the origin of the system, once and for all. According to Newton, Descartes' conception lacks this possibility in principle.

Newton posits the example of supposing that Jupiter occupies the same position after one year as it did the year before. Newton argues that, under Cartesian assumptions, this concept is even inexpressible. One could not define Jupiter's position relative to ether particles, as these particles themselves would have shifted. Their positions, in turn, would require further reference, leading to an infinite regress; nor would it be possible to refer to the fixed stars or the sun because vortices surrounding the sun and the fixed stars would influence the motion of these stars with their motions. For example, a comet passing through the solar vortex would influence the state of motion of the particles in the solar vortex and thus change the position of the sun. Therefore, the determination of the position of Jupiter would be relative to a sun that is moving in some uncertain way. Thence, if we claim to determine the motion of bodies on the basis of the positions of the surrounding bodies, identifying space and matter, space and extension, we cannot determine precisely what Descartes wanted to determine, i.e. the relative motion of bodies. This implies the impossibility satisfying the minimal requirement of any theory of motion and, specifically, of planetary motion: fixed a reference frame at time t_0, and indicated the position of two bodies, determine the reciprocal position of the two bodies for an arbitrary time $t_1 > t_0$. Newton writes:

But that this may be clear, it is first of all to be shown that when a certain motion is finished it is impossible, according to Descartes, to assign a place in which the body was at the beginning of the motion; it cannot be said whence the body moved. And the reason is that according to Descartes the place cannot be defined or assigned except by the position of the surrounding bodies, and after the completion of a certain motion the position of the surrounding bodies no longer stays the same as it was before. For example, if the place of the planet Jupiter a year ago be sought, by what reason, I ask, can the Cartesian philosopher define it? Not by the positions of the particles of the fluid matter, for the positions of these particles have greatly changed since a year ago. Nor can he define it by the positions of the Sun and fixed stars. [...]. Truly there are no bodies in the world whose relative positions remain unchanged with the passage of time, and certainly none which do not move in the Cartesian sense: that is, which are neither transported from the vicinity of contiguous bodies nor are parts of other bodies so transferred. And thus there is no basis from which we can at the present pick out a place which was in the past, or say that such a place is any longer discoverable in nature. For since, according to Descartes, place is nothing but the surface of surrounding bodies or position among some other more distant bodies, it is impossible (according to his doctrine) that it should exist in nature any longer than those bodies maintain the same positions from which he takes the individual designation. And so, reasoning as in the question of Jupiter's position a year ago, it is clear that if one follows

Cartesian doctrine, not even God himself could define the past position of any moving body accurately and geometrically now that a fresh state of things prevails, since in fact, due to the changed positions of the bodies, the place does not exist in nature any longer. Now as it is impossible to pick out the place in which a motion began (that is, the beginning of the space passed over), for this place no longer exists after the motion is completed, so the space passed over, having no beginning, can have no length; and hence, since velocity depends upon the distance passed over in a given time, it follows that the moving body can have no velocity, just as I wished to prove at first. Moreover, what was said of the beginning of the space passed over should be applied to all intermediate points too; and thus as the space has no beginning nor intermediate parts it follows that there was no space passed over and thus no determinate motion, which was my second point. It follows indubitably that Cartesian motion is not motion, for it has no velocity, no definition, and there is no space or distance traversed by it. So it is necessary that the definition of places, and hence of local motion, be referred to some motionless thing such as extension alone or space in so far as it is seen to be truly distinct from bodies. (Newton, 1666, 1962, pp. 129–131).

This quotation, jointly with the bucket experiment and with our previous considerations allows us to conclude what follows:

Newton believed to have demonstrated the existence of absolute space.

The bucket experiment alone and the above quotation alone are not enough to prove our thesis. But if we combine the two arguments, we arrive at the desired demonstration. It is a reworking of the reasoning we developed before the quotation: Let us admit Descartes' assumptions: (1) the movement of a body is determined in relation to that of the neighbouring bodies; (2) space is identified with extension, that is, in Descartes' view, with matter; (3) no empty space exists; (4) in the universe there is a permanent circulation depending upon the reciprocal contact of all the particles. As we have seen, given these assumptions, no reference frame can be established. For its points cannot be determined once and forever with respect to the supposed initial configuration of the frame itself. Suppose Descartes is correct: when body *B* moves, the positions of the surrounding bodies also change, precisely because they are influenced by the motion of *B*. According to Descartes' assumptions – where space cannot be separated from bodies, it becomes impossible to establish a reference frame that permits the analysis and prediction of physical phenomena.

As Newton envisioned, let us suppose we are in the year *T* and we wish to determine Jupiter's position in the year *T*−1. According to Newton, under Descartes' conception, Jupiter's past position cannot be determined relative to ether particles, since those may have shifted. Their positions would then need to be defined relative to yet other bodies, leading to an infinite regress. Therefore, Jupiter's position in the year *T*−1 cannot be established. However, astronomical theories allow us to determine it, knowing the place Jupiter occupies in the year *T*. This contradicts Descartes' fundamental assumption that space cannot exist independently of matter.

Thus, Newton argued that this possibility revealed the existence of an absolute reference system independent of the bodies' locations. Such an absolute reference frame is absolute space, which cannot be identified either with matter or with the relative positions of bodies. Furthermore: (1) the bucket experiment demonstrates that not all motions can be explained as motions relative to surrounding bodies; (2) motion must be referred to some space, which cannot be merely the immediate vicinity of the moving bodies. It must be absolute space, to which absolute motions determined in the bucket experiment are referred[7].

These complex and often indirect lines of reasoning constitute the path by which Newton believed he had demonstrated the existence of absolute space. This does not imply that Newton's proof is conclusive, nor that he arrived at the concept of absolute space solely through formal demonstration.

Surely his theological and metaphysical convictions, which are a priori and independent of empirical demonstration, played a fundamental role.

Nevertheless, from a logical standpoint, Newton did not merely assume the existence of absolute space. He was convinced that he had demonstrated it.

According to Newton, the claim that one can define Jupiter's position relative to nearby bodies—given appropriate laws—is ineffective, for the reasons already explained. Furthermore, Cartesian physics did not provide such quantitative laws: not only did it not in fact do so, but its entire structure with its complicated and, at the same time, ill-defined vortex mechanism, its distinction between real motion and common or vulgar motion, with its erroneous explanation of mechanisms involving the principle of inertia, with its total lack of quantification, not only failed to provide a plausible explanation of phenomena, but gave a completely erroneous and misleading one. Newton, then, not only critiqued specific claims of Cartesian doctrine but aimed to dismantle its core premise: that motion is determined solely by the relative positions of nearby bodies, and that space itself is purely relational. It is likely that Newton's critical engagement with Cartesian physics helped lead him toward a fuller understanding of the principle of inertia and of centrifugal forces—both of which, in his framework, were inseparable from the concept of absolute space.

The one we have reported and commented on is Newton's main objection against Cartesian physics and comes after a series of eleven other objections, divided into two groups of three and eight objections respectively. Some of them will be analysed as they are closely related to our context.

The objection belonging to the first group concerns the general concept of Cartesian motion, i.e. the difference between true motion and common or vulgar motion. Newton recalls that, for Descartes, the true motion of a body *m* is only the translatory motion that occurs relatively to bodies that are in *direct contact* with *m*. All other motions are motions in the common or vulgar sense. It is the doctrine that leads Descartes to claim that, in the true and authentic sense, the earth is stationary because it is stationary with respect to the particles of the vortex surrounding it. Newton writes:

> For he [Descartes] says that speaking properly and according to philosophical sense the Earth and the other Planets do not move, and that he who declares it to be moved because of its translation with respect to the fixed stars speaks without reason and only in the vulgar fashion (*Ivi*, p. 124).

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[7] See Bussotti & Lotti, 2022, pp. 202–212. We also discussed the position of authors who have opinions different from ours as to the nature of absolute motion and absolute space in Newton in *ivi*, pp. 214–235. See Belkind, 2007, 2012, DiSalle, 1991, 1994, 1995, 2002, 2006, 2020, Gould 1962, Huggett2008, Kochiras, 2012, McLaughlin, 2008, Rynasiewicz, 1995, 2004, Sklar 1974, Stein, 1967, Toulmin, 1959, Zylbersztajn, 1994.

What Newton objects to Descartes, then, concerns an even more fundamental aspect of the problem than the existence of absolute space: by separating, as Descartes does, a scientific sense from a vulgar sense of motion, it becomes impossible for him to treat scientifically the relative motions of objects that are not in immediate contact. Thus, in Descartes' doctrine, it becomes problematic to treat even a broad class of relative motions, not only the supposed absolute motions, but, for example, motions relative to fixed stars. Ergo, among other things, there would be no possibility of explaining centrifugal forces, because, even those who, as Mach, rejected Newton's concept of absolute space, resorted to the fixed stars to explain centrifugal force.

The first objection of the second group highlights a paradox (in Newton's eyes a contradiction) of the Cartesian theory: suppose we have a solid body that moves with respect to the bodies in immediate contact with it and is, therefore, animated by true motion. Then, adhering to the Cartesian doctrine, we should conclude that only those particles of the body that have at least a portion of their surface in contact with the surface of the body are animated by true motion, while all internal particles are not, because their reciprocal positions with respect to close bodies do not change. Newton considers this to be absurd (*ivi*, p. 126).

The fourth objection of the second group is interesting. It highlights a further problem of Cartesian physics:

> It also follows from the same doctrine that God himself could not generate motion in some bodies even though he impelled them with the greatest force. For example, if God urged the starry heaven together with all the most remote part of creation with any very great force so as to cause it to revolve about the Earth (suppose with a diurnal motion): yet from this, according to Descartes, the Earth alone and not the sky would be truly said to move (Part III, Art. 38).1 As if it would be the same whether, with a tremendous force, He should cause the skies to turn from east to west, or with a small force turn the Earth in the opposite direction. But who will imagine that the parts of the Earth endeavour to recede from its centre on account of a force impressed only upon the heavens? Or is it not more agreeable to reason that when a force imparted to the heavens makes them endeavour to recede from the centre of the revolution thus caused, they are for that reason the sole bodies properly and absolutely moved; and that when a force impressed upon the Earth makes its parts endeavour to recede from the centre of revolution thus caused, for that reason it is the sole body properly and absolutely moved, although there is the same relative motion of the bodies in both cases. And thus physical and absolute motion is to be defined from other considerations than translation, such translation being designated as merely external. (*Ivi*, pp. 127, 128).

Newton poses a problem, which is, in a different form, the same as in the bucket experiment: considering only relative translational motions it is impossible to explain the centrifugal forces. This, from a physical point of view, is a fundamental objection to Descartes, who believed that physics could only be based on the translational motions of neighboring bodies. Newton made the choice to use absolute space as reference for such apparent forces.

To summarize, Newton's concept of absolute space serves three primary functions:

1) It provides a privileged reference system, isomorphic to all inertial systems. The crucial distinction is that, unlike abstract inertial frames, Newton considered absolute space a real, physical entity. This was both a formal and substantive property in his framework.
2) It underpins the validity of the inertia principle and explains phenomena such as centrifugal forces.
3) As will be seen in the next section, it plays a central role in explaining the shape of celestial bodies.

To conclude our considerations on the nature of absolute space it is also appropriate to consider the other experiment Newton proposed in the *Definitions*: the two-globes experiment. In the bucket experiment, Newton relied on an apparent force—the centrifugal tendency of the water—to argue for absolute motion. However, being a profoundly rigorous thinker, he sought to frame absolute motion within a more comprehensive physical context, namely in a context where absolute motion could be deduced both from its causes and effects and not only from its effects, as in the bucket experiment. Because centrifugal force is an apparent force, it can serve only as an intermediary for detecting the effects of absolute motion—not as its physical cause. Newton, thus, posed the following key question: is it possible to find a real, not an apparent, force, which causes an absolute motion? Newton was aware this to be difficult task. For the portions of absolute space in which the absolute motions take place do not fall under our senses. Nevertheless, he proposed a solution in the form of a thought experiment: consider two globes connected by a rope and rotating about their common centre of gravity. Owing to inertia, each globe tends to move tangentially away from the centre, producing tension in the rope. This tension is a real force pulling the globes toward their mutual centre. Furthermore, if equal forces are applied to opposite sides of the globes during rotation, the tension will either increase or decrease—reflecting the change in absolute motion. They indicate the increment or the decrement of the absolute motion as well as its direction, or determination (*determinatio*, as Newton wrote). The rope's tension, being independent of surrounding bodies, is an absolute quantity. As such, the experiment remains valid even in an idealized vacuum containing only the two globes and the connecting rope.

Newton recognized the difficulty of determining absolute motion directly. The idea of using the fixed stars as a reference frame was not foreign to his conceptual horizon. It was not a novelty by Mach. In the two globes experiment, Newton argued: add the fixed stars to the hypothetical universe composed only by the two globes and the rope linking them. The motion of the globes relative to the fixed stars does not suffice to determine whether absolute motion is attributable to the globes, the stars, or both. Nonetheless, the rope's tension indicates the globes' absolute motion—enabling, in principle, the inference of the absolute kinetic state of the fixed stars. In *De gravitatione*, Newton wrote what follows:

> As if it would be the same whether, with a tremendous force, He [God] should cause the skies to turn from east to west, or with a small force turn the Earth in the opposite direction. But who will imagine that the parts of the Earth endeavour to recede from its centre on account of a force impressed only upon the heavens? (Newton, 1666, 1962, p. 128).

This passage is extraordinarily similar to what Mach writes:

> […] the motions of the universe are the same whether we adopt the Ptolemaic or the Copernican mode of view. Both views are, indeed, equally *correct;* only the latter is more simple and more *practical.* (Mach, 1919 p. 232).

Therefore, Newton perfectly understood that all motions could be only relative and that the objection against Descartes' idea that all movements must be referred to neighbouring objects shows the inadequacy of Descartes' doctrine, but not that of the relativistic conception. For Newton himself conceived the idea that the motions could be referred to the fixed stars or to the skies, as he wrote in *De Gravitatione*. Then why did he conceive the existence of absolute space to explain centrifugal forces and, hence, inertia? First of all, Newton considered inertia as a property inherent to bodies, which is invariable in any possible universe. Instead, in Mach's view inertia also depends, at least in principle, on the whole mass of the universe, it is not an intrinsic property. Namely in Newton, inertia is not a relative quantity. Furthermore, it is problematic to assume the fixed stars as an inertial reference frame because they are agitated by several irregular movements. Therefore, if one looks for a system which can be more or less regarded as inertial with respect to the movements on the Earth or in the solar system, the fixed stars are adequate, but they are not if one searches a precise determination of inertia. This is why Newton thought of absolute space.

Now the question is: did the whole set of reasonings we have presented authorize Newton to establish the existence of absolute space? The answer is in the negative. To use a modern expression, Newton was legitimate to claim that inertia must be referred to the whole class of inertial reference frames. For the supposed absolute space is dynamically indistinguishable from any other inertial reference frame. All inertial reference frames are dynamically equivalent. Newton was aware of this situation as the celebrate Corollary 5 to the *Axioms or Laws of Motion* states:

> When bodies are enclosed in a given space, their motions in relation to one another are the same whether the space is at rest or whether it is moving uniformly straight forward without circular motion. (Newton, 1999, p. 423).

Then, why did he resort to absolute space? In the universe there are objects and events happen. There is no reference frame. This is an abstract concept we use to assign a recognisable position to the objects and to develop our calculations. Despite his extraordinary discoveries, Newton was, anyway, a man of the 17th century and he felt the need to anchor the notion of inertial reference frame – that he knew very well, although he did not use such expression – to something more concrete, namely to an entity existing in the universe and that might represent, in his view, a concrete prototype of inertial reference frames. This entity was absolute space. As Lange, the inventor of the expression "inertial reference frame", correctly argues:

> According to his explanations, he [Newton] bases the law [of inertia] on a certain coordinate system that he calls the "absolute, homogeneous, infinite and immovable" space. This "absolute space" is real and not merely something conceived, though admittedly it is not accessible to our imperfect human sensory perception. It consists of indiscernable absolutely fixed points arranged next to each other; and it is only by comparison of the bodies with these "in themselves" fixed points, not by comparison

> with other matter, that the nature of the positions and motions of the bodies are to be recognized. (Lange 1885, 2014, p. 252).

It is worth noting that the physicists continued to use the notion of absolute space without any problem until the end of the 19th century when the criticisms of Neumann, Lange himself and Mach clearly showed how unsatisfactory this concept was. Inertia continued to be a problematic notion, as the long series of investigations developed from the end of 19th century onwards testify[8].

Let us now move to a further question: the way in which Newton thought that the movement with respect to absolute space determined the shape of celestial bodies.

4 Absolute Space and the Shape of Celestial Bodies

According to Newton, the motion of a body relative to absolute space determines its physical shape—especially in the case of celestial bodies. Suppose, initially, that a body is perfectly spherical. When it rotates about an axis through its centre, centrifugal force acts unevenly across its surface. Then, taking as poles the extremes of the diameter around which the rotation takes place and as equator the maximum circle perpendicular to the axis, the speed of rotation will be greater at the equator, lesser as the absolute value of latitude increases, until it is zero at the poles. Since no real body is perfectly rigid—and assuming the rotation is not so rapid as to tear the body apart—the equatorial particles will be pushed outward more than those at higher latitudes, deforming the sphere into a rotational ellipsoid. Excluding body-specific factors, the general shape of celestial bodies subject to such rotational motion is that of an oblate (rotational) ellipsoid. The question is: with respect to what does the body rotate? Newton's answer is: the rotation must be defined with respect to absolute space.

In Newton's view, if a celestial body—such as the Earth—were stationary while the firmament rotated around it, then terrestrial observations would remain unchanged. However, there would be a crucial difference: the Earth would retain a perfectly spherical shape, since it would not be rotating relative to absolute space. This reasoning mirrors the logic of the bucket experiment and can be considered an application of it: centrifugal forces determine the motion relative to absolute space, which in turn, due to the principle of inertia, determines the general form of objects.

Newton does not merely set out a theoretical treatment of his ideas but refers to precise physical phenomena. Newton first formulates a general principle in Proposition XVIII, Theorem XVI of Book III of the *Principia*, where he writes:

> *The axes of the planets are smaller than the diameters that are drawn perpendicularly to those axes.* If it were not for the daily circular motion of the planets, then, because the gravity of their parts is equal on all sides, they would have to assume a spherical

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[8] Between the end of the 19th century and the beginning of the 20th century there were several investigations on the concept of inertia. See Bussotti & Lotti, 2022, pp. 225–235. Here, we have no room to develop such topic. See Barbour and Pfister (eds.) (Barbour 1995), Föppl 1904a, 1904b, Frank, 1909, Frank & Rothe, 1911, Friedlander & Friedlander 1896, Lange, 1885a, b, c, 1886, 1902; Neumann, 1870; Pfister, 2014; Reissner, 1914, 1915; Streintz, 1883. Several more recent investigations were also developed in the second half of the 20th century. But for them we refer to the bibliography of the section on Newton in Bussotti & Lotti, 2022, pp. 247–251.

> figure. Because of that circular motion it comes about that those parts, by receding from the axis, endeavor to ascend in the region of the equator. And therefore if the matter is fluid, it will increase the diameters at the equator by ascending, and will decrease the axis at the poles by descending. Thus the diameter of Jupiter is found by astronomical observations to be shorter between the poles than from east to west. By the same argument, if our earth were not a little higher around the equator than at the poles, the seas would subside at the poles and, by ascending in the region of the equator, would flood everything there. (Newton, 1999, p. 821).

In the next proposition (proposition XIX, problem III, ibid., pp. 648–653) Newton determined the size of the earth's diameter at the equator and the poles by the following reasoning: he reports the linear measurements of a degree of latitude found in four separate places, one in England and three in France, by Richard Norwood, Jean Picard, Gian Domenico Cassini and his son Giacomo Cassini respectively. From these measurements Newton deduced that, if the earth were a perfect sphere, its radius would be 19,615,800 Parisian feet[9] and its circumference 123,249,600 feet. He then calculated the force with which a heavy body falls at Paris and found that it causes a free-falling body to complete $2173\frac{7}{9}$ lines in one second. If the Earth is treated as a perfect sphere with a radius of 19,615,800 Parisian feet, one can calculate the centrifugal force acting at the equator. Newton found that the ratio of gravitational force that causes a heavy body to fall at Paris to the centrifugal force at the equator is 2174 to 7.54064. However, this force exerted at Paris is not the entire force of gravity at the latitude of Paris, but the force of gravity minus the centrifugal force at Paris. Now, the ratio of the centrifugal force at the equator to the centrifugal force at Paris can be calculated and is 7.54064 to 3.267. Thus, the total gravitational force at Paris is $2174+3.267=2177.267$. Ergo, the gravitational force in Paris is related to the centrifugal force at the equator as 2177.267 to 7.54064, i.e. as 289 to 1. To understand the following reasoning it is necessary to specify that, given the latitude of Paris, the ratio 289 to 1 is taken as the ratio between the average gravity on the earth's surface and the centrifugal force at the equator.

In the second part of the proof Newton analysed the situation in which the earth is not a sphere (Fig. 2).

Newton imagined that *ACQqca* is a channel filled with water, where *Cc* is the centre of the earth, *Qq* the pole and *Aa* the equator. Along the channel *AaCc*, which runs from the equator to the centre of the earth, the water is subjected to the centrifugal force exerted on the equator, while in the other channel *QqCc*, which is excavated along the axis of rotation, no centrifugal force is exerted. Furthermore, based on Corollary of proposition XCI of the first book[10], Newton found that, if the earth were made of uniform matter and was stationary

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[9] The Parisian foot used by Newton is equivalent to 0.324839 m. If the measurement is transformed into meters and then into kilometers, the result is a length of the earth's radius (assuming, at this stage of the reasoning, that the earth is a sphere) of approximately 6372 km, a value that is quite reliable, considering that the average radius of the earth is today calculated as 6371 km.

[10] In the mentioned Corollary Newton calculates the force by which a spheroid (let us recall that spheroid is synonymous of rotating ellipsoid) attracts a body placed on the extension of an axis of the spheroid (Newton, 1999, pp. 616–617). As we will see, the editors of the Geneva Edition of the *Principia* (Newton, 1687, 1726, [1822], III, pp. 60–62, footnote (r)*) gave a detailed explanation of the way in which Newton used this Corollary in the proposition we are analysing. For a description of the Geneva Edition see the works Pisano/Bussotti and Bussotti/Pisano in the Bibliography. Specifically on the notes concerning Proposition XCI of the first book, see Pisano & Bussotti, 2022.

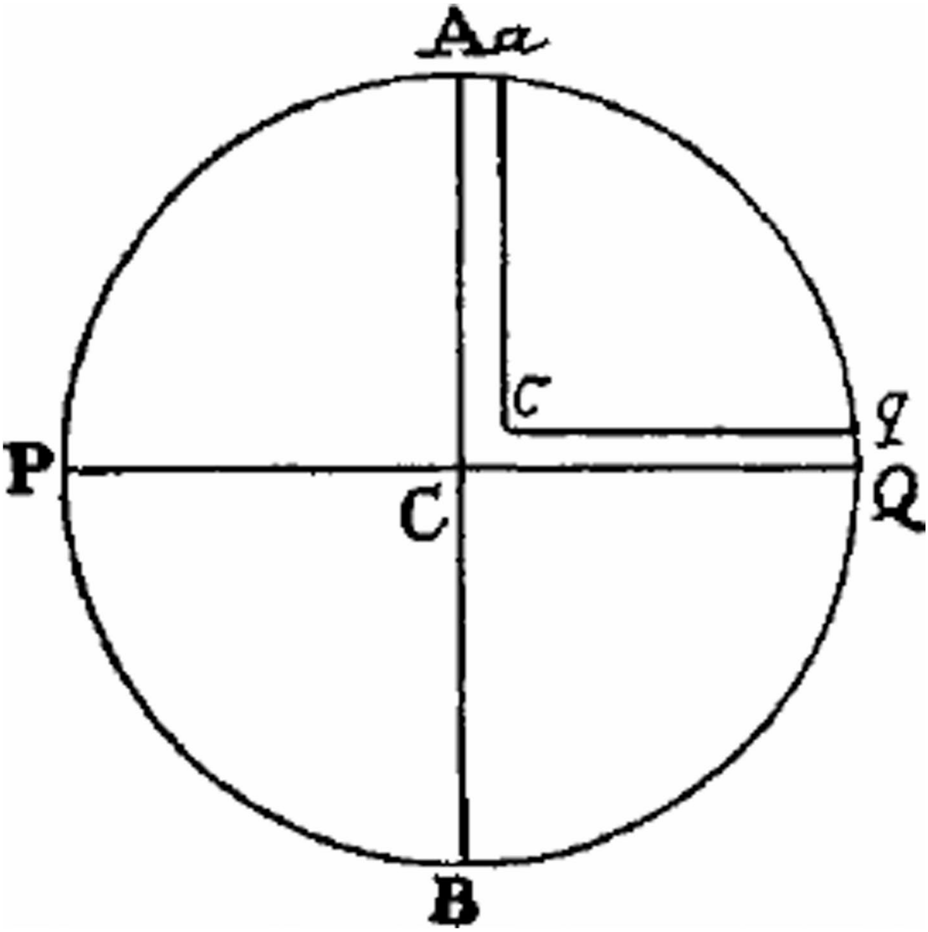

Fig. 2 The diagram used by Newton's in the second part of the reasoning. Newton, 1999, p. 823

with respect to absolute space and if the axis *PQ* were at the diameter *AB* as 100 to 101[11], then the effective gravity of the earth - which, it is worth remembering, is no more supposed to be spherical, but spheroidal, specifically it is an ellipsoid of rotation - in *Q* would be equal to the gravity of a sphere of radius *QC* in the same point *Q* as 126 is to 125. If, on the other hand, we consider point *A*, the ratio of the gravities is reversed and the gravity at locus *A* of the ellipsoid generated by the rotation of the ellipse *APBQ* around *AB* is to the gravity in locus *A* of the sphere of radius *AC* as 125 to 126.

Some explanations are necessary: Newton is considering four solids. We will first analyse the situation under the assumption that the solids do not rotate on themselves:

1) The ellipsoid of rotation along the minor axis *PQ*, which is, in fact, the earth;

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[11] It is to highlight for the continuation of Newtonian argument that the ratio 100 to 101 also indicates the ratio of the mass water in the *QqCc* channel to that in the *AaCc* channel.

2) the sphere of centre *C* and radius *QC*, which is smaller than the ellipsoid, so it is correct to claim that gravity at the same point *Q* is greater in the ellipsoid than in the sphere;
3) The ellipsoid of rotation about the major axis of the earth *AB*;
4) The sphere of diameter *AB*, which is greater than the ellipsoid. Thus gravity at the same point *A* is greater in the sphere than in the ellipsoid.

Newton continued arguing that gravity at place A on the earth is the mean proportional between gravity in the ellipsoid 3) and gravity in the sphere 4)[12]. With other similar arguments, he concluded that the acceleration of gravity at place Q on the earth is to that at place A as 501 is to 500[13]. Since the ratio between the masses of water in the 'equatorial' and in the 'polar' channels were set as 101 to 100, the ratio of the respective weights will be $500\times\ 101:501\times\ 100=505:501$.

This scenario reflects a stationary ellipsoid with respect to absolute space. In reality, however, the Earth rotates. It is necessary, therefore, to consider the effects of the centrifugal force. Newton reasoned as follows: if the centrifugal force due to the diurnal motion along the branch *ACca* were to the weight of the water in the canal as 4 to 505, the actual weight of the water in the canal *ACca* would be 505-4 = 501, therefore the water in the two canals *ACca* and *Qcq* would be in equilibrium. We have seen that, assuming the Earth to be a sphere, the centrifugal force is to the weight as 1/289 and assuming the Earth to be an ellipsoid in which the ratio of minor axis to major axis is 100 to 101, the ratio centrifugal force/weight is 4/505. This enables us to find the value of the polar axis and equatorial axis of the earth. For, the centrifugal force 4/505 causes the height of the water in the branch *ACca* to exceed that in the branch *QCcq* by a hundredth part of the whole height (in fact, the ratio of the heights was supposed to be 101 to 100). Now, to find which difference in height between the polar axis and the equatorial axis implies the ratio centrifugal force/gravity equal to 1/289 it is sufficient, indicating this ratio with *dh*, to set the proportion

$$\frac{4}{505}:\frac{1}{100}=\frac{1}{289}:dh.$$

Thus, we obtain $dh=1/229$, as Newton pointed out. Ergo, the Earth's equatorial diameter is to its polar diameter in the ratio of 230 to 229. If, therefore, the mean radius of the earth is assumed, according to Picard's calculation, equal to 19,615.800 Parisian feet = 6372 Km, we obtain that the equatorial radius is 19,658,600 Parisian feet, i.e. about 6386 Km and the polar radius of 19,573,000 Parisian feet, i.e. about 6358 Km. Today the equatorial terrestrial radius is calculated to be about 6378 Km and the polar radius about 6356 Km.

Let us summarise Newton's reasoning:

a) The earth is first supposed spherical and the relationship between the centrifugal force of bodies at the equator and the force of gravity at the latitude of Paris is determined.

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[12] The proof is taken for granted by Newton. As we will see, it can be found in the notes of the Geneva Edition, where the editors added all the details (Newton, 1687, 1726, 1739–1742, [1822], III, p. 63, footnote (s)80).

[13] Here Newton gives the details of the calculation. In the Appendix we will refer to the very detailed explanation given by the editors of the Geneva Edition (Newton, 1687, 1726, 1739–1742, [1822], III, pp. 63–64, note (t)*).

b) In the second phase of the reasoning, the earth is not supposed to be spherical, but an ellipsoid in which the ratio of the equatorial radius to the polar radius is 101 to 100. Four solids are determined and the ratios of their gravities at particular points are analysed.
c) The ratios of water weights in the equatorial and polar channels are deduced. The centrifugal force required for water in the two channels to be in equilibrium is calculated.
d) The true centrifugal force is considered. It is compared with the force required for equilibrium. The true shape of the earth or, at least, the true relationship between the equatorial radius and the polar radius, is deduced.

4.1 Commentaries

1) Newton deduces the ratio 289 to 1 between gravity at Paris and centrifugal force at the equator from the assumption that the earth is a rotating sphere, but in reality his final aim is to prove that the earth is not a sphere, and he will use this value in the final part of the demonstration treating it as it were referred to an ellipsoid. It is clear, therefore, that from a purely logical point of view, this step is not legitimate. But physics is not logic, or rather it has its own logic, which is, however, different from formal logic and the logic of physics demands to know what error is made by applying to an ellipsoid such as the earth a value found for a sphere. Among others achievements, the editors of the Geneva Edition show that in Newton's proportion 287 should be replaced by 287.67. The results are practically identical, from the physical point of view, to those obtained by Newton (Newton, 1687, 1726, 1739–1742, 1822, footnote (o)81, III, p. 59. See our Appendix).
2) In the steps we have indicated with (a) and (b) Newton uses two objects which cannot exist as physical realities, namely: a rotating sphere and two non-rotating ellipsoids. What kind of entities are they? We call them physical-mathematical fictions which are part of the argumentative structure, but not of the world. They are ideal objects that have no physical reality, but serve as instruments of reasoning. They are fictions, not just mathematical or physical because, unlike a mathematical entity, such as, for example, a pure sphere, they have physical properties, such as gravity, but, unlike a physical object, their concrete existence is not possible in principle. These linguistic and conceptual tools, these *fictiones*, are used in various circumstances by Newton and represent in physics and mathematics a type of procedure that is of great importance[14].
3) We have discussed this proposition in detail because it clearly illustrates the central role of absolute space in Newton's physics: it is the instrument and at the same time the reference system Newton considered essential in order to explain the form of the celestial bodies, that is, to understand and interpret the physical effects of centrifugal forces. The other instrument associated with absolute space is the principle of inertia, because, as has already been explained, the reference to centrifugal forces is nothing but a reference to the principle of inertia. Considerations concerning other propositions by Newton in which absolute space and centrifugal forces play a crucial role could be added. Significant examples regard the gravitation of the planets with respect to the sun, the gravitation of the Moon with respect to the earth and of the satellites of Jupiter

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[14] On this topic, see, e.g. Bussotti, 2015, pp. 51–53, Pisano & Bussotti, 2017.

and Saturn with respect to the two planets (propositions I-VI of the third book of the *Principia*). But the situation has already been sufficiently clarified by the example concerning the shape of the earth. Let us therefore see how the criticism of the concept of absolute space unfolded.

5 Critiques of Absolute Space: Leibniz and Mach

From the beginning of the 18th to the end of the 19th century, most physicists accepted the concept of absolute space. Their discipline had achieved such successes that seemed to be no reason to doubt this concept, however naive it might have been from a philosophical point of view. However, at the beginning of this period, when Newtonian physics was not yet the sole paradigm, and in the end, when considerations connected with the philosophy and methodology of physics intersected with problems within this discipline, Newtonian conceptions of absolute space (and time) were criticised in several respects. Leibniz was probably the philosopher-scientist who, at the end of the 17th and beginning of the 18th century, presented the most acute critique of Newtonian concepts. Mach was the author whose considerations concerning absolute space influenced, according to Einstein himself, the birth of the theory of relativity. It is therefore useful to propose and comment on their criticism of Newton's absolute entities[15]. It is appropriate to recall that, although Leibniz's and Mach's observations had several similarities, the context in which they were developed was different: as to Leibniz, his criticisms of Newton's concepts of absolute space and time are a part of his complex and stratified metaphysics. Instead, Mach aimed at eliminating all the metaphysical aspects of science. His enquiry belongs to the epistemological picture typical of the positivism that he developed, jointly with Avenarius, in the empirio-criticism. In this paper we cannot deal with such a wide topics, so that we restrict to present Leibniz's and Mach's critical observations to Newtonian concepts.

5.1 Leibniz's Criticism

In Leibniz the connection between the different parts of his theoretical edifice is so complex that it is difficult to speak of a specific section without taking into account the whole of his speculation. We will try, anyway, this enterprise.

As for Leibniz's criticism of Newton, it is well known that it comprises a plurality of aspects: from the supposed immediate action at a distance, to the controversy against absolute time and space. Leibniz criticizes several aspects of Newton's concept of space: from the idea of space as *sensorium Dei* to its logical status, to its physical properties. All these issues are interrelated in Leibniz's thought. The essential source to detect this topic will be the Leibniz-Clarke correspondence (Leibniz & Clarke 1715–1716. English translation 2000). Some ideas will also be taken from the *New Essays on Human Understanding* (Leibniz, 1703. English translation 1996) and from the *Dynamica de Potentia et Legibus Naturae Corporeae* (Leibniz 1689–95, 2023).

In point 4 of the third letter to Clarke, Leibniz clarifies his conception of space and time:

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[15] A great physicist who expressed remarkable opinions on circulatory motion, the concept of the relativity of space, and inertia was Christiaan Huygens. See, e.g., Bussotti, 2023, Bussotti & Lotti, 2022, pp. 149–177, Mormino, 1993, Stan, 2016.

> As for my own opinion, I have said more than once that I hold space to be something purely relative, as time is that I hold it to be an order of coexistences, as time is an order of successions. For space denotes, in terms of possibility, an order of things that exist at the same time, considered as existing together, without entering into their particular manners of existing. And when many things are seen together, one consciously perceives this order of things among themselves. (Leibniz 1715–1716, 2000, p. 14).

This relativistic conception of space and time does not, however, imply that space should be identified with material extension or with bodies, as Descartes believed. In the *New Essays on Human Understanding* it is made clear that space is not a body (Leibniz, 1703, 1996, II, XIII, §§ 19–21, pp. 150–151 and IV, VII, § 13, p. 423). Leibniz believes that even if nothing is fixed in the universe, the concepts of place and position do not lose their value, as long as one is able to record all the relative changes among bodies (*ivi*, II, XIII, § 8, p. 149). The whole of these considerations is clarified in the fourth letter to Clarke, where we find the following illuminating explanations:

> The author contends that space does not depend on the situation of bodies. I answer: It is true, it does not depend on such or such a situation of bodies, but it is that order which renders bodies capable of being situated, and by which they have a situation among themselves when they exist together, as time is that order with respect to their successive position. But if there were no creatures, space and time would be only in the ideas of God. (Leibniz 1715–1716, 2000, p. 27).

Thus, space appears as the order of all possible configurations of objects, not only of those currently realised, but also of those that can be realised. In other words, it is an ideal entity that is not identified with extension, or with a merely abstract system of reference, or, even less so, with Newtonian absolute space, but is seen as the set of all possible configurations and relations between things and simultaneous events. Leibniz thus has a very advanced conception of space that seems to prefigure, on a mathematical level, a set of ideas that will be typical of topology (*analysis situs*)[16]. Leibniz enriches his conception with further profound and interesting elements, in particular what he calls 'a kind of definition' is truly remarkable:

> Place is that which we say is the same to *A* and to *B* when the relation of the coexistence of *B* with *C*, *E*, *F*, *G*, etc. agrees perfectly with the relation of the coexistence which *A* had with the same *C*, *E*, *F*, *G*, etc., supposing there has been no cause of change in *C*, *E*, *F*, *G*, etc. It may be said also, without entering into any further particularity, that place is that which is the same in different moments to different existent things when their relations of coexistence with certain other existents which are supposed to continue fixed from one of those moments to the other agree entirely together. (*Ivi*, fifth Leibniz's letter, p. 46).

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[16] As works on Leibniz and topology, see De Risi, 2007 (where there is also an abundant bibliography on the subject) and De Risi, 2015. Among the numerous works dedicated to Leibniz's concepts of space and time see Arthur, 1998, 2021, Bouquiaux, 2008, Bussotti & Lotti, 2022, pp. 276–288, Crockett, 2008, Garber, 2009, Lariviere, 1987, McDonough, 2007, Mogens Laerke, 2016, Puryear, 2012, Roberts, 2003.

Leibniz then clarifies the meaning of his last statement by referring to the principle of the identity of indiscernible, and explains that, however much A and B agree in position, they must nevertheless differ in some respect in their relation to C, E, F, G, otherwise one would have two distinct objects that are completely indiscernible:

> To conclude, I have done here much like Euclid, who, not being able to make his readers well understand what ratio is absolutely in the sense of geometers, defines what are the same ratios. Thus, in like manner, in order to explain what place is, I have been content to define what is the same place. (*ivi.*, p. 47).

Given these refined and advanced characterizations of space, what does Leibniz have to say against Newtonian absolute space? His line of reasoning is based on the *principle of sufficient reason* accompanied by the *principle of the identity of indiscernibles*. The following passage shows, among other things, clearly what, for him, is to be understood when it is said that space concerns the order or relationship between bodies:

> I say, then, that if space was an absolute being, something would happen for which it would be impossible that there should be a sufficient reason -which is against my axiom. And I prove it thus: Space is something absolutely uniform, and without the things placed in it, one point of space absolutely does not differ in any respect whatsoever from another point of space. Now from this it follows (supposing space to be something in itself, besides the order of bodies among themselves) that it is impossible there should be a reason why God, preserving the same situations of bodies among themselves, should have placed them in space after one certain particular manner and not otherwise-why everything was not placed the quite contrary way, for instance, by changing east into west. But if space is nothing else but this order or relation, and is nothing at all without bodies but the possibility of placing them, then those two states, the one such as it is now, the other supposed to be the quite contrary way, would not at all differ from one another. Their difference therefore is only to be found in our chimerical supposition of the reality of space in itself. But in truth the one would exactly be the same thing as the other, they being absolutely indiscernible, and consequently there is no room to inquire after a reason for the preference of the one to the other. (*ivi.*, third letter, p. 15).

As a corollary to this, Leibniz adds a series of other considerations that also take into account the problem of vacuum: if space were an absolute reality, it would be immense, immutable and eternal in all its parts, and therefore there would be an infinity of eternal things besides God (ibid., fourth letter p. 23). (ibid., p. 323). Claiming that there is an empty space in nature is to attribute imperfect production to God, violating the *principle of sufficient reason*. Since matter is more perfect than vacuum, there must be as much fullness insofar as it is more perfect than emptiness. Therefore, no emptiness can exist because the perfection of matter is to emptiness as being is to nothing (ibid., pp. 23–24).

In his correspondence with Leibniz, Clarke often gives the impression not to fully understand Leibniz's arguments and his replies seems sometimes inadequate. However, there is a passage in which Clarke simply writes: regardless of all other arguments, Newton demonstrated with bucket experiment that an absolute motion can be determined and that this motion can in no way be referred to the relative positions of bodies. So, beyond various principles and persuasive arguments, what can be objected to Newton's proof and how can the forces of recession from the axis be explained? It is worth referring to what Clarke writes:

> It is largely insisted on by Sir Isaac Newton in his *Mathematical Principles* (definition 8) where, from the consideration of the properties, causes, and effects of motion, he shows the difference between real motion, or a body's being carried from one part of space to another, and relative motion, which is merely a change of the order or situation of bodies with respect to each other. This argument is a mathematical one, showing from real effects that there may be real motion where there is none relative, and relative motion where there is none real; it is not to be answered by barely asserting the contrary. (ibid., Clarke's fourth reply p. 31).

Therefore, Clarke argues that, in order to replace Newton's conception of absolute space with Leibniz's relationalist one, it is necessary to explain in another way the physical phenomena that Newton elucidated in terms of absolute space, but neither in Leibniz nor in other authors does Clarke note a valid alternative to the Newtonian explanation.

Clarke's considerations help to enucleate the hiatus that exists between the way Newton and Leibniz conceived space. Regarding the problem of how to interpret centrifugal forces, Leibniz's most interesting reference is probably found in a long unpublished work, reworked in the course of almost ten years, between 1689 and, at least, 1695, the already mentioned *Dynamica de Potentia et Legibus Naturae Corporeae*. Here in the context of the doctrine of the equivalence of hypotheses and in relation to the idea that all non-rectilinear motions are composed of infinitesimal rectilinear motions, Leibniz refers to Newton's ideas. In proposition 19 of the third part Leibniz argues that, if a ship is moving on the sea, all phenomena can be explained by assuming that the ship is stationary and that all the rest of universe is revolving around the ship; precisely, these would be two equivalent hypotheses. He writes that a celebrate man, Newton, believed to distinguish space and absolute motions from relative space and relative motions, relying upon the effects of circular motion. But if one accepts the equivalence of hypotheses and the idea that all phenomena must be explained on the basis of the laws of motion, then one will see that motions are relative[17].

In the next proposition, 20, Leibniz tries to prove the cohesiveness of bodies on the basis of the assumption of the equivalence of hypotheses and of the idea that motion and space are relative notions. The demonstration is very long and confusing, with a vocabulary in which the meaning of words is not always stable, there is no clear distinction between what is experimentally deduced, what is hypothesised and what is inferred as a demonstration. The set of ideas expounded in the third part of the *Dynamics* cannot be interpreted as a valid interpretation of Newton's bucket experiment. There are several hints that might prefigure an answer, but Leibniz's assertions in themselves do not provide sufficient arguments for a possible explanation of phenomena other than the one proposed by Newton. It should be

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[17] Leibniz had a purely relativistic conception of space and time. He believed, however, that true motions existed. We, however, have no criterion for distinguishing them from relative motions. On what Leibniz meant by true motion there is a vast literature with differing interpretations. For references to literature see De Risi, 2012, in particular paragraphs 5–9 (which are also useful for the content) and notes, and Bussotti, 2015, footnote 54, pp. 147–148.

noted that in the correspondence with Clarke, Leibniz does not refer to the results obtained in *Dynamics*, although they precede the correspondence with Clarke.

Let us summarise: the problems that led Newton to develop his notion of absolute space are problems of physics. Newton then clothed the physical concept of absolute space with a series of properties that are not physical, such as that of being the *sensorium Dei*, which, in effect, are naive and unsatisfactory from a philosophical point of view, so much so that the well known criticism of Leibniz - which is not taken up here – are effective on this point. How is Leibniz's polemic against absolute space as a physical entity structured? First of all, as we have seen, he resorts to the *principle of sufficient reason*. The first objection is that such a principle is a mere philosophical axiom whose validity in a physical context is far from being evident. For the sphere of validity and the specific modes of application of each principle must be delimited; a principle cannot be used as a universal picklock to tackle problems in very different fields without clarifying its use: on various occasions Leibniz (see, for example, Leibniz 1686?, 2006, p. 2; Leibniz 1715–1716, 1967, Leibniz's second letter, p. 7) argues that Archimedes resorted to the *principle of sufficient reason* to arrive at the conclusion that the equality of the static moment determines the equilibrium of the lever. There is no sufficient reason why the weight placed to the right or left of the fulcrum should prevail.

One can make various considerations as to how Archimedes arrived at his discovery, but anyone would admit that a reflection on the most immediate experiences with a lever or a series of instinctive knowledge (even without entering into the complex relationship between experience and instinctive knowledge) guided Archimedes in his research. If we want to summarise this procedure by tracing it back to a principle which, however, applied to science, remains heuristic and is a gnoseological-formal principle, not a content principle, we can say, with Leibniz, that Archimedes used the *principle of sufficient reason*.

But as far as the criticism of absolute space is concerned, Leibniz expounds an argument that concerns the arrangement of bodies in the entire universe and that takes the form of the lack of sufficient reason for God to prefer a certain arrangement of bodies to the opposite. The first, rather obvious criticism to this argument, is the anti-scientific nature of criticising any physical concept by referring to God. In Newton absolute space is interpreted metaphysically in reference to God, but he does not use the reference to God to physically demonstrate the reality of absolute space. Rather he proposes the bucket experiment in conjunction with the ideas expounded in *De gravitatione.*

Furthermore, even admitting that Leibniz's argument can be explained without recourse God, the problem remains that Leibniz refers to a purely hypothetical and theoretical situation such as the arrangement of all bodies in the universe, i.e. of the universe as a whole, a situation that we can never experience directly, even in a supposed mental experiment. It is a frequent and serious mistake to want to deduce something about the physical world through pure reasoning, disengaged from any possible experience. We are far from a bucket suspended to a rope!

Leibniz's use of the *principle of sufficient reason* in this context implies a *metabasis eis allo ghenos* with respect to the use attributed to Archimedes: from physics we have moved to a typically metaphysical reasoning disguised as physical reasoning.

Leibniz's other arguments against absolute space also seem to us to be based on sometimes naively metaphysical reasoning, such as when he claims that matter is more perfect than vacuum.

In conclusion: the constructive part of the concept of space in Leibniz is profound and interesting. It leaves open many potentialities for physical and mathematical research.

Undoubtedly, it is more in line with the concept of space-time of modern physics than Newton absolute space. On the other hand, this concept remains purely theoretical. There is no physical application, so that the critical part against absolute space as a mere physical entity misses the point. A conception like Leibniz's leaves open the problem of the physical explanation of centrifugal forces.

5.2 Mach's Criticism

Mach's criticisms of Newton's concepts of absolute time and space acquire their own pregnancy in relation to the period in which they were formulated. We are summarizing a complex set of issues, aware of the possible inaccuracies that the synthetic description of such a general framework may entail: from the moment the *Principia* were written, after an initial critical phase, the successes of Newtonian mechanics - and we refer, of course, not only to Newton's results, but to all of mechanics up to and beyond the middle of the 19th century - were such that, gradually, even the discussions on its most unconvincing assumptions, in particular the immediate interaction at a distance and the concepts of absolute time and space, tended to die down. In Newton's eyes, the physical status of interaction at a distance was quite different from that of absolute time and space. Indeed, in a famous letter to Bentley in 1693, Newton explicitly argues that only a fool can believe in the physical reality of immediate interaction at a distance[18], but since we do not know gravity's origin, we might accept this force as acting at a distance rather than make more complex, experimentally unverifiable and equally implausible hypotheses as to physical explanations for the origin of gravity. This is the meaning of the famous motto *Hypotheses non fingo*.

However, after Newton, it was generally accepted that interaction at a distance together with absolute space and time were the assumptions of the theory, implicitly accepted to be inseparable. Faraday's studies on electricity and magnetism, in which the great English physicist introduced the concept of the lines of force of a field and solved a number of problems by considering that electric and magnetic actions propagate with a finite velocity, were placed in a theoretical-mathematical framework by Maxwell. There are, therefore, forces, albeit not mechanical ones, which do not act with immediate interaction at a distance.

Mach emphasises this fact in the section of his *Mechanics* dedicated to Newton, where we read:

> Faraday's unbiassed and ingenious conceptions and Maxwell's mathematical formulation of them again turned the tide in favor of the forces of contact. Divers difficulties had raised doubts in the minds of astronomers as to the exactitude of Newton's law, and slight quantitative variations of it were looked for. After it had been demonstrated, however, that electricity travelled with finite velocity, the question of a like state of affairs in connexion with the analogous action of gravitation again naturally arose. (Mach 1883, 1919, pp. 534–535).

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[18] Newton wrote to Bentley on 25 February 1693: "That gravity should be innate, inherent and essential to matter, so that one Body may act upon another at a Distance thro' a *Vacuum*, without the Mediation of any thing else, by and through which their Action and Force may be conveyed from one to another, is to me so a great Absurdity, that I believe no Man, who has in philosophical Matters a competent Faculty of thinking, can ever fall into it". (Newton 1692-3, 1756, pp. 25–26).

Thus, Mach argues, the existence of contact actions in the electromagnetism made physicists' minds permeable to conceptualise the problem of action at a distance for gravity too.

Leaving aside all the nineteenth-century discussions on the luminiferous aether, there was a specific problem of electromagnetism, since in this branch of physics there was no principle exactly equivalent to the principle of inertia in mechanics: given two bodies, it was not indifferent which of the two was considered stationary or in uniform rectilinear motion. Einstein, in *On the Electrodynamics of Moving Bodies*, makes this very clear, referring to a situation that already existed at the time when Mach wrote the first edition of *Mechanics* (1883). In fact, Einstein writes:

> It is known that Maxwell's electrodynamics - as usually understood at the present time - when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion. For if the magnet is in motion and the conductor at rest, there arises in the neighbourhood of the magnet an electric field with a certain definite energy, producing a current at the places where parts of the conductor are situated. But if the magnet is stationary and the conductor in motion, no electric field arises in the neighbourhood of the magnet. In the conductor, however, we find an electromotive force, to which in itself there is no corresponding energy, but which gives rise - assuming equality of relative motion in the two cases discussed - to electric currents of the same path and intensity as those produced by the electric forces in the former case[19].

It is understandable that problems related to the principle of inertia are almost necessarily connected with the status of absolute space. Therefore, when Mach wrote his *Mechanics*, there were already all the prerequisites for a critical reappraisal of the basic concepts of Newtonian mechanics because of problems within physics. Mach's merit is that he clearly and unambiguously formulated his own positions, which were certainly due to his empiricist philosophy, but are incomprehensible without considering the developments of physics.

Mach clarifies that his criticism does not concern the content of Newton's mechanics because, as he himself states, "Since his [of Newton] time no essentially new principle has been stated" (Mach 1883, 1919, p. 187):

> Newton's sense of what fundamental concepts and principles were required in mechanics was admirable. The form of his enunciations, however, as we shall later indicate in detail, leaves much to be desired. (Mach 1883, 1919, p. 201).

In order to understand the meaning of Mach's criticism of Newton's concept of absolute space, it is convenient to refer to the path he followed in his criticism Newton: he begins his argument by criticising the Newtonian definition of mass as the product of volume and density and as expressing the quantity of matter in a body. Mach argues that density is not

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[19] The translation we use of Einstein, 1905 is freely available at https://users.physics.ox.ac.uk/~rtaylor/teaching/specrel.pdf. It is edited by John Walker.

a physical quantity that can be assumed without a previous definition and proposes the following interesting definition of mass obtained by abstraction:

> All those bodies are bodies of equal mass, which, mutually acting on each other, produce in each other equal and opposite accelerations. (*Ivi*, p. 218).

Mach argues that his concept of mass does not derive from any theory and depends only on experience. It renders useless the Newtonian concept of quantity of matter and the principle of action and reaction, which becomes a datum already inherent in the definition of mass (*ivi*, pp.218–222). Mach means what follows: it is ascertained from experience that bodies that collide with each other undergo variations in speed. After the collision, it can happen that the instantaneous acceleration of the two bodies is different, or that it is equal. If it is equal, the two bodies are said to have the same mass. The concept of mass thus divides bodies into classes of equivalence. Now the problem faced by Mach is to show that mass, as defined by him, has the essential characteristics attributed by Newton to mass itself. In particular: (1) demonstrating that if two bodies B and C, acting on a third body A, behave as equal masses, then they will also behave as equal masses when acting on each other; (2) the possibility of measuring mass through weight.

Particularly interesting is the basic idea, by virtue of which he feels the need to prove a property, such as 1): experience is the guide to physical research and every deduction from facts of experience, even the most intuitively obvious one, cannot be drawn unless it is proved by other experiences or mathematical reasoning.

With regard to the criticism to Newton's definition of mass, it is true that this definition, from the point of view of logical criterion, is objectionable because neither density nor quantity of matter are notions on which a definition can be based without before defining these concepts themselves. However, Mach sometimes seems to forget what he himself repeatedly states, namely that Newton was faced with difficulties that would have been insurmountable for anyone else, and his definition of mass, with its logical imperfection, serves a very precise purpose: to separate the concept of mass from that of weight in a clear and indisputable manner. It also seems to us that Mach's definition is not based only on experience data, but on experience data plus two hundred years of Newtonian physics, thanks to which it was understood that the fundamental quantity in physics is mass multiplied by acceleration. What Mach writes, in this case, is useful for clarifying logical-formal foundations of physics, but not for understanding its historical origin and the genuine conceptual problems from which this discipline evolved. While Mach really tries to understand the intentions of scientists, their difficulties and, therefore, their inevitable uncertainties when he speaks of Stevin, Galileo or Huygens, he does so much less when he speaks of Newton. Therefore, Mach enters very well into the internal logic of the discourse of Stevin, Galilei or Huygens, but not into that of the Newtonian discourse. What we are arguing seems to be confirmed by what Mach writes against Newton's absolute time:

> It is utterly beyond our power to measure the changes of things by time. Quite the contrary, time is an abstraction, at which we arrive by means of the changes of things; made because we are not restricted to any one definite measure, all being interconnected. (*Ivi*., p. 224).

And again:

> A motion is termed uniform in which equal increments of space described correspond to equal increments of space described by some motion with which we form a comparison, as the rotation of the earth. A motion may, with respect to another motion, be uniform. But the question whether a motion is in itself uniform, is senseless. With just as little justice, also, may we speak of an "absolute time" of a time independent of change. This absolute time can be measured by comparison with no motion; it has therefore neither a practical nor a scientific value; and no one is justified in saying that he knows aught about it. It is an idle metaphysical conception. (*Ivi*, p. 224).

What Mach writes is plausible, but it does not enter the logic that led Newton to introduce absolute time, motion and space. From the historical point of view, Newton, as we have seen, was in a very difficult situation: the mathematical means, which he himself had created, were still not very malleable and of uncertain reliability; the physicists were, for the most part, Cartesian, with erroneous conceptions of inertia, with a theory based on vortices that Newton himself had demonstrated not to be stable. What should he have done? To explain that time, space and motions are relative and, at the same time, to develop the whole mathematical apparatus of his new physics? In part he did, because almost all the motions treated in the *Principia* are, in fact, relative motions. But when it came to explaining the origin of centrifugal forces and deal with inertia, Newton resorted to absolute space.

In the historical circumstances in which Newton worked, his choice was probably the most economical - in Mach's own sense - and in terms of the prospectives opened up for research in physics. Mach seems to have overlooked these problematic aspects, related to the basic concepts of mechanics, with which Newton clashed. In this sense, Mach has got little into the inner problems concerning the construction of mechanics as a new science. What Mach writes against absolved space will make this statement clear.

Regarding absolute space and the bucket experiment, he argues that it is true that the rotation of water does not produce centrifugal forces with respect to the walls of the bucket. However, it cannot be deduced from this that it does produce them with respect to absolute space, but with respect to the earth and all other bodies in the universe. In particular, Mach refers to the fixed stars:

> Let us now examine the point on which Newton, apparently with sound reasons, rests his distinction of absolute and relative motion. If the earth is affected with an absolute rotation about its axis, centrifugal forces are set up in the earth: it assumes an oblate form, the acceleration of gravity is diminished at the equator, the plane of Foucault's pendulum rotates, and so on. All these phenomena disappear if the earth is at rest and the other heavenly bodies are affected with absolute motion round it, such that the same relative rotation is produced. This is, indeed, the case, if we start ab initio from the idea of absolute space. But if we take our stand on the basis of facts, we shall find we have knowledge only of relative spaces and motions. […]. The universe is not twice given, with an earth at rest and an earth in motion; but only once, with its relative motions, alone determinable. […] Newton's experiment with the rotating vessel of water simply informs us, that the relative rotation of the water with respect to the

> sides of the vessel produces no noticeable centrifugal forces, but that such forces are produced by its relative rotation with respect to the mass of the earth and the other celestial bodies. (*Ivi*, pp. 231–232).

A classic Machian idea and a new proposal are expressed: the classic idea is that the principles of physics must remain on the ground of facts and that metaphysical entities must not be introduced. If this remains a precept against thinking analogical, i.e., against the idea of extending, without proof of any kind (experimental or mathematical), properties which are true for phenomena that occur in given circumstances to phenomena that occur in circumstances assimilated to the previous ones, without actually having proved that they are so, then what Mach maintains is unexceptionable and is an excellent criterion of scientific prudence. But if one claims that the interpretation of the facts flows immediately from the facts themselves, then we would have a collection of facts, not a theory with predictive purposes. Furthermore, in Newton's interpretation, absolute space was an entity whose existence had been proved experimentally. Moreover, from the existence of absolute space an explanation for certain physical effects was achieved, at least according to Newton. Therefore, one is not legitimate to claim it to be a metaphysical entity; while, of course, one is legitimate to criticize Newton's conclusions. Had it not been for the theory of relativity, we would certainly still today have physicists and philosophers who would believe in absolute space and time and physicists and philosophers who would agree with Mach. These are points of view that are more or less fruitful and appear more or less plausible depending on the epoch, but on which there can be no definitive word without recourse to new experimental evidence or new physical-mathematical conceptions.

Let us now analyse Mach's new proposal: to refer centrifugal forces not to absolute space, but to all masses in the universe (in fact, we have seen that Newton too considered a similar hypothesis, but, after all, dismissed it). Mach argues like this: inertia exists because the totality of the masses of the universe, which, with respect to a mass *n*, can be considered distant, exert a zero force on *n*, which, therefore, has an average acceleration of zero with respect to these masses:

> Instead of saying, the direction and velocity of a mass μ in space remain constant, we may also employ the expression, the mean acceleration of the mass μ with respect to the masses *m*, *m*', *m*", … at the distances *r*, *r*', *r*", … is = 0, or $\frac{d^2\left(\frac{\sum mr}{\sum m}\right)}{dt^2} = 0$. The latter expression is equivalent to the former, as soon as we take into consideration a sufficient number of sufficiently distant and sufficiently large masses. […]. The considerations just presented show, that it is not necessary to refer the law of inertia to a special absolute space. On the contrary, it is perceived that the masses that in common phraseology exert forces on each other as well as those that exert none, stand with respect to acceleration in quite similar relations. We may, indeed, regard all masses as related to each other […]. The great distances between masses that stand in no especial force-relation to one another, change proportionally to each other. (*Ivi*, pp. 234–236).

What Mach offers is a statistical interpretation of the principle of inertia; it holds in a limit and statistical sense because the masses of kind *m* exert an action on the mass *n*, but the sum of their actions tends to 0. The idea that the principle of inertia is a limiting principle is not in contradiction with Newton's assertion: he was well aware of the non-existence of completely inertial motions in the current universe. This is the reason why, for Newton, absolute space guarantees the principle of inertia independently of any mass. Instead, Mach relates the principle of inertia to the masses that actually exist in the universe. The problem is that the reference to their existence is only apparently more direct than Newton's reference to absolute space, because Mach is forced to consider all the masses in the universe and this, within classical physics, is certainly no less problematic than the concept of absolute space. If, for example, the total mass of the universe was infinite, what guarantee would we have that it would make sense to speak of a null action of the masses on a body and, more generally, how would we determine such an action experimentally, just to remain within the Machianian perspective? But even if the total mass were finite, the effect of very distant bodies, beyond instrumental controllability, remains a concept that is very difficult to determine.

On the other hand, it is Mach himself who realises the problematic nature of his formulation of the inertia principle:

> We have attempted in the foregoing to give the law inertia a different expression from that in ordinary use. This expression will, so long as a sufficient number of bodies are apparently fixed in space, accomplish the same as the ordinary one. It is as easily applied, and it encounters the same difficulties. In the one case we are unable to come at an absolute space, in the other a limited number of masses only is within the reach of our knowledge, and the summation indicated can consequently not be fully carried out. It is impossible to say whether the new expression would still represent the true condition of things if the stars were to perform rapid movements among one another. (*Ivi*, p. 237).

Mach's criticism of entities such as absolute space and time, for him metaphysical notions, led him to conceive of a concept such as "the masses of entire universe", which is no less problematic. The situation will change when Einstein developed the theory of general relativity, but the immediate perceptibility (a point Mach often insists on) of the basic notions of physics would certainly not be regained. With his characteristic intellectual honesty, Mach does not hide the difficulties that even relativist theories encounter, in particular when, in the seventh edition of his work (1912), with regard to the concept of inertia, he comments the ideas of Neumann and Lange. Of course, as we mentioned at the beginning of this Sect. 5.2, problems within physics led to a reinterpretation and critique of Newtonian concepts. The solution to these problems only came through a new theory.

Finally, it is appropriate to recall Mach's criticism of the formulation of the principle of inertia (Newton's law I) as an independent proposition. For Mach argues that the principle of inertia is merely a special case of Newton's law II, according to which the change of motion is proportional to the imparted motive force $\boldsymbol{f}=m\boldsymbol{a}$. Mach therefore states that the principle of inertia is superfluous because it is already included in Newton's law II when the impelled force is zero (*ivi*, pp. 241–244). This is not true and Newton is right to state the first law, because $\boldsymbol{f}=m\boldsymbol{a}$ only in inertial reference frames, namely in the frames determined by the first law that, therefore, is the very core of Newton's mechanics.

6 As a Conclusion: Some of Einstein's Views on Newton's Absolute Space and Time

While on many occasions Einstein emphasized the inadequacy of Newtonian space and time for his new relativistic approach, he often underlined that, given the era in which Newton founded modern physics and the internal logic of that physics, their introduction is understandable, albeit problematic in some respects.

Einstein had an enormous admiration for Newton:

> It is necessary to commemorate this luminous spirit who, as no one before and after him showed the way to the Western thought, research, and the practical formation. He is not only the brilliant creator of special guiding methods, he has also mastered the empirical elements known in his time in a unique way, and his spirit has appeared marvelously ingenious in mathematical and physical argumentation. For all these reasons he is worthy of our high veneration. But this noble figure has even greater importance than that due to his authority as a teacher because fate placed him at a turning point in the development of the human spirit. To realise this exactly, we must not forget that, before Newton, there was no well-founded system of physical causality capable of grasping the deepest features the world of experience[20].

With regard to Newton's study of celestial motions and the discovery that gravity on Earth is the same force responsible for the rotation of the planets around the sun and the tides, Einstein wrote:

> […] Newton was able to explain the motions the planets, satellites and comets down to the minutest details, as well as the ebb and flow and the precession of the earth: a unique work of deduction. The realisation that the cause of the movements of celestial bodies is identical with gravity that is familiar to us from daily experience also had an admirable effect[21].

As for the understanding that instantaneous quantities are the fundamental quantities in physics and that a new mathematics was therefore needed, which Newton himself devised and applied to physics, Einstein argued:

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[20] Einstein, 1927, p. 273: "Da ist es Bedürfnis, dieses leuchtenden Geistes zu gedenken, der wie kein anderer vor und nach ihm dem abendländischen Denken, Forschen und praktischen Gestalten die Wege gewiesen hat. Er war nicht nur ein genialer Erfinder einzelner führender Methoden, sondern er beherrschte auch das zu seiner Zeit bekannte empirische Material in einzigartiger Weise, und er war wunderbar erfinderisch bezüglich der mathematischen und physikalischen Beweisführung im einzelnen. Aus all diesen Gründen ist er unserer hohen Verehrung würdig. Diese Gestalt bedeutet aber dadurch noch mehr, als es der ihr eigenen Meisterschaft entspricht, daß sie vom Schicksal an einen Wendepunkt der Geistesentwicklung gestellt wurde. Um dies lebhaft zu sehen, müssen wir uns vergegenwärtigen, daß es vor NEWTON kein geschlossenes System physikalischer Kausalität gab, das irgendwie tiefere Züge der Erfahrungswelt wiederzugeben vermochte". Our translation.

[21] Einstein, 1927, p. 274: "Kometen bis in feine Einzelheiten zu erklären, ferner Ebbe und Flut, die Präzessionsbewegung der Erde, eine deduktive Leistung von einzigartiger Großartigkeit. Besonders wunderbar mußte auch die Erkenntnis wirken, daß die Ursache der Bewegungen der Himmelskörper identisch ist mit der uns aus der alltäglichen Erfahrung so geläufige Schwere." Our translation.

> In order to give his system mathematical form at all, Newton had first to invent the concept of the differential quotient, and to draw up the laws of motion in the form of total differential equations — perhaps the greatest intellectual step that it has ever been given to one man to take[22].

Finally, we refer to this general remark by Einstein concerning a comparison between Newton's theory and his own theory of relativity:

> No one must think that Newton's great creation can be overthrown in any real sense by this or by any other theory. His clear and wide ideas will forever retain their significance as the foundation on which our modern conceptions of physics have been built. (Einstein, 1919).

If we then analyse what Einstein writes about Newton's absolute space, we realise how he emphasises Newton's merit in having understood that not all physical elements that can be ascertained in the motion of a body can be reduced to local events and measurements in space (Einstein speaks of "observable geometric quantities") and time. From the understanding of this state of affairs, then, derives the introduction of absolute space and time, which, writes Einstein, are entirely adequate concepts within Newton's physics. Einstein explains very well how, for Newton, absolute space is a physical reality, not merely a mathematical entity. The concluding remark in the following quotation is an almost epistemological rather than properly physical criticism:

> Although Newton's efforts to present his system as necessarily conditioned by experience and to introduce as few concepts as possible that could not relate to the direct data of experience are recognised, he nevertheless formulated the principle of absolute space and absolute time. He has often been reproached for this in our days. But exactly on this point, he was particularly consequent. He had recognised that the observable geometrical quantities (rooms of material points between them) and their movement in time do not completely characterise movement from a physical point of view. He proves this deduction with the celebrate bucket experiment. Therefore, besides masses and their distances variable in time. Something else exists which is crucial for the events. He also recognises, if his laws of motion make sense, then space must have a kind of physical reality, a reality of the same nature as material points and their distances. This precise knowledge equally demonstrates Newton's wisdom and the existence of a weak side to his theory. For the logical formulation of it would certainly be more satisfactory without this vague conception[23].

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[22] Einstein, 1931. In that year, Einstein published the article "Maxwell's influence on the developent of the conception of physical reality", in *James Clerk Maxwell: A Commemoration Volume*, pp. 66–73. This paper is now freely available without pagination at https://d-meeus.be/physique/Maxwell-Einstein-en.html. We used this free version.

[23] Einstein, 1927, pp. 274–275: "Trotzdem man allenthalben das Streben NEWTONS bemerkt, sein gedankensystem als durch die Erfahrung notwendig bedingt hinzustellen und möglichst wenig auf Erfahrungsgegenstgäde nicht unmittelbar beziehbare Begriffe einzuführen, stellt er den Begriff des absoluten Raumes und den der absoluten Zeit auf. Man hat ihm dies in unserer Zeit öfter zum Vorwurf gemacht. Aber gerade in diesem Punkte ist NEWTON besonders konsequent. Er hatte erkannt, daß die beobachtbaren geometrischen Größen (Abstände der materiellen Punkte voneinander) und deren zeitlicher Verlauf die Bewegungen in physikalischer Beziehung nicht vollständig charakterisieren. An dem berühmten Eimerversuch beweist er

Einstein therefore argues: in the whole of his physics Newton seeks to trace concepts back to a basis of phenomenal evidence. This operation is impossible for the notions of absolute space and time. Thence, they have a status that seems to undermine the coherence, also epistemic and methodological, of the Newtonian edifice. We shall see that Einstein argues that there is no possible reference to perception in the laws formulated with respect to absolute space. This is indisputable: no one can perceive absolute space. However, for Newton, there was indeed a reference to experience, as seen at length during this article.

There is also a very significant passage that shows how Einstein understood in depth the physical significance that Newton attributed to space:

1) it is not entirely determined by mechanical phenomena, i.e. relative motions do not determine absolute space;
2) it is the guarantor of the validity of the inertia principle

Completely consistent with what is said in the first part of the excerpt that we quote below, Einstein also highlights how, in fact, the idea of absolute space as a mere container of phenomena expresses a conception of the physicists subsequent Newton, up to Faraday and Maxwell, rather than of Newton himself. Once again, what Einstein writes is illuminating:

> For dynamics cannot manage with the concepts of the mass point and the (temporally variable) distance between mass points alone. In Newton's equations of motion the concept of acceleration plays a fundamental part, which cannot be defined by the temporally variable intervals between points alone. Newton's acceleration is only thinkable or definable in relation to space as a whole. Thus to the geometrical reality of the concept of space a new inertia-determining function of space was added. When Newton described space as absolute, he no doubt meant this real significance of space, which made it necessary for him to attribute it a quite definite state of motion, which yet did not appear to be fully determined by the phenomena of mechanics. This space was conceived as absolute in another sense also; its inertia-determining effect was conceived as autonomous, i.e., not to be influenced by any physical circumstance whatever; it affected masses, but nothing affected it. And yet in the minds of physicists space remained until the most recent time simply the passive container of all events, playing no part in physical happening itself. Thought only began to take a new turn with the wave theory of light and the theory of the electromagnetic field of Faraday and Clerk Maxwell. (Einstein, 1934, pp. 474–475).

Einstein, in a paper of 1952, specifically dedicated to the problem of space, highlights another functional aspect of Newton's absolute space. Einstein's way of expressing is very concise, but what he means is clear and can be interpreted as follows: if we do not pose the ontological problem of the nature of Newtonian absolute space, but only analyse it from a functional point of view, it is not necessary to think of it as being at rest, it can be thought

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diesen Umstand. Es gibt also außer den Massen und ihren zeitlich variablen Abständen noch etwas, was für das Geschehen maßgebend ist : dieses „Etwas" faßt er als die Beziehung zum „absoluten Raum" auf. Er erkennt, daß der Raum eine Art physikalischer Realität besitzen muß, wenn seine Bewegungsgesetze einen Sinn haben sollen, eine Realität von derselben Art wie die materiellen Punkte und deren Abstände. Diese klare Erkenntnis zeigt ebenso NEWTONS Weisheit wie auch eine schwache Seite seiner Theorie. Denn der logische Aufbau der letzteren wäre gewiß befriedigender ohne diesen schattenhaften Begriff". Our translation.

of as being in a state of uniform rectilinear motion. From the point of view of the relationship between absolute space and the real physical world, this view seems implausible, but it perfectly captures Newton's idea that absolute space is the guarantor of the inertia principle, and the state of rest cannot be distinguished from uniform motion even in absolute space. Einstein continues claiming that Newton felt some discomfort as to absolute space. However, he mentions no passage of Newton's work that justifies his idea. Einstein then concludes by asserting that the introduction of absolute space was the only viable alternative in Newton's time:

> It is characteristic of Newtonian physics that it has to ascribe independent and real existence to space and time as well as to matter, for in Newton's law of motion the idea of acceleration appears. But in this theory, acceleration can only denote "acceleration with respect to space". Newton's space must thus be thought of as "at rest", or at least as "unaccelerated", in order that one can consider the acceleration, which appears in the law of motion, as being a magnitude with any meaning. Much the same holds with time, which of course likewise enters into the concept of acceleration. Newton himself and his most critical contemporaries felt it to be disturbing that one had to ascribe physical reality both to space itself as well as to its state of motion; but there was at that time no other alternative, if one wished to ascribe to mechanics a clear meaning. (Einstein, 1952. English translation 1954).

We conclude with a quotation from Einstein, where he points out how, in special relativity, the Newtonian concept of absolute space has been transformed into four-dimensional continuum, which, although in a very different context from Newtonian space and within a general conception in which many of the basic assumptions of classical physics have been abandoned, maintains the same characteristic of absoluteness:

> physical space was thus increased to a four-dimensional space which also included the dimension of time. The four-dimensional space of the special theory of relativity is just as rigid and absolute as Newton's space (Einstein, 1934, pp. 476–477).

Appendix

In this appendix we will analyse four notes to Proposition XIX, Book III of *Principia*. They are useful because they offer many mathematical details that Newton gave for granted and they are paradigmatic about how the editors of the GE worked[24]. The first and longest note we examine concerns Newton's assertion that, considered the ratio between the polar diameter PQ of the earth and its equatorial diameter AB as 100 to 101 and considered a sphere whose diameter is equal to PQ, then the ratio of earth's gravity at the point Q to the gravity of Q on the sphere is as 126 to 125 (Fig. 3). The second note (Fig. 4) concerns the proof that gravity in the place A of the earth is mean proportional between gravity in the rotational

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[24] This appendix concerns only technical notes of the Geneva Editions. We do not enter the scientific, methodological, epistemic and political reasons that caused the realization of such a huge editorial enterprise. On these complex issues we refer to Bussotti and Pisano, 2014a, b, 2025; Pisano and Bussotti 2016b, 2016b, 2017a, 2017b, 2020, 2022, 2025, 2030; Pisano, Bussotti and Belotti 2025.

ellipsoid around the major axis of the earth (ellipsoid 3) of our running text) and the sphere whose diameter is the major axis of the ellipsoid 3) (sphere 4)). The third and fourth notes (Fig. 5) regard the proof that gravity acceleration in the place *Q* of the earth is to gravity acceleration in the place *A* of the earth as 501 to 500.

Let us now move to the first note by Calandrini (Newton 1867, 1726, 1739–1742, [1822], III, note (r)*, pp. 60–62). The author of the note can be recognized by the asterisk in the indication of the note. (Fig. 3)

Calandrini argues as follows: in both images of Fig. 3, the line *PAQB* represents the earth's meridian. In the first image *QDPQ* is the sphere of centre *C* and radius *QC*. In the second image *PAQB* is the rotation ellipsoid that Newton images to be described by the rotation of the earth's meridian around the equator and *AED* is the sphere with radius *AC*. According to Corollary 2, Prop. XC, Book I (see Pisano and Bussotti, 2022), if the circles perpendicular to the revolution axes whose radiuses are *FG* and *fg* (this holds for both images in Fig. 3) are drawn, the attractions on the points *Q* and *A* belonging to those two circles are resp. $1-\frac{QF}{QG}$, $1-\frac{QF}{Qg}$, $1-\frac{AF}{AG}$, $1-\frac{AF}{Ag}$. Set *CQ* or *CD* equal to *b*, and *AC* or *CE* equal to *r*. The two abscissas *QF* and *AF* be indicated by *x*. Therefore, in the image on the left and in that on the right, it will be resp.

$$\begin{cases} FG^2=\frac{r^2}{b^2}\times\ (2bx-x^2) \\ Fg^2=2bx-x^2 \end{cases} \begin{cases} FG^2=\frac{b^2}{r^2}\times\ (2rx-x^2) \\ Fg^2=2rx-x^2 \end{cases} .$$

To these squares add QF^2, or AF^2 or x^2. You will obtain

$$QG^2=\frac{r^2}{b^2}\times\ 2bx-\frac{r^2-b^2}{b^2}x^2,\quad Qg^2=2bx$$

$$AG^2=\frac{b^2}{r^2}\times\ 2rx+\frac{r^2-b^2}{r^2}x^2,\quad Ag^2=2rx$$

Set $r^2-b^2=m$. The attractions of these two circles will be

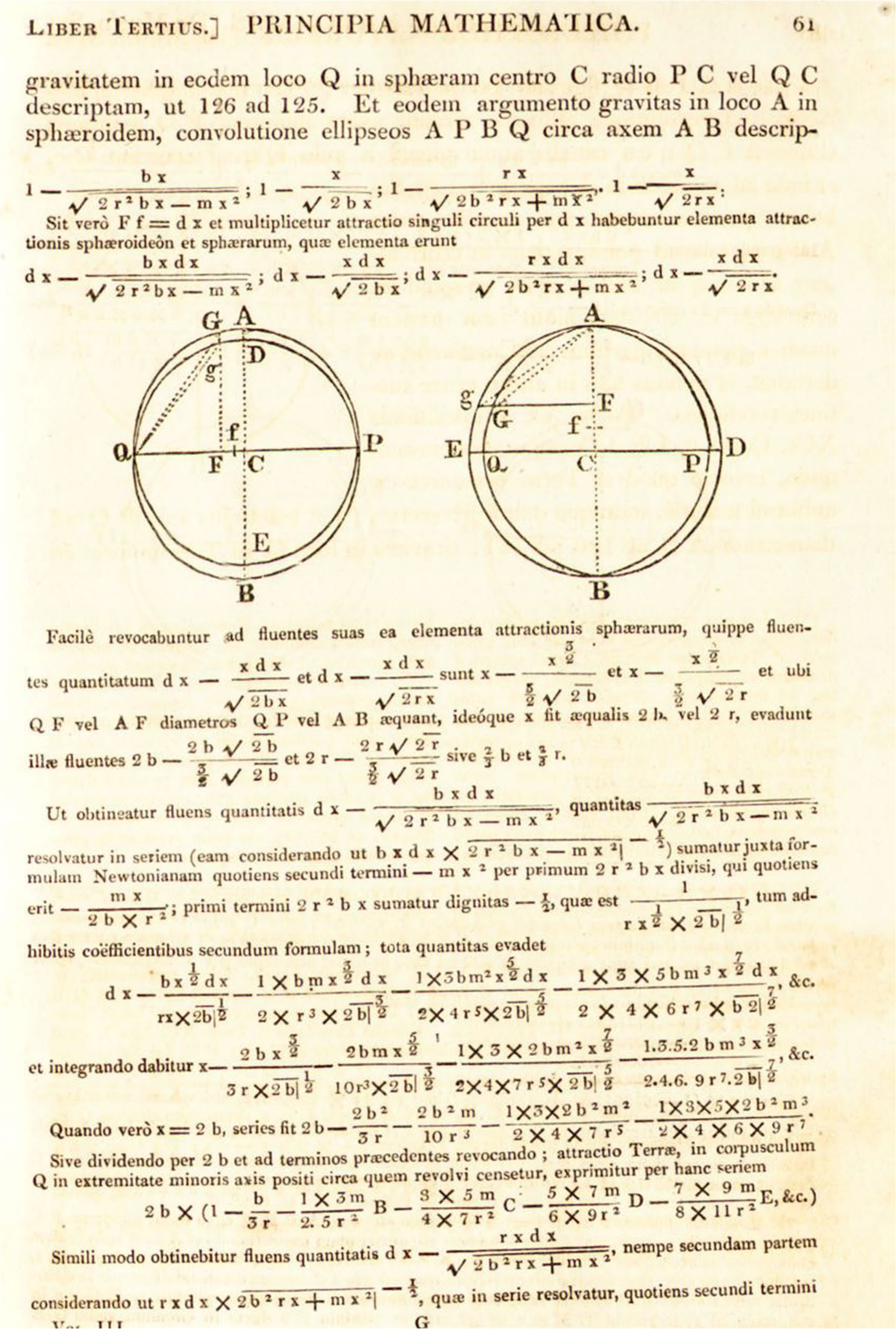
$$1-\frac{bx}{\sqrt{2r^2bx-mx^2}};\ \ 1-\frac{x}{\sqrt{2bx}};\ 1-\frac{rx}{\sqrt{2b^2rx-mx^2}};\ \ 1-\frac{x}{\sqrt{2rx}}$$

If $Ff=dx$ and the attraction of a single circle is multiplied by *dx*, one will obtain the elements of the attraction of the spheroids and of the spheres whose elements will be

$$dx-\frac{bx\,dx}{\sqrt{2r^2bx-mx^2}};dx-\frac{x\,dx}{\sqrt{2bx}};\ dx-\frac{rx\,dx}{\sqrt{2b^2rx-mx^2}};\ \ dx-\frac{x\,dx}{\sqrt{2rx}}$$

It is

$$\int dx-\frac{x\,dx}{\sqrt{2bx}}=x-\frac{x^{\frac{3}{2}}}{\frac{3}{2}\sqrt{2b}};\ \int dx-\frac{x\,dx}{\sqrt{2rx}}=x-\frac{x^{\frac{3}{2}}}{\frac{3}{2}\sqrt{2r}}$$



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gravitatem in eodem loco Q in sphæram centro C radio P C vel Q C descriptam, ut 126 ad 125. Et eodem argumento gravitas in loco A in sphæroidem, convolutione ellipseos A P B Q circa axem A B descrip-

$$1-\frac{bx}{\sqrt{2r^2bx-mx^2}};\ 1-\frac{x}{\sqrt{2bx}};\ 1-\frac{rx}{\sqrt{2b^2rx+mx^2}}.\ 1-\frac{x}{\sqrt{2rx}}:$$

Sit verò F f = d x et multiplicetur attractio singuli circuli per d x habebuntur elementa attractionis sphæroideôn et sphærarum, quæ elementa erunt

$$dx-\frac{bxdx}{\sqrt{2r^2bx-mx^2}};\ dx-\frac{xdx}{\sqrt{2bx}};\ dx-\frac{rxdx}{\sqrt{2b^2rx+mx^2}};\ dx-\frac{xdx}{\sqrt{2rx}}.$$

Facilè revocabuntur ad fluentes suas ea elementa attractionis sphærarum, quippe fluentes quantitatum $dx-\frac{xdx}{\sqrt{2bx}}$ et $dx-\frac{xdx}{\sqrt{2rx}}$ sunt $x-\frac{x^{\frac{3}{2}}}{\frac{3}{2}\sqrt{2b}}$ et $x-\frac{x^{\frac{3}{2}}}{\frac{3}{2}\sqrt{2r}}$ et ubi Q F vel A F diametros Q P vel A B æquant, ideóque x fit æqualis 2 b, vel 2 r, evadunt illæ fluentes $2b-\frac{2b\sqrt{2b}}{\frac{3}{2}\sqrt{2b}}$ et $2r-\frac{2r\sqrt{2r}}{\frac{3}{2}\sqrt{2r}}$ sive $\frac{2}{3}$ b et $\frac{2}{3}$ r.

Ut obtineatur fluens quantitatis $dx-\frac{bxdx}{\sqrt{2r^2bx-mx^2}}$, quantitas $\frac{bxdx}{\sqrt{2r^2bx-mx^2}}$ resolvatur in seriem (eam considerando ut $bxdx\times\overline{2r^2bx-mx^2}|^{-\frac{1}{2}}$) sumatur juxta formulam Newtonianam quotiens secundi termini $-mx^2$ per primum $2r^2bx$ divisi, qui quotiens erit $-\frac{mx}{2b\times r^2}$; primi termini $2r^2bx$ sumatur dignitas $-\frac{1}{2}$, quæ est $\frac{1}{rx^{\frac{1}{2}}\times\overline{2b}|^{\frac{1}{2}}}$, tum adhibitis coëfficientibus secundum formulam; tota quantitas evadet

$$dx-\frac{bx^{\frac{1}{2}}dx}{rx\times\overline{2b}|^{\frac{1}{2}}}-\frac{1\times bmx^{\frac{3}{2}}dx}{2\times r^3\times\overline{2b}|^{\frac{3}{2}}}-\frac{1\times3bm^2x^{\frac{5}{2}}dx}{2\times4r^5\times\overline{2b}|^{\frac{5}{2}}}-\frac{1\times3\times5bm^3x^{\frac{7}{2}}dx}{2\times4\times6r^7\times\overline{b2}|^{\frac{7}{2}}},\ \&c.$$

et integrando dabitur $x-\frac{2bx^{\frac{3}{2}}}{3r\times\overline{2b}|^{\frac{1}{2}}}-\frac{2bmx^{\frac{5}{2}}}{10r^3\times\overline{2b}|^{\frac{3}{2}}}-\frac{1\times3\times2bm^2x^{\frac{7}{2}}}{2\times4\times7r^5\times\overline{2b}|^{\frac{5}{2}}}-\frac{1.3.5.2bm^3x^{\frac{3}{2}}}{2.4.6.9r^7.\overline{2b}|^{\frac{7}{2}}}$, &c.

Quando verò x = 2 b, series fit $2b-\frac{2b^2}{3r}-\frac{2b^2m}{10r^3}-\frac{1\times3\times2b^2m^2}{2\times4\times7r^5}-\frac{1\times3\times5\times2b^2m^3}{2\times4\times6\times9r^7}$.

Sive dividendo per 2 b et ad terminos præcedentes revocando; attractio Terræ, in corpusculum Q in extremitate minoris axis positi circa quem revolvi censetur, exprimitur per hanc seriem

$$2b\times(1-\frac{b}{3r}-\frac{1\times3m}{2.5r^2}B-\frac{3\times5m}{4\times7r^2}C-\frac{5\times7m}{6\times9r^2}D-\frac{7\times9m}{8\times11r^2}E,\ \&c.)$$

Simili modo obtinebitur fluens quantitatis $dx-\frac{rxdx}{\sqrt{2b^2rx+mx^2}}$, nempe secundam partem considerando ut $rxdx\times\overline{2b^2rx+mx^2}|^{-\frac{1}{2}}$, quæ in serie resolvatur, quotiens secundi termini

G

Fig. 3 The figures and the calculations used by Calandrini to prove that if $PQ : AB = 100 : 101$ gravity in Q on the earth is to gravity in Q in the sphere with centre C and radius QC as 126 to 125

If *QF* or *AF* are equal to the diameters *QP* or *AB*, so that *x* is equal to $2b$ or $2r$, those fluents (integrals) will be

$$2b-\frac{2b\sqrt{2b}}{\frac{3}{2}\sqrt{2b}}=\frac{2}{3}b;\ \ 2r-\frac{2r\sqrt{2r}}{\frac{3}{2}\sqrt{2r}}=\frac{2}{3}r$$

To obtain $\int dx-\frac{bx\,dx}{\sqrt{2r^2bx-mx^2}}$, Calandrini develops in series the quantity $bx\,dx\times\ (2r^2bx-mx^2)^{-\frac{1}{2}}$. After this operation the integrand will be

$$dx-\frac{bx^{\frac{1}{2}}dx}{r(2b)^{\frac{1}{2}}}-\frac{1\times\ bmx^{\frac{3}{2}}dx}{2\times\ r^3\times\ (2b)^{\frac{3}{2}}}-\frac{1\times\ 3bm^2x^{\frac{5}{2}}dx}{2\times\ 4r^5\times\ (2b)^{\frac{5}{2}}}-\frac{1\times\ 3\times\ 5bm^3x^{\frac{7}{2}}dx}{2\times\ 4\times\ 6r^7\times\ (2b)^{\frac{7}{2}}}-\dots$$

By integrating one has

$$x-\frac{2bx^{\frac{3}{2}}}{3r(2b)^{\frac{1}{2}}}-\frac{2bmx^{\frac{5}{2}}}{10r^3\times\ (2b)^{\frac{3}{2}}}-\frac{1\times\ 3\times\ 2bm^2x^{\frac{7}{2}}}{2\times\ 4\times\ 7r^5\times\ (2b)^{\frac{5}{2}}}-\frac{1\times\ 3\times\ 5\times\ 2bm^3x^{\frac{9}{2}}}{2\times\ 4\times\ 6\times\ 9r^7\times\ (2b)^{\frac{7}{2}}}-\dots$$

If $x=2b$, this series is written as

$$2b-\frac{2b^2}{3r}-\frac{2b^2m}{10r^3}-\frac{1\times\ 3\times\ 2b^2m^2}{2\times\ 4\times\ 7r^5}-\frac{1\times\ 3\times\ 5\times\ 2b^2m^3}{2\times\ 4\times\ 6\times\ 9r^7}-\dots,$$

so that the attraction exerted by the earth on a corpuscle *Q* located around the extreme of the minor axis, along which it is thought to rotate, will be

$$2b\left(1-\frac{b}{3r}-\frac{1\times\ 3m}{2\times\ 5r^2}B-\frac{3\times\ 5m}{4\times\ 7r^2}C-\frac{5\times\ 7m}{6\times\ 9r^2}D-\frac{7\times\ 9m}{8\times\ 11r^2}E-\dots\right)\tag{1}$$

where the values of the constants $B,C,D,E,\dots$ can be easily calculated.

Through completely analogous reasoning Calandrini calculated $\int dx-\frac{rxdx}{\sqrt{2b^2rx+mx^2}}$. Eventually, by setting $x=2r$, one has in this case, the expression:

$$2r\left(1-\frac{r}{3b}-\frac{3m}{2\times\ 5b^2}B-\frac{3\times\ 5m}{4\times\ 7b^2}C-\frac{5\times\ 7m}{6\times\ 9b^2}D-\frac{7\times\ 9m}{8\times\ 11b^2}E-\dots\right)\tag{2}$$

Calandrini now recalls that r=101, b=100, so that $r^2-b^2=m=201$, $r^2=10201$. Let us replace these values in 1). With the used approximation, Calandrini reached this result which he wrote in a very expressive form:

$$2b\times\ 1-.66006600$$

$$-.00390177$$

$$-.00004118$$

$$-.00000052$$

$$-.00000001.$$

$$2b\times\ (1-.66400948)$$

This result can also be written as $2b\times\ .33599052$. The sphere's attraction was $2b/3$. Ergo, gravity in the place *Q* of the earth will be to gravity in the sphere with centre *C* and radius *QC* as 1.00797156 to 1. This result is obtained multiplying both terms by 3 and dividing by $2b$. Such value is almost as 1008 to 1000, namely as 126 to 125, which was to prove.

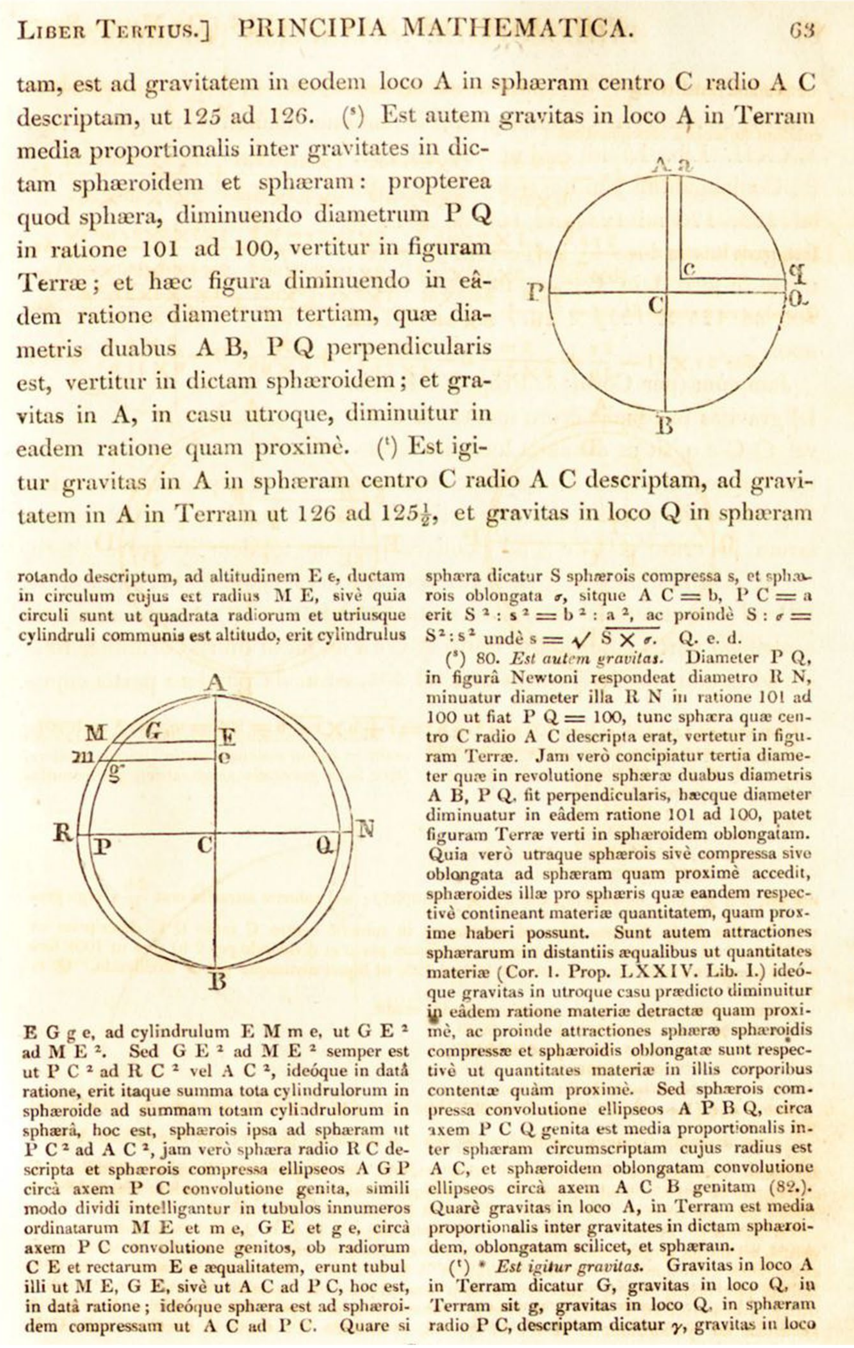
With a similar reasoning, resorting to expression 2), Calandrini showed that gravity in *A* in the rotation ellipsoid around the major axis is to gravity in *A* in the sphere of radius *AC* as 125 to 126.

Let us now see (Fig. 4) how Jacquier and Le Seur proved that gravity in the place *A* of the earth is mean proportional between gravity in the same place of the rotation ellipsoid around the major axis of the Earth *AB* and the sphere whose diameter is *AB*.

In Fig. 4, the image above is by Newton, while that below by the editors. The diameter *PQ* in Newton's figure corresponds to the diameter *RN* in the editors' figure. Let us reduce *RN* in the ratio 101/100, so that *PQ*= 100. Then the sphere described with centre *C* and radius *AC* is transformed into the earth. Consider now a third diameter that, while rotating the sphere, be perpendicular to *AB* and *PQ*. Be this diameter reduced in the ratio 101/100. The earth's shape is so transformed into that of an oblong spheroid. Both spheroids, the compressed, which is bigger and represents the earth, and the oblong one, are almost spherical. Therefore, as to gravity attraction, they can be regarded as almost equivalent to spheres having their same masses. The spheres' attractions to points located at the same distances from their centres are as the spheres' masses (Corollary 1, Proposition LXXIV, Book I. See also Pisano & Bussotti, 2022). Thence, in both our cases, gravity almost diminishes as the quantity of matter lost in the two described operations which makes to pass from the sphere to the earth and from the earth to the smaller rotation ellipsoid. This means that the attractions of the two rotation ellipsoids compared to that of the sphere are almost as the quantity of matter of these three bodies. The mass of the spheroid given by the rotation of the ellipse *APBQ* around its smaller axis *PQ* (which represents the earth) is mean proportional between that of the sphere circumscribing it, whose radius is *AC*, and that of the oblong spheroid generated by the rotation of such ellipse around the axis *AB*. Therefore, the gravity exerted by the earth in point *A* is mean proportional between that exerted by the oblong spheroid and the sphere Finally, let us see that gravity on the earth in the place *Q* is to gravity on the earth in the place *A* as 501 to 500 (Fig. 5).

The notes are by Calandrini, as the presence of the asterisk testifies. Name *G* the gravity in *A* and *g* the gravity in *Q* on the earth. In the sphere with radius *PC* (in Fig. 5 see image above which was Newton's) be γ the gravity in *A*, while in the spheroid generated by the rotation of the ellipse *APBQ* around the axis *AB*, be *V* the gravity in *A*. Finally, be Γ the gravity in *A* exerted by the sphere of radius *AC*.

Because of the property proved in the note we have just analysed, it is $g:\gamma\ =126:125;\ V:\Gamma\ =125:126,\ V:G=G:\Gamma$. Therefore, it is $V:G=G:\Gamma\ =125:125\frac{1}{2}=125\frac{1}{2}:126$. Thus, one has these three ratios:



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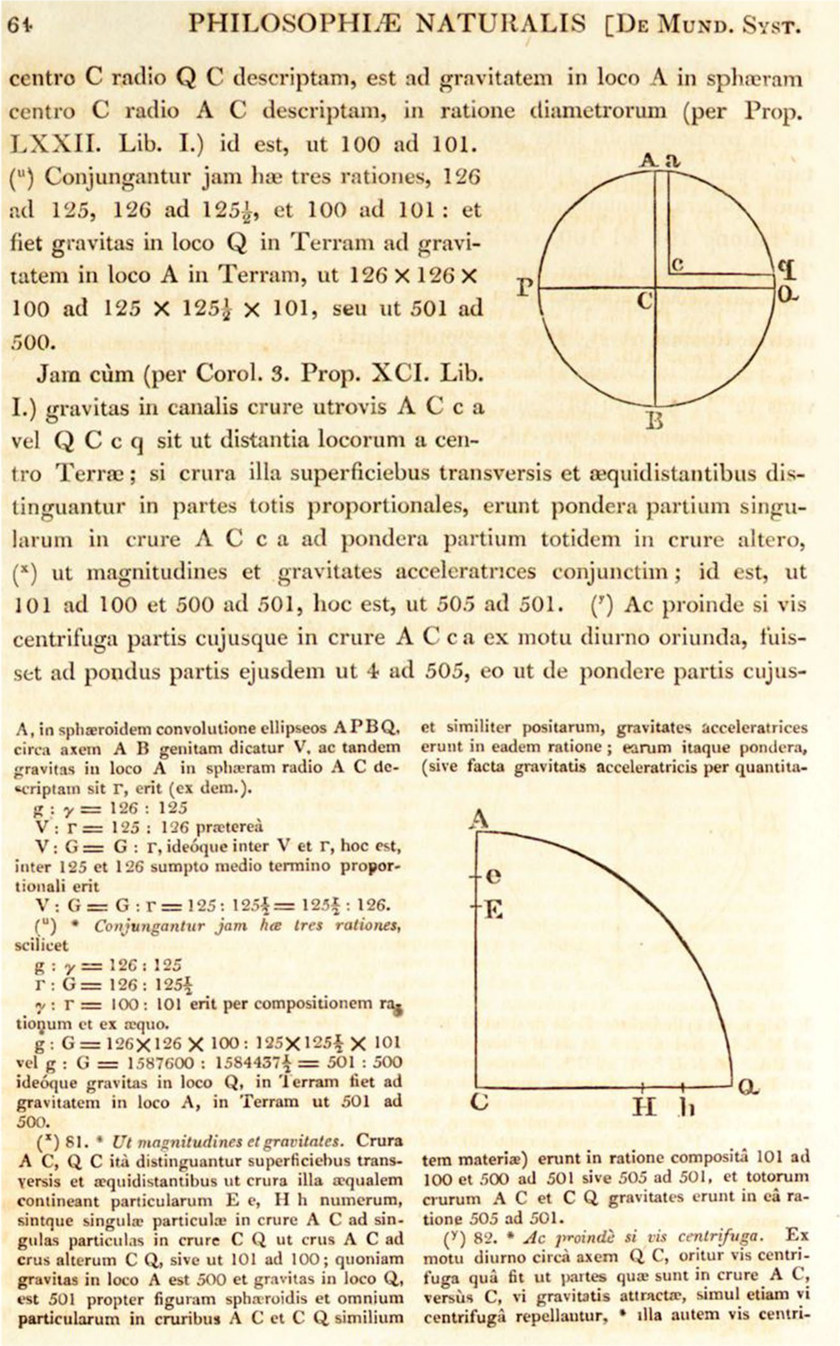
tam, est ad gravitatem in eodem loco A in sphæram centro C radio A C descriptam, ut 125 ad 126. ([s]) Est autem gravitas in loco A in Terram media proportionalis inter gravitates in dictam sphæroidem et sphæram: propterea quod sphæra, diminuendo diametrum P Q in ratione 101 ad 100, vertitur in figuram Terræ; et hæc figura diminuendo in eâdem ratione diametrum tertiam, quæ diametris duabus A B, P Q perpendicularis est, vertitur in dictam sphæroidem; et gravitas in A, in casu utroque, diminuitur in eadem ratione quam proximè. ([t]) Est igitur gravitas in A in sphæram centro C radio A C descriptam, ad gravitatem in A in Terram ut 126 ad $125\frac{1}{2}$, et gravitas in loco Q in sphæram

rotando descriptum, ad altitudinem E ϵ, ductam in circulum cujus est radius M E, sivè quia circuli sunt ut quadrata radiorum et utriusque cylindruli communis est altitudo, erit cylindrulus E G g e, ad cylindrulum E M m e, ut GE^2 ad ME^2. Sed GE^2 ad ME^2 semper est ut PC^2 ad RC^2 vel AC^2, ideóque in datâ ratione, erit itaque summa tota cylindrulorum in sphæroide ad summam totam cylindrulorum in sphærâ, hoc est, sphærois ipsa ad sphæram ut PC^2 ad AC^2, jam verò sphæra radio R C descripta et sphærois compressa ellipseos A G P circà axem P C convolutione genita, simili modo dividi intelligantur in tubulos innumeros ordinatarum M E et m e, G E et g e, circà axem P C convolutione genitos, ob radiorum C E et rectarum E e æqualitatem, erunt tubul illi ut M E, G E, sivè ut A C ad P C, hoc est, in datâ ratione; ideóque sphæra est ad sphæroidem compressam ut A C ad P C. Quare si sphæra dicatur S sphærois compressa s, et sphærois oblongata σ, sitque $AC = b$, $PC = a$ erit $S^2 : s^2 = b^2 : a^2$, ac proindè $S : \sigma = S^2 : s^2$ undè $s = \sqrt{S \times \sigma}$. Q. e. d.

([s]) 80. *Est autem gravitas.* Diameter P Q, in figurâ Newtoni respondeat diametro R N, minuatur diameter illa R N in ratione 101 ad 100 ut fiat $PQ = 100$, tunc sphæra quæ centro C radio A C descripta erat, vertetur in figuram Terræ. Jam verò concipiatur tertia diameter quæ in revolutione sphæræ duabus diametris A B, P Q, fit perpendicularis, hæcque diameter diminuatur in eâdem ratione 101 ad 100, patet figuram Terræ verti in sphæroidem oblongatam. Quia verò utraque sphærois sivè compressa sive oblongata ad sphæram quam proximè accedit, sphæroides illæ pro sphæris quæ eandem respectivè contineant materiæ quantitatem, quam proxime haberi possunt. Sunt autem attractiones sphærarum in distantiis æqualibus ut quantitates materiæ (Cor. 1. Prop. LXXIV. Lib. I.) ideóque gravitas in utroque casu prædicto diminuitur in eâdem ratione materiæ detractæ quam proximè, ac proinde attractiones sphæræ sphæroidis compressæ et sphæroidis oblongatæ sunt respectivè ut quantitates materiæ in illis corporibus contentæ quàm proximè. Sed sphærois compressa convolutione ellipseos A P B Q, circa axem P C Q genita est media proportionalis inter sphæram circumscriptam cujus radius est A C, et sphæroidem oblongatam convolutione ellipseos circà axem A C B genitam (82.). Quarè gravitas in loco A, in Terram est media proportionalis inter gravitates in dictam sphæroidem, oblongatam scilicet, et sphæram.

([t]) * *Est igitur gravitas.* Gravitas in loco A in Terram dicatur G, gravitas in loco Q, in Terram sit g, gravitas in loco Q, in sphæram radio P C, descriptam dicatur γ, gravitas in loco

Fig. 4 Jacquier's and Le Seur's reasoning to explain Newton's assertion. Newton, 1687, 1726, 1739–1742, III, p. 63, note (s)80



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centro C radio Q C descriptam, est ad gravitatem in loco A in sphæram centro C radio A C descriptam, in ratione diametrorum (per Prop. LXXII. Lib. I.) id est, ut 100 ad 101. (u) Conjungantur jam hæ tres rationes, 126 ad 125, 126 ad $125\frac{1}{2}$, et 100 ad 101 : et fiet gravitas in loco Q in Terram ad gravitatem in loco A in Terram, ut $126 \times 126 \times 100$ ad $125 \times 125\frac{1}{2} \times 101$, seu ut 501 ad 500.

Jam cùm (per Corol. 3. Prop. XCI. Lib. I.) gravitas in canalis crure utrovis A C c a vel Q C c q sit ut distantia locorum a centro Terræ; si crura illa superficiebus transversis et æquidistantibus distinguantur in partes totis proportionales, erunt pondera partium singularum in crure A C c a ad pondera partium totidem in crure altero, (x) ut magnitudines et gravitates acceleratrices conjunctim; id est, ut 101 ad 100 et 500 ad 501, hoc est, ut 505 ad 501. (y) Ac proinde si vis centrifuga partis cujusque in crure A C c a ex motu diurno oriunda, fuisset ad pondus partis ejusdem ut 4 ad 505, eo ut de pondere partis cujus-

A, in sphæroidem convolutione ellipseos APBQ, circa axem A B genitam dicatur V, ac tandem gravitas in loco A in sphæram radio A C descriptam sit Γ, erit (ex dem.).

$g : \gamma = 126 : 125$

$V : \Gamma = 125 : 126$ præthereà

$V : G = G : \Gamma$, ideóque inter V et Γ, hoc est, inter 125 et 126 sumpto medio termino proportionali erit

$V : G = G : \Gamma = 125 : 125\frac{1}{2} = 125\frac{1}{2} : 126.$

(u) * *Conjungantur jam hæ tres rationes,* scilicet

$g : \gamma = 126 : 125$

$\Gamma : G = 126 : 125\frac{1}{2}$

$\gamma : \Gamma = 100 : 101$ erit per compositionem rationum et ex æquo.

$g : G = 126 \times 126 \times 100 : 125 \times 125\frac{1}{2} \times 101$

vel $g : G = 1587600 : 1584437\frac{1}{2} = 501 : 500$

ideóque gravitas in loco Q, in Terram fiet ad gravitatem in loco A, in Terram ut 501 ad 500.

(x) 81. * *Ut magnitudines et gravitates.* Crura A C, Q C ità distinguantur superficiebus transversis et æquidistantibus ut crura illa æqualem contineant particularum E e, H h numerum, sintque singulæ particulæ in crure A C ad singulas particulas in crure C Q ut crus A C ad crus alterum C Q, sive ut 101 ad 100; quoniam gravitas in loco A est 500 et gravitas in loco Q, est 501 propter figuram sphæroidis et omnium particularum in cruribus A C et C Q similium et similiter positarum, gravitates acceleratrices erunt in eadem ratione; earum itaque pondera, (sive facta gravitatis acceleratricis per quantitatem materiæ) erunt in ratione compositâ 101 ad 100 et 500 ad 501 sive 505 ad 501, et totorum crurum A C et C Q gravitates erunt in eâ ratione 505 ad 501.

(y) 82. * *Ac proindè si vis centrifuga.* Ex motu diurno circà axem Q C, oritur vis centrifuga quâ fit ut partes quæ sunt in crure A C, versùs C, vi gravitatis attractæ, simul etiam vi centrifugâ repellantur, * illa autem vis centri-

Fig. 5 Calandrini's proof that gravity on the earth in the place *Q* is to gravity on the earth in the place *A* as 501 to 500. Newton, 1687, 1726, 1739–1742, III, pp. 63–64, notes (t)* and (u)*

$$\begin{cases} g:\gamma &= 126:125 \\ \Gamma\ :G &= 126:125\frac{1}{2} \\ \gamma\ :\Gamma &= 100:101 \end{cases}$$

By composing and *ex aequo*, it is:

$$g:G = 126\times\ 126\times\ 100:125\times\ 125\frac{1}{2}\times\ 101 = 1587600:1584437\frac{1}{2} = 501:500$$

Hence, it is proved that gravity in the place Q of the earth is to gravity in the place A of the earth as 501 to 500.

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Declarations

Competing interests The authors declare no competing interests.

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