A Landscape of First-Order Linear Temporal Logics in Infinite-State Verification and Temporal Ontologies

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Abstract
We provide an overview of the main attempts to formalize and reason about the evolution over time of complex domains, through the lens of *first-order temporal logics*. Different communities have studied similar problems for decades, and some unification of concepts, problems and formalisms is a much needed but not simple task.

Keywords
Verification of infinite-states systems, temporal knowledge representation and reasoning, first-order temporal logics

1. Introduction

Linear temporal logic (LTL) is a well-known extension of classical propositional logic by means of time-related modalities, such as *'tomorrow'*, *'eventually'*, *'always'*, and *'until'* [1], that are used to formulate temporal specifications interpreted on the linear order of the natural numbers, or on finite initial segments thereof. Applications of LTL in computer science and artificial intelligence range from model checking [2] to declarative business process modelling [3, 4], through program verification and synthesis [5, 6], as well as temporal conceptual modelling and reasoning [7, 8, 9, 10].

First-order linear temporal logic (FOLTL), in turn, lifts LTL from propositional to first-order logic [11, 12, 13, 14, 15, 16]. Unfortunately, its increased expressive power comes at a high computational cost, compared to its propositional counterpart. As FOLTL contains first-order logic, its validity problem is clearly undecidable. Not only that: the set of valid formulas in the full FOLTL language is not even recursively enumerable, and this result holds already for the fragments where only *two variables*, or only *unary predicates* (so-called *monadic*) are allowed [11].

Such limitations, however, have not impaired the research on FOLTL-based formalisms that combine reasoning over a time dimension, on the one hand, and over an object dimension, on the other. Given this two-dimensional essence, FOLTL is indeed a natural choice when it comes

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to modelling and verification of dynamic systems in presence of data that evolve over time, or whenever the task is to formalise, and reason about, the temporal properties of objects in a changing domain. Over the last couple of decades, multiple research trends that involve FOLTL and its fragments have been developed. Based on their primary focus, we attempt to categorise such diverse directions in terms of the following (neither mutually exclusive, nor collectively exhaustive) family resemblances: on the one hand, research approaches that focus on *modeling and verification of infinite-state systems*; on the other hand, a line of study oriented towards *temporal ontology-based knowledge representation and reasoning*.

Given the differences in goals and objectives between these communities, it is a non-trivial task to provide detailed comparisons of the respective results within a uniform framework. As a first step in this direction, we use the privileged viewpoint of FOLTL to illustrate formalisms from the two communities—by focusing, in particular, on two recent representatives thereof, namely, *LTL modulo theories* and *temporal free description logics without the rigid designator assumption* (RDA), respectively.

2. The Multiform First-Order Linear Temporal Logic Background

In FOLTL, on the syntactic level of the first-order component, *function* and *constant symbols* can be either introduced by suitable definitions starting from predicate symbols [12], or taken as primitive elements of the language [17, 13]. FOLTL both with and without equality has also been considered [11]. Semantically, other than the choice of structure representing the designated flow of time (e.g. reals, integers, natural numbers, or initial segments of the latter), a number of options are available. Regarding the domain of interpretation, a standard solution is to equip models with a *constant domain*, which is shared by all time points. This assumption is the most general, since semantics based on varying or expanding domains can be simulated in constant domain models by means of a suitable *existence* predicate [11, 14].

When it comes to the interpretation over time of the non-logical symbols (i.e., constant, function, and predicate symbols, excluding equality), it is possible to either allow the extension of function and predicate symbols (other than equality) to change from one instant to another, making them *flexible*, or to impose instead a *rigidity* condition that fixes the same interpretation at all times [17]. This is in contrast with the assignment of variables, and with the interpretation of constants, in a FOLTL model, since their mapping into elements of the domain typically is not relative to any time point [13]. When a rigidity condition is imposed on constant and function symbols, they are said to behave as *rigid designators* [14], meaning that they are forced to designate the same object at all time points.

In order to drop such a rigid designator assumption, the syntactic and semantic machineries need to be somewhat modified. Several approaches in *first-order modal* and *intensional logic*, other than in FOLTL [18, 12, 13, 14], have devised mechanisms to introduce constant and function symbols, as well as (possibly quantifiable) variables, as *flexible individual concepts* that can change their interpretation over time. Standard examples of non-rigid designators in a temporal setting are time-dependent *definite descriptions*, such as ‘the value that variable x will take tomorrow’. First-order modal formalisms involving definite description as terms of the language have been investigated as well [19, 20, 21, 22]. In light of these FOLTL features, in the following we illustrate formalisms that have been proposed in the communities of infinite-state verification and of temporal ontology-based knowledge representation, respectively.

3. A Landscape of First-Order Linear Temporal Logic Fragments

Modelling and verification of infinite-state systems. A significant line of research in verification of data-driven systems has focussed on the extension of LTL with *concrete domains* [23]. Concrete domains are relational structures, such as the natural or the rational numbers equipped with the the binary relations of strict linear order and identity over them. By replacing proposition letters with atomic constraints over terms of the language, expressed by means of the relational signature, LTL with concrete domains can express comparisons between term values that are allowed to change over time. For instance, taking as concrete domain the linear order of the natural numbers, the formula $\Box(x < \bigcirc x)$ can be used to express that, at all future time points, the value of x will always be less than the value of x at the subsequent instant (represented by $\bigcirc x$). In this setting, a variable like x plays the role of a non-rigid designator with a time-dependent interpretation over the constant domain. The interpretation of relational symbols, on the other hand, is rigid, hence fixed at all time points.

More recently, *LTL modulo theories* (LTL$^{\text{MT}}$) has been introduced as a fragment of FOLTL, to leverage SMT-based techniques (see the BLACK framework [24]) for applications in the context of verification of *data-aware processes* [25, 26, 27, 28, 29, 30, 31] and, in general, infinite-state systems [32, 33, 34, 35]. Syntactically, LTL$^{\text{MT}}$ is obtained by replacing LTL propositional letters with purely first-order formulas, and similarly allowing for "timed" variables of the form $\bigcirc x$, used to refer to the value of x at the next time point [36]. As such, it allows to express temporal constraints and perform reasoning tasks under the axioms of given first-order theories, such as linear integer or rational arithmetic, or equality and uninterpreted functions. For instance, given the first-order theory of linear integer arithmetic, with $+$ as function and $\leq$ as relation symbols (plus the natural definitions for $=$ and $<$), we can state the value of variable x is initially set up to 0 and it is incremented by one until it reaches the value of 42, with the formula $x = 0 \wedge ((\bigcirc x = x + 1) \,\mathcal{U}\, x = 42)$. Whenever the underlining first-order theory is decidable (as in the examples mentioned above), satisfiability in LTL$^{\text{MT}}$ is semi-decidable, and several decidable fragments have been identified as well in this setting [37]. While the first papers introducing this formalism have considered only *rigid* predicates and functions over a common domain, some ongoing work on LTL$^{\text{MT}}$ is moving in the direction of dropping this assumption.

Temporal ontology-based knowledge representation and reasoning. The study of so-called *monodic* fragments of FOLTL has shown that decidability of the satisfiability problem can be regained by combining a decidable *purely first-order* fragment (such as the two-variable, or the monadic one) with a restriction on temporal operators that allows their application only over formulas with *at most one free variable* [38, 11, 39]. As an example,

$$\exists x \mathsf{Alive}(x) \wedge \Box \forall x (\mathsf{Alive}(x) \rightarrow \bigcirc(\neg \mathsf{Alive}(x) \wedge \exists y (\mathsf{isRememberedBy}(x, y) \wedge \mathsf{Alive}(y))$$

is a monodic two-variable formula, stating that someone is currently alive and, at any time point in the future, anyone alive will not be anymore so in the next generation, but will be remembered by someone alive [40].

Strictly related to monodic fragments are *temporal description logics* [40], obtained by combining description logic (DL) formalisms (that can be seen as fragments of first-order logic) with linear temporal operators. For instance, the formula given above can be rephrased in a temporalised extension of the standard DL $\mathcal{ALC}$ by means of an *assertion* stating that Alice, a, is alive, $\mathsf{Alive}(a)$ and with the following *concept inclusion*, holding at all points in time, to capture the second conjunct above:

$$\Box(\mathsf{Alive} \sqsubseteq \bigcirc(\neg\mathsf{Alive} \sqcap \exists\mathsf{isRememberedBy}.\mathsf{Alive})).$$

For these formalisms, a quite neat complexity picture is available (we do not provide here a full survey of the results; see [11, 41, 8, 42, 40, 43, 44, 45, 46, 47] for more details). The satisfiability problem is ExpSpace-complete in case of the two-variable monodic, as well as the monadic monodic fragments, and the same holds for the fully temporalised version of $\mathcal{ALC}$ [11, 41]. For other temporal extensions of $\mathcal{ALC}$ that involve syntactic restrictions on the application of temporal modalities or on the DL dimension, the complexity lowers down to NExpTime- or ExpTime-complete [40, 43]. Finally, for temporal extensions of lightweight DLs in the *DL-Lite* and $\mathcal{EL}$ families, introduced for temporal conceptual and data modelling, the complexity of reasoning can be tamed even in NP or in NLogSpace [45, 48].

However, to the best of our knowledge, despite the extensive research on temporal extensions of DLs [49, 8, 40, 50], non-rigid designators have received little attention in this field. In an attempt to fill this gap, a recent paper [22] extends the *free DLs* introduced in the non-modal case [51, 52] to a temporal setting, by: (i) *introducing definite description terms* of the form ιC (read as 'the object that is C'), where C is a concept, alongside the standard *individual names*; (ii) *dropping the RDA*, allowing terms to be flexible individual concepts across states (or even not to designate anything at all, hence being a language "free" from existential assumptions on names). In [22], it is shown (by adjusting a proof from [53]), that, without the RDA, the formula satisfiability problem becomes undecidable for temporal free DLs (even without definite descriptions) interpreted on linear time structures based on the natural numbers, or on initial segments thereof (while it is known to be decidable, over the same structures, for standard temporal DLs without definite descriptions and with the RDA [11, Theorem 14.12]).

4. Towards a Unified Perspective

As it is often the case, different communities may have different perspectives on a class of closely related problems. In this context, we believe that an attempt at unifying techniques and formalisms might be beneficial to both communities. In this position paper, we notice the need of such an effort. On the one hand, techniques from the formal verification community might help knowledge representation with efficient reasoning techniques, given suitable fragments of FOLTL. On the other hand, existing work about FOLTL from the knowledge representation community might shed light on foundational issues of the formalisms used for verification of infinite-state systems, improving our understanding of the matter. A future initial step to establish a bridge between the two communities may be that of investigating the connections between temporal DLs without the RDA, on one side, and $\mathrm{LTL}^{\mathrm{MT}}$, on the other.

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