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How to Increase Earnings by Exploiting the Veblen Effect

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***Abstract*— The price/demand curve in a scenario characterized by the so-called Veblen effect is analytically identified as a modification, due to a variable that we call *hankering*, of the classical monotonic (price/demand) curve. Differentiating this function with respect to price and using a classical time variation of price, we obtain a control dynamical system. We exploit this model to analyse the possible trajectories and the equilibrium points. Based on these results, we design a strategy that maximizes the earnings of the seller.**

Index terms: Dynamic models in economy, Veblen effect, bi-stability.

I. Introduction

Since it was first theorized by Thorstein Veblen [14] the effect bearing his name has been widely studied by scientists from the economic and behavioural disciplines (see, e.g. [2], [4], [7], [10], [8], [11], [9], [1], [3], [5], [13], [12]), although only some of them investigate it from a mathematical point of view. Veblen effect (VE) basically consists in the fact that the demand for a good may, for different reasons, contradict the classical price/demand curve, which describes the demand as a decreasing function of price, and exhibits an increasing behaviour, at least in a limited interval.

Along the lines of the works by Leibenstein [6], we assume that the price/demand curve in a scenario characterized by the VE is an "S" shaped function. We analytically describe this function as a modification, due to a variable called *hankering*, of the classical monotonic price/demand curve.

Subsequently, time is introduced by differentiating the above function with respect to price and using the classical time-variation of price (a function of demand and supply). This accessory step allows us to obtain a time dynamical system which can be exploited, in particular, to (i) analyse the time-evolution of price and demand, (ii) design a control strategy, and (iii) study the robustness of the control strategy with respect to model uncertainties or exogenous disturbances.

Herein, in particular, referring to the dynamical system obtained as above, we analyse the possible trajectories of the state, depending on the initial conditions. Under some reasonable assumptions on the price variation, the system is proven to admit, for a constant input, two equilibrium states, characterized by the same value of the demand but different values of the price.

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As a consequence of this result, we propose an input strategy that, starting from an equilibrium point on the price/demand curve, drives the state to another equilibrium point associated with the same demand but to a higher price. In addition, we show that this strategy can be further improved driving the state to a different equilibrium point which, although characterized by a smaller price, is associated with a higher demand so that the overall earning is higher.

Summarizing, the contribution of this paper is twofold

- we propose an analytic model for the VE;
- assuming that this model can reasonably describe a real scenario, we show how a producer or a seller can profitably exploit the VE to increase the earnings.

The paper is organized as follows. Section II describes a mathematical model of the variation of the demand as a function of price which contributes to give insight on the VE. Section III analyses the trajectories of the dynamic model and proposes a selling strategy that leads to a maximization of the earnings. Section IV applies the results to a numeric example while conclusions are drawn in Section V

II. The model

This section is dedicated to analyse how VE affects the equilibrium of the price/demand dynamics and how it can be profitably exploited by a firm in order to maximize the earnings.

A. The demand as a function of price

First of all we propose a model to describe how the demand changes with respect to price. Let p denote the price of a good and let D denote the demand of it. Besides the positiveness of both these variables, it is reasonable to suppose that, in a natural context (hence neglecting the VE) and fixed any other condition, the demand decreases as the price increases. Another reasonable assumption is that the demand tends to zero asymptotically as the price tends to infinity, since there is always someone keen to spend a large amount of money to buy a good. These requirements are fulfilled, for instance, by an exponential decay[1] which can be written as a function $D : [0, +\infty) \to [0, +\infty)$ defined by[2]

$$D(p) = D_0 e^{-\alpha p}\,, \tag{1}$$

where $\alpha > 0$ is the rate of decay and D_0 is the value for $p = 0$. This function is depicted in Figure 1 for $\alpha = 0.2$ and $D_0 = 10$. It is worth noting that, in principle, when

[1]An exponential is also an easy-to-handle function.

[2]Note that both price and demand are positive variables.

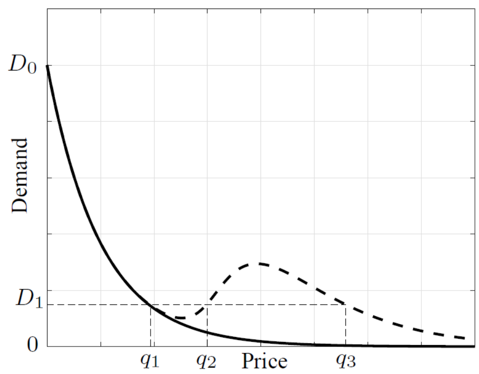

Fig. 1. Variation of the demand as a function of price. The solid line represents equation (1) while the dashed line represents equation (4).

the price tends to zero the demand should ideally tend to infinity, a behaviour which is not captured by the model (1). However, we assume that the real price is $\widetilde{p} = p + p_0$, where p_0 is a lower bound which may be, for instance, the minimum selling price guaranteeing parity between earnings and expenses.

To model the modification of the above dynamic due to the VE, we introduce a new variable, called "hankering" and denoted by H, which is zero when the price is less than a threshold p_1 and increases linearly above this threshold (see Figure 2). The hankering can be modeled as a function $H : [0, +\infty) \to [0, +\infty)$ defined as:

$$H(p) = \begin{cases} 0\,, & \text{if } p < p_1\,, \\ \gamma(p - p_1)\,, & \text{if } p \geq p_1\,. \end{cases} \tag{2}$$

where $\gamma > 0$ is a gain factor, or, in order to have a smooth function, by

$$H(p) = \log(1 + e^{\gamma(p-p_1)})\,. \tag{3}$$

Both functions are reported in Figure 2. The effect of the

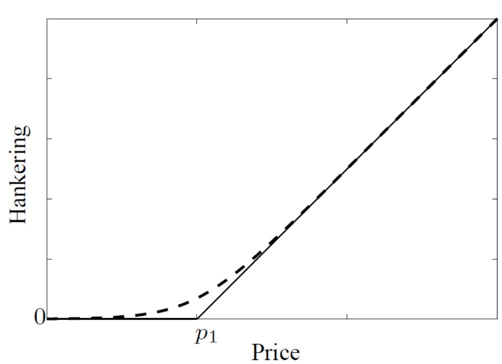

Fig. 2. Hankering as a function increasing in time over a given threshold. Equation (2) is depicted with a solid line while equation (3) is depicted with a dashed line.

hankering on the demand is that of modulating the amplitude of the decay (1) so that, when considering the VE, the total demand is expressed by

$$D_{VE}(p) = D_0 e^{-\alpha p}\,(1 + \beta H(p)) \tag{4}$$

where $\beta > 0$ is a gain factor. Function (4) is depicted in Figure 1 for $H(p)$ as in equation (3) with $\gamma = 1$ and $\beta = 3$. Looking at the Figure, one may note that for some values of the demand there are three possible values of the price. As an example, the value D_1 is reached at $p = q_1$, $p = q_2$ and $p = q_3$. Clearly, a seller would benefit more of a selling price equal to q_3; is there any way in which the seller can act so that, according to the market law, the price settle to the maximum value? Before trying to give a positive answer to this question, we need to introduce the dynamical behaviour of the price and of the demand.

B. The variation of price in time

As far as the price is concerned, we suppose that its variation in time is the sum of two contributions. The first one accounts for the demand/supply law and is proportional to the difference between the two: if the demand is higher than the supply, the price increases, if the demand is less than the supply the price decreases. The second term is an additional term associated with the possibility of the seller to modify the price. The overall model is captured by the differential equation

$$\dot{p}(t) = a(D(t) - S(t)) + u(t)\,, \tag{5}$$

where S is the supply, $a > 0$ is a gain factor and u is the control input determined by the seller.

C. The demand as a dynamic variable

Putting together equations (4) and (5), we obtain the time variation of the demand as

$$\begin{aligned} \dot{D}_{VE}(t) &= D_0 e^{-\alpha p(t)}\left[-\alpha - \alpha\beta H(p(t)) + \beta H'(p(t))\right] \times \dot{p}(t) = \\ &= D_0 e^{-\alpha p(t)}\left[-\alpha - \alpha\beta H(p(t)) + \beta H'(p(t))\right] \times \\ &\quad \times \left[a(D_{VE}(t) - S(t)) + u(t)\right]\,. \end{aligned} \tag{6}$$

In the next section we focus on the dynamical system consisting in equations (5) and (6), we analyse its equilibrium points as well as their stability and we show that, under some mild assumption on u, the seller can force the market to choose, for the same value of demand, the higher price among the three admitted.

III. Main result

For the sake of simplicity, we suppose, without loss of generality, $a = 1$ and we introduce the variables $x_1 = p$ and $x_2 = D_{VE}/D_0$ and the function $f : \mathbb{R}^+ \to \mathbb{R}$ defined by

$$f(x) = e^{-\alpha x}\left[-\alpha - \alpha\beta H(x) + \beta H'(x)\right]\,. \tag{7}$$

Then, equations (5) and (6) can be written as

$$\dot{x}_1 = x_2 - v\,, \tag{8}$$

$$\dot{x}_2 = f(x_1)(x_2 - v)\,, \tag{9}$$

where $v = aS - u$.

Remark 1: Equations (8) and (9) show that the price of a good, x_1, cannot be fixed directly by the producer. In fact, its value is associated with the equilibrium point of the trajectory which, in turn, is determined by the condition that the demand x_2 equals v, which is the variable controlled by the producer.

Remark 2: Although x_1 and x_2 come from quantities that are allowed to assume only positive values (demand and price, respectively), in the following discussion we do not introduce any zero-saturation so that x_1 and x_2 may assume also negative values. However, the control described in Section III-A guarantees that their values are always positive.

Let us begin with an analytic result.

Theorem 1: Let H as in (3) and let $x_1(0) = x_{10}$ and $x_2(0) = x_{20}$ be given. For $v \equiv \overline{v}$ (constant) the admissible trajectories of system (8)-(9) belong to the sets associated with the following cases.

1) If $x_{20} = \overline{v}$ then $x_1(t) = x_{10}$ and $x_2(t) = \overline{v}$ for all t.
2) If $x_{20} \neq \overline{v}$ and

$$\overline{v} > x_{20} - e^{-\alpha x_{10}}(1 + \beta Y(x_{10})) \triangleq w_H(x_{10}, x_{20}) \,, \tag{10}$$

then x_2 tends to $\overline{v}$ and x_1 tends to a value $\overline{x}_1$ such that

$$e^{-\alpha \overline{x}_1}(1 + \beta H(\overline{x}_1)) = \overline{v} - w_H(x_{10}, x_{20}) \,, \tag{11}$$

3) In all the other cases x_1 tends to $+\infty$ and x_2 tends to $w_H(x_{10}, x_{20})$. □

Proof. Case 1) is immediate once noting that in this case both time-derivatives in (8) and (8) vanish. To prove cases 2) and 3), note that since (6) has been obtained from (4) the phase portrait turns out to be a curve defined by

$$x_2 = x_{20} + \int_{x_{10}}^{x_1} f(\tau) d\tau = $$
$$= x_{20} + \left[e^{-\alpha \tau}(1 + \beta H(\tau)) \right]_{x_{10}}^{x_1} \,. \tag{12}$$

More precisely, the phase portrait is the graph of $e^{-\alpha x_1}(1 + \beta H(x_1))$ shifted upwards or downwards according to the value of $w_H(x_{10}, x_{20})$. Now, let x_{1m} and x_{1M} denote the relative minimum and maximum, respectively, of D_{EV}. It is not difficult to see that $f(x)$ is positive if $x \in (x_{1m}, x_{1M})$ and negative otherwise (see Figure 3). As a consequence, the following four cases are possible.

(a) If, at some time instant t, $x_1(t) \in (x_{1m}, x_{1M})$ and $x_2(t) > \overline{v}$, then $x_1(t)$ and $x_2(t)$ are both increasing (see Figures 4 and 5).
(b) If $x_1(t) \in (x_{1m}, x_{1M})$ and $x_2(t) < \overline{v}$, then $x_1(t)$ and $x_2(t)$ are both decreasing (see Figures 5 and 6).
(c) If $x_1(t) \notin (x_{1m}, x_{1M})$ and $x_2(t) > \overline{v}$, then $x_1(t)$ is increasing while $x_2(t)$ is decreasing (see Figures 4, 5 and 6).
(d) If $x_1(t) \notin (x_{1m}, x_{1M})$ and $x_2(t) < \overline{v}$, then $x_1(t)$ is decreasing while $x_2(t)$ is increasing (see Figures 4, 5 and 6).

The above conditions are maintained until x_2 reaches, if possible, the value $\overline{v}$ thus proving the case 2) of the statement. However, if condition (10) does not hold, then $x_2(t) > \overline{v}$ for all t so that x_2 never reaches the value $\overline{v}$. Eventually, case (c) holds indefinitely, what corresponds to case 3) of the statement. ■

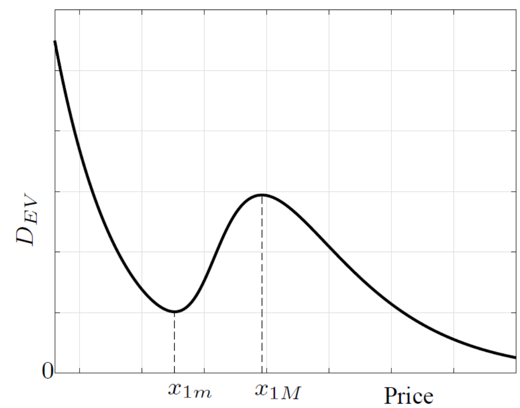

Fig. 3. An enlargement of Figure 1 showing local minimum and maximum of D_{EV}. Function f, which is proportional to the derivative of D_{EV} with respect to p, is positive between the two extrema and negative elsewhere.

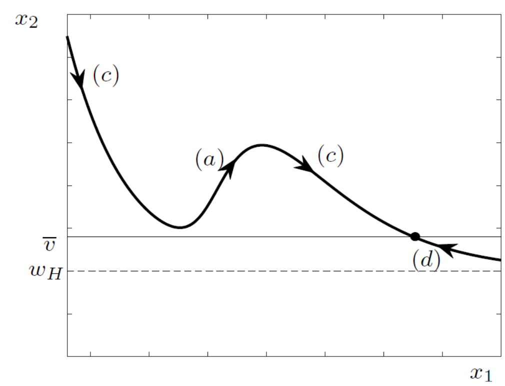

Fig. 4. A situation with one stable equilibrium point.

Theorem 1 allows us to understand what happens to the variables x_1 and x_2 in a general context. Now we come back to the question posed at the end of Section II-A and we show how the above analysis can be used to exploit the Veblen effect in order to set the price to a higher value without changing the demand.

A. Improving earnings without increasing the demand

First of all, we assume that at the initial time instant the market is settled in some "stable" point, namely that current price and demand identify a point in the curve depicted in Figure 1. As a consequence, $w_H(x_{10}, x_{20}) = 0$. Then, to establish a first result, let D_m and D_M denote the values of the relative minimum and maximum, respectively, of the static price/demand curve (see Figure 1) divided by D_0.

Assumption 1: The initial conditions are such that

- $x_{20} \in (D_m, D_M)$,
- letting $q_1 < q_2 < q_3$ the three values of price such that $D_{EV}(q_1) = D_{EV}(q_1) = D_{EV}(q_1) = D_0 x_{20}$ (see again Figure 1), $x_{10} = q_1$.

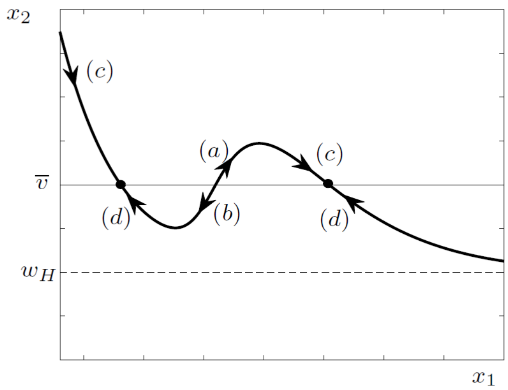

Fig. 5. A situation with two stable equilibrium points.

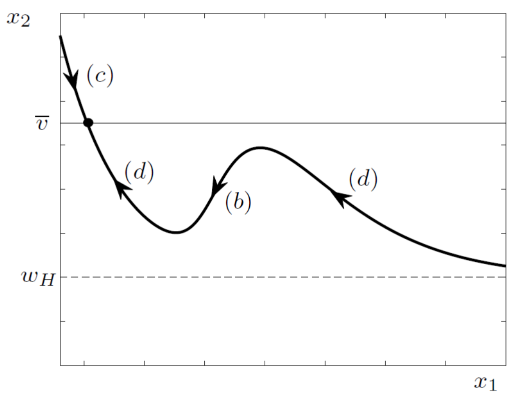

Fig. 6. Another situation with one stable equilibrium point.

Proposition 1: If Assumption 1 holds, then there exists a control strategy such that $x_1(t) \to q_3$ and $x_2(t) \to x_{20}$. □

Proof. With reference to Figure 5, we want to find a control strategy that steers the state of the dynamic system (8)-(9) from (x_{20}, q_1) (the bullet on the left in the figure) to (x_{20}, q_3) (the bullet on the right). Let v^* be defined by

$$v^*(t) = \begin{cases} D_m/2\,, & \text{if } x_1(t) \leq x_{1M}\,, \\ x_{20}\,, & \text{otherwise.} \end{cases} \tag{13}$$

Since $x_{10} < x_{1M}$ by assumption, $v^*(0) = D_m/2$ and for the first positive time-instants the scenario is the one depicted in Figure 4. The state will run along the curve according to the criteria described in case (c) (statement of Theorem 1) until $x_1 = x_{1m}$ then case (a) until $x_1 = x_{1M}$. After this point, according to (13), v^* switches back to the value x_{20} so that the scenario is again the one depicted in Figure 5. The state runs along the curve as in the case (c) until $x_1 = q_3$. ■

B. Maximizing earnings

In the previous section we have seen that there exists a control strategy steering the price to a higher value while keeping the same (final) value of the demand. However, this result can be further improved by properly selecting the final value of the control signal v^*. To see how this can be done, we first observe that, since at the equilibrium the demand equals the supply and both equal the sold quantity, the maximum of the earnings is associated with the price corresponding to the maximum of the product between p and D_{EV}. This maximum, as shown in Figure 7, occurs at a price slightly higher than x_{1M}. Let x_{1B} denote its value.

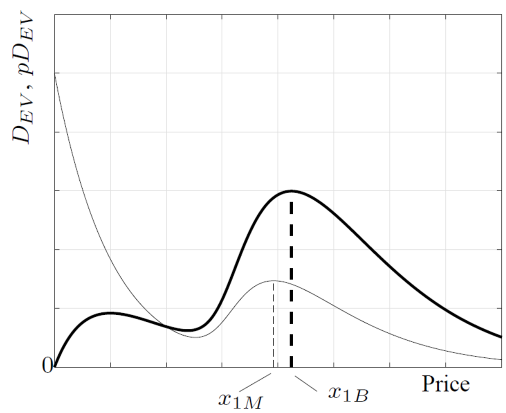

Fig. 7. Total earnings as a function of price (bold line). The maximum occurs at a price x_{1B} slightly higher than x_{1M}.

Hence, the control strategy v_B steering the price to x_{1B} is defined by

$$v_B(t) = \begin{cases} D_m/2\,, & \text{if } x_1(t) \leq x_{1M}\,, \\ x_{1B}\,, & \text{otherwise.} \end{cases} \tag{14}$$

IV. Simulations

In this section we show how to implement the strategy described above and what results could be obtained. We assume, for the model (4)-(5), the following values: $D_0 = 10$, $a = 1$, $\alpha = 0.2$, $\beta = 3$, $\gamma = 1$ and $p_1 = 15$ and we suppose that at $t = 0$ the system is at the equilibrium corresponding to $x_{10} = 10$ and

$$x_{20} = e^{-\alpha x_{10}}\left(1 + \beta H(x_{10})\right) \simeq 0.1381\,. \tag{15}$$

The price/demand curve (4) is depicted in Figure 8 where the initial condition is also reported (with an asterisk) as well as the (normalized) product demand×price (with a dashed line). For these values it is easy to compute $x_{1,m} \simeq 12.61$, $x_{1,M} \simeq 19.6$ and $x_{1,B} \simeq 21.9$ and $D_m \simeq 0.1$, so that (14) becomes

$$v_B(t) = \begin{cases} 0.05\,, & \text{if } x_1(t) \leq 19.6\,, \\ 21.9\,, & \text{otherwise.} \end{cases} \tag{16}$$

The time evolution of the price, of the demand and of their product as a consequence of the control strategy (16) is reported in Figure 9. It can be noted that, after an initial interval in which the demand is decreasing (centre plot) and the earning (bottom plot) is almost constant, both demand and price increase so that earning increase as well until its maximum.

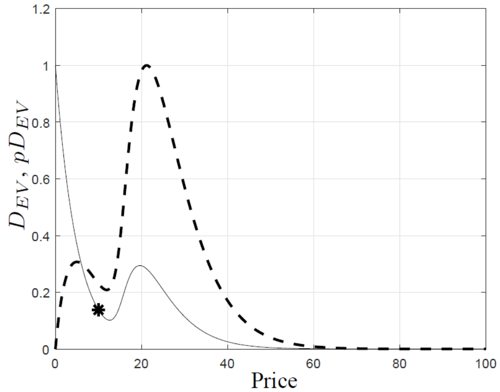

Fig. 8. Price/demand (solid line) and price/gain (dashed line) curves. The initial equilibrium is reported with an asterisk.

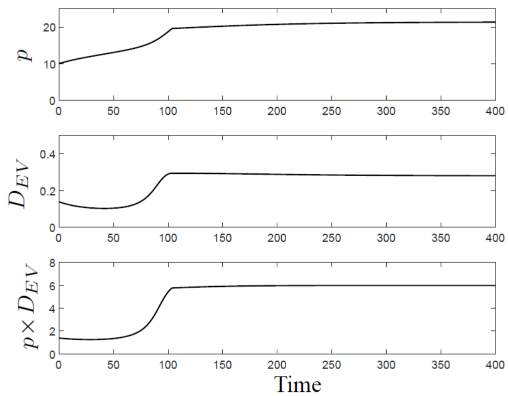

Fig. 9. Time histories of price (top plot), demand (centre plot) and their product (bottom plot) when applying the strategy (16).

A. An approximated strategy

A sub-optimal solution, leading to the same final values of the price and of the demand but in a longer time, might be the following one. Set the value of v to 0.05 for a time "long enough" to allow the price to increase beyond $x_{1,M}$ and then to set the value to 21.9. Application of this strategy led to the results reported in Figure 10 (note that the horizontal axis refers to a larger interval). The higher value of v, namely 21.9, has been applied only once the price has reached the equilibrium corresponding to the lower value 0.05. In this way the strategy could be robust to parameter uncertainties.

V. Conclusions

We have shown that an counterintuitive effect occurring in the context of the price/demand settling can be explained with a dynamic model, which leads to interesting results concerning the equilibrium points and the admitted trajectories. In turn, these results can be exploited by a possible seller to improve the earnings by applying a suitable price-setting strategy. Admittedly, the model is only hypothetical; however, it provides a good insight on the consumers' behaviour and on the possible optimal selling strategies. Moreover, a dynamic model as the one described above can be exploited to analyse other characteristics such as the effect of uncertainties on the model or of exogenous disturbances, what could be the subject for future investigations.

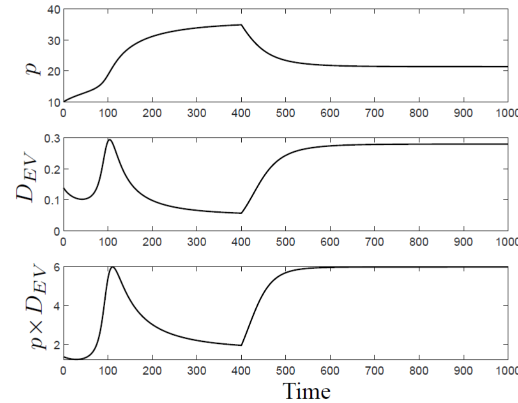

Fig. 10. Time histories of price (top plot), demand (centre plot) and their product (bottom plot) when applying the approximated strategy.

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