UNIVERSITÀ
DEGLI STUDI
DI UDINE

PhD Course in :

“Mathematical and Physical Sciences”

Cycle 38°

Title of the thesis

“Developing a learning path on electromagnetic waves for high school students”

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Year 2026

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"From a long view of the history of mankind—seen from, say, ten thousand years from now—there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics. The American Civil War will pale into provincial insignificance in comparison with this important scientific event of the same decade".

[Richard Feynman in Feynman et al., 1965]

Abstract

A learning path on electromagnetic waves was developed for high school students. The teaching proposal was elaborated according to two different versions, through Design-Based Research methods. Thus, it consisted of several preliminary steps: identification of the key ideas characterizing the classical theory of the electromagnetic field (and choice of the learning objectives), analysis of students' learning difficulties (literature review), analysis of the main (Italian) school textbooks presenting electromagnetic waves. The teaching efficacy followed some experimentations in Italian schools, and it was carried out by taking into account the results of tutorials and questionnaires, according to different mixed methods (pre-test vs post-test, experimental group vs control group, etc.). Teaching implications will be discussed, together with limitations and possible further development of the research.

Chapter 1

Introduction

This research work has as final objective the development of a teaching proposal on electromagnetic waves for high school students (13th grade). The study is anchored to the Italian national context, as in many other countries the physics (electromagnetism) school curriculum ends with the study of electromagnetic induction phenomena. Despite this, some of the phases of the research have been carried out in collaboration with research groups from other countries (in particular Spain and Argentina) and other Italian universities, in order to provide a broader horizon to the thesis, and to allow possible future developments in an international context. This brief introductory chapter has the role of providing a summary of the research work carried out and of clarifying its context, in order to make the reading easier.

The theoretical framework of educational reconstruction we adopted (that is the Entangled Physics Education Research Methods for Paths, which will be illustrated in Chapter 2) focuses the attention on several preliminary phases necessary for the elaboration of the learning path, which a specific chapter of the thesis is dedicated to: this theoretical framework fits well our composite experimental conditions, as we will illustrate later. In Chapter 3 we will discuss a historical reconstruction – on an epistemological basis – of the classical theory of the electromagnetic field, starting from the introduction of the concept of field, till the Hertz experiments which allowed to recognize the generation and detection of electromagnetic waves, according to a propagation mechanism following the Maxwell's theoretical predictions. Of course, the purpose of this reconstruction is not an overview of the historical development of the theory, but the identification of the key ideas that will characterize the construction of the learning path.

Chapter 4 reports an in-depth discussion of the learning difficulties that literature has shown to be recurrent in the context of electromagnetism. They concern many aspects, including: the concept of field and its distinction from that of force (with comparison between the adoption of a Newtonian or Coulomb model); the relationships of (electric and magnetic) fields with their sources (i.e., their characterization, and the causal relationships occurring between the variables that characterize one and the other); the conceptual meaning of integral operators (flux and circulation) and the role they play in Maxwell's equations; the principle of superposition for fields and the distinction between analogous situations analyzed between the electrical and magnetic contexts; the relationships between the quantities characterizing these two contexts and the electromagnetic field as a single entity; the transition from the static to the dynamic context; the phenomenon of electromagnetic induction and the conceptual meaning of the quantities characterizing it, the comparison between the descriptions on a microscopic or macroscopic basis of the same phenomenon (an example is the generation of the current induced in a circuit); the three phases of the wave nature of the electromagnetic field (generation, propagation and detection). Moreover, for the peculiar purposes of our research, the results related to the Concept Inventories proposed in the literature have also been briefly discussed. This need arises from the search for standardized questions to be submitted as test-out (and possibly test-in) to evaluate the teaching effectiveness of the learning path (or possibly to probe learning difficulties on a control sample). We anticipate that the literature we analyzed – in particular the Concept Inventories – provide few examples of questions that can be used in the context of electromagnetic waves, so we chose to develop items that would require a preliminary validity study: this led us to use mixed modalities (two-tier test: multiple choice answer + open answer as justification) which may provide more information on the actual reliability of the measuring instrument.

Furthermore, although the population we are addressing is made up of (Italians) high school students, due to their similarities many results that literature reports for undergraduates can be assumed. In this regard, the teaching proposals illustrated in Chapter 5 mainly address this population, identifying some premises – rather than alternatives – for our learning path on electromagnetic waves. Indeed, the structure of the content knowledge of the classical electromagnetic theory – as discussed at the end of Chapter 4 – and the conceptual leap characterizing the transition from the static to the dynamic context, require a reworking of the conceptual referents for the specific purpose of dealing with the propagation of the electromagnetic field: it represents a learning objective which may be inconsistent with the structure of the knowledge students own till that point.

This inconsistency is also highlighted by the discussion in Chapter 6, where the most popular textbooks on electromagnetism in Italian schools are analyzed, according to a widespread methodology in Science education for this kind of analysis: the goal is to relate the structure and its (formal and representational) content with the structure of students' knowledge and their learning difficulties, in order to select another source of information for the development of the teaching proposal. Chapter 7, on the other hand, aims to deepen two peculiar aspects that will characterize our learning path, which – unlike the previous steps – do not play a specific role in the general process of educational reconstruction: they are the mathematical formalism and the representations' formalism. Indeed, the role of conceptual referent played by the integral operators of vector fields (flux and circulation) makes it necessary to investigate the typical students' performances in their application, as well as with respect to the interpretation of three-dimensional representations (e.g. wave profile, field lines, vector plots, etc.) that students often encounter for the first time in the context of electromagnetism (in particular, when studying the field propagation in the form of waves).

After these preliminary steps, in Chapter 8 we get to the heart of the discussion through the illustration of the methods, i.e. the characteristics of the (experimental and control) samples which characterized the several phases of the research, the tools we used (i.e. the two versions of the questionnaire submitted to evaluate the teaching effectiveness, whose original versions in Italian are reported in Appendix A), and the (qualitative and quantitative) analysis' methods used in examining the answers to the questionnaires and tutorials. The latter were submitted to the students during the activities, which constitute the learning path described in Chapter 9. As in the case of the questionnaire, it also consists of two different versions, which partially differ in terms of learning methods and objectives: in particular, in the second version changes have been made considering of the results that emerged from the first implementation. For this reason, in Chapter 9 we have only described the modifications, which can also be deduced in a more concise way from the comparison between Tables 9.1 and 9.2 – which report the "stimulating questions" that we used during the tutorials and activities – and Figure 9.10 which represents the rationale of the final version of the learning path. Chapters 10 and 11 will respectively report the results and their discussion – answering the research questions defined in Chapter 8 – which also illustrate the reasons leading to the modifications we mentioned above.

In the final chapter, beyond summarizing the main results, we will discuss the limits of the research and its possible developments, as well as the links with other teaching and research activities that characterized the PhD course, in order to make the context in which the work was carried out more clear. Indeed, while we seriously considered the difficulties characterizing the development of a teaching proposal on a rather complicated topic – and about which little research has been done – we also believe that it can represent a first step to obtain possible preliminary indications supporting more organic research studies.

The learning path that we are going to develop has foundations with respect to many aspects: both from the point of view of physics (relevance of the topic for the dynamic treatment of the electromagnetic field and as a conceptual bridge towards modern physics) and from that of educational reconstruction (little research has been carried out on the topic so far), but also considering the possible applications (numerous for each region of the electromagnetic spectrum) and the social perspective (e.g. considering the discussion about electromagnetic pollution, which is becoming increasingly important in the mass media). We conclude by observing that we refer to a "learning path" – rather than a "teaching-learning sequence" – in order to highlight the relevance of learning as an individual process dependent on the style of each student, who can search for his (or her) own conceptual trajectory in the absence of pre-set reasoning paths.

Chapter 2

Theoretical framework

For several decades, research in physics education has found with numerous evidences and about multiple aspects that students have ideas related to the evolution and causes of physical phenomena, as well as about the relationships between the quantities characterizing them, which often precede their study (Hallound & Hestenes, 1985). Such pre-conceptions can be influenced by phenomenological experience – for example in the area of mechanics (Clement, 1981) – or they have a spontaneous characterization: the latter is a recurring case in the area of electromagnetism, as students do not have direct experience with characterizing quantities such as the (electric and magnetic) fields. Such spontaneous ideas imply specific learning difficulties that we will analyze later according to the results of the research literature, which obviously depend on the age of the students' population involved. Conceptual nuclei, on the other hand, are often independent of the learning context, so they can be generalized while considering the need for different teaching strategies – depending on the composition of the students' audience – to transform these alternative conceptions into currently accepted scientific ideas.

However, this requires a conceptual change operation that is often difficult to implement, even though different strategies may be considered. In fact, non-common sense ideas are not usually isolated conceptions, but they rather form real mental models, whose formation and meaning can be interpreted in different ways. According to the theoretical framework "knowledge in pieces" (diSessa, 1993), the models adopted by students strongly depend on the particular request that is posed, activating specific cognitive resources. As observed by Jelicic and colleagues (2017), in our case – i.e. dealing with electromagnetic waves – it is presumable to make the assumption of such a theoretical framework, considering the abstract characterization of the physical quantities describing electromagnetism and which students have no direct experience of.

Literature (Planinic et al., 2006) specifically suggests how "students are less likely to have strong preformed concepts in electromagnetism than they are in mechanics" as "electromagnetic concepts and phenomena are generally not a part of students' everyday language and experience"; furthermore, this would be especially noticeable in high school students, "since the domain of electromagnetism is not something that they would have much contact with before instruction in high school, except very briefly in elementary school". This students' population could form a conceptual basis, including "some mental models of physics phenomena which can later be further expanded and refined". Nevertheless, most "physics education research studies on students' understanding of electromagnetism concepts involve college or university students, and only seldom high school students": thus, we chose to address this population in order to get insight about these mental models related to the propagation of the electromagnetic field.

According to this theoretical framework (diSessa, 1993), the phenomenological primitives (p-prims) constitute basic cognitive elements that determine conceptual learning and the development of knowledge, whose inappropriate activation can produce incorrect explanations. The declination in the specific context of the p-prims is in fact fundamental, as they are not correct or incorrect concepts a priori. For example, about the topics we will discuss, it may be relevant to analyze the p-prim "equilibration", which consists in considering that "a return to equilibrium is the natural result of removing a disequilibrating influence". An application of this principle may concern the change in the configuration of electric and magnetic fields as the sources' characteristics change, since we are interested in the transition from static to dynamic conditions. As we will explore in the following chapters, this concept can be framed in different ways, determining different conclusions on the evolution of the system. Thus, the possible investigation about the different declinations of students' models is attractive from the educational perspective.

However, the context in which this study takes place is peculiar: the population is made up of Italian high school students (18 years old, 13th grade) who have already completed the study of electromagnetism with their curricular teachers – with the exception of the part concerning electromagnetic waves – and who turn out to have a rather solid average preparation in physics and mathematics (in the next paragraph we will deepen this aspect). Consequently, it is plausible to adopt a model of conceptual change that starts from the detection of learning problems through the clarification of the students' reasoning which, by exploiting the internal inconsistency – especially among the implicit elements – of the mental model adopted, allows to overcome learning difficulties and to promote conceptual learning related to the scientific models according to a gradual and continuous approach (Vosniadou, 2008).

This is why we will report an extensive analysis of the research literature on learning difficulties students usually face in electromagnetism, especially those relevant in the dynamic context. Once again, we emphasize that the peculiarity of the population addressed allows us to assume the conceptual obstacles that literature reports about undergraduate students. In fact, high school students in Italy can choose between several curricula: in particular, they can attend the "scientific", where mathematics and physics play a central role (7 out of 30 hours). In this regard, the ministry of education issued some "national indications", which represent guidelines regarding the general competences and specific learning objectives that should be pursued (rather than programs as lists of topics to be covered for each discipline).

Regarding electromagnetic waves, these indications for the scientific curriculum report verbatim: "The student will face the study of electromagnetic waves, their production and propagation, their effects and their applications in the various frequency bands". Thus, it is evident that explicit attention should be paid to the different phases of the phenomenon – generation, propagation and detection – as well as to the effects and its applications of the same, in order to consider the interesting aspects for the social perspective (e.g. about problems such as electromagnetic pollution related to the different bands).

The identification of recurrent learning difficulties and the related strategies of conceptual change, only represent some of the aspects considered in the theoretical framework of educational reconstruction that we adopted. It is the *Entangled Physics Education Research Method for Paths*, later EPERMP (Michelini et al., 2023). Its representation is provided in Figure 2.1: it represents a reinterpretation of the *Model of Educational Reconstruction* (Duit et al., 2012), in which the learning objectives are selected – without reductionism and in the perspective of a vertical curriculum – through a preliminary focus on the conceptual nuclei, in order to focus the didactic attention on the interpretative nodes. The learning path is therefore developed according to a series of formative intervention modules in accordance with Design-Based Research methods, which provide for several cycles of intervention-evaluation-review.

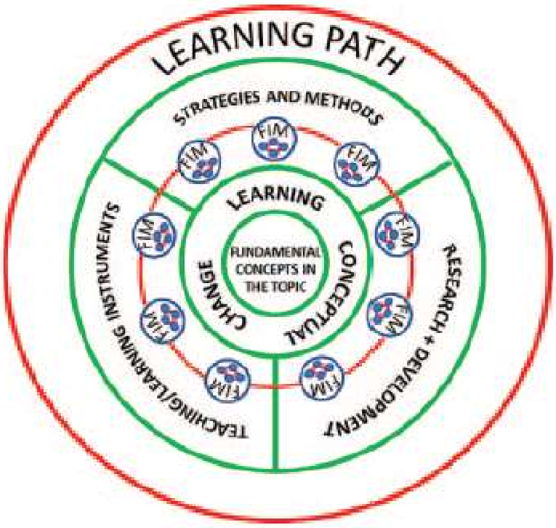

Figure 2.1: Representation of the Entangled Physics Education Research Method for Paths (EPERMP, from Michelini et al., 2023) theoretical framework.

The phases that characterize the development of the learning path – from a global perspective – involve, in addition to the educational reconstruction of the content knowledge of the theory in problematic terms which characterized the historical-epistemological development of the theory and the analysis of learning difficulties, also the analysis of the teachers' perspective. It can be framed in different ways, for example by taking into account the typical structure of textbooks – both from the point of view of the content knowledge and the representations used for the different phenomena – most widely adopted in schools. In fact, they are usually an important resource for teachers, which influences their teaching practices and can potentially be linked to students' learning difficulties. In addition, in this way a proxy variable of traditional teaching is analyzed, in a simpler and more quantitative way it than would occur with interviews with teachers.

The analysis of the textbooks' content knowledge makes it possible – at least in principle – to verify their internal consistency in terms of structure and organization. It is a relevant aspect as it could be reflected in the teaching method, and consequentially in the structure of the students' knowledge. The importance of this variable is highlighted in the literature (Bagno & Eylon, 1997) as an epistemological resource, according to the classification of the cognitive resources by Hammer and colleagues (Hammer, 1994). In particular, Hammer reports that often "some students' knowledge may remain fragmented, because, in part, they do not expect it to be coherent". Consequently, despite the fact that teachers – and sometimes even textbooks – are often induced to form and evaluate conceptual resources, without considering the possible role that epistemological resources – i.e. students’ metacognitive beliefs about learning and knowledge in relation to the specific topic – play in determining students’ alternative conceptions. Obviously, this does not diminish the importance of the purely conceptual resources, necessary in the activity of problem solving.

In the following chapters we will deepen the conceptual problems involved in the analysis of the behavior of electromagnetic waves. In general, Kesonen and colleagues (2011) observe that – based on the previously reported studies – students can experience several kinds of difficulties in understanding the concepts of the electric and magnetic fields and their interrelations; in particular, “most of the studies concerning these fields, however, focus on stationary situations, with the result that dynamic situations are rarely examined: hence, more investigations are needed to uncover students' conceptions of the dynamic situations in which the electric and magnetic fields are changing”.

As pointed out above, a peculiarity will also concern the population under investigation, as most of the research literature on the subject concerns university students. A notable exception is the study by Bagno & Eylon (1997), conducted on a large sample of 17-18 year old students who had already completed the study of electromagnetism. In a similar way, we will deal with classes with experienced teachers and good academic performance, which make them assimilable to the level of introductory students. For this reason, it will be possible to take on learning difficulties similar to those found in the literature also for our samples, according to the discussion carried out in Chapter 4.

Chapter 3

Conceptual nuclei of the electromagnetic field's theory

In this chapter we are going to analyze the historical development of the classical theory about the electromagnetic field, in order to identify the key ideas, namely, the epistemic steps which characterize the conceptual nuclei of the topic we are investigating. Indeed, a global view of the electromagnetic field's theory – whose conceptual characterization also emerge from its historical analysis – is necessary in order to deal with the dynamic context and the propagation of electromagnetic waves, which is the final aim of the learning path. Furthermore, it may provide some suggestions to enable students to retrace the main stages of the pathway leading to the interpretation of the conceptual nodes (Etkina et al., 2021) or other general strategies for building physics identity on an epistemological basis (Monk & Osborne, 1997). Nevertheless, here we are not going to address the quantum development of theory, as it is beyond our goals.

3.1 Historical development of the electromagnetic field's theory

The theory of electromagnetic phenomena in classical physics had a great development over the 19th century. In their epistemological reconstruction for educational purposes, Zuza and colleagues (2018) show how, until that moment, the Newtonian vision had dominated physics: it was characterized by instantaneous forces acting at a distance, in which the physical quantity "field" (e.g. gravitational) was used as a simple mathematical artifact, expressing the action of the force per unit of mass (in general, the generating unit of the field). As a result, efforts were made to provide the same quantitative foundations for the theories of electricity and magnetism (despite the introduction of repulsive forces), but the noncentral character of the magnetic force discovered by Oersted and Ampère was one of the results which "spurred Faraday to imagine a role for the surroundings in which the interaction took place". According to the authors' historical reconstruction, another "driving force for the development of the field theory came from the search for underlying unity of natural forces, culminating in the law of the conservation of energy formulated in 1840". So, "throughout the 19th century a shift away from a Newtonian world view took place, even though for a long time there was no experimental evidence to favor one view over the other"; this paradigm shift required the adaptation of some descriptive variables in order to keep the more important conceptual referents: for instance, "to be consistent with mechanics, linear and angular momentum were ascribed to the field so as to not violate momentum conservation".

The formalization of the theory about electromagnetic phenomena had the culmination in 1873, when Maxwell reported in his "A Treatise on Electricity and Magnetism" the details of the theory, according to an analytic approach (Suárez et al., 2022). Nevertheless, his search for a mechanical model of the propagation medium that would account for electrical and magnetic interactions had not yet been definitively abandoned, according to the Faraday's model of continuous action along the medium (Steinle, 2005). Apart from their interpretation, there was a general agreement in the physicists' community at that time regarding the fact that the system of Maxwell's equations contained the whole theory of electromagnetic phenomena, at least on a formal level: using Hertz's words, "Maxwell's theory is Maxwell's System of equations" (Hertz, 1892).

The most important step in reaching the final version of the Maxwell's equations is the identification of the displacement current, as it is related to experimental evidences whose phenomenology is difficult to detect, and it completes the conceptual leap between electrostatics and electrodynamics placing the symmetry the electric and magnetic fields. As Suárez and colleagues (2022) show, the displacement current was first introduced by Maxwell in "On Physical Lines of Force" (1861): "thanks to the introduction of this contribution he obtained a version of the continuity equation similar to the one currently used, even though it is worth mentioning that he considered it similar in nature to conduction current". Indeed, in "A Dynamical Theory of the Electromagnetic Field" (1865), he describes his view of the physical meanings of displacement current as follows: "Electrical displacement consists in the opposite electrification of the sides of a molecule or particle of a body which may or may not be accompanied with transmission through the body […] The variations of the electrical displacement must be added to the currents p, q, r to get the total motion of electricity".

Some years later, Maxwell published his main work, "A Treatise on Electricity and Magnetism" (1873), in which he states that – like conduction currents – the temporal variation of the electric displacement generate a magnetic field. Furthermore, the authors (Suárez et al., 2022) suggest that Maxwell's hypothesis about the existence of the ether – as a special medium where the fields' propagation take place – implies that "he considers the displacement current a consequence of the variation of the electric displacement in any mechanical medium, and therefore it is always associated with a movement of bound charges, unlike the currently accepted view". Considering the current model, the authors also observe that "the suppression of the mechanical ether presents several problems for the interpretation of the displacement current, especially when analyzing the problems of the generation and propagation of electromagnetic fields in a vacuum" because it "makes the vacuum displacement current a simple term directly proportional to the rate of change of the electric field". As we will discuss later about the sources of the electromagnetic waves, the fact that displacement currents can occur in a vacuum is an important aspect to take into account to understand why they are not sources of magnetic fields.

However, some experimental evidences were still missing at this stage. Indeed, in "On Physical Lines of Force" (1861) Maxwell made explicit that the variations of the electromagnetic field propagate as a wave, with the electrical and magnetic components being perpendicular to each other and lying in the wave's plane (Callegaro et al., 1998). Maxwell also proved the formula of the velocity of propagation, having the same value that was known at that time for light propagation from the experiments of Weber and Korlausch (Darrigol, 2000). Furthermore, as the transversal nature of light was already known, in "A Dynamical Theory of the Electromagnetic field" (1865) Maxwell suggested that light was made up of undulations of the same medium whose oscillations cause electrical and magnetic phenomena. Nevertheless, he did not provide any evidence for generating and/or detecting electromagnetic waves.

This empirical evidence came from the Hertz's experiment in 1887, when he used a simple device to prove the existence of electromagnetic waves (D'Agostino, 1975). It consisted of an open RLC circuit as transmitter, generating sparks between the capacitor's plates, and of a series of open spires as detector. Changing the distance between the two systems the spark's intensity is changing, and assuming the wavy nature of the propagation of the field according to the change in amplitude and phase, Hertz was able to measure a value for the propagation velocity that was still consistent whit that of the light. In the following year, he improved many aspects of the device, being able to detect radiation with different wavelengths.

In doing so, Hertz empirically proved that the electromagnetic waves propagate through dielectrics, while they are mainly dissipated or reflected by conductors: Hertz exploit this behaviour in order to stretch the optical path of the wave, thus measuring a more precise value for the propagation velocity. Furthermore, he carried out a series of parallel experiments based on the analogy between light and electromagnetic waves, demonstrating that phenomena such as prism refraction, circular slit diffraction or interference occur in any case.

The crucial aspect of the experiment was that, exploiting sparks' phenomenon, Hertz was able to demonstrate that the microscopic sources of electromagnetic waves are accelerated charges. Furthermore, he was able to maximize the spark received by the detector device changing the parameters of the generating circuit (D'Agostino, 1975), showing that the motion of the source affects the characteristics of the propagating wave.

Rather than deriving the relationship between the electric and magnetic fields from a mechanical model, Hertz postulated the system of Maxwell's equations – in matter – in a new formalism: it was more similar to the vector formalism we currently adopt, due to Oliver Heaviside (Sengupta & Sarkar, 2003). These equations referred to a reference system at rest with respect to the ether, satisfying all the fundamental hypothesis of any wave-theory of light. Indeed, in the free ether they provided a wave equation for the fields' components (Callegaro et al., 1998):

$$(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})E = 0$$

Where E represents any component of the fields. Thus, another fundamental step was carried out about the unification of the electric and magnetic field, whose components follow the same law in propagating over time. From this equation we also observe that the propagation velocity is constant, namely, it does not depend on the motion of the source. To apply the Maxwell's equations in the case of moving and deformable bodies, Hertz recognized that it is necessary to specify the state of motion of ether, but he was not able to discern the ether's state from the state of matter. Indeed, according to the Fresnel's wave theory of light, the ether was supposed to be at rest in the media and moving with a speed dependent on their refractive index in the media (Bellone, 1990).

The limits of the ether's hypothesis were also given by the specific characteristics that were requested: in particular, it had to be rigid enough to allow a very fast propagation. On the other side, its existence was consistent with some other phenomena. For instance, this explained the phenomenon of optical aberration and Fizeau's experiments on the speed of light in water (Darrigol, 2000). For these reasons, the hypothesis of the existence of an elastic medium that supported the propagation of electromagnetic waves, as for other wave phenomena occurs, persisted for a long time. According to this hypothesis, the speed of electromagnetic waves predicted by Maxwell's equations should have been that relative to a reference system at rest with respect to the ether (Steinle, 2005).

Among the several experiments were carried out to investigate the existence of ether, the one from Michelson and Morley (1887) played a crucial role, as using an interferometer they were able to evaluate its effects with greater precision (Buchwald

& Fox, 2013). Based on the hypothesis of Galilean relativity – namely, that the velocity of light in ether is the sum of the velocity of light in the Earth's reference system and the Earth's velocity with respect to ether – it showed that it was not possible to measure a shift in the interference fringes once the interferometer is rotated. As the prediction for the shift's magnitude was bigger than the precision of the instruments they were using, this represented the most important proof refuting the ether's existence. Nevertheless, the experiment was repeated many times during the following years (Earman et al., 1982), as its results just allow to conclude that the relative speed between ether and Earth is much smaller than the value expected. For this reason, several ad-hoc hypothesis were made explicit – the main ones due to Fitzgerald and Lorentz (Cassidy, 1986) – to save the ether's theory, but it was not definitively rejected at this stage.

However, the Maxwell's theory was saved because of his efforts – and that of his successors – to make the theory independent of the ether's existence. In particular, the mechanisms of continuous action imagined by Faraday (Steinle, 1995) already disappeared in "A Dynamical Theory of the Electromagnetic Field", where Maxwell clarified that the fields acts directly on specific points in matter (Berkson, 2014). This interaction takes place according to the Lorentz force law, whose expression was given by Lorentz in the case of electric convention through an immovable ether (Darrigol, 2000). It completes the system of Maxwell's equations, specifying how an electromagnetic field generated by a specific charge distribution acts on another charge distribution. The Lorentz force law implies fundamental features that could be identified as key ideas regarding the classical fields' theory, that is, they exert forces only on their sources without self-interaction (Zuza et al., 2018).

According to Scaife and Heckler (2010), it took a number of years for physicists to understand the nature of the magnetic force, finally getting the straightforward expression $\vec{F} = q\vec{v} \times \vec{B}$: this process started from the early observations of Oersted in the 1820s until Maxwell's first theoretical description and the formulations of Heaviside and Lorentz, together with the appearance of the right-hand rule in the late 1800s, now commonly used to determine the direction of the magnetic force.

In 1892, Lorentz also showed that the analytic interpretation of Maxwell's equations was the only possible one, assuming that the ether is immobile and unperturbed by the movement of charged bodies (Berkson, 2014). At this stage, the electric and magnetic fields were still different entities, until Einstein resolved the asymmetries appearing when applying Maxwell's equations to moving bodies (Einstein, 1905). The Minkowski's definition of the electromagnetic tensor (Earman et al., 1982) made explicit that, from the relativistic point of view, the electric and magnetic fields form a single physical entity. Thus, it changes according to the Lorentz's transformations (Cassidy, 1986) between inertial systems of reference, having its own sources defined by the Maxwell's equations – charge and current distributions – which establish the values of the field according to the delay due to the propagation of its variations. The theory of special relativity finally rejected the ether's hypothesis to support this propagation; using Einstein's words in the paper "On the electrodynamics of moving bodies" (1905):

> "*We want [...] to introduce another postulate, apparently incompatible with the first, that light in empty space always propagates with a speed independent of the state of motion of the emitting source. [...] These two conditions are sufficient to arrive at a simple and contradiction-free electrodynamics of bodies in motion, based on the same Maxwell theory formulated for bodies at rest. The introduction of a "luminiferous ether" will prove superfluous since, according to the concept to be developed, neither an "absolutely immobile space" endowed with special properties is introduced, nor is a velocity vector assigned to a point in empty space in which electromagnetic processes occur.*"

3.2 Key ideas for the learning path development

The historical development of the theoretical framework that we have illustrated allow us to define some key ideas corresponding to the epistemic steps characterizing the theory of electromagnetic waves. At this stage, the wavy nature of the propagation of the electromagnetic field requires a specific relationship between its two components – electric and magnetic – but a single entity in the form of a tensor cannot be introduced. Thus, we defined the following key ideas:

1. The electromagnetic phenomena show a wavy nature, propagating both in matter and in vacuum as a single entity (Maxwell, 1873).
2. The sources of electromagnetic waves are accelerated charges (Hertz, 1892).
3. The interaction between electromagnetic waves and matter is described by the Lorentz force (Lorentz, 1895).

Thus, the key ideas we identified also represent the three phases of the wavy phenomena – generation, propagation and detection – although we showed that their conceptual development and their mutual relationships did not follow a linear pathway. Furthermore, taking into account the different aspects characterizing the wavy nature of electromagnetic phenomena – emerging from the epistemological development considering both the analytical and empirical perspective – and considering the current formulation of the theory, each key idea will be linked to different learning objectives (which are the variables we can actually define and measure according to the students' learning outcomes), which the learning path will be based on: we will come back on this point in Chapter 9.

Chapter 4

Learning Difficulties in Electromagnetism

There is a large literature on the learning difficulties of high school students about electromagnetism, which focuses mainly on the distinction between the concepts of (electric and magnetic) field and force, the superposition principle, the concepts of current and potential difference in solving direct current circuits, and the phenomenon of electromagnetic induction (see the review of Chapter 5 in Van Kampen & De Cock, 2023). On the other hand, there is no extensive literature on the electromagnetic field and its propagation. In the perspective adopted in this research work, this represents a limitation, as an outdated line of research (Eylon & Ganiel, 1990; Rainson et al., 1994) shows that there is a missing link in students' reasoning – and in traditional teaching – between the domain of electrostatics and that of electrodynamics. On the one hand, some physical quantities seem to take on different conceptual meanings depending on the specific context: for instance, in dealing with circuits, the electric field is replaced by the electric potential, without pointing out the link between these two quantities. On the other hand, students do not seem to be able to formulate coherent reasoning in non-equilibrium contexts: an example provided by Rainson and colleagues (1994) is that of the formulation of hypotheses regarding the changes in the characteristics of the electric field generated by a charged body which occur when a point charge is approached to it.

In dealing with electromagnetic waves, the fundamental physical entities are the electric and magnetic fields. For this reason, together with the lack of connection pointed out by the research literature between the static and dynamic contexts in the interpretative models adopted by students, we deemed it necessary to proceed with the analysis of the research literature on typical learning difficulties in electromagnetism, focused on the concepts that will represent the referents of the learning path that will be presented later. It should be noted that the population we are going to address for the literature review is made up of high school and undergraduate students of introductory physics' courses involved in STEM degree programs (e.g. engineering, chemistry, mathematics or physics). The differences between these two types of sample will be emphasized only in special cases: we made this choice because the two populations do not seem to be very dissimilar in this context (we will come back on this aspect in Chapter 8), and also because the literature that deals exclusively with secondary education in this context is less consistent and more outdated.

4.1 The static context: the concepts of force and field

The first learning difficulties emerge in dealing with the concept of field, as evidenced by the literature regarding the electric field (Furió & Guisasola, 1997), that clearly shows the students' preference in using the Newtonian model of action at a distance, which they are accustomed to in the context of mechanics. As the authors pointed out, this does not represent a specific learning difficulty, but rather an ontological and epistemological obstacle tracing the transition to the Maxwellian model, which also occurred in the epistemological development of the theory. This aspect related to the cognitive conflict between theories can be framed in a series of learning difficulties, that have to do with the inability to distinguish between forces and fields in specific contexts – both in terms of intensity and direction – applying or not applying the Coulomb model depending on the specific context, or even making explicit reasoning dealing with aspects of different models (e.g. when asked to represent the new configuration of a field of a field when a source is added, many students tend to include interactions between charges in terms of forces in their reasoning). As a natural consequence, the confusion between forces and fields leads to the difficulty in identifying the fields' sources (Zuza et al., 2018), and therefore in applying the superposition principle, which is fundamental to define the vector character of the electric and magnetic fields.

4.2 The fields' operators and sources

In this context, Viennot and Rainson (1992) show that some students argue that electric fields cannot exist inside insulators because their charges are unable to move: this indicates that they are focusing on the concept of force between charges, rather than on electric field and its superposition with external fields (as the absence of the effect implies, according to students' reasoning, the absence of the cause). In a subsequent research article, Rainson and colleagues (1994) show how the results they got regarding the students' conceptions about the cause-effect relationship between fields and their sources can be interpreted through two different categories: in the first case, students accept the existence of a cause (the electric field) only when they can see its effect, such as the motion of charges. As a result, when students are asked to identify the sources of electric fields in specific contexts, the authors find answers such as "voltage, battery, charges or currents". In particular, in the latter case there is a reversal of the cause-effect roles, showing one of the reasons why electrostatics and the treatment of electrical circuits often constitute two completely separate domains for students (Benseghir & Closset, 1996).

In the second case, the cause-effect relationship is identified between the two terms of an equation, where the quantity on the left is often thought – by students – as generated by the quantity on the right, adopting an analogy reasoning (e.g. as the second law of dynamics). This result is particularly relevant because Maxwell's equations, which describe the link between electric and magnetic fields and their sources, in their integral form – that is the one high school students are usually introduced to – present a link between two operators acting on the fields – flux and circulation – and (just) some of their sources: in particular, the charges inside the volume through which the flux of the electric field is calculated and the currents passing through the surface along which the circulation of the magnetic field is calculated. Undergraduate students should be aware that external charges and currents have to be included among the sources of the fields, even though they do not contribute to the calculation of their operators (namely, flux and circulation): nevertheless, even in standard problems' contexts, literature shows that they often fail to identify the characteristics of the electric and magnetic fields generated by a set of sources including internal and external (with respect to the volume or surface we are considering) charges or currents (Guisasola et al., 2008), or they do not distinguish between the operators and the fields which they act on (Li & Singh, 2018).

In particular, Guisasola and colleagues (2008) show how most students apply a mechanical reasoning based on strategies such as the "functional fixedness" – which implies the non-identification of the presence of fields where operators take on zero value – and the "functional reduction" – which leads to global conclusions starting from local values of physical quantities – thus pointing out the absence of conceptual meaning attributed to the flux and circulation. Indeed, they are usually introduced in teaching practices as simple calculation tools, in order to indirectly derive the expressions of the electric and magnetic fields when the characteristics of the field lines are already known. In addition, Campos and colleagues (2023) report, among others, an even more tricky learning difficulty, which they identify as a mechanism of "authority fallacy": since Gauss's law – providing information about the electric flux – and Ampère's law – providing information about the magnetic circulation in the static case – are physical laws of general validity, students often think they can be applied to derive the fields' expressions in any situation, without considering the possible symmetry of the sources generating them. It could be argued that this idea is supported by the arbitrariness of the surface or curve that are used for the application of these laws: thus, the students' mental models may transfer to these geometric entities the symmetry properties that the sources should possess in order to apply these laws in a simple way. Since this study is based on the solution of inverse problems that exploit symmetry arguments of the Gauss and Ampère laws, the authors have also been able to implement comparative analysis between the electrical and magnetic contexts: among the results obtained, it is pointed out that the analysis of currents can be more difficult for students than that of point charges, which may explain why in the context of Ampère's law there is a greater prevalence of the "authority fallacy" category. Other differences between the two contexts emerge between magnetic field and circulation (Hernandez et al., 2023): the phenomenographic approach shows how the difficulties are parallel between the two contexts – namely, the same categories emerge – but the categories' frequencies differ significantly.

4.3 The superposition principle and the differences between the electric and magnetic contexts

Also when dealing with the fields' superposition principle, students show similar categories about conceptual understanding in the electrical and magnetic contexts, reporting a clear correlation between the correct application of the superposition principle and understanding the concept of field (Li & Singh, 2017). Analyzing the students' drawings, the authors also point out how this occurs in the case where they master a robust representation of the fields, especially when they are able to compare and create two different kinds of representations (such as vector graphs and field lines): because of this interaction, the students seem to be able to provide a physical interpretation of the phenomenon. In general, vector charts seem to be easier for students to be interpreted: however, we will come back on these aspects in Chapter 7.

The similarities between electric and magnetic fields are inherent to the theory of electromagnetism, as Maxwell's equations show. From the students' point of view, these similarities can be attributed primarily to elementary phenomenological aspects, such as polarity (positive-negative for charges, north-south for magnets) and the occurrence of both attractive and repulsive forces between them. Exploiting previous results reported by literature about students' spontaneous ideas (magnetic poles carry electric charges, magnetic fields can be generated by stationary charged particles, etc.), Scaife and Heckler (2011) argue that there is sufficient evidence for such similarities to be the cause of students' difficulties in clearly distinguishing between electric and magnetic phenomena. Examples of this kind of difficulties are the confusion between electric and magnetic fields (Sağlam & Millar, 2006), and the identification of the force acting on a point charge in the direction of the field lines or the magnetic poles, according to the simple mental model conceiving magnets attracting all objects directly to their poles (Borges & Gilbert, 1998). In the latter case, the direction attributed to the force can be in or out of the magnet, depending on the students' interpretation of the right-hand rule and their awareness of the non-commutativity of the vector product: according to Scaife and Heckler (2010), these aspects reveal persistent (also after teaching) and unsystematic signs' errors.

Continuing the exploration of the comparison between the two contexts, Scaife and Heckler (2011) also observed another symmetry in the "interference" between electrical and magnetic concepts from the students' point of view: it is related to the fact that, after studying magnetostatics, many students identify the direction of the force acting on a charged particle in motion in an electric field as perpendicular to the field itself, despite the fact that before instruction a significant fraction of students answered these questions correctly. These results clearly show how the interference between the two contexts strongly depends on which one has been taught more recently, even though in the long term the more intuitive concept of electric force (as it is parallel to the corresponding field) will probably prevail in memory. However, this dependence of reasoning on teaching timing led the authors to hypothesize that students are aware of the differences in behavior between electric and magnetic fields, but they hardly recognize them as fundamental in solving problems related to the forces acting on charges.

In this regard, Campos and colleagues (2021) report that reasoning that presents interference between the two contexts is more frequent when students have to deal with the magnetic field. On the contrary, another kind of interference in reasoning – between the concepts of field and force – is prevalent when an electric field is present. The authors attribute this asymmetry to the contents of traditional teaching, which focuses the treatment of electrostatics on interactions, to a much greater extent than in magnetostatics: indeed, the electric field is often introduced through Coulomb's law of force, while the magnetic field is introduced through the Biot-Savart's law, which is independent of the concept of force. Similarly, while the study of the interaction between two point charges is a fundamental phenomenon, the one between currents is usually considered as an appendix, and the one between magnets is often completely neglected.

4.4 The Concept Inventories in Electromagnetism

As we specified above, the analysis of the learning difficulties and reasoning of students in solving direct current circuits would deserve a specific chapter, which goes beyond our objectives. However, it is important to observe that the most widespread Concept Inventories on electromagnetism in the research literature – i.e. the "Brief Electricity and Magnetism Assessment" (BEMA: see Chabay & Sherwood, 1997) and the "Conceptual Survey of Electricity and Magnetism" (CSEM: see Maloney et al., 2001) – contain many questions about resistive circuit resolution and about the concepts of (direct) current and potential difference. These are standardized tests with more than 30 multiple-choice questions, designed to investigate students' knowledge about the basic concepts covered in introductory electromagnetism courses at undergraduate level. Both BEMA (Ding et al., 2006) and CSEM (Saarelainen et al., 2006) have been extensively validated in the literature with large and different samples, demonstrating their reliability and discriminatory power according to the tools of the Classical Test Theory (De Vellis, 2006), even though some items may need to be modified in order to improve their differential functioning (Ding, 2014). They can also be used for different purposes, for example for the comparison of effectiveness between different curricula (Kohlmyer et al., 2009) or as a pre- and post-test in the evaluation of a single teaching sequence (Maries et al., 2022). About the limits of these instruments, Saarelainen and colleagues (2006) have shown that additional methods – such as interviews – are necessary to deepen students' explanations, which are not necessarily in a one-to-one relationship with the answers' options, thus the choice of the correct answer in a question may hide the use of an inappropriate reasoning model.

Like the other Concept Inventories, the BEMA and the CSEM are not one-dimensional tests: according to previous results, Planinic (2006) has shown that the conceptual area significantly more difficult for students to deal with – among the six provided by the CSEM – is the one concerning electromagnetic induction. It represents the bridge phenomenon with the dynamic context, which we are going to investigate in the next paragraphs with reference to the learning difficulties and spontaneous ideas of students that have emerged from literature.

This review of the Concept Inventories also allowed us to verify that there are no items related to time-dependent fields we can adapt and use for the development of an instrument measuring the learning outcomes after implementing our teaching proposal about electromagnetic waves: thus, we chose to elaborate some questions on purpose, which we will describe in Chapter 8.

4.5 The dynamic context: the Newtonian and Maxwellian models

The learning difficulties analyzed in the static case had at their core the concept of field, which the students usually face for the first time in the context of electrostatics. As observed by Zuza and colleagues (2018), the Newtonian (force) and Maxwellian (field) models can be considered to have different but not contradictory ontological and epistemological statuses, regarding the interpretation of the interaction between charges (despite the Maxwellian model has a greater explanatory power). Furthermore, the development of the field theory requires the Newtonian model as a prerequisite, as the electric field is defined from the expression of the Coulomb's force describing the interaction between two stationary point charges. Nevertheless, the Newtonian model becomes inadequate in explaining the non-instantaneity of the electromagnetic field's propagation, while in the previous theory the forces act instantly. From this point of view, in the electrodynamic context the concept of field takes on its relevance and a new meaning: thus, the learning difficulties regarding the confusion between forces and fields are not surprising, as – at least for the electrostatic field – students may interpret it as a mere and superfluous mathematical artifact, along the lines of the historical development followed by the concept.

In this regard, Bagno and Eylon (1997) observe that students' explanations often attribute a "static" nature to the (electric) field, namely, it is considered as a structure existing in space and exerting forces on charges, that is immutable when new sources (in this case charges) become part of the region of space in which the field acts. Thus, the fields represent fixed properties of space: this model may be influenced by the use that traditional teaching propose for this concept, not mentioning the time which the field needs to propagate in space – moving away from its source – until it reaches an equilibrium configuration. In addition, students may consider the additional charges with respect to the original source to be equivalent to the test charges, which are used to define the electric field. As a result, in this conceptual model, the only way to change the field is not adding (or removing) sources, but changing the geometric configuration of the pre-existing sources' distribution.

As we mentioned above, in the dynamic case there is a conceptual leap between the instantaneous model of forces acting at a distance, and the field model, whose variations propagate in space at finite speed. In this context, therefore, it is easier for the field to no longer be considered as a mere mathematical artifact, thus taking on a new conceptual meaning. Despite this, the emergence of new physical characteristics and new relationships between the electric and the magnetic fields reasonably make electrodynamics a source of notable learning difficulties, in relation to several phenomena: the main one, both for the time that is dedicated to it in teaching – for its conceptual relevance and because of the numerous possible applications – is the phenomenon of electromagnetic induction.

4.6 Electromagnetic induction

A large research literature has reported similar learning difficulties – even after teaching – both for high school students (Loftus, 1996) and undergraduate introductory students (Park, 2006; Galili et al., 2006), concerning different aspects related to the application of the Faraday-Neumann-Lenz law: the determination of the direction of the induced current, its difference with the electromotive force, the recognition of the temporal variations of the magnetic flux as its cause, the difference of the "induced" electric field with the electrostatic one, and the consequent difference between electromotive force and potential difference. In addition, other specific learning difficulties are reported in literature (see the review of Chapter 5 in Van Kampen & De Cock, 2023) that we are not interested in analyzing for our purposes, or others that we have already discussed (e.g. the conceptual confusion between fields and their operators, in this case between the magnetic field and the magnetic flux).

It is interesting to analyze the difficulties' categories we have reported. The determination of the direction of the induced current is related to the energy conservation, but this interpretation – that often students know – creates confusion as the statement "the induced magnetic field (or current) opposes the change in the (flux of) the external magnetic field" is often interpreted as "the induced magnetic field (or current) is in the opposite direction to the inducing field", rather than in the opposite direction with respect to the change in the inducing field (Bagno & Eylon, 1997). The recurrent confusion between induced current and electromotive force is instead linked to the difficulty in recognizing the phenomena of electromagnetic induction when a real circuit detecting its effects is not present.

Identifying the cause of the induced current in the temporal variations of the magnetic flux represents a difficult task for students, as it consists of two different components. The first is to recognize the cases in which the flux varies (Maloney et al., 2001) and to evaluate the magnitude of the effects produced by the rate of change of this variable (Sağlam & Millar, 2006). A notable case is the one reported by Thong and Gunstone (2008), who show how students can identify a direct proportionality between the induced current and the one flowing in the inducing circuit: also in this case, the statement would be correct only with reference to the rate of change of the inducing current.

The second component is related to the recognition of the mechanism through which the changing magnetic flux can cause the motion of the charges in the circuit. Jelicic and colleagues (2017) show how students adopt different alternative mental models for this purpose – as they are often characterized by internal incoherence due to their specific construction – in order to justify the specific phenomenon they are dealing with: for instance, for some students the external magnetic field acts directly on the conduction electrons, while others identify the cause in the strength of interaction between the magnet and the magnetic poles of the coil in the experiment they are dealing with. Moreover, for our purposes it is interesting to observe how literature report recurrent students' reasoning predicting that the "agent" – which we can interpret as the cause of the phenomenon – and the object on which it acts, must be in contact for the phenomenon – namely, the electromagnetic induction – to occur (Loftus, 1996), or that the distance between the "inducing" and the "induced" circuit influences the effect of the phenomenon, for example regarding the brightness which a light bulb connected to the second circuit turns on with (Munley, 2004). This is interesting because there are other cases (in electromagnetism) in which students identify cause-effect relationships in contexts where they do not occur (Sağlam & Millar, 2006), as we will discuss later.

Finally, we have identified the last class of misconceptions that have emerged in the literature regarding the phenomenon of electromagnetic induction as those related to the concept of electromotive force. First of all, we think it is necessary to clarify its meaning, since the literature has reported persistent difficulties about it also for teachers (Mulhall et al., 2001). Following the explanation of Chabay and Sherwood (2007), the electromotive force quantifies the non-conservative actions (chemical reactions) which take place in a battery, in terms of the work done per unit of charge to produce and maintain the potential difference – spatially separating charges of different polarity between the battery terminals – that allows the current to flow. Garzón and colleagues (2014) use this definition to clearly distinguish this concept from that of potential difference, which measures the work used up by charges when they move from one point to another in the circuit, it being performed by conservative forces. The authors underline how this distinction – which students are often unaware of – is conceptually important in the same way as the distinction that is placed in traditional teaching between work and potential energy in the mechanics' context. In our case, this difference is even more relevant, as it represents the loss of the conservative property of the electric field in the dynamic case, which will be fundamental for the purpose of determining the characteristics of the electromagnetic field's propagation. Indeed, the use of the term "induced electromotive force" hides this result: literature show that the same learning difficulties identified in the context of direct current circuits are also recurring in the context of electromagnetic induction (Zuza et al., 2016). Furthermore, the confusion with the induced current does not allow us to clarify that the electromotive force would also be measured in vacuum.

In any case, reasoning in terms of fields in the analysis of electromagnetic induction phenomena is seldom recurrent among students. Thong and Gunstone (2008) report that many students are unable to explain the phenomenon by using the electric field in the context of a simple circuit with a bulb that lights up: rather, they use the concepts of potential drop, potential difference, or electromotive force to justify their reasoning.

4.7 Comparison between different contexts (micro-macro, electric-magnetic)

The analysis of the interpretations provided by students regarding electromagnetic induction phenomena is also paradigmatic because it allows to investigate the relationships that are identified by them between macroscopic descriptions (in terms of flux) and microscopic ones (in terms of forces): according to Guisasola and colleagues (2013), students often do not clearly distinguish between the different aspects, but they tend to prefer macroscopic interpretations – despite the equivalence of the two models in each situation – even when they are explicitly asked to adopt a microscopic description of the phenomenon. This difficulty can be easily related to those identified above, in particular with those related to the concept of electromotive force and about recognizing the existence of an "induced" electric field.

Electromagnetic induction represents the first and most relevant case among the phenomena related to the relationships between the electric and magnetic fields, but it is of great interest for us to analyze other contexts in which conceptual interference between these two quantities occurs, which recurrent learning difficulties have been reported in the literature about. For example, Guisasola and colleges (2004) show how most high school students identify a single class of sources – namely, the charges – for both the electrostatic and magnetostatic fields, regardless of their state of motion: also after instruction, many of them include charges at rest among the possible sources of magnetic fields, using a model by analogy that tends to confuse the two fields. Turning to the dynamic case, Bagno and Eylon (1997) show that most students also include changing magnetic fields among the sources of electric fields, but they do not recognize the symmetry characterizing the Maxwell's equations. Indeed, less than 5% of students state that a changing electric field generates a magnetic field. This asymmetry is probably due to the fact that the Ampère-Maxwell's law usually takes much less time than Faraday's law in traditional teaching, as it is much more difficult to show its phenomenological effects.

According to Guisasola and colleagues (2004), the model by analogy we have just mentioned implies considering the magnetic interactions as central forces, and the consequent confusion between field and magnetic force. This lack of distinction has also been found in other articles, together with another recurrent learning difficulty in this context: Bagno and Eylon (1997) have shown that many students believe that the path of a point charge in a magnetic field is always circular. This consideration is independent of the speed of the charge, which in the model adopted by the students can also be zero: Zuza and colleagues (2018) have shown that many students believe that the magnetic field also acts on stationary charges, despite the fact that they are not among the sources of the magnetic fields. It is easy to hypothesize that this erroneous generalization is due to the fact that, in most of the problems usually presented to students, the initial velocity of the charge is perpendicular to the direction of the magnetic field. Furthermore, another consequence of the model's adoption by analogy consists in justifying the nature of magnets by the fact that their poles own charges of opposite sign.

4.8 The fields' sources

The above considerations can also be extended to the moving charges. Kesonen and colleagues (2011) show that a significant fraction of undergraduate students recognize that a charge in uniform motion generates a magnetic field, but they do not mention the electric field at all, even in cases where they correctly identified the characteristics of this field when the charge was considered at rest. Some of these students have stated an equivalence between uniformly moving charges and direct currents in a conductor, which may be one of the reasons why they have not identified any electric fields.

Turning to the case of (uniformly) accelerated charges, the same study shows that, although most students consider them to be sources of both electric and magnetic fields, only a small minority recognize that these fields will vary over time. In particular, several students seem to consider the electric field as a stable property of the charges – according to the conceptual model of the field as a "fixed property" of the space, which we have described above – asserting that only the magnetic field will change over time. Moreover, the relationship of mutual orthogonality between the two fields seems to be unknown to the majority of the sample.

With regard to the relationships between the fields and their sources, the considerations reported by Suárez and colleagues (2022) are very relevant. Here the authors observe that the sources of electromagnetic fields are basically the charges, represented in Maxwell's equations by the quantities charge density and current density. Nevertheless, the language used by high school textbooks – and in some cases also for texts adopted at undergraduate level, with the remarkable exception of Chabay and Sherwood (2015), where this idea is explicitly refuted – suggests that electric fields can be generated by time-varying magnetic fields, and vice versa: in this case, the two components of the electromagnetic field should be counted among the sources of the other component. However, this interpretation is not correct, as Maxwell's equations represent exclusively a system that allows – at least theoretically – to calculate the electric and magnetic fields generated at every point in space and at every instant of time by the sources distributions. In this link lies the fundamental importance of this system of coupled differential equations, which have to be completed by the expression of the Lorentz force with the aim of determining how these fields act on charges other than the sources.

As a proof, let's consider that Maxwell's equations – at least in their integral form, that is the only one high school students are introduced to – represent relationships between the values of the electric and magnetic fields at different points in space and at the same instant of time. This implies that there is no time interval separating the cause from the effect, which would violate – as they are spatially separated – the classical interpretation of the principle of causality. The evidence of this delayed action emerges in Jefimenko's equations, which express the links between the electric and magnetic fields – calculated at the position $\vec{r}$ and at the instant t – and their sources at the position $\vec{r}'$ at an earlier instant t' (named v the propagation speed of the field, it is $t' = \mathrm{t} - \frac{|\vec{\mathrm{r}} - \vec{\mathrm{r}}'|}{v}$). More generally, fostering the idea that "correlation does not imply causation" is of great value, both in physics and in all experimental sciences.

4.9 The electromagnetic field

The previous considerations about the relationship between fields and their sources provide a support in favor of the existence of a single physical entity – the electromagnetic field – in accordance with the classical theory presented in physics textbooks. However, in traditional teaching – at least at high school level – there are usually not enough reasons to introduce this new terminology, at the expense of the physical quantities – electric field and magnetic field – introduced in the static context.

As a proof, the literature reports other specific learning difficulties related to the interconnection between the two fields, in addition to those we already discussed above. In this regard, it is important to illustrate the results of a research article by Ambrose and colleagues (1999), which – although it is dated – provides one of the (few) examples of research about students' reasoning in the electrodynamic context, by analyzing the conceptions relating to the two fields when they explicitly represent two components of the same entity – the electromagnetic wave – which propagates over space and time.

In this regard, a premise with respect to the concept of propagation velocity in this context is useful. The results obtained by Zuza and colleagues (2018) show that, when asked questions about the speed of the electromagnetic field, undergraduate students show many difficulties in recognizing that it is a finite value, or in justifying why the propagation is not instantaneous. These difficulties also depend on the context in which the question is asked: they are prevalent when it does not explicitly refer to the fields, just mentioning a signal carrying information at a distance. Indeed, this context can elicit non-scientific reasoning based, for example, on erroneous phenomenological perceptions or on technological – and not physical – limitations: because of this contingency, the information may not be transmitted with infinite speed, without any reference to the natural limit of the speed of light. The situation changes when referring to the interaction between charges: however, in this case another difficulty category may emerge, linked to the possible use of the Newtonian (instantaneous) model of interaction.

The article of Ambrose and colleagues (1999) moves from a different perspective: it arises from the observation that some important learning difficulties identified about wave optics – in particular regarding the prediction of interference and diffraction patterns – can be caused – at least partially – by the lack of understanding of the nature of light as an electromagnetic wave (Wosilait et al., 1999). Indeed, as Kesonen and colleagues (2011) observe, optics and electromagnetism support each other, "thus creating a continuum in which the meaning of the concept of field can develop from the model of interaction towards that of the electromagnetic nature of light". In addition, many students show serious problems in dealing with the basic characteristics of wave phenomena, such as wavelength, frequency and phase difference (Wittmann et al., 1999). In any case, to deal with phenomena such as polarization, the scalar wave model of light is no longer sufficient, thus it is necessary to develop the transverse (electromagnetic) wave model.

4.10 The electromagnetic waves and their detection

For the reason illustrated above, Ambrose and colleagues (1999) aimed at identifying the main learning difficulties in dealing with (plane) electromagnetic waves. The results show three main categories, partially overlapping, which they described as follows: 1) failure to interpret formal representations of a plane electromagnetic wave; 2) failure to apply the Lorentz force law to interactions involving electromagnetic waves; 3) failure to recognize that the electric and magnetic fields in an electromagnetic wave are interdependent. We will come back later to the first category in a specific chapter, analyzing the relationship between learning and the representations of the learning object. Here we just remark that the class of plane waves is the only one that is explored in traditional teaching (at least at a not too advanced level), as it is possible to consider wave packets consisting of several plane (monochromatic and linearly polarized) waves overlapping to form the resulting wave.

Let's analyze the other two categories identified by the authors. In the case of the application of the Lorentz force to the reception of electromagnetic waves, we are going to make explicit some premises from the physical point of view. Indeed, conductivity is usually considered as a scalar quantity, so that the current can always be thought as collinear to the electric field. In addition, the context of the questions we will analyze is very simple: students have to deal with antennas receiving signals, represented as straight wires – rather than as closed circuits – and not as radiating bodies. These simplifications are intended to focus conceptual attention on recognizing that, even in the case of time-varying electric and magnetic fields, it is the expression of the Lorentz force that determines how they act on free charges in matter. Such interactions are mostly faced in traditional teaching related to static contexts, which may lead students to think that the characteristics of these interactions change in the dynamic case, according to that conceptual leap we have previously discussed between the two contexts. In these cases, in order to maximize the current circulating in the antenna, it will be necessary to orient it in the direction of the waves' polarization: thus, the electric field will provide its maximum contribution, while the contribution of the magnetic field can always be neglected, as it is perpendicular to the direction of the antenna (according to the expression of the Lorentz force).

This study reports very unsatisfactory results: only 5% of students before instruction and 15% in the post-test provide a correct explanation to the question "in which direction should the antenna (of a portable radio) be oriented to get the best signal's detection?". Furthermore, it does not appear that an experimental demonstration of detecting a radio signal through a long straight antenna helped students to answer the question correctly. It should be noted, however, that in this case the difficulty in identifying the Lorentz force as a conceptual referent of the interaction can overlap with the difficulties in interpreting the graphic representation of the electromagnetic wave, whose profile is represented in the question to indicate the direction of the fields' oscillation. The most common incorrect answer consists in identifying the direction of best detection with that of the wave propagation, as in this case "a maximum value for electric and magnetic fields would be obtained". These students do not recognize the transverse character of electromagnetic waves. Another common answer is to choose any direction in the plane perpendicular to the direction of waves' propagation: in this case students do not seem to be able to distinguish between the effects of electrical and magnetic interactions.

4.11 The electromagnetic waves and their propagation

Regarding the interdependence of electric and magnetic fields in the dynamic case, this can be pointed out in several cases. The questions posed by the authors focus on cases in which such interdependence implies that both fields must be absorbed or transmitted together, in particular when an electromagnetic wave passes through a polarizing filter. For example, when students are asked what would happen placing a filter in front of a slit, some answers point out reasoning that recognizes the decrease in brightness of the pattern on the screen, but they also provide an incorrect justification. For instance, some students state that a beam of light consists of waves moving in different directions, and only those moving in the direction parallel to that of the filter's axis will be transmitted. However, it is not always clear which is the entity he is referring to: for example, when one of the students is asked to consider the case in which the electric field is parallel to the axis of the filter, he replies that "all the electric field" will be transmitted, while "all the magnetic field" of the incident wave will be absorbed. This concept of independent fields is also recurrent in the post-test: almost half of the sample states that one or the other field – depending on the direction of the filter – will be completely transmitted, while the other will be completely absorbed.

Interestingly, this third cluster of difficulties is the most difficult to tackle, as it was the only one among the three identified that required the authors to modify the tutorial – used during instruction – before it became successful (i.e. before freshmen's post-test results were equal to or better than those of graduate students in the pre-test). This modification required focusing on both the electric and magnetic fields transmitted by the filter, in order to elicit the understanding of their inseparability. As a proof, Kesonen and colleagues (2011) have also found – regarding the same type of questions – answers that point out a lack of understanding about the symmetry of the relationships that link the electric and magnetic fields. For example, a student asserts that the (magnetic) field transmitted through the polarization filter, changing over time, is able to generate a new (electric) field, which has the same overall characteristics as the one that has been absorbed.

4.12 The structure of the content knowledge in electromagnetism

We conclude this chapter with some important and general remarks emerging from the study by Bagno and Eylon (1997). In this case, the request submitted to the students consists of three short recall tasks, respectively named as "free" ("Summarize in a few sentences the main ideas of electromagnetism according to the order of their importance. Don't use formulae"), "cued" ("Next to each of the following concepts, write as many relationships as possible that include the concepts of electric and magnetic field") and "contextual" ("Suggest as many ways as possible to produce an electric field and a magnetic field"). In general, the results clearly show that students lack a coherent organization of the structure of knowledge, both in terms of concepts and relationships, which hinders the process of cognitive retrieval of information, that is fundamental for problem solving.

Such a structure would allow to recognize Maxwell's equations and the Lorentz force as the fundamental relationships in electromagnetism. On the contrary, many students affirm the fundamental character of several other relationships, in particular the Ohm's law (which establishes the link between current, potential difference and the resistance of a conductor).

In addition, a comparison of the three types of recall tasks shows that the majority of students who refer to Faraday's law as a fundamental relationship is actually an artifact. Indeed, many of them mention only Lenz's law or the appearance of an induced electromotive force in specific cases, without mentioning the relationship between the circulation of the electric field and the temporal variations of the magnetic field's flux (which was fully reported by 10% of the sample). Moreover, the students' summaries do not reflect the symmetry between the electric and magnetic fields, since (according to the results of the literature previously reported, regarding the knowledge of the Ampère-Maxwell's law) only 5% of the sample also refers to the generation of magnetic fields from changing electric fields.

In general, the recall patterns adopted by students in the "free" and "cued" tasks are quite similar, suggesting that such summaries fairly accurately reflect the content of the information stored in memory, and that the missing information is not due to difficulty in the retrieval process. On the contrary, the “contextual” task helped the students to remember most of the relationships: however, considerations such as those reported regarding the description of the Faraday’s law lead to the conclusion that these kind of relationships are often not considered according to their generality, but only regarding specific aspects or with respect to particular cases. The search for this generality is one of the objectives of our research, in order to try to provide students with a general framework that, in the context of electrodynamics, would develop the conceptual meaning of the entities and relationships that students have explored during instruction.

Chapter 5

Teaching proposals on electromagnetism

In this chapter we are going to discuss some teaching proposals or simple implications and suggestions reported in literature about electromagnetism, in order to address the learning difficulties we identified in the previous chapter. Thus, we are going to present some proposals reported by literature representing cues or hypothesis for the development our learning path: with this purpose, we selected those studies aligning with our learning objectives or prerequisites.

As we mentioned above, we are adopting a global perspective of the theory to develop our learning path – in order to overcome the learning difficulties related to the conceptual leap between the different contexts we identified – thus in this case the distinction we made between the static and dynamic contexts disappears. This perspective also consists of analyzing students' understanding of the structure of the theory we are investigating, in order to elaborate teaching activities aiming at developing this kind of knowledge.

5.1 A global perspective for electromagnetism

The global perspective (see Bagno and Eylon, 1997) implies knowing what are the central ideas in a domain (in the case of electromagnetism they are related to some representation of Maxwell's equations and the Lorentz force), since these are the ideas that students are likely to remember and use in the future. For this reason the authors of this study proposed an "integrative" instructional approach, focused on the construction of a hierarchical concept map. This map is constructed by students in five stages ("solve": they solve a set of problems that highlight the central ideas in the domain; "reflect": they reflect on the conceptual basis of their solutions; "conceptualize": they perform activities that deal with relevant conceptual difficulties; "apply": they implement complex applications; "link": they link their activities to the evolving concept map). Students solve problems and add the concepts and relationships that are used in the problems to the map. As a result, an organized structure is formed linking conceptual knowledge and how it is used in problem solving. When new concepts are added to the map, the relevant conceptual issues and difficulties are processed, thus the conceptual knowledge is naturally linked to the structure. Moreover, the hierarchical design of the map at different levels of detail is helpful for recalling and problem solving.

The students' performances when learning according to the integrative approach was compared with that of students in two comparison groups: an isolated treatment of conceptual difficulties and a standard review of electromagnetism. The authors found an overall advantage of students in the integrative approach over the students in the control groups about all aspects (recall, conceptual understanding and problem solving). There was also a transfer effect: students in the experimental group learned how to identify important ideas and relationships in the presentation of an unfamiliar topic. Considering the fact that the time spent by students in the three groups was about the same, they recommended to adopt the integrative approach.

5.2 The key concepts in electromagnetism

Coming to the analysis of the specific concepts, the first one that students are introduced to when studying electric and magnetic phenomena is the field: to improve its conceptual understanding, Guisasola and colleagues (2008) suggest to design teaching sequences stimulating students to analyze the fields' distribution for different charges or currents configurations. This analysis should encourage students to take into account all the sources and to avoid functional reduction based on the formula. These activities will also encourage students to discuss what the pattern would be if only the charges or currents enclosed by the surface or path were taken into account, in order to address the difficulties we discussed about flux and circulation.

Beyond the field idea, in the previous chapter we discussed the learning difficulties related to other central concepts. The first one is the electromotive force: Garzon and colleagues (2014) suggest to emphasize its non-conservative nature in order to lower the conceptual barrier when students are dealing with electromagnetic induction. Indeed, in traditional teaching they are used to consider circuits as being driven by conservative forces, so when they deal with this new situation, they still expect that a potential difference is required for the current to flow. On this basis, Zuza and colleagues (2020) presented a teaching-learning sequence about the concept of electromotive force, which represents an interesting case-study about the assessment of students' learning outcomes in electromagnetism after experiencing non-standard pathways.

Furthermore, Kesonen and colleagues (2011) suggest that the learning difficulties related to electromagnetic induction reported in literature may be a consequence of teaching this phenomenon in such a way that the main focus is placed on electric circuits and induced currents or voltages, rather than on the electric field in the circuit. In turn, this may lead to an insufficient understanding of the general meaning of the

Faraday's law. Hence, they recommend that the integral form of Faraday's law and its qualitative meaning should be clearly emphasized as another aspect of the symmetry connection between the electric and magnetic fields. Also later, in teaching the model of the electromagnetic plane wave, the symmetry connections between the electric and magnetic fields need to be emphasized.

Following this approach, the authors also suggest that the Ampère–Maxwell's law should be highlighted as another part of the symmetry connection between the electric and magnetic fields. To achieve this, the charging process of a plate capacitor may provide an appropriate example to emphasize that a changing electric field generates a magnetic field, and that no charges are needed for the interaction between electric and magnetic fields. Thus, this particular example could also emphasize the relevance of field-based thinking in electromagnetism.

In the same study, the authors also suggest to highlight the relationship between the fields and their sources in the dynamic context, thus going beyond the traditional approach by discussing fields' generation only in the static case. In particular, the charge moving in uniform motion may be an appropriate example for elaborating the interrelations between the electric and magnetic fields, despite they observe that this approach is not usually included in textbooks. Following this line, it may also be useful to explicitly show students how an accelerated charge produces an electromagnetic wave, that is independent of its source and moving at the speed of light: we will deepen this aspect in our teaching proposal.

It is important to underline that, regarding this aspect, different approaches are reported in literature. Indeed, while Kesonen and colleagues (2011) recommended – in the context of moving point charges – to define the magnetic field starting from the changing electric field, Suárez and colleagues (2022) highlight that instruction should include some examples' discussion allowing the recognition that considering the electric and magnetic fields as disjoint (e.g. cause-effect) entities leads to contradictions. For instance, if we keep the context of a point charge moving in uniform motion ($\vec{v}$ representing its velocity), the magnetic field in a point whose position in relation to the charge is represented by the vector $\vec{r}$, is given by the expression: $\vec{B} = \frac{\mu_0 q \vec{v} \times \vec{r}}{4\pi r^3}$. It could be derived both from the Biot-Savart law for a field generated by a generic current distribution, or from the Ampère-Maxwell law. If we assign a cause-effect meaning to both the relationships and we analyze the situation applying the former, we would derive that, because the point charge is in motion, it generates a magnetic field around it. Nevertheless, if we apply the latter, we would conclude that the time varying electric field is the cause of the magnetic field. If both interpretations were valid, we would have to consider both contributions for the calculation of the magnetic field, that of course would lead to an incorrect result.

The authors also show that a similar discussion occurs for a magnet moving in uniform motion with respect to an inertial frame of reference R. In this case we could apply both the Faraday's law and the Ampère-Maxwell's law to the same closed curve, asserting that the variations of one field have generated the other, depending on the equation we are analyzing. However, both causal relationships between the fields cannot be valid simultaneously, as both fields are generated at the same time and they share the same cause. Indeed, if we consider the reference system where it is at rest (R'), only the magnetic field will be detected, while a simple change in the reference system would enable us to observe both the fields. It is possible to determine the relationship between the fields in R and R' with the Lorentz transformations. Indeed, using these equations, we can derive the relationship between the two fields in the system R, that is $\vec{\mathrm{E}} = \vec{\mathrm{B}} \times \vec{\mathrm{v}}$ (as $\vec{\mathrm{v}}$ represents the velocity of the magnet). An analysis of this equation allows us to recognize the absence of cause-effect relationships between its different terms: a moving magnet generates both an electric field and a magnetic field, related by this equation. They are actually components of the same entity – the electromagnetic field – and, "as it makes no sense to think that a component of one vector is the cause of another, neither does it make sense to think that a component of the electromagnetic field be the cause of the other".

There is an important aspect of this discussion that should not be underestimated: these examples require a quite advanced mathematical formalism, so that they could not be discussed in depth with high school students. We will come back later on this aspect, trying to adopt another point of view and developing a qualitative discussion. At the moment, let's just remark that some physics curricula (as in Italy) include the possibility to introduce students to the special relativity theory through the Lorentz transformations (or their weak relativistic approximation): in this case, the last examples of a magnet moving in uniform motion may be included in teaching with some variations on the theme, showing that these fields are mathematically related by these transformations, with no cause-effect relationships between them.

5.3 The unified approach for the electromagnetic field

Finally, Suárez and colleagues (2022) discuss a more advanced example related to electromagnetic induction. When dealing with this kind of problems in introductory physics courses, electric field functions are generally determined using Faraday's law, without checking or discussing whether the solution also verifies the Ampère-Maxwell's law. This could lead students to believe that the solutions of the fields only have to verify one of the Maxwell's equations. In order to show that this is not the case, we can consider a circular solenoid whose current varies with time. To determine the electric field at a distance smaller than the radius of the solenoid, we can apply Faraday's law to a closed curve C_1, assuming that the magnetic field inside the solenoid is uniform. The expression obtained is strictly valid in situations where the current is constant or increases linearly with time: if we apply the Ampère-Maxwell's law to the electric field obtained through the Faraday's law, we can easily understand its validity framework and reveal the way in which Maxwell's equations are linked. Indeed, in the case in which the magnetic field is uniform, the circulation along a closed curve C_2 should vanish. Thus, since the calculations show that the magnetic field circulation is directly proportional to the second time derivative of the current, the only way that the electric field obtained through Faraday's law is a solution of Maxwell's equations is that the current intensity linearly changes over time. When assuming this, the rate of change of the electric field flux becomes zero, decoupling Faraday's and Ampère-Maxwell's laws, which allows us to determine the electric field through the application of Faraday's law alone.

A unified treatment of the electromagnetic field would avoid conceptual confusions such as those just presented. For this purpose, it is essential to develop teaching-learning sequences allowing students to understand that Maxwell's equations do not provide information about the fields' sources; rather, they describe relationships between different quantities, determining their temporal evolution. This can be done not only in situations where time-varying electric and magnetic fields are present (as in the examples we discussed), but also when analyzing magnetostatic and electrostatic problems where fields' expressions are determined by Gauss' or Ampère's laws. Thus, emphasis should be placed on the fact that the fields must simultaneously verify all the Maxwell's equations, confirming that they form a system of coupled equations. Along these lines, both in static and dynamic contexts, efforts should be made to discuss problems where hypothetical configurations of electric and magnetic fields are analyzed to verify Maxwell's equations. Another possible approach to the Gauss' and Ampère's laws, which can be easily extended to the Ampère-Maxwell's law, is to analyze the field lines configurations for different distributions of charges and currents: these problems would foster students to reflect on fields' sources, avoiding the emergence of causal linear reasoning.

Moving the discussion about the interdependence of the electric and magnetic fields to the specific context of electromagnetic waves, Ambrose and colleagues (1999) discussed some tutorials' proposals aimed at overcoming the learning difficulties they have previously identified (see Chapter 4). Some of them are related to the electromagnetic waves' representations, thus we will discuss them in Chapter 7. They found that polarization might be a useful context for addressing this difficulty. Indeed, the students find that the orientation of the polarizing filter affects the amount of light transmitted; they account for their observations by assuming that the polarizing filter has a transmission axis, and that the component of the incident electric field that is parallel to this axis is transmitted. They also develop a general expression relating the ratio of the amplitudes of the transmitted and incident electric fields, to the angle between the incident electric field and the transmission axis of the polarizer. In the second version of the tutorial, the students are also asked to consider the incident and transmitted magnetic fields. They find that the amplitudes of the electric and magnetic fields are reduced by the same factor. Thus, beyond inferring that neither field can be transmitted alone, they can also experience an activity fostering the development of the model of light as an electromagnetic wave.

So far, we have discussed some proposals about how to deal with the interrelationships between the electric and magnetic fields: the following step is to deal with the propagation of the single entity they form together. Despite this topic may be not considered mandatory in the physics curricula in many countries for introductory students, Zuza and colleagues (2018) suggest that the discussion of the electromagnetic field's propagation would provide "an important stepping stone towards understanding electromagnetic radiation, electro-optical and magnetooptical effects, and special relativity later in the physics curriculum". We that this remark could also be extended to secondary education (at least in some cases), as we will discuss in developing the learning path.

Chapter 6

Textbooks' analysis

In this chapter we are going to illustrate an analysis of the most widespread high school Italian textbooks presenting electromagnetism: in particular, we will focus our analysis on the chapters dedicated to electromagnetic waves. As this part of the research was carried out in collaboration with the research group in physics education at the University of Buenos Aires, we are also going to analyze a sample of Argentine textbooks, in order to compare the Italian results with a country where electromagnetic waves are not included as a mandatory topic in the physics curriculum for high school students.

As we mentioned in a previous chapter, the textbook analysis represents an important step in the framework we adopted, in order to take into account the teachers' view and trying to relate the learning difficulties we identified through the literature review to some sort of missing pieces in traditional teaching: thus, it will represent a fundamental source informing the development of the learning path. In this chapter we are going to present some important results provided by literature from textbooks' analysis in the context of electromagnetism, then the theoretical framework which our analysis is carried out according to will be illustrated. The results will be reported and discussed according to the specific research questions which guided this part of the research.

6.1 Literature review

The results of the Physics Education Research (PER) about students learning difficulties should represent – especially at the upper secondary level – an important source of information for developing textbooks helping students to overcome them. As Van Kampen and De Cock (2023) observed, the research about teaching and learning electricity and magnetism has not had the same impact as that on mechanics. For example, the Force Concept Inventory is quite known outside the PER community, and the corresponding diagnostic tests CSEM and BEMA (see the discussion in Chapter 4) much less so. Furthermore, there is some evidence that some findings on the teaching and learning of mechanics have affected some changes in mainstream textbook presentations. Apart from electric circuits, where qualitative bulbs and battery questions are now part and parcel of textbook treatment, large differences between today's textbooks and those from the 1970s cannot be detected.

For this reason, some of the learning difficulties' sources which literature has reported about electromagnetism can be detected in the most widespread textbooks. For instance, Bagno and Eylon (1997) found that most students' perceive the electric field as a mathematical artifact because it is merely presented as a force applier. Also, the problems usually deal with static situations such as: "Four charges are fixed in the four corners of a rectangle; find the resultant electric field", and they do not illuminate the dynamic nature of the electric field. Even in problems dealing with charged particles entering a region with a constant electric field, students are never asked to find the new field, while they are usually asked about the path of the particle, its velocity or energy, and so on.

Also the organization of the content knowledge structure is a fundamental aspect – as we discussed in Chapter 4 – for the process of information retrieval in problem solving. Although some of the textbooks attempt to locally organize the information (within a single chapter) by providing a summary or a table, there are no comprehensive attempts to organize the information at a global level. This can explain the difficulty of students in producing a global view of the information by themselves, since there is no conceptual structure to support their retrieval. For instance, in most textbooks Ohm's law is central, either in the presentation of theory or in the exercises: thus, it is not surprising that Ohm's law is considered by students to be a fundamental relationship in electromagnetism, at the same level of Maxwell's equations (Bagno & Eylon, 1997).

These equations involve fields, thus their treatment is influenced by how textbooks deal with the concept of field itself. Zuza and colleagues (2018) point out that usually the electric field is used to define and calculate the electric potential, sometimes in parallel with using the Coulomb force law to calculate the electric potential energy, reinforcing the idea of force per unit charge exerted on a test charge. Then, when introducing Gauss's law, the electric field is thought of as permeating space; suddenly, test charges and forces are no longer involved in the discussion. In this way the textbooks' approach to the development of the theory of electric interactions superficially aligns with its historical development, but it lacks of conceptual resources that may help students familiarize with these new concepts. The situation is less ambiguous for magnetic interactions, where phenomenology is the starting point too, but the asymmetry between the source (magnets or currents) and the object experiencing the source (compass needles) often results in a hybrid of force-based and source-based conceptualizations.

About the fundamental relationships involving fields, several remarks may be pointed out. The textbooks usually emphasize the idea that a change in magnetic field is related to the production of an electric field, while the idea of an induced electromotive force is emphasized (Guisasola et al., 2013). At the same time, the induced magnetic field is presented as an element of a complicated integral, while Lenz's law is usually presented in a concrete way (Suárez et al., 2023): the only excuse for the existence of this magnetic field seems to be the need for completeness in Maxwell's equations.

In Chapter 4 we also observed that many textbooks do not properly describe the (supposed) cause-effect relationship involving the electric and magnetic fields. About this aspect, Kesonen and colleagues (2011) pointed out that some over-idealizations – which can cause specific learning difficulties – such as the equivalence between the charges moving in uniform motion and the direct currents, were found in the standard introductory and upper secondary school textbooks when discussing the Biot-Savart law, which describes the magnetic field generated by a moving charge. Although textbooks often emphasize that a charge is also the source of an electric field, there are still many figures where only the magnetic field is shown when a charge is moving uniformly.

Of course, the textbooks usually report a lot of images and representations about the topic of the discussion. In the context of electromagnetic waves, these aspects deserve a specific discussion – as these kind of representations hide a lot of implicit information – so we will postpone these remarks to the following chapter.

6.2 Theoretical framework

The analysis of the content knowledge presented in school science textbooks is essential to understand how scientific concepts are communicated to learners, and how they align with educational goals. Several theoretical frameworks have been employed in science education research to evaluate the textbooks' content (Chiappetta and Fillman, 2007). These frameworks provide a structured approach to examine the representation of scientific knowledge, its pedagogical appropriateness, and its alignment with curriculum standards and scientific accuracy. The choice of the theoretical framework often depends on the research objectives and the specific aspects of textbooks' content being analyzed. A combination of frameworks may be employed to provide a more comprehensive analysis, capturing both the cognitive and contextual dimensions of science education. By leveraging these theoretical approaches, researchers can identify strengths and gaps in textbooks, ultimately informing the curriculum development and improving science education practices.

The theoretical framework we adopted follows the indication of many papers about the analysis of textbooks in science education (Niaz & Fernández, 2008) when dealing with a specific theory – in our case the classical theory of the time-dependent electromagnetic field – namely, when the purpose of the study is not to analyze a text as a whole. In these cases, the analysis is split according to the definition of some criteria, which summarize the content knowledge of the theory we are analyzing. Each criterion is linked to some specific indicators, which represent the main conceptual issues related to the criteria. In Table 6.1 the criteria and a synthetic description of the indicators we used in the analysis are reported: we will go into detail of their meaning in the next sections. In particular, the four criteria represent the phases of the wave phenomena (generation, propagation and reception), and a specific section about the representations was included as literature shows how they are crucial for developing the conceptual understanding of electromagnetic waves (Ambrose et al., 1999).

The analysis will be carried out according to a three-tiers scale as reported by Niaz and Fernández (2008), where the levels are defined as follows:

1. *Satisfactory (S)*: the treatment of the concept is considered satisfactory for the set of indicators, and the educational implications are also drawn.
2. *Mention (M)*: some aspects of the criterion are mentioned without any explicit elaboration.
3. *No mention (N)*: the issues involved in the criterion are not mentioned at all.

Following this analysis, each criterion may be evaluated according to the same scale, depending on which level is prevalent among its indicators.

Table 6.1. Definitions of the criteria and indicators we used in the analysis of textbooks presenting electromagnetic (EM) waves. Each criterion will be classified according to the three-tiers scale depending on which level is prevalent among the indicators.

Criteria	Indicators
Sources of electromagnetic waves	• Microscopic sources EM waves as accelerated charges. • Characterization of the macroscopic sources EM waves. • Quantities characterizing the sources of EM waves. • Quantities characterizing the EM waves which depend on their sources.
Propagation of electromagnetic waves	• Propagation in vacuum. • Propagation in matter.

	• Relationships between the propagation velocity and other quantities.
Interaction of electromagnetic waves with matter	• Mechanism of reception of EM waves based on the Lorentz force or radiation pressure. • Description of the electromagnetic spectrum.
Representations of electromagnetic waves	• Different kinds of representations for different phenomena.

6.3 Research questions

The aim of this study is to analyze how the content knowledge about the classical theory of the time-dependent electromagnetic field is reported in school textbooks. Following the definition of the criteria we provided in the previous section, the specific research questions that guided our study are the following:

1. How do textbooks explain the relationship between electromagnetic waves and their sources?
2. How do textbooks explain the propagation of electromagnetic waves in empty space and in matter?
3. How do textbooks explain the interaction of electromagnetic waves with matter?
4. What kind of representations do textbooks use for electromagnetic waves?

These research questions cover a large part of the electromagnetic field theory, and almost all the aspects that are usually reported in the school textbooks.

6.4 Methods

We carried out the analysis with a sample of 18 textbooks, which represent the most frequently used resources for teaching electromagnetism in Italian and Argentine schools, according to the education ministry databases of both countries. This approach facilitates a comparative evaluation of the pedagogical methodologies adopted in these two nations, according to the selection methods most diffused in literature (Suárez et al., 2023).

Actually, some details are needed to specify the sample selection method. Indeed, Argentina is a federal country, and some provinces' curricula do not include a mandatory physics teaching. On the other hand, in Italy physics is included as a mandatory subject in almost all kind of high school curricula, and electromagnetic waves are explicitly mentioned in the national indications for teaching. Nevertheless, in Italy there is not a database indicating the most diffused textbooks adopted in high schools. Thus, we included in the sample the textbooks that our experience with teachers suggest to be the most diffused. In Figure 6.1 we report the complete sample of textbooks, both Italians and Argentines.

In order to answer the research questions, we used the rubric previously defined. The indicators also correspond to the learning objectives the students should achieve according to us. The criteria are reported in Table 6.2 and Table 6.3 with the corresponding results, according to the scale we defined in the theoretical framework.

6.5 Results

In this section we are going to discuss the results reported in Table 6.2 (Italian textbooks) and in Table 6.3 (Argentine textbooks) for each research question. At this stage we are going to consider the Italian and Argentine textbooks as a unique sample, while any significant differences according to the criteria we defined will be remarked at the end of the chapter. We purposely chose the same number of Italian and Argentine textbooks to foster this comparison. Furthermore, we observe that this set of research questions almost cover all the content knowledge presented by school textbooks about electromagnetic waves, thus providing confirmation of the epistemic reconstruction which led us to establish the analysis criteria.

RQ1: How do textbooks explain the relationship between electromagnetic waves and their sources?

11 out of 18 textbooks state that accelerated charges are the only sources of electromagnetic waves, while the others do not specify anything about the motion of the generating charges. Only 2 textbooks provide an explanation about why a charge in uniform motion cannot generate an electromagnetic wave. These 2 textbooks also relate the characteristics of the wave to those of the sources. The textbooks mostly provide examples about the oscillating dipole – e.g. reporting its electromagnetic fields lines – but at the same time they do not add information about the specificity of this case. Furthermore, the textbooks generally do not provide examples for macroscopic sources, but they usually just mention different sources for different parts of the electromagnetic spectrum.

13 out of 18 textbooks do not report the definition of the quantities characterizing the sources of electromagnetic waves – such as charge density and current density – that are generally defined in relation to static fields. This implies that there is a missing –

also at a qualitative level – link between the characteristics of the propagating field and the system of Maxwell equations, which explains the mechanism of propagation.

N°	Book	Total number of pages in the book	Total number of pages dedicated to OEMs	Total number of figures dedicated to OEMs	N°	Book	Total number of pages in the book	Total number of pages dedicated to OEMs	Total number of figures dedicated to OEMs
1	Il nuovo Amaldi per i licei scientifici.blu	309	60	50	1	Física y química I _ Serie nuevas miradas - Tinta fresca	255	53	107
2	La fisica di Cutnell e Johnson	213	34	79	2	Física _ Serie nuevas miradas - Tinta fresca	237	27	319
3	Le traiettorie della fisica	587	42	71	3	Physics – K.A. Tsokos – Seventh Edition – Cambridge 2023 (Digital	551	93	175
4	Fisica, Lezioni e problemi	574	29	67	4	Physics – Raymond A. Serway and Chris Vuille – Ninth Edition – Cengage 2012	982	298	156
5	Fisica, teorie, esperimenti	399	52	44	5	Physics – Allum and Morris – Hodder 2023	604	163	168
6	Quantum Green	535	42	55	6	Physics – Study Guide – Tim Kirk – Fourth Edition – Oxford 2023	205	40	101
7	Il Walker	358	34	47	7	Physics SL – Chris Hamper – Third Edition – Pearson 2023	408	117	126
8	Le risposte della fisica	227	36	49	8	Física – John Allum y Christopher Talbot – Segunda Edición – Vinces Vices 2015	564	169	272
9	Imago	375	40	43	9	Physics – Gregg Kerr and Paul Ruth – Third Edition – IBID 2008	572	76	163

Figure 6.1: List of Italian (on the left) and Argentine (on the right) textbooks we included in the sample. For each textbook the total number of pages, the number of pages dedicated to EM waves and the number of representations are reported in the following three columns.

RQ2: How do textbooks explain the propagation of electromagnetic waves in vacuum and in matter?

Half of the textbooks make explicit that electromagnetic waves do not propagate instantly, even though they do not report any example about how the electromagnetic field's propagation takes place. Indeed, an illustration would help students to understand how they delay effect – i.e. the fact that, if a variation in the characteristics of the source occurs, it will cause a variation of the values that the electric and magnetic fields take on in a point of the space after a well-defined time interval – that may represent a learning obstacle according to the literature about learning difficulties (Rettob et al., 2021; Suárez et al., 2024). Furthermore, the discussion of textbooks usually takes place in vacuum, thus they do not characterize the velocity as dependent from the propagation medium. This consideration is usually postponed to the study of optics, where the index refraction is defined, but it is not reported for the whole electromagnetic spectrum. In particular, there is only one textbook providing an explanation about the reason why the velocity of electromagnetic waves in any propagation medium is smaller than in vacuum. In the second case, the value is reported (c ≅ 300.000 km/s) without any reference to the limit it represents in physics. On the other hand, 4 textbooks state the velocity of electromagnetic waves – both in vacuum and in matter – does not depend on the motion of the source and on the motion of the observer – that is, on the relative motion between the observer and the source – but they do not make explicit that this aspect is not consistent with the classical mechanics.

About the relative motions we also observe that 9 textbooks report – without explanation, which sometimes is included in the chapter about mechanical waves – the formula related to the variation in the frequence due to the Doppler effect, thus providing further proof that we are dealing with wave phenomena. As a consequence of what we reported above, there is almost no evidence about the difference in propagation when the medium is a conductor or a dielectric. Indeed, electromagnetic waves are mostly reflected or absorbed by conductors, while they propagate through dielectric. This is a key aspect in the Hertz's experiment, and also represent another evidence of the wave characteristics of the phenomenon.

Finally, none of the textbooks reports that also the direction of the velocity of electromagnetic waves changes when the medium is changing. This is an important aspect, as it is linked to the transversal character of the waves, that 15 textbooks present by introducing the vector relationship between the electric field, the magnetic field and the propagation velocity. Nevertheless, in all cases no reason for this feature is provided, and also the correlation between the field's intensity emerging from the same relationships is not emphasized.

Table 6.2: Evaluation of the criteria used in the analysis of Italian textbooks. More details are provided in the discussion of the research questions.

Italian Textbooks	**Criterion 1: Sources**	**Criterion 2: Propagation**	**Criterion 3: Interaction**	**Criterion 4: Representations**
Il nuovo Amaldi per i licei scientifici.blu – terza edizione – Zanichelli	M	S	N	N
La fisica di Cutnell e Johnson – Zanichelli 2020	S	S	S	M
Le traiettorie della fisica – terza edizione – Zanichelli	S	M	N	N
Fisica, Lezioni e problemi – seconda edizione – Zanichelli	M	S	M	N

Fisica, teorie, esperimenti – SEI 2019	S	S	N	N
Quantum Green – SEI 2022	M	S	M	N
Il Walker – Pearson 2021	M	M	M	N
Le risposte della fisica – Le Monnier Scuola 2017	N	M	N	N
Imago – Mondadori Education 2023	M	N	N	N

Table 6.3: Evaluation of the criteria used in the analysis of Argentine textbooks.

Argentine Textbooks	**Criterion 1: Sources**	**Criterion 2: Propagation**	**Criterion 3: Interaction**	**Criterion 4: Representations**
Física y química I – Serie nuevas miradas – Tinta fresca	N	M	N	N
Física – Serie nuevas miradas – Tinta fresca	N	M	M	N
Physics – Seventh Edition – Cambridge 2023 (Digital Edition)	M	M	M	N
Physics – Ninth Edition – Cengage 2012	S	S	N	N
Physics – Hodder 2023	M	M	N	N
Physics - Study Guide – Oxford 2023	M	M	M	N
Physics SL – Pearson 2023	M	N	N	N
Física – Segunda Edición – Vinces Vices 2015	M	S	N	N
Physics – Third Edition – IBID 2008	M	M	M	N

RQ3: How do textbooks explain the interaction of electromagnetic waves with matter?

4 out of 18 textbooks provide an explanation of the interaction mechanism between electromagnetic waves and matter that can be considered complete at this level. Indeed, they report the typical example of an antenna receiving a signal: in this case the antenna is schematized as a long wire where free charges can move after interacting with the signal, according to the Lorentz force. This means that the magnetic field produces a force perpendicular to the wire – i.e. the direction along which the charges can move – so it can always be neglected. On the contrary, the electric field produces a force in the direction along which it is oscillating – that is fixed in the case of linearly polarized waves to which textbooks refer. As a consequence, the direction in which the antenna should be oriented to have the best signal's reception is that of polarization of the incident wave. In the other textbooks there is almost no mention of any kind of interaction between electromagnetic waves and matter, except for the concept of

radiation pressure. Indeed, half of the textbooks report the definition of this quantity, together with the reason why it is necessary to define it in describing the interaction with matter.

Only 2 textbooks made explicit – providing an explanation – which quantities are changing when changing the propagation medium. In doing so, they both consider the energetic description of electromagnetic waves, and the description of their propagation over space and time (in terms of wavelength, frequency and velocity).

A specific aspect that may be considered in answering this research question is also how textbooks introduce the electromagnetic spectrum. It is usually presented as a set of frequencies (or wavelengths) of electromagnetic waves, whose boundaries between the different regions – radio waves, microwaves, infrared, visible light, ultraviolet, X rays, Gamma rays – are not clearly defined, as they also depend on the kind of interaction occurring with matter. There is a large variety of phenomenological examples that are reported to characterize experimentally the regions of the spectrum, thus not allowing for any classification. One remarkable aspect is that these examples do not relate the regions with different kind of sources, but they usually report some of their characterizing properties (e.g. the penetration depth).

In many textbooks a specific chapter is dedicated to the light, where the phenomena of interference, diffraction and polarization are almost always reported (in 16 out of 18 cases). It may also be interpreted as a limitation of the textbooks, as they do not discuss the same phenomena for the other regions of the spectrum. Furthermore, in introducing the spectrum, the textbooks usually present the variety of applications of electromagnetic waves, from the communication systems to the biomedical applications.

RQ4: What kind of representations do textbooks use for electromagnetic waves?

The most used representations in all textbooks are those of wave profiles over space and time (see the representation reported in Figure 8.1 and the related discussion in Chapter 7). Nevertheless, they represent an important learning obstacle when representing plane and monochromatic waves, as literature show (Ambrose et al., 1999). On the contrary, other kinds of representations – such as wave fronts and field lines – are not very diffused. A remarkable aspect is that no difference emerge between the representations used to illustrate different kinds of phenomena – such as free propagation in vacuum or in other media, interference between weaves, diffraction through obstacles or polarization through filters. Thus, we observe a lack of the functional role of representations, that are often also disconnected from the text.

Interestingly, that related to representations is the only criterion for which we observe no significant differences between the Italian and Argentine textbooks. We did not use a specific measure as the sample is limited, but the ratings reported in Table 6.2 and Table 6.3 in this case are almost the same. On the contrary, considering the previous criteria, the Italian textbooks generally reported a greater frequency of the "satisfactory" evaluation.

6.6 Discussion

In this section we are going to discuss the results we got for each research question, which also provide some teaching implications.

RQ1: How do textbooks explain the relationship between electromagnetic waves and their sources?

The first aspect emerging from the analysis is related to the characterization of accelerated charges as the only microscopic sources of electromagnetic waves. The 2 textbooks that we mentioned provide an explanation based on the fact that, in order to generate an oscillating radiation, the source has to be an oscillating charge. Thus, the link between the wave and the source is not made explicit, as there is not a direct explanation of the equality between their frequencies. Furthermore, the difference between the quantities which directly depend on the source and those dependent on the propagation medium is not reported.

Since electromagnetic waves are not explicitly defined as a periodic variation of the fields, the students may interpret them as a generic case in which the electric and magnetic fields are time-dependent. Following this interpretation, also a single charge in uniform motion is able to generate non-static fields (even though not in all inertial systems of reference). This aspect is not clear in any textbooks, and it may produce a fundamental misinterpretation.

The missing link between electromagnetic waves and their sources is also evident by the fact that the case of plane wave fronts is introduced when the sources are absent (namely, when the propagating radiation is far away from them). This may excessively simplify the concept of independence from the source: there is a cause-effect relationship that is not made explicit, even though the radiation may continue its propagation also when the source is removed from the space.

RQ2: How do textbooks explain the propagation of electromagnetic waves in vacuum and in matter?

The reason why the waves' velocity changes when changing the medium can be identified in the refraction phenomenon, which students already know from geometric optics. Thus, the phenomenology should just be extended to the whole spectrum, also providing a confirmation that the velocity in vacuum reaches its maximum value. On the other hand, a parallelism with the concept of velocity in classical mechanics is ambiguous: as we reported in the results' section, the textbooks we analyzed do not emphasize that the independence of the propagation velocity on the relative motion between the observer and the source is actually a postulate of the special relativity theory, and it is incompatible with classical mechanics.

On the contrary, the aspects which characterize the wave behavior of the electromagnetic radiation – such as interference, diffraction, reflection, polarization – are sometimes reported. Furthermore, the fundamental relationship $\vec{E} = \vec{B} \times \vec{v}$ is introduced to characterize the transversal nature of waves, but the correlation between the fields' amplitudes is not emphasized. Thus, the students cannot recognize that the fields propagate in phase, that is a fundamental aspect in understanding how the values of the radiation change over space and time.

Finally, only one textbook mention the "luminiferous ether" hypothesis, which plays an important role in the historical and conceptual development of the electromagnetic field theory. Indeed, this discussion would make explicit that electromagnetic waves, differently from the mechanical ones, can propagate in vacuum. Furthermore, no textbooks make any mention of the packets of electromagnetic waves, namely about the difference between phase and group velocity (Buongiorno et al., 2021).

RQ3: How do textbooks explain the interaction of electromagnetic waves with matter?

The few textbooks dealing with the reception of electromagnetic waves focus on the interaction based on the Lorentz force, at least in the case of linearly polarized waves. This aspect is neglected by many other textbooks, but it is at the base of the operating mechanism of any technical application. The definition of the radiation pressure is more diffused, in order to underline that electromagnetic radiation also carries a specific momentum, which result in a pressure whose amount depends on the behaviour of the radiation (namely, if it is absorbed or reflected by matter).

Another remarkable lack in the textbooks we analyzed is the discussion about which among the characterizing quantities of the waves change when changing the propagation medium. The students should already know the definitions of these quantities from the phenomena involving mechanical waves, but the relationships between them may vary in this case. The textbooks make explicit that the propagation velocity (v) depends on the medium, and also that it is equivalent to the product of the wavelength (λ) and the oscillation frequency (ν): $v = \lambda\nu$. It may be important to remark that this formula does not directly imply that the wavelength and the frequency have to change too. Indeed, the oscillation frequency of – any – periodic wave only depends on the generating source, thus it does not change when changing the characteristics in the propagation. On the contrary, the variation in the velocity is related to the variation of the wavelength, which depends on the density of the medium. This consideration has an important implication: the colour of the visible light does not change when changing the medium, which is not a trivial aspect if we consider that students often relate the colour both to wavelength and frequency. Indeed, in most of the textbooks the regions of the electromagnetic spectrum are characterized both in terms of wavelength and frequency – since they usually refer to the propagation in vacuum – thus fostering a possible misconception for students.

Visible light is in all cases the band which the most extensive discussion is dedicated to. This may not make students aware of the fact that, regardless of their frequency, the electromagnetic waves experience the same phenomena – such as interference, diffraction, polarization – and the same characteristics regarding the propagation. In this case, a parallelism with the sound spectrum would help, as also in that case there is a central region which can be perceived by our senses, even though the whole set of frequencies may experience the same phenomena.

Nevertheless, there are some quantities which depend on the oscillation frequency. For instance, we reported that some textbooks declare that the energy carried out by the high frequency radiation is greater than that of the low frequency radiation: thus, for instance, X rays are able to pass through soft tissues and be absorbed by human bones – so we use them for radiography – or Gamma rays are able to damage the DNA of tumour cells – and for this reason they are considered as ionizing radiation. Actually, according to the classical theory of the electromagnetic field presented by school textbooks, the intensity of the waves is independent of their frequency: the direct proportionality between these quantities only emerges in the context of quantum theory with the photoelectric effect and the Planck relation. The lack of this consideration may foster a students' misconception, and also does not clarify the reason why the low frequency radiation may usually be considered as not harmful for human health.

RQ4: What kind of representations do textbooks use for electromagnetic waves?

In all textbooks the most diffused formal representations of electromagnetic waves are the wave profiles over space and time. Indeed, dealing with monochromatic and plane symmetric waves, this kind of representation is particularly powerful to summarize the information of the propagating fields. On the other hand, since the learning difficulties in their interpretation – as for other three-dimensional representations – is strongly supported by literature (Ambrose et al., 1999), the clarification of the implicit information related to the simmetry would help students in the interpretation of the related phenomena.

Furthermore, using different kinds of representations – e.g. wave fronts or field lines – may help students in relating and exptrapolating more information, and also in representing different phenomena – namely, different kinds of the radiation-matter interaction – with different representations.

Overall, this discussion summarizes some of the conceptual issues on which a teaching proposal may be based, aimed at consolidating the aspects deemed as missing from the analysis of the school textbooks.

Chapter 7

Formalisms in teaching physics

In this chapter two very relevant types of formalisms in teaching physics will be discussed: the mathematical formalism and the representations. Indeed, they can hinder or help students in developing conceptual understanding, depending on the cases and on the difficulty of their interpretations. As we are dealing with electromagnetic waves, these aspects play a crucial role: for instance, it is almost the first time that students will deal with three-dimensional representations – the plane wave profile over space or time – and with a system of differential coupled equations. Of course, as we are addressing secondary education, in the second case it will be essential to simplify the formalism, for example by presenting the Maxwell's equations in their integral form or using finite time intervals rather than time derivatives: however, the students' learning difficulties in dealing with coupled equations where some fields' operators – flux and circulation – play a role still occur.

We will deal with these aspects separately in the following two paragraphs, reporting a brief literature review, analyzing – in general and in the electromagnetic context – the learning difficulties related to the use of these instruments and some teaching proposals to overcome them.

7.1 Representations

The formal representations of a phenomenon are very important in physics, as students can extract different information from different kind of representations, in order to build up some mental models. Thus, one fundamental and transversal skill is to be able to implement transitions between different representations of the same phenomenon. This skill is strongly related to spatial ability (Kozhevnikov et al., 2007) and it can be improved with specific tasks depending on the physical context students are dealing with. For instance, when dealing with kinematics, the students can implement transitions from graphs, tables or equations representing the same motion (Ceuppens et al., 2018) in order to develop a more complete picture of the phenomenon. Literature show is desirable to foster this ability when students are very young (starting from secondary education). Of course, it requires quite simple contexts, for example dealing just with uniform motions (Planinic et al., 2012). In more complicated cases, the task can be limited to the recognition of the correct series of multiple representations (Susac et al., 2018), rather than implementing the transition by themselves. Furthermore, the transition can also be implemented between different aspect of the same phenomenon:

for example, the same motion can be illustrated in relation to the time trend of the position, velocity or acceleration.

In the case of electromagnetic waves, we will limit our analysis to the case of plane, monochromatic and linearly polarized waves, as it would allow a generalization later. We chose to deal with three different kind of representations: the wave profile over space or time (Figure 7.1), the wave front (Figure 7.2) and the field lines (Figure 7.3). The last is widespread especially in the static context, but in some specific cases it is also used to represent time-dependent fields (usually representing only the electric or magnetic field). All these formal representations are introduced to students in the context of electromagnetism, that means they never met them before instruction. Indeed, it is difficult to compare the electromagnetic wave profiles with those of mechanical waves: in the last case, the representation is not three-dimensional, and (above all) the oscillating quantities is a scalar, corresponding to something students can visualize (for example a wave propagating on a string) rather than an abstract vector (the electric or magnetic field). Nevertheless, the students may be familiar with the definition of some quantities, such as wavelength and period, starting from the transverse wave profiles' representations (if they were previously introduced to mechanical waves, that is not always the case).

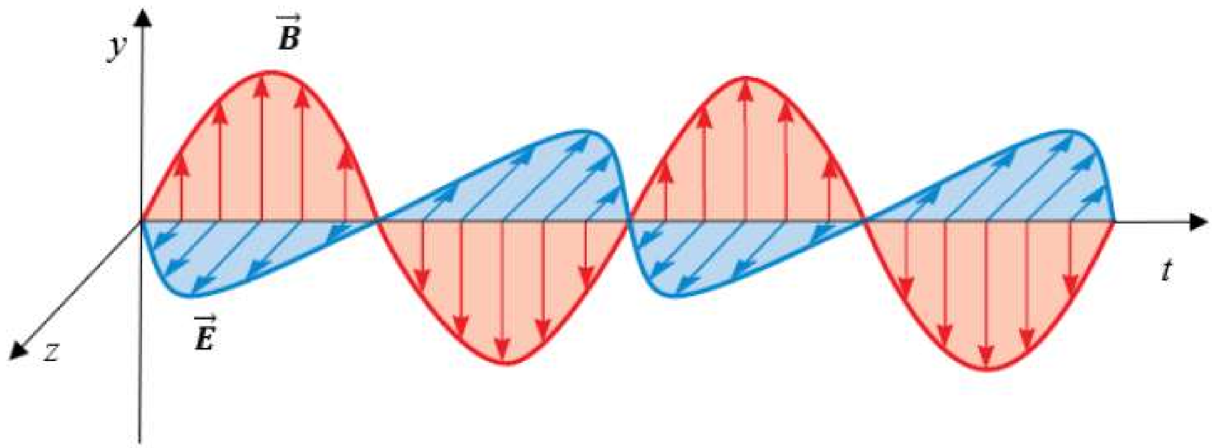

Figure 7.1: Typical (plane) electromagnetic wave profile representation over time (at a specific point). The same occurs if it is represented over space (at a specific instant). Sometimes the arrows representing the vector fields are not reported.

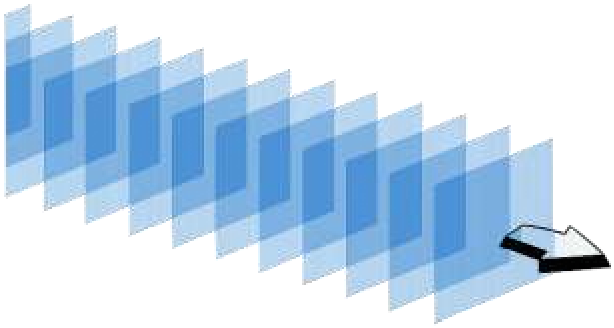

Figure 7.2: Simple representation of a (plane) wave front, without reporting any other detail. Sometimes the indication of the wavelength as the distance between two following wave fronts is reported.

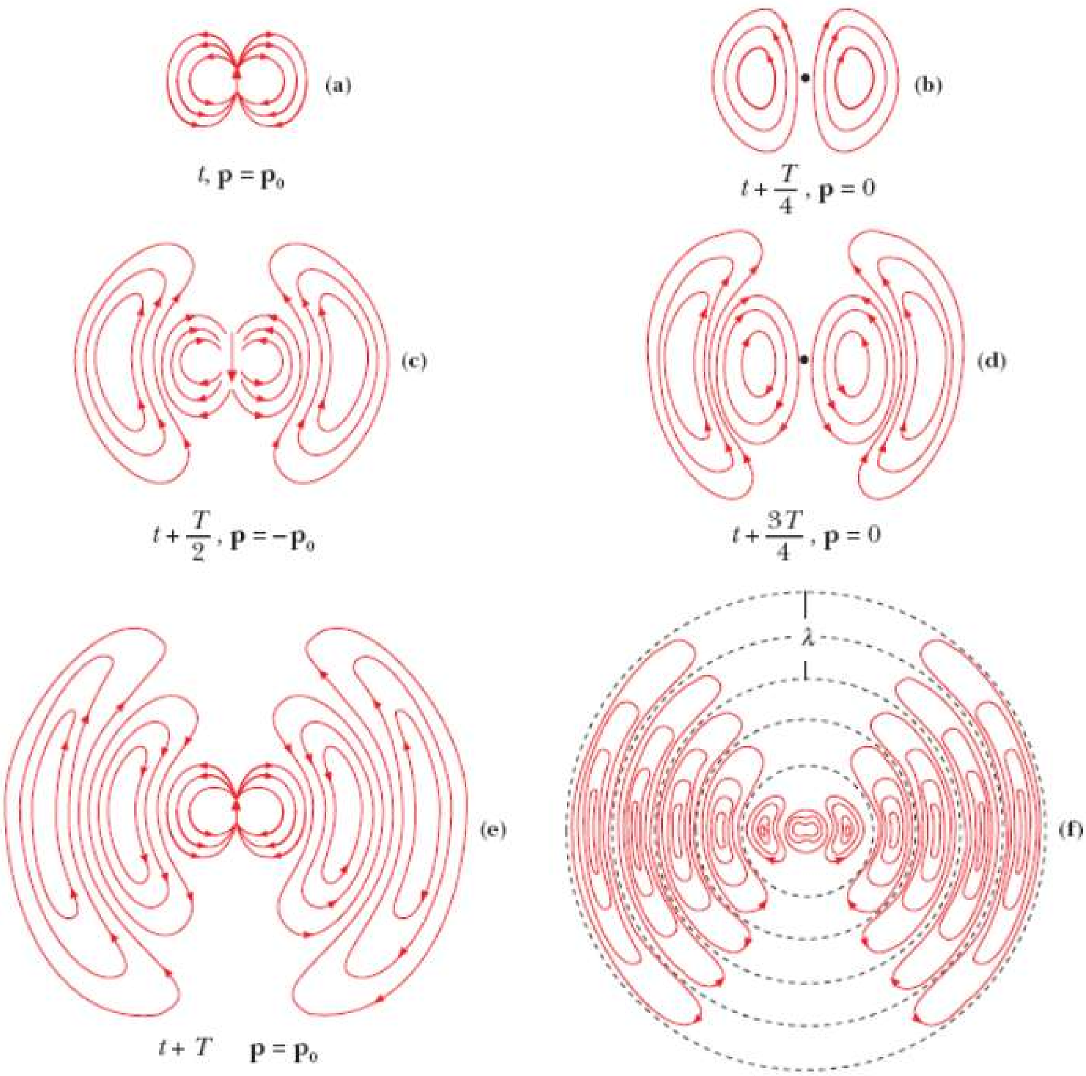

Figure 7.2: Schematic representation of the production and propagation, at different times, of the electric field lines produced by an oscillating electric dipole.

We are not going to consider other types of representations – such as the equipotential surfaces – as they are not usually widespread (and useful) in the electrodynamic context. Furthermore, also the algebraic formulas – for instance, the ones about the fields' expression in terms of time and position – may be considered as another kind of representation: we chose to not include them, as they are they are rarely used and superfluous for high school students, even though they may be introduced to familiarize with multivariable functions.

Turning to the analysis of learning difficulties related to the representations' interpretation, it is not easy to distinguish them from the conceptual difficulties, as the two are intertwined. In general, one option is to avoid possible complications by eliminating a particular representation from the introductory course. However, in the case of electromagnetic waves it is difficult to design a diagram that effectively demonstrates the variation in space and time of two propagating, oscillating, orthogonal vector fields. Thus, the most suitable approach is probably try to help students interpreting the existing formalism in such a way that their conceptual learning is enhanced. As Ambrose and colleagues (1999) pointed out, if such issues are not explicitly addressed, it may be impossible for students to relate the formalism to physical phenomena: helping students understand the representations should also help them to understand the concepts and to relate both the concepts and the representations to the real world. On the contrary, if an effort to help students understand the wave profile representation is not made, maybe the best choice is to not use it.

In the same study the authors elaborated tutorials and questions asking for students' interpretation of the wave profile representation over space or time. Sometimes the final aim of the question was to deal with the prediction of a phenomenon (e.g. polarization) or the application of a law (e.g. the Ampère-Maxwell's law): we will discuss the latter case later. Here we focus on the results' analysis of the items whose final aim was the representations' interpretation itself. When asking to rank the electric and magnetic fields' values at different points in space (compare with Figure 8.1), the students' performances do not report significant differences before or after traditional instruction: it means that it does address this crucial aspect. Only 10% of students in introductory courses provide the correct ranking. The most common detected error was to ascribe to the electromagnetic wave a finite spatial extent in the plane perpendicular to the direction of propagation. Many responses reflect the mistaken belief that the electric and magnetic fields are confined to the region within the sinusoidal curves. About 60% of the students state that the electric or magnetic field is zero at the point lying outside both sinusoidal curves. Students who treat the plane wave as having finite spatial extent often attribute special significance to the propagation axis shown in the diagram. About 35% have stated that the electric or magnetic fields are zero along this axis. Often these students think that the magnitudes of the electric and magnetic fields increase with distance from the propagation axis. Many of the errors related to the misinterpretation of the algebraic expressions seem to be related to a failure to distinguish between the *y* and *z* coordinates and the corresponding unit vectors. Other students have treated the magnitude as decreasing with distance from the x-axis. Some even seem to believe the fields are ‘‘emitted’’ from the x-axis. Some students who have correctly stated that the points have fields of equal magnitude – they all belong to a plane parallel to the wave front – have confused the vectors' and field-lines' representations. For example, one student wrote ‘‘Q=R=S because lines have the same spacing (the field is uniform below the curve). P=0 because there are no field lines above the curve.’’

Following these lines, in dealing with polarization there was a tendency to attribute a spatial extent to the amplitude of the wave. Some students drew diagrams of sinusoidal curves incident on a slit and based their reasoning on whether or not the light would fit through the slit. Some claimed that diffraction occurs only when the wavelength or amplitude is larger than the slit width because the light ‘‘has to bend in order to fit through’’ the narrow slit. These aspects clearly show that, at least in this context, learning difficulties may be the source of serious conceptual difficulties: this makes it even more urgent to address them with specific activities in teaching.

We expect that students' difficulties dealing with the wave fronts' representations are strongly related with those just discussed, and also with the identification of the wavelength and of the propagation direction. Unfortunately, in this context we did not find any research identifying these difficulties, probably because this kind of representation is not diffused in traditional teaching, as we found when analyzing textbooks in the previous chapter. On the contrary, a lot of research article focus on students' understanding of the fields' lines representations, both for the electric and magnetic fields and in the static and dynamic contexts.

Sağlam and Millar (2006) show that common misinterpretations include considering field lines as indicating a “flow”. Nevertheless, students' performances on the spatial rotation test were only weakly correlated with performances on the questions about electromagnetism, thus indicating that in this case the misinterpretation can be caused by traditional teaching – e.g. the simple analogy with fluids when describing field lines – or by students' spontaneous mental models.

These spontaneous ideas may include considering field lines as real physical entities, as detected by Scaife and Heckler (2010). They reported students' thinking about field lines as flowing objects. As a consequence, if a charge is drawn on a field line, it will experience a force; if it is drawn off the field line, it will not. Furthermore, it has been shown that when a particle is placed in either an electric or a magnetic field, students believe that its trajectory must follow the field lines (Tornkvist et al., 1993). Students have a tendency to assign a force parallel to the magnetic field lines, similar to the case with electric field lines (Maloney et al., 2001).

Similar results in the magnetic context were reported by Thong and Gunstone (2008). They mentioned some students reasoning considering field lines as real entities, that was expressed as a need for real contact of the field lines with the external coil for any electromotive force to be induced in the coil (the item's context was about electromagnetic induction). The same alternative conception has also been observed by Guisasola and colleagues (2004), where they identify some students' (about 15%) belief about the magnetic interaction which occurs as a consequence of the attraction and repulsion of the field lines.

Thus, although the same alternative conception were detected in different contexts, electromagnetic induction seem to be the one most suitable to elicit this kind of misinterpretation. As Jelicic and colleagues (2017) remarked, many introductory students explained induced current or electromotive force as being due to the presence of the (unchanging) magnetic field in the area considered. Students seemed to think that the field lines crossing the loop were the cause of the induced electromotive force: they confused field lines which cross the circuit with the variation in magnetic flux through the circuit.

Comparing the students' performances with field lines representation and more informal ones – such as the poles' representation in the case of a field generated by a bar magnet – Scaife and Heckler (2010) found that, despite students begin at the same low level of performance in both representations, they answer correctly more often in the field line representation than in the pole representation after instruction. This difference is due in part to more students believing that charges are attracted to magnetic poles than believing that charges are pushed along magnetic field lines. The authors also remarked that, although traditional instruction is fairly effective in teaching students to answer correctly up to a few weeks following instruction – especially for the field lines representation – some students revert to their initial reasoning several months after instruction.

When coming to the teaching proposals aimed at overcoming these learning difficulties or spontaneous ideas, Saarelainen and colleagues (2007) suggested that field lines' representation should include information about the vector they are referring to – such as the direction and strength – in addition to the dimension of the vector. For instance, this has been taken into account in the representations introduced by Chabay and Sherwood (2015). However, as we will deepen in the discussion of our teaching proposal, the crucial task may still be the recognition and conversion of several representations (Campos et al., 2023): in this context also small details seem to produce a significant effect on students' performances, such as the presence or absence of the field's source or using the vector plots coupled with the field lines.

7.2 Mathematical formalism

As detected by a well-established line of research, the introduction of formulas and other mathematical notations may impede, rather than promote, understanding of the basic physics principles (Albe et al., 2001). Indeed, Bagno and Eylon (1997) report that most students seem to represent the relationships only in mathematical form (even when they are explicitly asked to not use formulas) and do not have access to more qualitative representations that are important in experts' reasoning. Furthermore, even students who are capable of providing such a qualitative description of the knowledge do not do so spontaneously.

This aspect is crucial in the context of this thesis, as we are dealing with two different steps between physical quantities (and concepts) which also requires different mathematical formalisms (and skills): the first one consists of switching from force-based reasoning to field-based reasoning; the second one consists of switching from field-based reasoning to some sort of operators-based reasoning (namely, flux and circulation) in order to be able to describe the electromagnetic field's propagation in a way that is coherent whit the law of electromagnetism students are previously introduced to. The mathematical formalism and the related adaptations will be discussed when describing our teaching proposal in Chapter 9.

Regarding the two conceptual (and mathematical) steps we identified, some remarks in literature support our hypothesis. For example, Dunn and Barbanel (2000) argue that there is a strong conceptual difference between thinking about and solving the problems by using the Coulomb force acting on a charge or by using the electric field concept. The results of Saarelainen and colleagues (2007) agree with this observation, also finding that the concepts of electric and magnetic flux, as well as the use of flux and path integrals in learning Maxwell's equations, requires the application of vectors with relevant dimensions for the fields, surfaces and paths. Mastering these additional requirements of new mathematical techniques, and at the same time learning new physical concepts, seems to be reasonably difficult for the students.

At this point we are going to discuss possible strategies in order to deal with the differential operators – divergence and curl – characterizing the Maxwell's equation. We will not include them in the learning path, but we think it is important to argue their relevance for introductory students, as a development of our proposal for secondary education.

As we previously discussed, all the fundamental relationships in electromagnetism are represented by the Maxwell's equations (link between electric and magnetic fields, and their sources), the continuity equation (link between the sources charge density and current density) and the Lorentz's law of force (action of the electromagnetic field on charges). When discussing a teaching proposal, it is possible to introduce Maxwell's equations directly in their complete form (electrodynamics), in order to avoid problems of conceptual understanding related to electrostatics as a separate domain following different laws (Rainson et al., 1994). Therefore, the educational problem consists of providing conceptual meaning to the introduction of the integral operators (flux and circulation) acting on the fields, or their corresponding differential operators (divergence and curl).

In magnetostatics, flux can be introduced as a characteristic constant: starting from the observation that magnetic field lines are more distant when the field value is smaller, the study of the correlation between these two quantities leads to discover that the product between the area of a section and the fields' values related to this section is constant (Michelini & Vercellati, 2012). To demonstrate the corresponding Maxwell's equation, it will be sufficient to observe that (by definition of the "flow pipe") the flux is zero through the lateral surface of the pipe, therefore it must be equal (in absolute value) to the outgoing flux. Thus, the total flux of the magnetic field (considering, for example, the outgoing one as positive and the incoming one as negative) is always zero through any closed surface.

The integral form of the Maxwell's equations, however, does not allow the local reasoning of the differential form, namely, the association between the values of the sources and those of the fields at a specific point in space and at a given instant. Indeed, the literature shows that students, even after university courses in electromagnetism, have many difficulties in establishing where the divergence (or curl) of an electric or magnetic field cancels out starting from the field lines representation (Bollen et al., 2016). The importance of this association also lies in the fundamental difference between electromagnetic and gravitational interactions, according to the classical point of view (later disproved by general relativity): the electromagnetic field propagates at a finite speed, only depending on the medium (and not on the reference frame).

In the case of Maxwell's equation, the transition to the differential form is simplified by the fact that the flux of the magnetic field through any closed surface is constant (zero), that is it will be zero even through closed surfaces that enclose a volume V tending to zero; thus, the distribution of the magnetic field flux through closed surfaces is zero everywhere. The introduction of a vector operator representing the flux density, is defined by the following reformulation of the divergence theorem:

$$\vec{\nabla} \cdot \vec{B} \equiv \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = \lim_{V \to 0} \frac{1}{V} \int \vec{B} \cdot \vec{n}\, ds$$

Where the integral is extended to a closed surface with an outgoing normal vector (in the three-dimensional case). The proof of this theorem requires mathematical tools that high school students do not master, but a conceptual introduction is possible in the following terms. First of all, a reference system can be established for which the three components of the vector are constant with respect to the other two directions, in order to operate with total derivatives. At this point, the mathematical limit operation can be replaced with a reasoning related to the calculation of the divergence quantity for smaller and smaller volumes, evaluating the value towards which this quantity "converges". For the previous reasoning, it is clear that the flux will be greater the greater is the intensity between the incoming and outgoing field lines, compared to the closed surface we are considering. For each of the field's components, the derivative in the same direction as the component provides the index of variation (that is punctual, hence the need for the limit occurs) of the field's strength in that direction. The variation in the intensity of the incoming and outgoing field lines will obviously be directly proportional to the (three) dimensions of the volume considered, so multiplying the derivative by the volume, and adding over all the three dimensions, we will obtain the integral flux, that is the expression of the divergence theorem reported above.

This is not a rigorous reasoning, which aims to account for the introduction (according to a method of approximation) of the divergence operator in Maxwell's differential equations, an aspect that is not required in dealing with electrodynamics at high school level: thus, there is no explicit reference to the wave equation. However, the introduction of the divergence would allow to provide a second representation on which to base the reasoning, or rather a second referent to be implemented on a representation. We already discussed that the transition between different representations (or between different approaches for the same representation) is a powerful tool for enhancing conceptual understanding: in particular, with regard to field lines (or field vector representations) the literature reports examples of how students' performance improves significantly when both strategies (integral and differential, for flux and divergence operators) are introduced in a two-dimensional space (which a similar reasoning with respect to that reported for the three-dimensional space applies), and students are free to choose which strategy to implement (Klein et al., 2018).

Other limitations related to the use of divergence are provided by the fact that its conceptual introduction is given in Cartesian coordinates, when the symmetry of some charge distributions would require the use of a spherical or cylindrical coordinates' systems. Furthermore, the fact that the electric field's flux is not constant does not make it possible for the operator to diverge in the same way.

Finally, the introduction of the other conceptual operator of electromagnetism – the curl – is conceptually more complicated. On the one hand, this is given by the symmetry breaking with divergence – as the curl of the electric field is zero only in the static case – not aligning with the direct introduction of Maxwell's equations in the general case. On the other hand, a conceptual reformulation of the Stokes' theorem[1] is more complex than that reported for the divergence theorem (an example is reported in the "Berkeley Physics Course", 1965). Indeed, the curl of a vector field is still a vector, each surface having its own orientation in space, and the passage to the limit cannot be accomplished in the same way as in the case of divergence: since considering smaller and smaller circuits, it is possible to make the bounded area have any orientation. The dependence of the curl on the field's (derivatives) components along the other directions is intuitive, but the proof of the expression on this basis is quite complicated.

However, we can use some "rotation meters" for the electric field, such as the one described in the "Berkeley Physics Course" (1965): by placing positive charges at the ends of "insulating rays headed by a single hub", we would find a tendency of the device to rotate around its axis in regions where the curl of the electric field is not zero. Using a spring to brake the rotation, "the torque could be deduced from its winding", which would be proportional to the electric field's component in the direction of the rotation axis: the direction of the field's curl will be the direction of the axis for which the torsion is maximum. The problem in the experimental implementation of such a

[1] It represents the analogue of the divergence theorem, i.e. the one stating that the curl of a vector field is equal to the ratio between the circulation of the field and the area collected by the closed line of integration, to the extent that this area tends to zero.

device arises from the difficulty of using a "rotation meter" in a field significantly changing for distances equal to the size of the measurement system.

To address these difficulties, divergence and curl calculation simulators may be involved in teaching (Hahn & Klein, 2023; Hahn & Klein, 2023), in order to improve the conceptual association between differential operators and the vector representation of different fields. This would probably help in solving the problems mentioned (for the differential approach) in Bollen et al. (2016) and (for the integral approach) in Hernandez et al. (2023), in particular regarding the link inherent in Maxwell's equations between the electromagnetic field and its sources.

Nevertheless, the introduction of the conceptual meaning of these differential operators still establishes a relevant problem for the educational reconstruction (Bollen et al., 2015), as it occurs for other math prerequisites in introductory electromagnetism (Burkholder et al., 2021). However, at high school level, the conceptualization of the field through its own lines and the operationalization of flux and circulation represent sufficient tools for the treatment of the electromagnetic field's propagation in the form of waves.

Chapter 8

Methods

In this chapter we will describe the research methods used – according to the EPERMP framework illustrated in Chapter 2 – to develop the learning path and measure the effectiveness of the two versions, as well as to analyze the learning difficulties related to the topics covered for the spin-off concerning undergraduate students. Thus, the aim is to answer all the research questions listed in the next paragraph. In particular, the characteristics of the research samples, the testing tools used before, during and after the classroom activities, and finally the analysis' techniques will be described below.

8.1 Research Questions

The specific research questions that guided the central part of the research – after obtaining the results of the literature review (Chapter 4) and the textbook analysis (Chapter 6) – are the following:

1. How do high school students – following our learning path – interpret the generation process of electromagnetic waves?
2. How do high school students – following our learning path – interpret the propagation process of electromagnetic waves?
3. How do high school students – following our learning path – interpret the reception process of electromagnetic waves?
4. Are there any significant differences between the reasoning patterns of undergraduate and high school students?

The answer to the research questions will be linked to the learning objectives defined in Chapter 9.

8.2 Research Samples

The study is based on different types of (experimental or control) samples in the different phases of the research, which can be classified as follows.

1. Experimental samples for learning path trials:

1.1 65[1] high school students (13th grade): they have experimented the first version of the learning path, after completing the curricular study of electromagnetism – with the exception of the part regarding electromagnetic waves – with their teachers.

1.2 40[2] high school students (13th grade): they experienced the second version of the learning path, after completing the curricular study of electromagnetism with their teachers.

1.3 34[3] high school students (from 11th to 13th grade): in this case, the sample is made up of gifted students with several backgrounds, as some of them have already completed the curricular study of electromagnetism while others have not. In any case, they have experimented the second version of the learning path.

2. Control Samples:

2.1. 194[4] high school students (13th grade): they form the control group for the second experimentation of the learning path, as it was not possible to submit also the pre-test to the experimental sample.

2.2. 35[5] undergraduate students, who have already attended the electromagnetism course.

In all cases, these are convenience samples with similar characteristics – at least regarding the samples 1.1, 1.2 and 2.1– as they attend the same kind of ("scientific") curriculum, with a similar composition of the students' backgrounds (they came from the same geographical region).

Curricular teachers are not involved in the research activity, even though they participated as auditors in the activities during the experimentation and the submission of the pre-post-tests. Although they do not take part in either the design or the analysis

[1] They belong to 4 different classes, all attending the "scientific curriculum" (see Chapter 2) at the school "Galileo Galilei" in Trieste (Italy).

[2] They belong to 2 different classes, all attending the "scientific curriculum" at the school "Alessandro Tassoni" in Modena (Italy).

[3] They came from different Italian schools, and they were selected for attending a 7-days interdisciplinary summer school organized by our research group in physics education at the "University of Udine" (Italy).

[4] They came from 2 different schools ("Leonardo da Vinci" in Treviso and "Galileo Galilei" in San Donà di Piave: they are both Italian schools following the "scientific curriculum"). All of them had already completed the curricular study of electromagnetism with their teachers.

[5] They attended the second year of the bachelor degree courses in engineering at the university "Politecnico" of Milano (Italy). Later we will discuss why we involved this sample in the research.

of the results, they were previously informed about the content of the activities and the prerequisites that students need to possess.

The figure of the teacher-researcher was in all cases covered by a member of our research group[6], who played the role of the experimenter in all teaching activities. In this way, uniformity in teaching methods and a more adequate comparison with the control groups is also guaranteed.

8.3 Instruments

The instruments used during the experimentations consisted of:

- Teaching-learning activities with related "stimulating questions" used during the implementation of the two versions of the learning path (they will be described in Chapter 9). The total duration of the activities was 6 hours for the first experimentation cycle, and 8 hours for the second cycle.
- Questionnaire used as pre-post-test or as a pure diagnostic tool – without teaching activities – which consists of two different versions (developed by our research group[7]).

In any case, the tests were submitted in a single module, so that students could decide which questions to prioritize or spend more time with. As a result, no attention was paid to the order of the questions. No preliminary indications were provided on the difficulty and content of the single questions, but the participants had the opportunity to interact with the teachers during the administration for any clarifications with respect to the indications of the questions. In addition, it was explicitly made clear to them that these were not evaluation tests.

Below we describe the two versions of the questionnaire: the first was submitted as a pre-post-test for sample 1.1 and as a diagnostic test for sample 2.2, while the second version was submitted as a test-out for samples 1.2 and 1.3 and as a diagnostic test for sample 2.1. These choices are due to the experimental conditions, thus determining the analysis' criteria later described.

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[6] In all teaching activities the role of the experimenter was entrusted to the PhD student, with some interventions of the thesis supervisor.

[7] The first version of the questionnaire – also submitted to the sample of undergraduate students – was developed in collaboration with the research group in physics education of the "Universidad del Pais Vasco" (Spain).

8.3.1 First version of the questionnaire

The first version of the questionnaire consists of 6 multiple-choice items with 5 answer options, in which students are also asked to provide a reason for the selected option (the specific request is: “Briefly explain your answer”). The distractors were chosen based on possible students’ reasoning, considering the results of the textbooks’ analysis and preliminary discussions with teachers. Each item also investigates a specific learning objective, as we chose – to avoid submitting a questionnaire that is too long – not to include multiple questions for the same objective. In addition, the choice of the two-level questionnaire was made to apply the mixed methodologies (Dawadi et al., 2021) described in section 8.4, in order to obtain more complete results. At the same time, the answer options play a guiding role for students’ reasoning with respect to topics on which they may not have sufficient conceptual tools to produce coherent mental models. Furthermore, in all cases the last option is represented by the statement: “None of the previous alternatives”.

Below we analyze each item. The original version of the questionnaire is reported in Appendix A (it was submitted in Italian), while here we report the items in English. The questionnaires were submitted as paper-pencil tests for samples 1.1 (both as pre-test and post-test) and 1.3, while as online forms for samples 1.2, 2.1 and 2.2; in all cases it occurred under the supervision of a teacher, who requested individual answers for the validity of the research. They were also submitted anonymously, and all participants expressed their explicit consent for the use of the data for research purposes. Finally, no indication was given on the limit of characters to be used for the open-ended answers, but the duration of the test was set at 30 minutes for the first version of the questionnaire, and 50 minutes for the second version (taking into account the items’ number).

The first question investigates the interpretation of the representation of the plane electromagnetic wave profile (as reported in Figure 8.1): it is modified starting from a tutorial reported in the research paper by Ambrose and colleagues (1999). Indeed, an instantaneous representation of the electromagnetic field is provided, that is the most widespread type in Italian school textbooks (see the discussion in Chapter 6). It has some peculiar characteristics: 1) the electric and magnetic fields are represented in all their aspects by different colors; 2) a frame of reference is shown whose three axes are represented by the generic *xyz* labels, that are usually reported for an orthogonal set; 3) both fields are represented both by the wave profile and by some vectors, which represent the values which these quantities take on at the points belonging to the axis of the wave's propagation. All information (that is not contained in the representation) is reported in the text of the question, referring to the *y-z* plane as the symmetry plane.

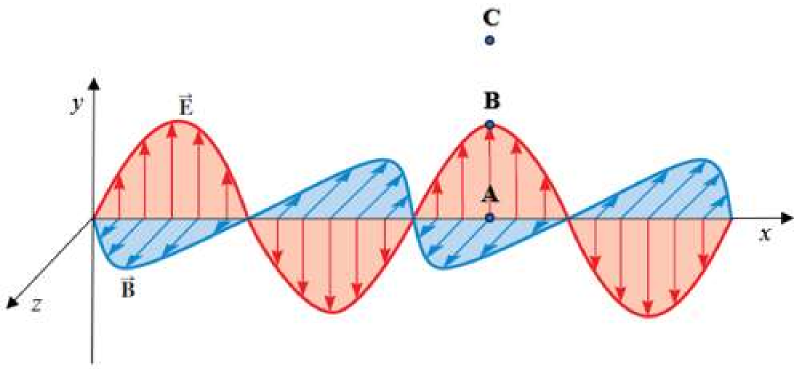
The figure also shows three points – A, B and C – in correspondence with three particular positions: the first (A) belonging to the axis of the wave's propagation, the second (B) at the maximum of the wave profile – in this case with reference to the portion representing the values of the electric field – and the third (C) lying outside the region delimited by the wave profile. All points share the same value of the x coordinate. Students are asked to indicate the correct order of the intensities for both fields at the three points, considering a different time instant from the one the representation refers to: they are separated by a time interval equal to a period (T) of the wave's oscillation.

The reference to two different time instants – the representation referring to $t = 0$ and the question referring to $t = T$ – was not present in the original version of Ambrose's tutorial, but it aims at inducing students to reason also on the basis of the temporal dimension that involves the waves' propagation (going beyond the spatial dimension alone): this aspect may also lead students to attribute a more precise meaning to an instantaneous representation, given the well-known learning difficulty – which already emerges in kinematics, but is widespread in various areas of physics – which consists in confusing the spatial and temporal representations of the same object in motion (Planinic et al., 2013). Another relevant difference with the original tutorial is that it does not provide an algebraic representation together with the graphical one, as the research population we refer to is not familiar neither with multivariable functions nor with "versors" (unit vectors labelling the axis) and vector functions.

Despite these differences, considering the period as time interval implies that actually the fields' representation has not changed, so students do not have to build up a new distribution of the fields' values in order to understand that both of them take on the same value at all the points we are considering. The other response options refer to (potential) specific alternative conceptions, in particular (according to the numbering shown in Figure 8.1): 1) the electric field is zero both at the points belonging to the propagation axis, and at the points lying outside the wave profile; 3) the (electric) field decreases when moving away from the propagation axis; 4) both fields decrease when moving away from the propagation axis.

Question 1

The figure shows an electromagnetic wave at instant $t = 0$. Both the electric and magnetic fields take on the same values in all the points belonging to a plane parallel to the y-z plane.



Given the period of the wave T, which of the following options represents the order of the intensities of the electric and magnetic fields in the points A, B, C at instant $t = T$?

1. $E_B > E_A = E_C$ and $B_A = B_B = B_C$
2. $E_A = E_B = E_C$ and $B_A = B_B = B_C$
3. $E_A > E_B > E_C$ and $B_A = B_B = B_C$
4. $E_A > E_B > E_C$ and $B_A > B_B > B_C$
5. None of the previous alternatives.

Briefly explain your answer.

Figure 8.1: Question n.1 of the first version of the questionnaire (text in English).

Figure 8.2 shows the second question, consisting of a dialogue between four students that focuses on the methods in order to generate electromagnetic waves, i.e. on the characterization of their sources (at least in the case of a single point charge). The text is expressed in a similar modality with respect to tutorials used by the research group in physics education of the University of Washington (e.g. Wosilait et al., 1999), as it requires expressing agreement or disagreement with students, who dialogue expressing mutually alternative positions.

The opinions expressed classify the different outcomes: student A believes that the motion of the source is not related to the characteristics of the electromagnetic field (in particular to its variability over time), and consequently it is also independent of the motion of the observer who detects the fields. Student B excludes the case of the charge at rest – in the chosen frame of reference – as a source, as it generates static fields; all other types of motion, on the other hand, are among the possible causes of the generation of electromagnetic waves. Student C excludes a further case – that of the uniform motion – by making an analogy with the previous case (generation of static fields) and limiting the possibilities to the case of accelerated motions. This recursive structure of necessary conditions is broken by student D, who explicitly refers to the role of the observer – that is, to the choice of the (inertial) frame of reference which the question refers to. Indeed, he includes the case of uniform motion of the source among the possible methods for generating an electromagnetic field varying over time in both its components. Indeed, he implicitly breaks the analogy with direct currents – which generate static magnetic fields – while observing that the motion of the source will not be the same for all observers; as a consequence, the characteristics of the generated field – in particular, its temporal variability – will not be uniquely determined. Thus, all possible conceptual classifications are reported, while partial answers can fall under option n.5 (“I disagree with all the students”).

Question 2

Four students discuss about the states of motion that a point electric charge has to take on with respect to an inertial frame of reference *S*, in order to generate an electromagnetic wave detectable in all inertial frames.

Student *A* states: "An electric charge at rest in *S* may generate an electromagnetic wave".

Student *B* replies: "I disagree, because a charge at rest does not generate time-varying electric and magnetic fields. Thus, we need the charge to move in uniform motion."

Student *C* replies: "I disagree, because, also in this case, the generated fields are constant. Thus we need an accelerated charge".

Student *D* replies: "I disagree, because also a charge in uniform motion may generate time-varying fields, but not in all inertial frames of reference. For this reason we need an accelerated charge".

Who do you agree with?

1. With student A.
2. With student B.
3. With student C.
4. With student D.
5. With none of the students.

Briefly explain your answer.

Figure 8.2: Question n.2 of the first version of the questionnaire (text in English).

The third question is shown in Figure 8.3, concerning the relationships of the propagation velocity of the electromagnetic field with the propagation medium and the oscillation frequency of the fields. Indeed, in the three proposed cases the variation of both these variables is observed, even though in the first two cases the medium is the same (in order to make the comparison with the propagation in a vacuum more evident). It is specified that the dielectric medium is simple in order to avoid any reference to dispersion phenomena. The positions where the fields are generated and detected is made explicit, in order to relate the reasoning with the relationships between the two phases delimiting the propagation mechanism. It is also specified that in all the proposed cases the electromagnetic waves share the same direction of propagation: in this way the reference to plane waves or other particular wavefront cases is avoided.

For simplicity's sake, the text refers only to the electrical component of the wave: thus, it will be possible to observe the possible emergence of inconsistent distinctions with the predictions about the differences with the propagation of the magnetic field. Furthermore, students are not required to explicitly indicate the propagation velocity in the different cases, but they rather refer to the time required for the propagation to occur: in this way it will be possible to observe the emergence of any references to the temporal parameters – e.g. period of oscillation of the wave – or to the general characteristics of propagation and the relationships between the variables involved. Finally, also in this case – as for the other questions – no numerical values are reported, but students are asked to indicate only some "order relationship" between the different cases reported. Thus, possible calculation errors are avoided, and a more adequate meaning is provided to the variables involved in the context of the question, even though it is necessary to consider the difficulty of handling the mathematical symbology related to the order relationships.

The proposed alternatives reflect the following conceptions: 1) the propagation velocity is independent of both the medium and the oscillation frequency of the fields (at least, of the electric field); 2) the velocity decreases as the frequency increases; 4) the velocity increases as the frequency increases. Of course, the third option is the correct one – maximum velocity in a vacuum, that is independent of the frequency – while no other plausible alternatives emerged during the elaboration of the questionnaire.

We also remark that, in order to justify the reference to the oscillation frequency, a sinusoidal trend for the fields was supposed, so that the students – without interpreting any visual representation – can refer to the simplest periodic trend. Furthermore, the reference to dielectric media is justified by the fact that in conductive media the propagation of electromagnetic waves is associated with reflection and absorption phenomena, even though this represent an implicit detail in the text of the question.

Question 3

Consider the following cases:

1. An electric field $\vec{E}_1(t)$ propagates in a simple dielectric medium D with an oscillation frequency equal to f_1.
2. An electric field $\vec{E}_2(t)$ propagates in a simple dielectric medium D with an oscillation frequency equal to $f_2 > f_1$.
3. An electric field propagates in a vacuum with an oscillation frequency equal to $f_3 < f_1$.

The three fields are generated in the same point O, having the same direction of propagation, and are detected in the point P. Given the time intervals t_i necessary for the fields $\vec{E}_i$ to be detected in P, which of the following options will be verified?

1. $t_1 = t_2 = t_3$
2. $t_2 > t_1 > t_3$
3. $t_2 = t_1 > t_3$
4. $t_2 < t_1 < t_3$
5. None of the previous alternatives.

Briefly explain your answer.

Figure 8.3: Question n.3 of the first version of the questionnaire (text in English).

The fourth question – shown in Figure 8.4 – also investigates the relationships of the propagation velocity with the other wave parameters, in particular with the velocities of the source and the observer. The question presents three different cases in which the relative velocity between these two entities varies both in sign – the observer can move closer or further away from the source – and in modality – the relative motion can be uniform or accelerated (or the objects can be at relative rest). In this way, all possible cases have been considered, and students are once again asked for a sorting criterion. Since this involves only three possible arrangements – differences based on the type of relative motion can only emerge from the qualitative analysis of the reasoning – we chose (by exclusion) to match the fourth option with the sentence "There is not enough information to answer". In this way, it is possible to investigate a specific case of alternative responses to the previous options, i.e. to verify whether students identify other variables to consider in determining the relationships analyzed.

Question 4

An observer at rest in a frame of reference S, equipped with a portable radio, receives a signal from a radio wave source. Consider the following three cases:

1. The source is at rest in *S*.
2. The source moves away from the observer with uniform motion with respect to *S*.
3. The source approaches the observer with uniformly accelerated motion with respect to *S*.

Given the propagation velocities v_i of the radio signal measured by the observer, which of the following alternatives will be verified?

1. $v_1 = v_2 = v_3$
2. $v_3 > v_2 > v_1$
3. $v_1 > v_2 > v_3$
4. There is not enough information to answer.
5. None of the previous alternatives.

Briefly explain your answer.

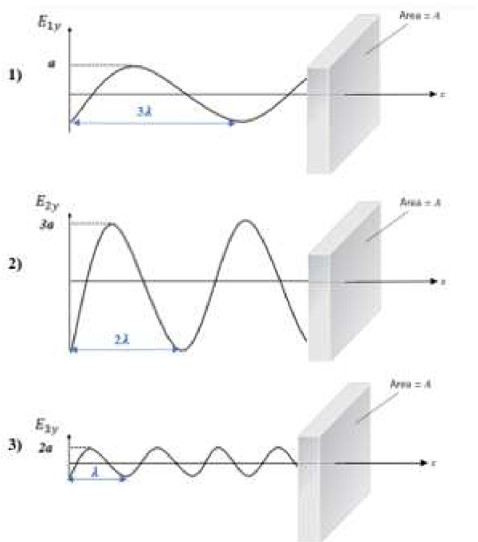
Figure 8.4: Question n.4 of the first version of the questionnaire (text in English).

Figure 8.5 reports the question concerning the relationship between the intensity of electromagnetic waves and its characteristic parameters (wavelength and amplitude of fields). The pattern of the item is similar to the previous ones: there are three different cases in which both parameters take on different values (again, to avoid numerical values the parameters take on multiple values in the different cases). Moreover, the reference is exclusively to the electric field, as a component of waves sharing the same direction of propagation. These elements help the reference to a conceptual framework common to the previous questions, in order to reinforce the internal coherence of the test and to preserve the possible comparison between the students' reasoning. In addition, the instantaneous representation of the wave profile as a function of the propagation direction also allows us to report the representation of the unit surface, in terms of which the intensity of the electromagnetic wave is explicitly defined (in the text of the question).

Again, the question requires ordering the different cases by direct reference to the quantity of energy per unit of time and surface, without using the term “Intensity”. The options reflect the following conceptions: 1) the intensity increases with the wavelength; 2) the intensity decreases as the wavelength increases; 3) the intensity is independent of the wavelength and the amplitude of the (electric) field; 4) the intensity increases with frequency. Actually, we remark that, since nonlinear relationships are potentially involved, the possible combinations exceed the number of options (there is no one-to-one correspondence).

Question 5

Consider the following 3 figures. They represent the components in the direction of propagation of three electric fields, which vary over time with a sinusoidal trend. They invest an area $A = 1\ m^2$ lying in the y-z plane. The amplitudes of the three fields are a, 3a and 2a respectively, while the wavelengths are 3λ, 2λ *and* λ respectively.



Given the amount of energy I_i that invests the surface A in 1 second, which of the following alternatives will be verified?

1. $I_1 > I_2 > I_3$
2. $I_3 > I_2 > I_1$
3. $I_1 = I_2 = I_3$
4. $I_2 > I_3 > I_1$
5. None of the previous alternatives.

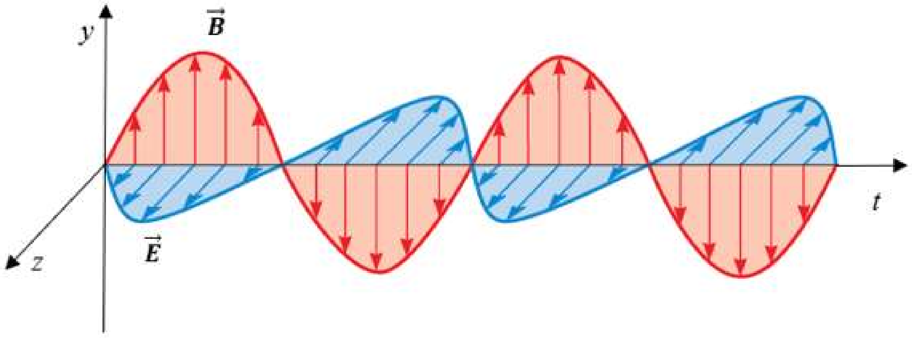
Briefly explain your answer.

Figure 8.5: Question n.5 of the first version of the questionnaire (text in English).

Finally, Figure 8.6 shows the last question, in which a plane wave profile as a function of time is represented, i.e. the distribution of the values of the electromagnetic field at a point as time varies: this representation is alternative and complementary to that of the first question, even though the details of the figure are the same (while the time axis takes the place of the *x* axis). It is also specified that it is a signal in the "radio wave spectrum", as it can be received by an observer equipped with a portable radio with a straight antenna. Students are asked to indicate – if there is one – the direction in which the signal should be received, i.e. the one in which the antenna should be oriented to get the best signal reception. The meaning of the expression "best reception" in this context is not further specified, leaving its interpretation to the students.

The answer options are not explicitly provided in terms of the fields' oscillation directions, but rather referring to the axes of the frame of reference. Thus, in order to probe their interpretation of this representation, the students have to identify the first three options with those, respectively, of oscillation of the magnetic field, of the electric field and of that of wave propagation. The directions of the fields' oscillation are reversed with respect to the figure of the first question, in order to avoid the repetition of mnemonic patterns. Finally, the fourth option reflects the possibility that this direction is not uniquely determined, or that it changes over time: to detect different possible interpretations, it is not specified whether this direction is – instant by instant – determinable starting from the values' distribution of the electromagnetic field, or it is indeterminate as it depends on other external parameters (in this case the students can specify them).

Question 6



The figure illustrates the time trend of an electromagnetic wave. An observer is equipped with a portable radio which he can detect this signal with. In which direction should the antenna of the portable radio be oriented to get the best possible reception?

1. In the direction of the y-axis.
2. In the direction of the z-axis.
3. In the direction perpendicular to the y-z plane.
4. The direction of best reception varies over time, so it is not possible to determine it.
5. None of the previous alternatives.

Briefly explain your answer.

Figure 8.6: Question n.6 of the first version of the questionnaire (text in English).

8.3.2 Second version of the questionnaire

The second version of the questionnaire consists of 10 items with the same characteristics as the first version (5 options + motivation request). However, in this case the last option will not always be represented by the statement "None of the previous alternatives". Some questions are taken from the first version, implementing some modifications. In particular, questions number 2, 4 and 5 have not undergone any modification, while question n.6 has been replaced by the tutorial about the brightness of light bulbs described in paragraph 9.1 (see Figure 9.6). Indeed, in the second experimentation cycle we preferred to use this question in the test-out rather than as a tutorial during the teaching activities – due to its greater complexity – replacing it with a very similar version of item n.6 from the first version of the questionnaire (see question n.17 in Table 9.2).

At the end of paragraph a Table (8.1) is added in order to relate the items with the learning objectives and the discussion of the correct answer.

Questions number 1 and 3 have undergone small changes. In particular, for the first item, the instant of time which the question refers to has been modified: it is no longer equal to the oscillation period of the wave (T), but to a half-period ($T/_2$). Thus, students are forced to consider a case in which the values' distribution of the electromagnetic field has varied with respect to that represented in the figure, so they are also induced to consider the temporal dimension of the propagation of the wave: nevertheless, the plane symmetry means that the fields will continue to take on the same value at all the points considered (although the value as changed from the initial one).

On the other hand, the modification for the third item exclusively concerns the answer options. The fifth option has been replaced by the following statement: "The third field comes last", as we wanted to consider the case for which propagation in vacuum is slower than all other propagation media. Indeed, from the answers to the corresponding question of the previous version, not many explicit references to the propagation velocity in a vacuum emerged, so it is appropriate to probe this aspect more deeply. Moreover, this modification brings out more complete reasoning with respect to the role of the propagation medium, which – at least referring to mechanical waves – is sometimes considered by students as an obstacle for propagation (Wittmann et al., 1999). Nevertheless, in the new context of electromagnetic waves different reasoning may emerge that – in analogy with mechanical waves – deny the possibility of propagation in a vacuum, or attributing to it peculiar characteristics which deserve to be investigated.

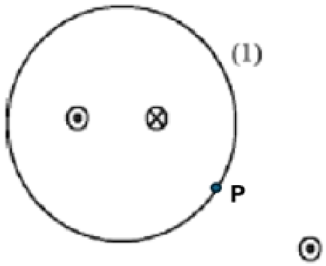
Finally, It is worth noting that in the second version we decided to transform the answer options into statements, rather than maintaining the use of the symbolic sorting that characterized the first version of the question. Indeed, the students' answers showed confusion due to the use of indices for the time variable, which led to inconsistency between the options selected and the motivations provided. Although this was not the only item that asking for some sorting of the same variable among different cases, in the third question this aspect plays a peculiar role, as it focuses on the propagation velocity, while the ordering refers to time. In order to keep the references to both variables avoiding such confusion, we decided to reformulate the first four answer options as follows (keeping the text of the question unchanged):

1. The three fields arrive together.
2. The third field comes first, then the other two together.
3. The three fields arrive according to the increasing order of frequency.
4. The three fields arrive according to the reverse order of frequency.

Four new items have also been added, which we describe below. Figure 8.7 shows a question related to the circulation of the magnetic field: in presence of three parallel wires crossed by current – flowing upwards or downwards – students are asked to indicate whether the magnetic field and the circulation cancel each other out or not, respectively at a point P and along a circular line represented in the figure. All possible alternatives are contemplated in the answer options, so the last option is represented by the statement: "There is not enough information to answer". It refers to an item reported in Guisasola et al. (2008), slightly modifying the structure of the question so as not to produce a predetermined statement – which it is required to motivate the agreement or disagreement about – and to bring out more complete reasoning both about the field and its circulation. For this purpose, a third wire – in an external position with respect to the closed line – was also added, in order to bring out the contribution of the concatenated currents.

The second question (item n.8) relating to the part of the preliminary activities – in particular regarding the phenomenon of electromagnetic induction – is taken from item n.29 of the Conceptual Survey of Electricity and Magnetism (Maloney et al., 2001): thus, we refer to the literature for the analysis of this item's structure.

Question 7



The previous figure shows three wires crossed by currents, which are orthogonal to the plane of the paper. The symbol $\odot$ indicates currents flowing upwards, while the symbol $\otimes$ indicates currents flowing downwards. We know that a wire carrying current generates a magnetic field.

What is the value of the magnetic field circulation along the circular curve (1) and the value of the magnetic field at the point P (belonging to the curve)?

A. The magnetic field is zero, while the circulation is not zero.
B. The magnetic field is not zero, while the circulation is zero.
C. Both the magnetic field and its circulation are zero.
D. Both the magnetic field and its circulation are not zero.
E. There is not enough information to answer.

Briefly explain your answer.

Figure 8.7: Question n.7 of the second version of the questionnaire (text in English).

The last two questions – shown in Figure 8.8 and Figure 8.9 – focus again on the relationships between source, field and observer. Thus, we want to distinguish more explicitly between these three agents, in particular between the motion of the observer and that of the source. In addition, we also aimed at probing the difference between a question asked in a specific context (n.9) and one referring to a generic situation (n.10), similar in its structure to questions asked during the activities – see Table 9.1 – but adding answer options which correspond to different conceptual categories.

In particular, item n.9 aims at probing the possibility that students identify a direct relationship between the propagation velocity of the electromagnetic field and the speed of the source generating it (instant by instant). The context is that of conduction electrons moving along a wire, and all the possibilities are listed: the propagation velocity of the field may be much greater or lower than that of the electrons, or they may coincide. Moreover, they could be different in general, but similar – at least if we look at the order of magnitude – in this specific case, as students may identify a peculiarity in this case of conduction current (for example, conceptually identifying the value of the speed of light thus generated with that of the charges in motion). Finally, there may be a more complex relationship between the two quantities, such as to be indeterminate in this case: this explains the meaning of option A ("There is not enough information to answer").

Question 9

Consider conduction electrons moving, under the action of a generator, in a metal wire. They will generate an electromagnetic field.

What can we say about the relationship between the speed which electrons move in the with wire and the propagation velocity of the field they generate?

A. There is not enough information to answer.
B. The electrons' velocity is much less than that of the field.
C. The field's velocity is much less than that of electrons.
D. The field's velocity is equal to that of the electrons.
E. The velocities are different, but they are of the same order of magnitude.

Briefly explain your answer.

Figure 8.8: Question n.9 of the second version of the questionnaire (text in English).

In question n.10 the point of view of different observers who measure the fields generated by a point charge is considered: all possible alternatives are listed about the fields' variations when the observer changes (only the electric field changes, only the magnetic field changes, no field changes, both of them change); finally, we took into account the possibility that these variations depend on the characteristics of the observer's motion (option E), which has to be specified.

Question 10

What happens to the electric and magnetic fields generated by a stationary charge when they are measured by an observer in uniform motion with respect to it?

A. Only the electric field changes.
B. Only the magnetic field changes.
C. Neither of them changes.
D. Both change.
E. It depends on the motion of the observer.

Briefly explain your answer.

Figure 8.9: Question n.10 of the second version of the questionnaire (text in English).

Table 8.1: Summary of the items with the related learning objectives and the description of the correct answers (EM stands for electromagnetic).

Item	Learning objective	Correct answer
1	Interpretation of the EM (plane) wave profile representation	2. $E_A = E_B = E_C \; and \; B_A = B_B = B_C$: At any instant of time, both fields take on the same value at all points considered as they belong to the same wave front.
2	Characterization of the EM waves' sources	4. $With \; student \; D$: The accelerated charges can be identified as the sources of EM waves as the radiative emission cannot depend on the frame of reference.
3	Identification of the relationships between the EM wave parameters (propagation velocity)	3. $t_2 = t_1 > t_3$: The propagation velocity of EM waves depends on the medium (taking on the maximum value in vacuum) but it is independent of the fields' oscillation frequency.
4	Identification of the relationships between the EM wave parameters (propagation velocity)	1. $v_1 = v_2 = v_3$: The propagation velocity does not depend on the source-observer relative velocity.
5	Identification of the relationships between the EM wave parameters (wave intensity)	4. $I_2 > I_3 > I_1$: The wave intensity is directly proportional to the square of the fields' amplitude, but is independent of their wavelength.

Item	Learning objective	Correct answer
6	Identification of the direction of (best) signals' reception	2. In the direction of the $z-$axis: According to the Lorentz force, the direction of best signals' reception coincides with that of the electric field oscillation.
7	Identification of the relationship between the magnetic field's circulation and sources	B. $The\ magnetic\ field\ is\ not\ zero, while\ its\ circulation\ is\ zero$: The circulation of the magnetic field depends only on the sources within the region of space enclosed by the line along which the circulation is calculated.
8	Identification of the magnetic flux as agent of the electromagnetic induction	C. I, II, IV : The induced current only occurs when the magnetic flux is varying over time.
9	Identification of the relationships between the EM wave parameters (propagation velocity)	$B. The\ electrions' velocity\ is\ much\ less\ than\ that\ of\ the\ field$ The fields' propagation velocity does not depend on the sources' velocity.
10	Identification of the relationship between the fields and the observers	$D. Both\ change$: The fields' values depend on the motion of the source, thus both fields' values change when the observer change.

8.4 Data Analysis

The data analysis was carried out according to mixed – qualitative and quantitative – methodologies. Indeed, the instruments consisted of two-tier questionnaires, so it is possible to combine different analysis' methods: in the first case, only the frequency of the different answer options is taken into account, thus calculating the items' difficulty indices and the possible presence of ineffective distractors (when chosen by less than 5% of the sample, as the criteria of the Classical Test Theory establish: see De Vellis, 2006). Thus, preliminary information can be obtained with respect to the reliability and validity of the questionnaires, which can be re-evaluated later through a new submission with possible changes in the distractors or in the questions' formulation.

The results of the first experimentation cycle were analyzed by comparing the results of the pre-test and post-test, categorizing the students' answers, and using the Bowker test (Field, 2013) as a statistical index in order to assess the significance of the variation in the categories' distribution. For the second cycle, the comparison was made between the control and experimental groups, using the same categorization method and the Chi-square test (Field, 2013) as a statistical index. The categories we extracted in both cases from the students' answers do not completely overlap with the answers' options, even though the results will show they are often similar.

In the latter case, a significant part of the answers of students belonging to the control group lack motivation, having they exclusively selected the option. However, this does not invalidate the analysis, as the greater size of this group allows a comparison partially limited to the multiple choices. It is also possible because one of the options always represents an "Other" category, in which reasoning not sharing common conceptual aspects with other categories may be included.

According to what we illustrated above, the analysis' method of open-ended answers consists of an iterative categorization that – in cases when the reasoning is not explicit, and can be interpreted according to alternative schemes – takes into account the selected answer (all participants who were included in the analysis have selected one, and only one, option for each item). The analysis' purpose is not to obtain a hierarchical categories' structure, but rather to specify the conceptual meaning underlying the different answer options, while verifying the possible emergence of new response profiles (or the unification of those we already considered).

These aspects differ from the methodology used for the undergraduate sample, where the phenomenographic approach (Guisasola et al., 2023) was applied for a two-tier questionnaire. Indeed, in this case the students' open answers were more complete, thus we tried to implement a more structured analysis, although we were aware of the limitations due to the limited sample size. As a consequence, we are not going to discuss the reliability of these results, also considering that – as for the other analysis' steps – the categorization was carried out by a single researcher or group of researchers.

Finally – as we are going to describe in Chapter 9 – during the teaching activities students were asked some "stimulating questions" in order to detect their progress in conceptual understanding: as the answers we got often fail to report students' reasoning, we limited the analysis by reporting just some recurring patterns, which may help in identifying the weak points of our teaching proposal.

Chapter 9

The learning pathway

In this chapter we will discuss the two versions of our teaching proposal aimed at introducing electromagnetic waves for high school students. The second version was developed following the learning outcomes obtained in the first trial. They were obtained through “stimulating questions” posed in class during the activities – included in some worksheets students had to complete during the activities – and by submitting a final questionnaire described in Chapter 8 (also consisting of two different versions). As we mentioned above, the analysis’ methods differ between the two versions, due to the different characteristics of the research samples and the experimental conditions. The results will be illustrated in Chapter 10, while in the Discussion (Chapter 11) the reasons for the differences between the two versions of the learning path will be clarified: here we are going to describe them without any further specifications.

We also observe that the teaching proposal will address the key ideas of the classical theory about the electromagnetic field that we have identified through its epistemological reconstruction, linked to the specific learning objectives that will be specified later. These objectives only partially overlap with those usually established in traditional teaching – for the context of secondary education in Italy – according to the national guidelines of the Ministry of Education (see Chapter 2) and the results of the textbooks’ analysis reported (see Chapter 6). Rather, the learning pathway aims to counteract the main learning difficulties emerging from the literature review (see the discussion in Chapter 4).

The learning path was developed including tutorials and experiments, as we will describe later. The experiments were usually carried out in a single (class) workstation, but students had the opportunity to manage the devices in order to answer the related questions. On the other hand, the tutorials follow the different sections of the pathway – where students are introduced to the topics through some examples – consisting of multiple-choice questions (see Table 9.2) students had to answer individually, followed by a class discussion.

9.1 Description of the first version of the learning path

9.1.1 Maxwell's equations

The first version of our teaching proposal moves from a brief summary of Maxwell's equations, which the students discussed extensively with their teachers. This reintroduction, beyond summarizing the content knowledge which equations hide, aims at focusing attention on some conceptual aspects, such as the fact that the operators – flux and circulation – in some cases may not undergo variations even though the fields have changed (following changes in their sources). Thus, students are presented with a simple situation in which a specific charge distribution (at rest) generates an electric field $\vec{E}$ at a specific point P in space. This field is associated with a flux ϕ_E through a spherical surface S. Subsequently, at a point R outside the volume enclosed by S, a point charge q is placed: students are asked to indicate whether – after the time necessary for a new state of equilibrium to be reached – the electric field in P and the flux through the surface S will change (compared to the previous configuration).

The Maxwell's equations are reintroduced exclusively in a vacuum and in integral form. Furthermore, time derivatives are not used: in fact, in the following application of these equations, we prefer to emphasize the conceptual aspects that will emerge by referring to finite time intervals (even though they should be small enough to guarantee with a good approximation the equations' validity), without introducing the use of formal tools – derivatives – which could represent an important learning obstacle. As a result, the Maxwell's equations will reintroduced as the following expressions[1]:

(9.1) $$\phi_B = 0$$

(9.2) $$\phi_E = \frac{Q}{\varepsilon_0}$$

(9.3) $$\Gamma_E = -\frac{\Delta\phi_B}{\Delta t}$$

(9.4) $$\Gamma_B = \mu_0 I + \varepsilon_0\mu_0\frac{\Delta\phi_E}{\Delta t}$$

Subsequently, after defining the current intensity, students are asked to describe the characteristics of the electric field generated by direct currents (i.e. constant in time)

[1] The notations have the meaning traditionally attributed by textbooks: for example, the letter ϕ stands for flux and the letter Γ stands for circulation.

outside the wire. This question aims at starting a discussion about the fields' sources (that will be completed later), as well as to recognize that the free charges in a conductor are not additional (namely, the conductor is a neutral body) and to distinguish the fields generated by the current from those that generate it: cause-effect relationships are often obscure for students in these cases.

9.1.2 Fields' sources

Following this line, the experimental evidence that allowed Ampère to formulate the principle of magnet-current equivalence (Blondel & Benseghir, 2017) is provided by: 1) a magnetic needle of a compass changing orientation when placed near a wire crossed by current; 2) a wire crossed by current being attracted or repelled by a magnet; 3) Two wires crossed by currents interacting like two magnets, attracting or repelling each other depending on the relative direction of the currents. Such associations are made exclusively on a macroscopic basis, renouncing a description in terms of microscopic currents that would not be rigorously correct. On the other hand, this is a necessary step to somehow link the experimental evidence on Maxwell's sources to the content of Maxwell's equations, which do not contain any quantity characterizing permanent magnets: we expect that this aspect may represent a learning obstacle for students' full awareness of cause-effect relationships.

After reintroducing the expression of the Lorentz force[2] as a tool for determining the fields' action on charges other than sources, students are faced with the problem of determining the expressions of the fields generated by a point charge at rest, starting from the Maxwell's system of equations. In this way, we want to highlight how these equations form a solvable system – beyond the mathematical difficulties – only once some information on the boundary and initial conditions is provided. For instance, this consideration turns into recognizing that if – as in this case – the magnetic field is identically zero, it cannot be derived from the equations – i.e. from the fact that the circulation along closed lines and the flux through closed surfaces are zero – but we need experimental observations (any charged body does not generate any magnetic field when it is at rest, as can be observed through any explorer, such as sensors or simple compasses). This also allows us to conclude that the only equation from which information on the electric field can be obtained is the Gauss' theorem (equation 9.2):

[2] By Lorentz force we mean the force exerted on the charges by both the electric field and the magnetic fields, given by the expression: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. The expression is justified by means of several experimental evidences (e.g. the force which two circuits interact with is proportional to the current flowing in the "test circuit" and to the length of the section considered).

however, in order to apply it, it is necessary to know the field lines' geometry distribution, since from this equation it is possible to obtain only the component of the field along the direction orthogonal to the surface through which the flux is calculated. Obviously, the expression obtained can be compared with the one known from the definition of electric field – and from the expression of the Coulomb force law – deducing that there are no other components of the field beyond the radial one. Such reasoning is useful to show how, even in the simplest case, Maxwell's system cannot be solved without taking into account other information about the fields.

In any case, the state of motion of the source depends on the reference frame adopted: indeed, if any inertial source-fixed frame of reference is chosen, it will move in a uniform rectilinear motion. Thus, students are asked if in all these cases the fields measured by different observers will always be the same. Therefore, students can recognize that a charge in motion constitutes some type of current, which will generate a magnetic field. Furthermore, since the position of the charge varies in time, even the electric field at any point in space can no longer be constant. Finally, for the previous considerations, the values of the fields must necessarily depend on the charge's velocity, and this of course applies to any sources' distribution. It turns out that, in transformations between two (inertial) frames of reference, it no longer makes sense to refer to the electric and magnetic fields separately, since by changing the observer's point of view it will be possible to measure a field that was previously undetectable: in this way we have identified one of the reasons why in the dynamic context we refer to the "electromagnetic field".

The scenario changes if we consider a charge in accelerated motion: indeed, in this case the motion will be accelerated in all inertial frames of reference, since acceleration is invariant. Thus, in these cases all observers will measure electric and magnetic fields varying over time, although their values will still depend on the frame of reference adopted. In any case, having to deal with a context that is no longer static from all points of view, we can no longer assume waiting a sufficiently long time interval for a state of equilibrium to be reached. We must take into account the time that is necessary for the fields to propagate in space moving away from the source, with a speed that we want to determine from Maxwell's equations: thus, we will make evident the completeness of the information that can be obtained from this system, with respect to the characteristics of the electromagnetic field.

9.1.3 Fields' propagation (ideal source)

For observations that will follow, it is useful to consider an ideal case of plane symmetry, in which the source consists of an infinite conducting plane crossed by currents flowing upwards (see Figure 9.1), immersed in a vacuum. Let's now focus on the analysis of the transient: at the instant $t = 0$ in which the currents start flowing on the plane, there is a variation of the magnetic field in the surrounding space, which was initially zero (when the source is absent). This will allow to analyze a particular case in which – with the necessary approximations – Maxwell's equations and the propagation of the electromagnetic field will take on an evident conceptual meaning.

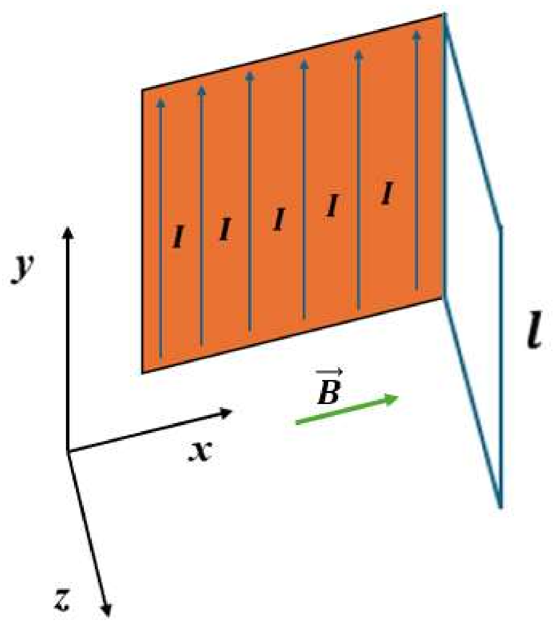


Figure 9.1: Infinite conducting plane crossed by uniform surface current (its representation consists of infinite wires of infinite length crossed by the same current of intensity I). The plane is parallel to the x-y plane (in the frame of reference we adopted). The resulting magnetic field is parallel to the x-axis. The figure also shows an ideal circuit parallel to the y-z plane, with a surface area S and length l of the side parallel to the y-axis.

First of all, we want to show that this field is directed as in Figure 9.1: for this purpose we implement a geometric reasoning, which is based on considering the plane as an infinite series of parallel wires (of infinite length) crossed by current. Indeed, students already know the geometry of the magnetic field lines generated by an infinite wire, so it is possible to proceed by representing the field at a point resulting from two wires that are at the same distance from it. Figure 9.2 shows how, in case the currents of the two wires were identical ($I_1 = I_2 = I$) and the distance between the wires and the point we are considering was such that the two fields ($\vec{B}_1$ and $\vec{B}_2$) were perpendicular to each other, the resulting magnetic field will be directed along the line connecting the two wires.

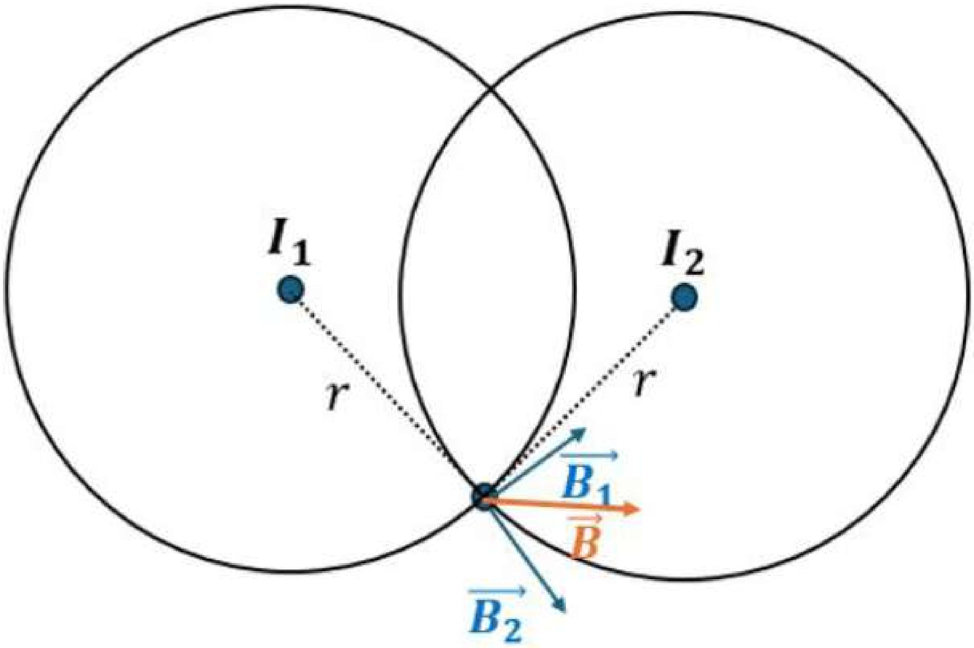


Figure 9.2: Representation of the field resulting from two wires of infinite length crossed by current, at a point equidistant from the wires themselves (at a distance r). The resulting magnetic field $\vec{B}$ is equal to the vector sum of the fields $\vec{B}_1$ and $\vec{B}_2$.

Of course, in order to determine the direction of the resulting field, we have to consider the contribution of all the wires' pairs that are equidistant from the point we are considering, which will generate pairs of magnetic fields that are not mutually orthogonal. In these cases, we take advantage of the fact that magnetic fields $\vec{B}_1$ and $\vec{B}_2$ have equal module: consequently, the total magnetic field $\vec{B}$ forms the same angle both with $\vec{B}_1$ and with $\vec{B}_2$. Consequently, for each pair of wires we are considering, a magnetic field will be obtained that is parallel to the x-axis: it follows that the magnetic field generated by an infinite plane crossed by a uniform (surface) current is parallel to the source plane, and it is also perpendicular to the direction in which the current flows. This reasoning is independent of the specific point which the field is being calculated at – since the source plane is infinite – as long as the distance from the plane is kept fixed. Therefore, we can conclude that this source generates a plane symmetrical magnetic field (with respect to planes parallel to those of the source).

Now we want to determine the speed which this field propagates in space with, assuming that it is constant. Thus, the field will be initially generated near the source, then moving away from it as time passes (we remark that this analysis can be implemented both in the region of space to the left or to the right of the source's plane). Students have already been reintroduced to Maxwell's equations as complete relationships between electric and magnetic fields and their sources: of course, the laws calculating the circulation are useful for our purpose, which we can apply in this case starting from Faraday's law. Indeed, it suggests that when temporal variations in the magnetic flux occur, an electric field will also exist in the surrounding space. For the reasons illustrated in Paragraph 4.8, we will not use the adjective "induced" to describe this electric field, which could be misleading since its source is represented only by the plane crossed by current.

Despite this, the characteristics of the electric field have to be determined by Faraday's law: in order to apply it in a simple way, students can recognize that it is appropriate to choose an ideal circuit that is parallel to the x-z plane, as shown in Figure 9.1. Indeed, in this way the magnetic field will be perpendicular to the circuit itself, and the calculation of the flux is trivial. Since we are focusing the analysis on the "transient", at the initial instant there is no magnetic field (as currents are not flowing yet on the plane). For our purposes, the "transient" will be represented by a very short – but finite – time interval (Δt), during which the current takes on a uniform value throughout the plane. As a consequence, after Δt, the magnetic field $\vec{B}$ generated near the plane will have moved away from the source by a distance equal to $v \cdot \Delta t$ (v representing its propagation velocity). Thus, considering the length of the side parallel to the y-axis of the ideal circuit we are applying the law to, the surface swept by the magnetic field during Δt will be equal to $S' = lv\Delta t$, and from Faraday's law the circulation of the electric field will be given by:

(9.5) $$\Gamma_E = -Blv$$

At the same time, if we also knew the direction of the electric field, we may write the first member of equation 9.5 in terms of the electric field, in order to obtain a relationship between the two fields. To determine this direction, we can use some qualitative reasoning about Faraday's law, which ensures that the electric field is directed along the y-axis and pointing downwards[3] (see Figure 9.3).

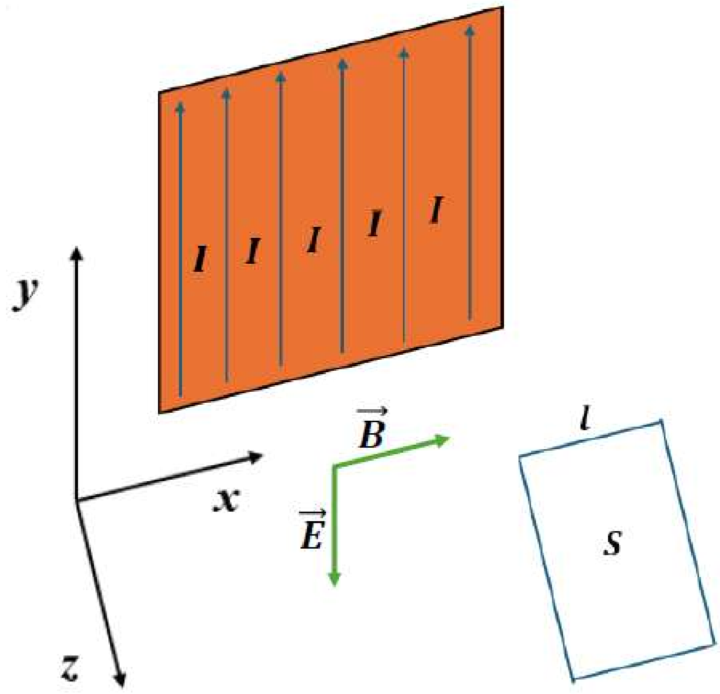


Figure 9.3: Infinite conducting plane crossed by uniform surface current (its representation consists of infinite wires of infinite length crossed by the same current of intensity I). The plane is parallel to the x-y plane in the adopted frame of reference. The resulting magnetic field is parallel to the x-axis, while the electric field is parallel to the y-axis (directed downwards). The figure also shows an ideal circuit

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[3] Faraday's law assures that the electric field belongs to a plane that is perpendicular to the magnetic field; moreover, it cannot have a component along the z-axis because it would be a "static" one: indeed, the circulation would be zero as its contributions along the sides of the ideal circuit which are parallel to the z-axis would cancel each other out, while those along the sides parallel to the y-axis would be zero. Furthermore, the electric field is directed downwards (negative direction of the y-axis) because if the circuit were real, the induced current – according to the Lenz's law – should flow in such a direction as to generate a magnetic field whose flux concatenated to the circuit compensates for the variations in magnetic flux that produced it. For this reason, in our case the current will have to flow counterclockwise, according to the right-hand rule (so the induced magnetic field would be directed in the opposite direction to the external one). This determines that the electric field must be directed downwards, as it has to be in accordance with the direction in which the current circulates on the side considered (i.e. the one close to the source, in which the fields have already been determined at that moment).

parallel to the x-z plane, with a surface S and length l of the side parallel to the x-axis.

At this point we can easily express the circulation of the electric field in terms of the field itself, by rewriting equation 9.5 in the following terms:

(9.6) $$El = Blv \; \rightarrow E = Bv$$

It is obtained by taking into account the fact that the circulation of the electric field – calculated according to the convention of the circuit's orientation in a counterclockwise direction – provides non-zero contribution only along the side near the source, since on the far side the fields are still zero at that instant[4], while the contributions along the sides parallel to the z-axis cancel each other out as equal and opposite[5]. Furthermore, the members of equation 9.5 have been considered in absolute value, by taking into account only the module of all the physical quantities involved (for our purposes, we are not interested in their signs).

By exploiting the Ampère-Maxwell's law, we can implement a reasoning mirroring the previous one, in which the roles of the two fields are reversed. Indeed, given the ideal circuit represented in Figure 9.3 – and considering that there are no conduction currents – the application of equation 9.4 leads in this case to the following relationship:

(9.7) $$Bl = \varepsilon_0 \mu_0 Elv \; \rightarrow B = \varepsilon_0 \mu_0 Ev$$

It is important to remark that the propagation velocity of the electric field (appearing in 9.7) must be equal to the propagation velocity of the magnetic field (appearing in 9.6), as we have already shown that they represent two components of a single physical entity (the electromagnetic field). These components are correlated through the relationships 9.3 and 9.4, thus they cannot propagate in space with different velocities.

Furthermore, comparing the relations 9.6 and 9.7, we actually used an incorrect notation: indeed, the module of the electric and magnetic fields are calculated at different points in the two cases. In 9.6 E and B represent the fields' values at the points

[4] For the sides parallel to the z-axis, a length greater than the distance covered by the fields in the time interval Δt was specifically chosen.

[5] To draw this conclusion, it is necessary to demonstrate that the electric field in this case also holds plane symmetry. It follows from Faraday's law, because if the magnetic field (or rather the temporal variations of its flux) are uniform in space, this property must also apply to the circulation of the electric field. If the geometric characteristics of the ideal circuit which we are applying equation 9.3 to have not changed, this implies that the electric field also holds plane symmetry, with respect to planes that are parallel to the source.

on the side of the circuit that is parallel to the y-axis and lying on the source plane (Figure 9.1), while in 9.7 they represent the fields' values at the points belonging to the side of the circuit that is parallel to the x-axis and lying on the source plane (Figure 9.3). However, all these points belong to the same plane, which is parallel to that of the source (in this case they coincide). We have previously shown that both the electric and magnetic fields in this case hold such a symmetry that they take on the same value at all points belonging to a plane that is parallel to the *x-y* one: this explain using this kind of notation, since both in equations 9.6 and 9.7 the values of E and B will be the same.

After recognizing that we got two equations relating the electric and magnetic fields through their propagation velocity, we want to determine its value. For this purpose, we can exploit equations 9.6 and 9.7; using the values of the characteristic constants in vacuum[6], we will derive:

(9.8) $$\frac{E}{v} = \varepsilon_0 \mu_0 E v \ \rightarrow v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 299792458 \, {}^{m}/_{s}$$

This value is usually represented by the letter *c*, so later we will keep this notation to indicate the propagation speed of the electromagnetic field in a vacuum. Thus, this value was derived from Maxwell's equations in the particular case of an infinite plane source, but it can be immediately generalized to any source.

Moreover, since the electromagnetic field is moving away from the source, this propagation velocity is directed along the z-axis of the frame of reference we adopted, so that the relation 9.6 – with general validity – can be reformulated in the following vector terms (emphasizing how the electric and magnetic fields form a triad of orthogonal vectors with their propagation velocity):

(9.9) $$\vec{E} = \vec{B} \times \vec{v}$$

9.1.4 Fields' propagation (real source)

So far, we have referred to an ideal situation in analyzing the "transient" phase. Now, we want to generalize this discussion to the case of a real source continuously producing a time-varying electromagnetic field. In these cases, we will no longer analyze what happens during the "transient" phase, but we will have to consider a source characterized by currents varying over time. For example, we can consider an

[6] Using the International System of Units, they are: $\varepsilon_0 = 8.854 \times 10^{-12} \, {}^{F}/_{m}$ and $\mu_0 = 4\pi \times 10^{-7} \, {}^{H}/_{m}$.

alternating current (RLC) circuit[7]: basically, this was what Hertz did in his famous experiment in 1887 (Buchwald, 2013). In this case, the source will be characterized by a periodic motion of the charges: hence, it is easy to hypothesize that the electromagnetic field will oscillate with the same frequency that characterizes the motion of the source (charges). In these cases, we use to refer to electromagnetic waves, whose characteristics can be derived from the generalization of the previous discussion to the cases of continuous (over time) fields' generation. These represent transverse waves (see relation 9.9) propagating in a vacuum with a speed of about 300000 km/s. We remark that dealing with the propagation in a vacuum also allows us to refute the widespread (among students) conception considering the medium as an obstacle for the propagation to occur.

The relation 9.8 allows to derive other fundamental properties related to the propagation of electromagnetic waves. Indeed, the expression obtained for their propagation velocity only depends on the constants (dielectric and magnetic) that characterize the medium[8]: this implies that the velocity varies with the propagation medium, but it does not change either with the variation of the fields' oscillation frequency, or with the variation of the sources' velocity with respect to the observer. We think it is important that the discussion with the students highlights these aspects, as they are associated with some typical learning difficulties arising from spontaneous ideas related to mechanical wave phenomena (see Chapter 4). Furthermore, since the relative constants (dielectric and magnetic) of the media are generally greater than 1, the maximum value of the propagation velocity is obtained in a vacuum.

In addition, the 9.9 relation shows another fundamental property: in an electromagnetic wave the fields oscillate in phase, so a maximum value of the electric field (at a specific point and at a specific instant) will correspond to a maximum value of the magnetic field (as a minimum value will correspond to a minimum value, and so on).

In particular, students may assume a direct (or inverse, depending on their interpretation) relationship between the propagation speed and the oscillation frequency through the wavelength, starting from the relationship $v = \lambda f$ (or by placing

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[7] Actually, this would require a more complex discussion, as it would be necessary to analyze the amount of charge that is deposited on the plates of a capacitor in the circuit as time changes (the capacitor can be schematized as an oscillating dipole), and how the electromagnetic field (generated by this source) propagates in space.

[8] In the case of simple dielectric media, the relation 9.8 is immediately generalized by multiplying the constants (dielectric and magnetic) of the vacuum with the relative constants of the medium (which are always represented by values greater than 1, except for diamagnetic materials for which $\mu_r \cong 1$).

an equivalence in visual-lexical terms between “a wave oscillating faster with greater frequency” and “a wave propagating faster”). As a consequence, it is important to discuss what happens when an electromagnetic wave passes from one medium to another: since frequency is, as discussed above, a property determined exclusively by the source, it will remain constant, while the wavelength (and velocity) will vary.

On the other hand, the expression 9.8 does not show any relationship between the propagation velocity and the value of the electric and magnetic fields, or other quantities depending on the frame of reference we adopted: the consequence is the independence of this quantity from the relative velocity between source and observer, which is in contradiction with Galileo’s transformations[9]. In this way, this fundamental property of electromagnetic waves is derived from Maxwell’s equations, rather than a simple statement with no relation with the electromagnetic theory which the students were previously introduced to.

The propagation mechanism of electromagnetic waves can still be difficult for students to understand despite the previous discussion, due to the “abstract” nature of oscillating fields (which cannot be visualized). Thus, some “stimulating questions” were posed aiming at provoking a discussion that clarifies this mechanism. An example is: “after varying the oscillation frequency of the source, how long does it take for a variation in the fields’ oscillation frequency to be measured at a point placed at a specific distance from the source?”.

9.1.5 Fields’ detection

Once the discussion about the generation and propagation of electromagnetic waves has been completed, we can proceed with the discussion of their detection mechanism. With reference to the Hertz’s experiment, we can schematize the receiving apparatus as a simple circuit – with characteristics similar to the generator apparatus – closed by a capacitor: when electromagnetic wave arrive, an induced current is generated in the circuit, and the charging process of the capacitor starts. When the charge deposited on the capacitor’s plates reaches a critical value, a spark is generated between the plates, signalling the waves’ reception[10]. Furthermore, in this context the phenomenon of resonance can be briefly discussed: the theory of alternating current circuits (which the

[9] Thus, this property was supposed by Einstein as a postulate of the special relativity theory.

[10] The intensity of the spark can also be increased by adjusting the distance between the capacitor’s plates. In addition, its intensity will vary periodically according to the trend over time of the electromagnetic waves we are receiving.

students use to be previously introduced) ensures us that the components of the receiving circuit can be chosen in such a way as to maximize the current intensity. This phenomenon plays a fundamental role in signals' reception: indeed, it allows the detection of the waves corresponding to the resonance frequency of the receiver, avoiding the collection of indistinct noise.

For this purpose, we submitted another tutorial taken (with some modifications) from the paper by Ambrose et al. (1999): in this case we are dealing with the reception of electromagnetic waves, which will complete the discussion of the three phases characterizing the waves' nature (following generation and propagation).

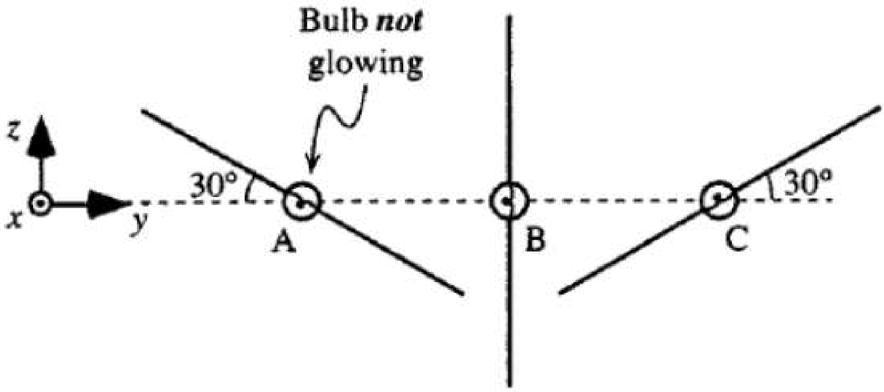

Figure 9.4: Figure taken from the paper by Ambrose et al. (1999). It represents a plane electromagnetic wave that is propagating in the direction out of the page, incident on the y-z plane where there are three light bulbs connected to different straight wires (oriented as in the figure). The text of the tutorial states that when the wave arrives, bulb A will not light up.

In the case illustrated in Figure 9.4, an electromagnetic wave is propagating in the direction of the x-axis (out the page) incident on a plane where three straight wires with different orientations connected to light bulbs lie. Knowing that bulb A does not light up, students are asked – through several intermediate questions – to compare the brightness of bulbs B and C after the wave arrives. Beyond the assumption of some hypotheses discussed in Chapter 4, students can recognize that the mediator of the interaction is given by the Lorentz force: since the force exerted by the magnetic field is always perpendicular to the charges' velocity (i.e. to the direction of the wire), this component can always be neglected. As a consequence, only the electric field provides an active contribution, so – if bulb A does not light up – the electric field will be perpendicular to this wire, even though it belongs to the y-z plane (since the wave is transverse and propagates in the direction of the x-axis). As a result, the electric field forms the same angle (of 30°) with both bulbs B and C, so the component of the electric field will be the same in both cases. As a result, the currents generated will have the same intensity, and the two bulbs will light up with the same brightness.

9.1.6 Energetic description of the field's propagation

At this point we are ready to face the discussion about energy, that is fundamental for the macroscopic description of the electromagnetic waves' propagation[11]. As mechanical waves, also electromagnetic waves carry energy without moving matter: moreover, in this case their propagation can also take place in a vacuum (which is the medium where we are describing the phenomenon). Students already know that electric and magnetic fields are associated with specific amounts of energy per unit volume, according to the following formula that expresses energy densities point by point[12]:

(9.10)
$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

However, rather than the energy density, the relevant quantity for the description of the electromagnetic waves' propagation is the intensity, since it provides a spatial and temporal location for the energetic analysis of the phenomenon: it can be defined as the amount of energy carried by the wave investing a unit surface – perpendicular to the direction of wave propagation – in one second. Moreover, it can be expressed in terms of fields by simple reasoning: we have already shown that in a time interval :Δt, an electromagnetic field propagates for a distance equal to $c\Delta t$; Considering any region of space through which the field is propagating, given its lateral surface S, the intensity I can be expressed through the intuitive relation:

(9.11)
$$I = \frac{uSc\Delta t}{S\Delta t} = uc$$

Furthermore, by placing the expressions 9.6, 9.8 and 9.10 in 9.11, after some trivial simplifications we will obtain the following relationship:

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[11] Nevertheless, in order to deal with the radiation-matter interaction, it would be necessary to overcome the classical electromagnetic theory by introducing a quantum description.

[12] Since students often lack an operational definition of the amount of energy associated with the fields, it can be defined as the work of the external forces necessary to bring into the current configuration the charges (or currents) which form the sources of those fields, starting from the configuration characterized by infinite reciprocal distances.

(9.12) $$I = \frac{EB}{\mu_0}$$

To emphasize the fact that energy propagates in the same direction in which the electromagnetic wave is propagating – and taking advantage of its transversal nature – the relation 9.12 can be reformulated in the following vector terms[13]:

(9.13) $$\vec{I} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$

It clearly shows how the intensity of electromagnetic waves depends on the square of the amplitude of the electric field (or magnetic field: according to the relation 9.6, they are directly proportional to each other), while it is independent of the oscillation frequency of the wave itself: we will come back on this aspect soon.

After these premises, the energetic description of the electromagnetic waves' propagation consists in the application of the law of energy conservation. Indeed, if we consider any region in vacuum in which an electromagnetic wave is propagating, this law implies that, if (during a time interval Δt) the total amount of energy U associated with the electromagnetic field present in that region is changing, it must be balanced by the variation of the flux ϕ_I (outgoing or incoming, depending on whether the energy contained in the volume is decreasing or increasing) of the intensity of the wave itself. Thus, we obtain:

(9.14) $$\Delta\phi_I + \frac{\Delta U}{\Delta t} = 0$$

Dimensional analysis and an example that clarifying the use of the sign + will help students account for this relationship. Although our discussion is limited to vacuum, it can be qualitatively generalized: when charges are present, the difference between the two quantities at the first member of 9.14 will be equal to the work that the electromagnetic field performs on them.

To provide a more concrete interpretation of the law of energy conservation in this context, let's come back to the case of an infinite plane crossed by currents. In order to no longer limit our analysis to the "transient phase", we are going to consider alternating currents. Thus, we will deal with a continuous source of electromagnetic

[13] Although 9.13 is often presented as the relation defining the "Poynting" vector (usually denoted by the letter $\vec{S}$), we believe that it is not necessary to introduce a different nomenclature than the one students may have already met in the context of mechanical waves (about intensity). In addition, in this way we have provided a proof – that is often absent in school textbooks – about why the Poynting vector is introduced in these terms. We also remark that, as the amount in 9.13 quickly changes over time following the fields' variation, we use to refer to the "average intensity".

waves, whose fields will respect the plane symmetry we presented above. In this case, a single “pulse” of electromagnetic wave (generated in a very short time interval) corresponds to an infinite plane – parallel to the source – in which both the electric and the magnetic field take on the same value: this single pulse represent a “wavefront”. Then, it will propagate in space moving away from the source, but – if we assume that we are in a vacuum – the energy contained in the wavefront will be preserved over time.

Another relevant aspect is that the size of the wavefront (in this ideal case) does not vary during propagation, as in this case it already has infinite extension at the initial instant. As a consequence, the energy carried by the wavefront has to be distributed on the same surface; thus, according to 9.14, the intensity of the electromagnetic waves does not vary with the distance from the source.

Another point of the discussion is that the geometry of the wavefront depends on the characteristics of the source, but in any case they will be perpendicular to the direction of propagation. Since the points belonging to a single wavefront oscillate in phase (at a specific instant, the fields assume the same value at all these points), the distance between two consecutive wavefronts corresponds to the wavelength[14].

Discussing plane waves, although they represent an ideal case, is of great relevance: indeed, they are usually introduced in traditional teaching, and it is also the case we will focus on in this last part of the learning path. This relevance can be attributed to several reasons: students can easily understand how, at a great distance from the source – i.e. in situations that are usually interesting to be analyzed – any wavefront can be geometrically approximated with a plane wavefront[15].

However, by varying the symmetries of the source, it is possible to obtain wavefronts that are spherical, cylindrical or referring to other geometric shapes. In these cases, a different situation is found compared to the case of plane waves: the size of the wavefronts increases as they move away from the source. As a consequence, to relation 9.14 shows how the intensity of the wave must decrease with distance, according to a trend determined by the particular geometry we are considering (for example, for spherical wavefronts the trend is $I \propto r^{-2}$).

[14] See the representation in Figure 7.2 (Chapter 7).

[15] Even though it goes beyond our purposes, this remark corresponded to the linearity of Maxwell’s equations, which implies that any solution of this system represents a superposition of plane waves with different characteristics (frequency, intensity, direction of propagation, etc.).

9.1.7 The electromagnetic spectrum

Before continuing with the analysis of plane electromagnetic waves, it is useful to observe that the propagation speed determined in 9.8 had already been calculated by Maxwell, who observed that it was compatible with the speed of light[16]. In addition, other experimental evidences are common to light and electromagnetic waves (e.g. interference and diffraction phenomena). As we remarked in Chapter 3, Maxwell had already hypothesized that light was made up of oscillating electromagnetic fields; On the other hand, it is important to discuss that not all electromagnetic waves constitute visible light. By analogy with the sound spectrum[17], there are regions of the electromagnetic spectrum – with lower or higher frequencies than the visible part – that differ according to the effects they produce in the interaction with matter. Students were introduced to a representation[18] describing some fundamental characteristics of the regions which the electromagnetic spectrum is usually divided into. Since the regions are defined according to the effects of the interaction, and frequency is not the only characteristic parameter of electromagnetic waves, the boundaries between the different regions of the spectrum are not clearly defined.[19]

Regarding the electromagnetic spectrum, a brief discussion can be addressed with the students, focused on the possible technological applications involving the different regions or the different types of sources, trying to stimulate attention on the diversification of the effects produced by different frequencies. For example, the use of radio waves in telecommunications can be attributed to the greater effects that diffraction phenomena have with longer wavelengths, thus providing signals a greater ability to overcome physical obstacles. High frequency radiation, on the other hand, is usually presented as “ionizing”, as it carries high energies (for example, it shows a higher ability to penetrate matter than low frequency radiation). However, this aspect may hide a contradiction with respect to the independence of the intensity of electromagnetic waves from the fields’ oscillation frequency (established by relation 9.13). It can be explicitly observed that dealing with the radiation-matter interaction

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[16] As we will discuss later, the speed of light was already known at that time, thanks to several experiments.

[17] Students should already be aware of infrasound and ultrasound, i.e. sounds characterized by particularly low (< 20 Hz) or high (> 20000 Hz) frequencies that are not audible to the human ear.

[18] This kind of representation can be found in almost every textbook: they use to report the 7 regions of the electromagnetic spectrum by indicating the order of magnitude of the wavelength, frequency and temperature of the objects at which that type of radiation corresponds to the wavelength of maximum emission.

[19] Nevertheless, we observed that school textbooks often report similar values.

requires a quantum description, which obviously goes beyond our purposes: by the way – looking at the historical development – one of the aspects that has challenged the classical theory of the electromagnetic field has been the interpretation of the photoelectric effect, regarding its dependence on the frequency of the incident radiation.

9.1.8 The representation of plane electromagnetic waves

After the description of the electromagnetic spectrum, let's go back to plane electromagnetic waves. We have not yet discussed the trend of the electromagnetic field over space and time: of course, it can be obtained by solving the Maxwell's equations system once the sources – as well as the initial and boundary conditions – are known. Even though the mathematical difficulties prevent us from proceeding in this way, for an electromagnetic wave (periodic signal) we can assume a simple sinusoidal trend[20]. Thus, if the wave holds plane symmetry, the electric and magnetic fields can be expressed as functions of time and position, according to the following expressions (relating to the fields represented in Figure 9.5[21]:

(9.15) $$\vec{E}(x,y,z,t) = E_0 \sin(kx + \omega t)\,\hat{z}$$

(9.16) $$\vec{B}(x,y,z,t) = B_0 \sin(kx + \omega t)\,\hat{y}$$

A mathematical proof of these equations is reported in Appendix D.

[20] Although students may consider sinusoidal signals as the simplest example, the hypothesis may also be justified on a rigorous basis. In fact, according to the Fourier's theorem, any periodic signal can be expressed as the sum of sinusoidal functions with different frequencies and amplitudes.

[21] The + sign in the argument of the sinusoidal function (in relations 9.15 and 9.16) is due to the fact that the electromagnetic wave represented in Figure 9.5 is propagating in the negative direction of the x-axis; in case of propagation in the opposite direction, it would be necessary to change it to the minus sign (this property can be determined from the representation by applying equation 9.9).

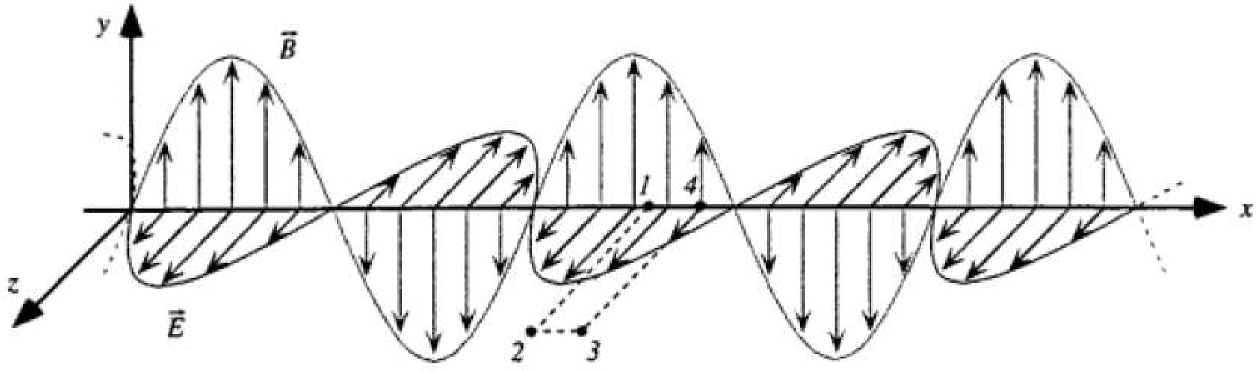

Figure 9.5: Figure taken from the paper by Ambrose et al. (1999). It represents the profile of a plane electromagnetic wave propagating in the negative direction of the x-axis. The points 1-2-3-4 form a rectangle, that will be used in a tutorial for calculating the circulation of the electric field.

The relations 9.15 and 9.16 express the synthesis of all properties discussed so far for plane electromagnetic waves, and the constants (k and ω) appearing in these relationships are directly related to the characteristic parameters (wavelength and period).

As we discussed in Chapter 7, representations of wave profiles (such as the one reported in Figure 9.5) represent a significant obstacle for conceptual learning: moreover, their misinterpretations may generate specific learning difficulties. On the other hand, their interpretation requires a high cognitive load: beyond being one of the few three-dimensional representations that high school students encounter in studying physics, it hides lots of information. Indeed, their captions often omit that it represents the fields' trend at a fixed instant – or at a fixed point in space, when the representation is given as a function of time – only reporting the fields' values at the points belonging to the propagation axis: it implies the symmetry property whereby the fields' values are the same at all points belonging to planes perpendicular to the propagation direction. In addition, there are some graphic elements that might confuse students, such as arrows representing vector fields below the wave profile, or axis' labels that might suggest a spatial extension attributed to the wave amplitude.

In order to overcome these learning difficulties, after explicitly discussing these details with students, they were submitted a tutorial taken from the paper by Ambrose and colleagues (1999), where they are asked to apply the Faraday equation (9.3) to the rectangular circuit shown in Figure 9.5: in this way, we also reiterate that plane electromagnetic waves are particular solutions of Maxwell's equations, thus these relationships have to be verified also in this case.

In particular, students will be able to calculate the sign of the electric field circulation along the rectangle considered – relating it to that of the second member of relation 9.3 – thus making inferences about the information that this provides on the temporal trend of the electromagnetic field propagation. Indeed, in the case analyzed, this sign tells us whether or not the magnetic field is increasing over time at the[22] points inside the area enclosed by the integration curve (of course, this applies to all points). Thus, students will be able to determine whether the magnetic field at a point P (that is any point inside the area of the rectangle) will take on a larger or smaller value at an instant immediately following the one which the representation refers to. This question will guide students to relate the elements deriving from spatial and temporal propagation, highlighting in particular how the representations contain implicit information, and finally allowing them to master them better.

Table 9.1 summarizes the "stimulating questions" submitted during the first version of the learning path, in order to guide students towards the achievement of the specific learning objectives and to collect information about their spontaneous reasoning: thus, we will also extract information for the revision (second version) of the learning path.

Table 9.1: List of activities characterizing the first version of the learning pathway, with related "stimulating questions" and learning objectives.

Learning objectives	Activities' description	Stimulating questions
Characterization of the (electric and magnetic) fields' sources	Reintroduction of Maxwell's equations	[Referring to the addition of external sources to Gauss surfaces in the application of relation 9.2] 1. Does the electric field change? 2. Does the flux of the electric field through the Gaussian surface change?
	Definition of the current intensity	3. Does a direct current flowing through a conductor generate an electric field (outside the conductor)?

[22] The direct information is actually related to the magnetic flux, but the fact that the integration surface and the direction of the magnetic field do not vary over time means that the information related to the field itself.

Learning objectives	Activities' description	Stimulating questions
Relationships between the fields' characteristics and their sources	Determining the fields generated by a point charge	4. Is it possible to derive the electric and magnetic fields generated by a stationary point charge from Maxwell's equations? If so, how? 5. Considering a new inertial frame of reference, will the fields generated by the charge be changed? If so, how? 6. What characteristic must the motion of the source-charge satisfy in order to determine an electromagnetic field varying over time in all inertial frames of reference?
Characteristics of electromagnetic field propagation	Determination of the propagation velocity of the electromagnetic field generated by an infinite conducting plane at the instant in which a uniform current starts flowing	7. [Knowing the directions of the electric and magnetic fields generated by the source we are considering] How can we apply Faraday's and Ampère-Maxwell's laws in this case?
	Determination of the delay effect in the propagation of electromagnetic waves	8. Assuming that, at the instant t^*, the oscillation frequency of the sources (charges) of an electromagnetic wave is changing from the value f_1 to the value f_2, will the frequency of the fields detected at a point P (at a distance

		d from the source *S*) change? 9. If so, is this change instantaneous or does it require a finite time interval? In this case, calculate it taking into account that the propagation direction of the wave forms an angle of 30° with the direction $\overline{SP}$ [information reported by a figure].
Learning objectives	**Activities' description**	**Stimulating questions**
Electromagnetic waves' detection mechanism	Determination of the modalities of electromagnetic waves' reception by means of circular and rectilinear antennas	10. What happens when an electromagnetic wave arrives on a ring-shaped conductor (closed by the plates of a capacitor)? 11. How could the periodic trend of electromagnetic waves be verified with such a device? 12. [Given the situation depicted in Figure 9.5] Compare the brightness of bulbs B and C. Justify your answer by indicating whether the bulbs light up.
Interpretation of the plane wave profile representation propagating over space	Application of Maxwell's equations in the case of electromagnetic waves starting from the representation of plane wave profiles	13. [Given the representation in Figure 9.5] Will the circulation of the electric field along the rectangle be positive, negative, or zero? 14. Will the magnetic field at point P (inside

		the area of the rectangle) take on a larger or smaller value at an instant immediately following that the representation refers to?

9.2 Description of the second version of the learning path

In this section we are going to describe only the modifications with respect to the first version of the learning path: a summary of the final version of the rationale is reported at the end of the chapter in Figure 9.10.

9.2.1 Fields' operators: the flux

The second version of the learning path starts with some preliminary activities related to the concept of field and the meaning of its operators (flux and circulation).

Let's start with the exploration of the (magnetic) properties of space through a compass (Michelini & Vercellati, 2012). We can observe that, in the absence of magnets, the compass needle at a point is always oriented in the same direction (also when rotating the compass); on the contrary, when magnets are present, the needle starts oscillating and its orientation changes (the same effect, as described above, is obtained by replacing the magnet with a circuit crossed by current). Thus, the surrounding space is modified through a property that we will call "magnetic field", which is able to make the magnet interact with other objects.

If, on the other hand, we use a set of compasses to explore space, we obtain a composition like the one shown in Figure 9.7. It will provide a representation of the field lines generated by a magnet in that plane, whose characteristics can be investigated[23]. Thus, we can recognize that there are no intersections between the field lines, that they seem to be closed – even though we do not know how they are directed where the source is placed – and that the distance between the homologous points of two consecutive lines does not seem to be constant (we will come back on this point shortly).

[23] An alternative modality to obtain a representation – that is easier to analyze – is to use a single compass and to move it in different positions, marking a segment representing the direction of the needle in every position. In addition, a three-dimensional representation of the field lines can be obtained by arranging the iron filings around a hollow cylinder, which the magnet is placed inside.

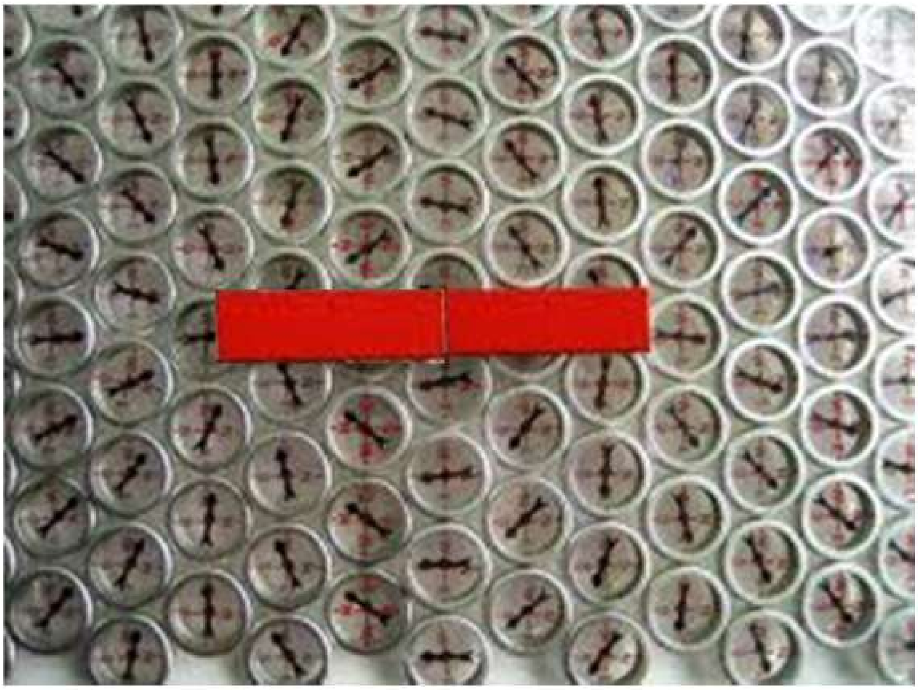

Figure 9.6: Carpet of compasses surrounding a magnet, providing a two-dimensional representation of the field lines.

The vector nature of the magnetic field can be easily demonstrated using two magnets and a compass: in fact, by arranging the magnets in mutually perpendicular positions, it is possible to verify that by approaching or moving away one of the two sources, the orientation of the compass needle changes according to the rules of vector composition. Of course, this quantity does not represent a force, as the trajectory of a ferromagnetic object will be directed towards the magnet (i.e., it will not follow the field lines).

Coming back to the field lines' representation, through a sensor we measure that the magnetic field is not constant along a line, but its intensity increases as it approaches the poles. Furthermore, if we measure the magnetic field in different points at the center between two field lines (i.e. being equidistant from the two lines), we find that it is inversely proportional to the square of the distance between the lines. This constant depends on the (pair of) field lines we are considering: nevertheless, by renormalizing the spatial metric – i.e. specifically choosing the distance between the lines – we can obtain a universal constant. As a consequence, we can define a constant quantity equal to the product of (the magnitude of) the magnetic field and the surface of the "flux tube" insisting between two field lines: by setting this relation in vector terms, we will obtain the usual definition of flux ($\phi_B \equiv \vec{B} \cdot \vec{S}$[24]), through which this operation will be introduced as the characteristic constant of the magnetic field.

9.2.2 Fields' operators: the circulation

After the introduction of the magnetic field and the concept of flux, we have to deal with the introduction of the electric field and the circulation operator[25]. By analogy with what we did previously, we introduce the electric field as a property of the space generated in the region surrounding a charged body[26]. The role of the explorer compass will in this case be entrusted to two metal plates with plastic handles (see Figure 9.7): if we place the plates side by side, and then we separate them at a point near the charged body, using a charge sensor we can observe how the two plates – that are initially neutral – have been charged with charges of opposite[27] sign, and by exchanging the plates also the measured charges will be of opposite sign. Moreover – beyond the experimental difficulties – it is possible to show how the amount of charge induced depends on the relative orientation of the plates: in particular, it can be observed that there is an orientation for which this quantity is minimal, and it is perpendicular to the orientation in which the charge is maximized. These observations is useful to characterize the property of the space – i.e. the electric field – inducing the charges as a vector quantity, as it shows properties of direction and sense.

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[24] This definition corresponds to the case of a uniform field $\vec{B}$ at the points of the surface S: it allows us to obtain the relation 9.1 as a direct consequence.

[25] These preliminary activities resume the approach of Prof. Francesco Dalla Valle – for a long time president of the Italian Association of Physics teachers (AIF: https://www.aif.it/il-prof-dalla-valle-ci-ha-lasciato/): however, in this regard no research papers have been published which we can refer to.

[26] The body can be an insulator charged by rubbing: in order to carry out the measurements (described below) without the body being discharged, a conducting body connected to a generator can also be used.

[27] Ideally, the amount of charge induced on the first plate should be exactly opposite to that induced on the second, but this result is very difficult to be experimentally obtained. In order to carry out this measurement, we used the PASCO charge sensor (see Appendix B).

Figure 9.7: A pair of metal plates with plastic handles (on the left) used to explore the electric properties of the space surrounding a charged body. For a rough measurement, it may be a PVC tube charged by rubbing it with a microfibre cloth (on the right).

To characterize it as a vector quantity, it is necessary to prove that the electric field also verifies the vector sum rule. By analogy with the magnetic field, we will use two more sources – two steel balls of different sizes connected to a generator, to make sure they are charged – to experimentally proceed with this verification, which requires the use of an electric field sensor (such as the type shown in Figure 9.8).

Figure 9.8: Electric field sensor working to verify the vector sum regular and to calculate the circulation along a generic curve.

The sensor allows the electric field component to be measured along the direction of the light indicator at its base. Thus, students can build an experiment in which they verify the vector sum rule, arbitrarily choosing the way in which to proceed[28]. For example, it is possible to measure the electric field at a point in space when only the first or second source is (alternately) placed, then measuring it at the same point when both sources are present. We remark that, in order to determine the direction of the electric fields we are measuring, we have to rotate the sensor while keeping it fixed at a specific point, in order to detect the direction in which the (measured) component is maximum: of course, this represents the direction of the electric field. Once the measurements of the component fields have been obtained, the value of the resulting field can be compared through the graphic method with the expected value (according to the vector composition rule).

Using metal plates, we previously showed that they are charged when an electric field is present, so their union forms an overall neutral body. Thus, the electric field can act

[28] An alternative modality by using a single source is to measure the components of the electric field at the same point while the orientation of the sensor changes. By placing a protractor at its base, it is possible to obtain the trend of this component as a function of the angle that they form with the electric field vector: if this function shows the trend of a cosinusoid, the vector sum rule is verified.

on the charges by moving them: it performs work on them[29]. In general, the work performed by the fields depends on the path taken by the moving object, unless it is a conservative field[30]. Thus, students can design a simple experiment to verify whether the electric field – in static conditions, i.e. when the source is stationary – is conservative or not. This property can be verified by showing that the work along a closed curve is zero, i.e. that the circulation of the electric field is zero.

To carry out this experiment, we have to approximate the closed curve with a broken line[31]. Thus – regarding Figure 9.8 – we want to demonstrate the relationship:

$$C_E(\Gamma) = E_1 l_1 + E_2 l_2 + \cdots E_{12} l_{12} = \sum_{i=1}^{12} E_i l_i = 0 \quad (9.17)$$

Where E_i represents the component of the electric field in the direction of the corresponding side (calculated at its midpoint) whose length is l_i. The measurement of the circulation (reported in appendix B) shows how it provides values compatible with zero within the measurement errors: this is verified also when the integration curve and the source of the field (and its position in space) change.

Thus, we have introduced the circulation as a characteristic constant of the electric field, similarly to what we did with the flux for the magnetic field: at this point we can check if the same properties are also verified by inverting the role of the fields. In particular, an experiment similar to the previous one can be carried out to study the behavior of the magnetic field circulation: the measurement shows that this quantity depends on the relative position between the sources and the integration curve, since in 9.4 the term *I* represents only the contribution of the currents enclosed by the circuit. On the other hand, if we consider permanent (stationary) magnets or the Earth's magnetic field as sources, the circulation turns out to be zero. Within the experimental limits[32], we can state that the circulation of the magnetic field is proportional to the

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[29] The sensor also shows how the electric field is parallel to the force exerted on the charges, so the work done by the electric field corresponds to the work of the corresponding force by less than a proportionality factor.

[30] Students are usually already familiar with the concept of conservative quantity in relation to the gravitational force (or field).

[31] The segments of the broken line should not be too long (otherwise the field may vary too much among its points) nor too short (it would greatly increase the number of segments forming the closed curve). Thus, we made explicit the operational definition of the circulation, in case the field is not uniform in the region we are considering.

[32] This measurement actually involves many experimental complications. Indeed, for currents' intensity we usually measure in educational laboratories (1-10 A), the magnitude of the Earth's magnetic field is similar to the magnitude of the magnetic field generated by the current at small distances (a few centimetres) from the circuit in which the current flows (we used vertical wires about

intensity of the "enclosed" currents, whose sign takes into account the direction in which they flow.

9.2.3 The electromagnetic induction

To complete the reintroduction of Maxwell's equations, we can deal with the phenomenon of electromagnetic induction by means of a simple apparatus consisting of a double-sided magnet (see Figure 9.9). Inside the magnet, we can immerse a circuit (consisting of numerous spiral windings) connected to a galvanometer: it signals the induced current. Using (quite) accurate instruments, we can show that the induced current occurs in the following three cases: 1) when the magnetic field varies over time (this happens by varying the distance between the two sides of the magnet); 2) when moving the surface of the circuit immersed in the magnetic field (i.e. continuously moving the circuit inside and outside the region of space delimited between the sides of the magnet); 3) by varying the orientation between the magnetic field and the area of the immersed circuit (i.e. by rotating the circuit). Considering all possible cases, it is necessary to hypothesize the existence of an electric field responsible for the induced current, which would be detected also when the circuit is not present (its presence may be detected by means of a voltmeter). Thus, the measured quantity corresponds to the circulation of the electric field[33], which is (experimentally) directly proportional to the speed of variation of the magnetic flux enclosed by the (real or ideal) circuit we are considering[34].

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2 meters long). Taking into account that we can measure – through the sensor – the component of the magnetic field in a specific direction, we tried to partially remedy this situation by measuring at points where the Earth's field (whose direction can be easily detected by means of a compass) is perpendicular to the field generated by the wire (whose field lines we know by assuming that the wires have infinite length).

[33] In this regard, the reader can refer to the discussion reported in Chapter 4 about the distinction between potential difference and (induced) electromotive force.

[34] This is true when considering short time intervals, as we are not going to introduce the formalism of (temporal) derivatives for Maxwell's equations. Taking into account the direction in which the induced current flows (or the electromotive force is determined), we obtain Faraday's law (relation 9.3).

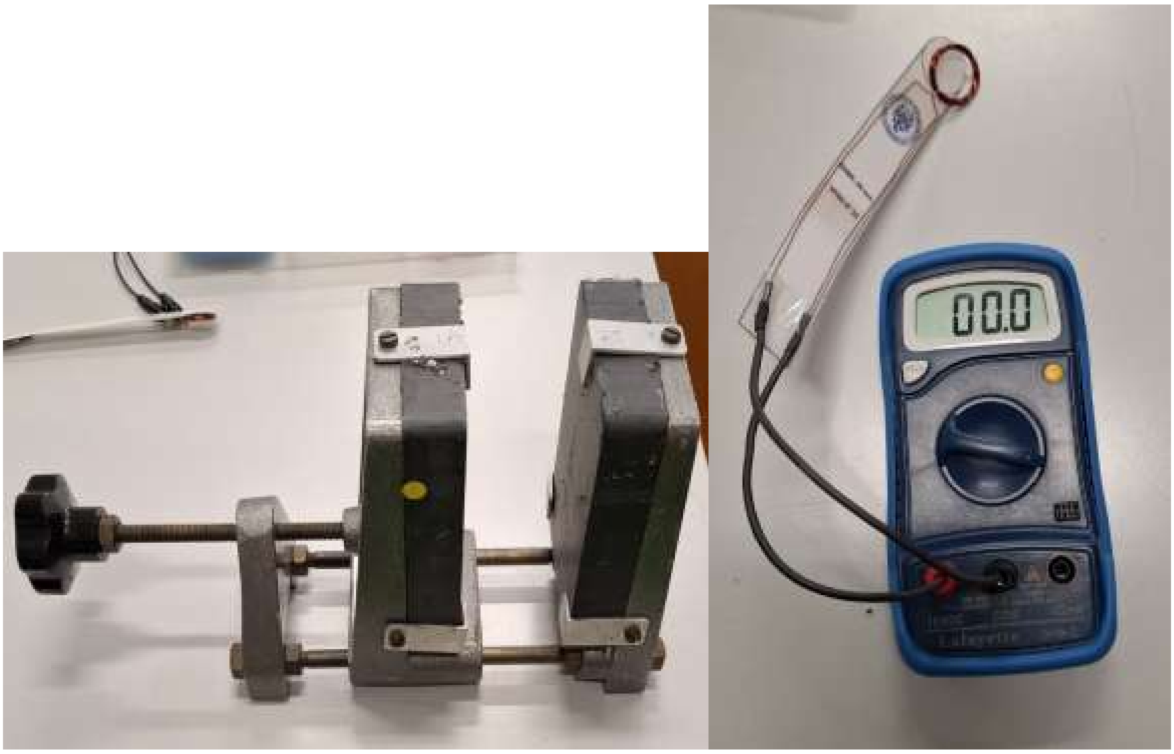

Figure 9.9: Devices we used for the electromagnetic induction experiment: double-sided magnet with adjustable distance (on the left) and resistive circuit connected to a multimeter (on the right).

As we already discussed, the Faraday's law points out a correlation between electric and magnetic fields through their operators (flux and circulation), which is completed by symmetry by the Ampère-Maxwell's law[35]. These relationships can be exploited to determine the propagation velocity of the electromagnetic field in the case of a source consisting of an infinite plane crossed by uniform (surface) current, as in the first version of the learning path. The preliminary activities, in this case, will hopefully have provided the students with a more precise conceptual meaning to flux and circulation – which represent the conceptual referents in the analysis of the propagation phenomenon – as well as an adequate justification of their introduction as peculiar characteristics of the fields.

[35] As we know, the law about the magnetic field circulation is modified in the dynamical case according to relation 9.4. The introduction of the additional term in our path follows the traditional reasoning present in textbooks (see for example Cutnell and Johnson, 2010). It takes up the Maxwell's explanation, where the circulation of the magnetic field is calculated across the edge of different surfaces containing (or not containing) the spatial region between the capacitor's plates (being charged). This leads to an inconsistent conclusion, that is resolved by assuming the existence of the "displacement current".

9.2.4 The electromagnetic field

However, this discussion is preceded by a more in-depth discussion regarding the fact that the electric and magnetic field actually form a single physical entity – the electromagnetic field – whose propagation properties we are going to study. Indeed, by resuming the discussion – addressed in the first version of the learning path – regarding the fields generated by a point charge, we now add a further step relating to the transformation of fields between two inertial frames of reference. We have previously observed how the same source generates different fields for different observers. However, we remarked it without referring to the fundamental reason why it requires to suppose the existence of a single entity: indeed, these fields can act on charges – different from the sources – exerting forces given by the Lorentz formula (in the case of point charges). If we consider the two (electric and magnetic) components of this force, both will depend on the frame of reference: in particular, a generic inertial observer will measure a force exerted by the magnetic field, which will be absent in the frame that is moving with the source. Apparently, this contradicts the invariance of the force between inertial frames (which characterizes classical mechanics): in order to save this theory, it is necessary to hypothesize that the transformations (between two inertial frames) of the (electric and magnetic) fields are mutually linked to each other, so that the overall (electromagnetic) force remains unchanged[36].

Thus, electric and magnetic fields can be considered – like Suárez and colleagues (2022) suggest – as two components of the same physical entity, being correlated with each other through Maxwell's equations, by means of the integral operators (flux and circulation). Indeed, we have already observed that relations 9.3 and 9.4 show how these two fields cannot be linked by a cause-effect relationship, and this constitutes the second element in favor of the unity of the electromagnetic field. The third – and last – aspect we have discussed in this regard is related to the sources' unification: under dynamic conditions, we can no longer distinguish between the sources of electric fields and those of magnetic fields, as any distribution of charges (or even magnets) will always generate an electromagnetic field. In the discussion with students, these three elements are separately presented, and then they are explicitly related each other to

[36] Actually, we know that the theory of special relativity replaces Galileo's transformations with Lorentz's transformations, thus establishing the laws of transformations for the tensor quantity "electromagnetic field", which cannot be derived through the apparatus of Newtonian mechanics. Nevertheless, the previous considerations are verified within the classical limit approximation: in our opinion, they are relevant from the educational point of view in providing a complete framework for the electromagnetic fields' description.

support our thesis (before proceeding with the discussion related to the characteristics of the electromagnetic field propagation).

9.2.5 Electromagnetic field's sources

In this second version of the teaching proposal we also used multiple-choice "stimulating questions"[37], which can elicit alternative reasoning (that were detected from the results of the first version) and a consequent discussion among peers that – under the supervision of the teacher – should guide students towards the correct interpretation of the phenomenon. An example is question n.10 reported in Table 9.2, concerning the possible modalities to generate an electromagnetic field. In particular, students have to select the possible sources, choosing among a charge (virtually stationary), a magnet (virtually stationary), a charge in motion or the charge-magnet set (thus each source alternately generates the two components, which will subsequently form a single entity propagating in space). This question – that is preliminary to the discussion on the electromagnetic field propagation, aims to strengthen students' understanding of the aspects discussed so far about the relationship between the fields.

Another example of a multiple-choice question will be discussed after characterizing the accelerated charges as sources of electromagnetic waves (as described in the previous paragraph): in this case, questions n. 8 and 9 of Table 9.1 will be followed by the reinforcing question – n.14 in Table 9.2 – related to the termination of the source's motion, which since that moment will no longer produce electromagnetic waves. Thus, students are asked what will happen about the electromagnetic field detected at any point at a specific distance from the source, by indicating whether it will cancel instantaneously or it will continue taking on the same values because of the independence of the field propagation from the source (or it will cancel out starting from a specific moment onwards). This question aims to complete the description of the "delay effect", investigating all its conceptual dimensions.

After this discussion, a phenomenological section has been inserted regarding the measurement of the speed of light (as a visible region of the electromagnetic spectrum). A brief historical discussion has been conducted, pointing out that until the seventeenth

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[37] These questions have 4 possible answer options: each student is provided with 4 cards of different colors, which can be shown when choosing the option. Students are asked to show – at the same time – only one card; thus, a class discussion starts under the teachers' guide, whose modalities depend on the resulting options' distribution.

century there was debate about whether or not such speed was infinite, while later several experiments converged towards the currently known value: the complete discussion is reported in Appendix C.

9.2.6 Electromagnetic waves' detection

The last difference with the first version of the learning path is represented by the tutorial used to deal with the detection phase of electromagnetic waves. Indeed, the one we previously submitted is often difficult for students to deal with, especially because of the geometric reasoning it requires. Thus, we chose to replace it with question n.17 (reported in Table 9.2), where the representation of a plane wave profile is provided – as a function of time – asking in which direction the antenna of a portable radio should be oriented in order to get the best possible reception of this (radio) signal. It represents a variant of a tutorial reported by Ambrose and colleagues (1999), but some answer options have been added: the antenna may be oriented in the direction along which the electric field or the magnetic field oscillate, or in the propagation direction; furthermore, students may establish that this direction varies over time following the periodic trend of the signal. Thus, although the context is simplified, the conceptual content of the question remains unchanged.

Finally, "stimulating questions" n.18 deals with the phenomenon of resonance, in the context of radio wave reception. Thus, we want to investigate students' conception of this phenomenon, in particular regarding the reason why it is possible to select a specific signal to be detected: this operation may be attributed to a change in the properties of the signal that is propagating, or in the parameters of the receiving circuit, or even students may report other alternative conceptions.

Table 9.2 summarizes all the "stimulating questions" used during the second version of the educational proposal (including preliminary activities) to guide students towards the achievement of specific learning objectives, and to collect information on their spontaneous reasoning.

Table 9.2: List of activities characterizing the second version of the learning pathway, with related "stimulating questions" or tutorials (multiple-choice questions followed by a class discussion) and learning objectives.

Learning objectives	Activities' description	Stimulating questions or Tutorials
Characterization of the concept of (electric) field	Exploration of the electric properties of the space surrounding a charged body with a pair of metal plates	[After placing the plates side by side at a point near the charged body] 1. Are the two plates charged? Why? 2. Does the combination of the two plates keep the same properties after the measurement? 3. Does the property of space we identified show a scalar or vector nature? How can we experimentally verify it?
Conceptual characterization of circulation	Study of the conservative nature of the electrostatic field	4. Is the electric field conservative? How can we determine it experimentally? What are the variables to consider when exploring this aspect?
Characterization of the sources of electric and magnetic fields	Definition of the current intensity	5. Does a direct current flowing through a conductor generate an electric field outside it?
Characterization of the entities acting in electromagnetic induction phenomena	Determination of the magnetic field flux as a conceptual referent of the phenomenon	6. In the general case, can the external magnetic field have determined the induced current?
Relationships between the characteristics of fields and their sources	Determining the fields generated by a point charge	7. [After determining the electric and magnetic fields generated by the point charge in an

		inertial frame of reference in which it is stationary] If we consider a new inertial frame of reference, will the fields generated by the charge be modified? If so, how? 8. Consider the forces that these fields exert on charges other than the source. Will they depend on the frame of reference? 9. If we consider a charge in accelerated motion, what does it change?
	Unification of electromagnetic field sources	10. [*Question with options*] How can we generate an electromagnetic field? A. A charge and a magnet are necessary together, so that the former generates the electric component and the latter generates the magnetic component of the field. B. We only need a single charge. C. We only need a single charge in motion. D. We only need a magnet.
Learning objectives	**Activities' description**	**Stimulating questions or Tutorials**
Characteristics of electromagnetic field propagation	Determination of the propagation velocity of the electromagnetic field generated by an infinite conducting	11.[Once the directions of the electric and magnetic fields generated by the source we are

	plane at the instant in which a uniform surface current starts flowing over it	considering are known] How can we apply the Faraday's and Ampère-Maxwell's laws to the case we are considering?
	Determination of the "delay effect" in the propagation of electromagnetic waves	12. If at the instant t^* the oscillation frequency of the charges forming the sources of an electromagnetic wave changes its value (from f_1 to f_2), will the frequency of the fields detected at a point *P* (at a distance *d* from the source *S*) change? 13. If so, is this change instantaneous or does it require a finite time interval? In this case, calculate it taking into account that the propagation direction of the wave forms with an angle of 30° with the direction $\overline{SP}$ 14. [Regarding the previous question; *question with options*] At a specific instant t* the source stops oscillating. From this instant, what is the value of the electromagnetic field detected in P? A. It will cancel out instantly, because in the absence of sources no field will be detected.

		B. It will take the same values (compared to the case of an oscillating source), because the propagation of the field is independent of the source. C. Initially it will continue assuming the same values, then it will cancel out after a specific instant. D. We do not have enough information to answer, as the values the fields take on only depend on the propagation medium.
Learning objectives	**Activities' description**	**Stimulating questions or Tutorials**
Electromagnetic wave detection mechanism	Determination of the reception modalities of electromagnetic waves by means of circular and rectilinear antennas	15. What happens when an electromagnetic wave arrives on a ring-shaped conductor (which is closed by a capacitor)? 16. How can the periodic trend of the electromagnetic waves be verified with such a device? 17. [Given the representation of a plane wave profile as a function of time; *question with options*] In which direction should the antenna of a portable radio be oriented in order to get the best possible

		reception of this radio signal? A. In the oscillation direction of the magnetic field. B. In the oscillation direction of the electric field. C. In the propagation direction of the electromagnetic wave. D. The direction of best reception varies over time, following the periodic trend of the signal. 18. [*Question with options*] Why does an antenna (receiving all the radio waves) makes you hear only one radio channel, and you have to change the frequency to change the radio channel? A. Because in this way you change the frequency of the received signal. B. Because in this way the parameters of the receiving circuit are changed. C. It is an operation related to the technical functioning of the radio, and it is not based on any physical principle or phenomenon. D. Because this operation compensates for the variation in the

		received frequency due to the Doppler effect.
Learning objectives	**Activities' description**	**Stimulating questions or Tutorials**
Interpretation of the representation of the plane wave profiles propagating over space	Application of Maxwell's equations in the case of electromagnetic waves, starting from the representation of a plane wave profile	19. [Given the representation in Figure 9.5] Will the circulation of the electric field along the rectangle be positive, negative, or zero? 20. Will the magnetic field at point P (inside the area of the rectangle) take on a larger or smaller value at an instant immediately following that of the representation?

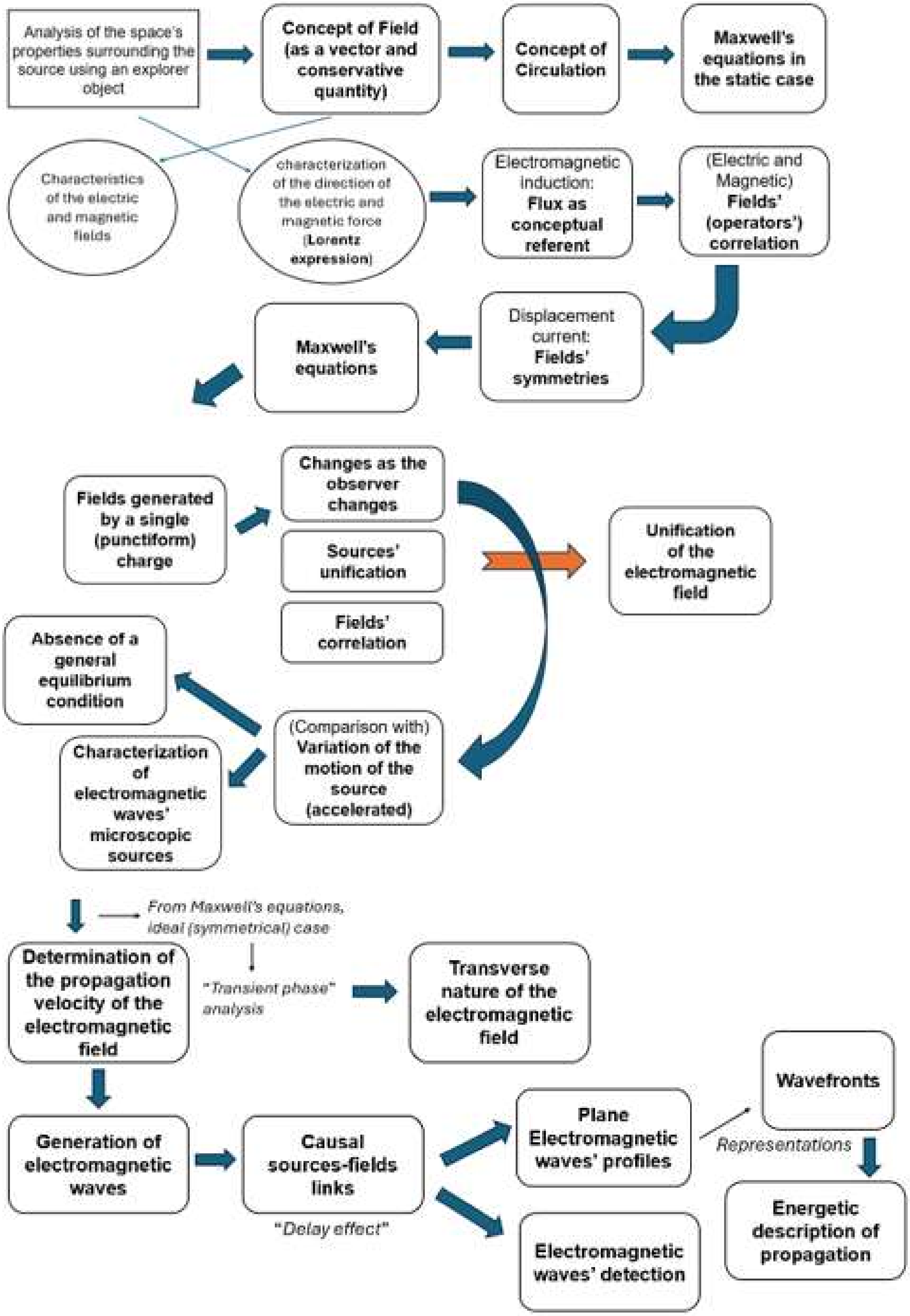


Figure 9.10: Summary of the rationale of the (second version of the) learning pathway.

Chapter 10

Results

In this chapter we are going to report the results of the two experimentation of the learning path described in Chapter 9, according to the methods illustrated in Chapter 8. They relate to the questionnaires (submitted before or after the teaching activities) described in Paragraph 8.2, including some observations relating to the answers provided to the "stimulating questions" asked during the activities and reported in Tables 9.1 and 9.2. These results will provide indications about the aspects that were modified in the second version, and about other details that can be improved for possible future applications, according to the discussion reported in Chapter 11.

10.1 Results of the first version of the learning path

In the following, each figure shows a bar graph illustrating the comparative pre-test and post-test results for the research sample n. 1.1 (N = 65, see Paragraph 8.2), representing the operational categories that characterize all response profiles. They take into account the justification provided by students, as described in Chapter 8: at this stage, the selected option influences the analysis only when complete information is missing in the open answers, thus making the categorization more difficult. The results take into account all – and only – the answers of the students who have completed both tests, choosing at least the answer option for each question.

In Figure 10.1 the results of the first question show that the emerging categories confirm the classification of the answer options, despite the largest category is represented by alternative answers: they result in very different interpretations, and they are often inconsistent with respect to the question (i.e. in our opinion, they cannot be further classified). This category also doubles its size in the post-test, showing that the interpretation of the representations of the plane wave profile is a difficult task for students (the difficulty index is 29%). Nevertheless, the Bowker test still shows a fairly significant change after treatment ($\chi^2[10] = 18.04, p \cong 0.05$), due to the lower prevalence of alternative reasoning in the answer options: in particular, a drastic drop in the number of students identifying the absence of the (electric) field in the region above the waveprofile is observed (21 students in the pre-test vs 6 in the post-test). We remark that most students who correctly identify the field values at the different points make some explicit reference to their common belonging to the same wavefront ("the points lie on the same plane, as a consequence the values of the fields will be equal").

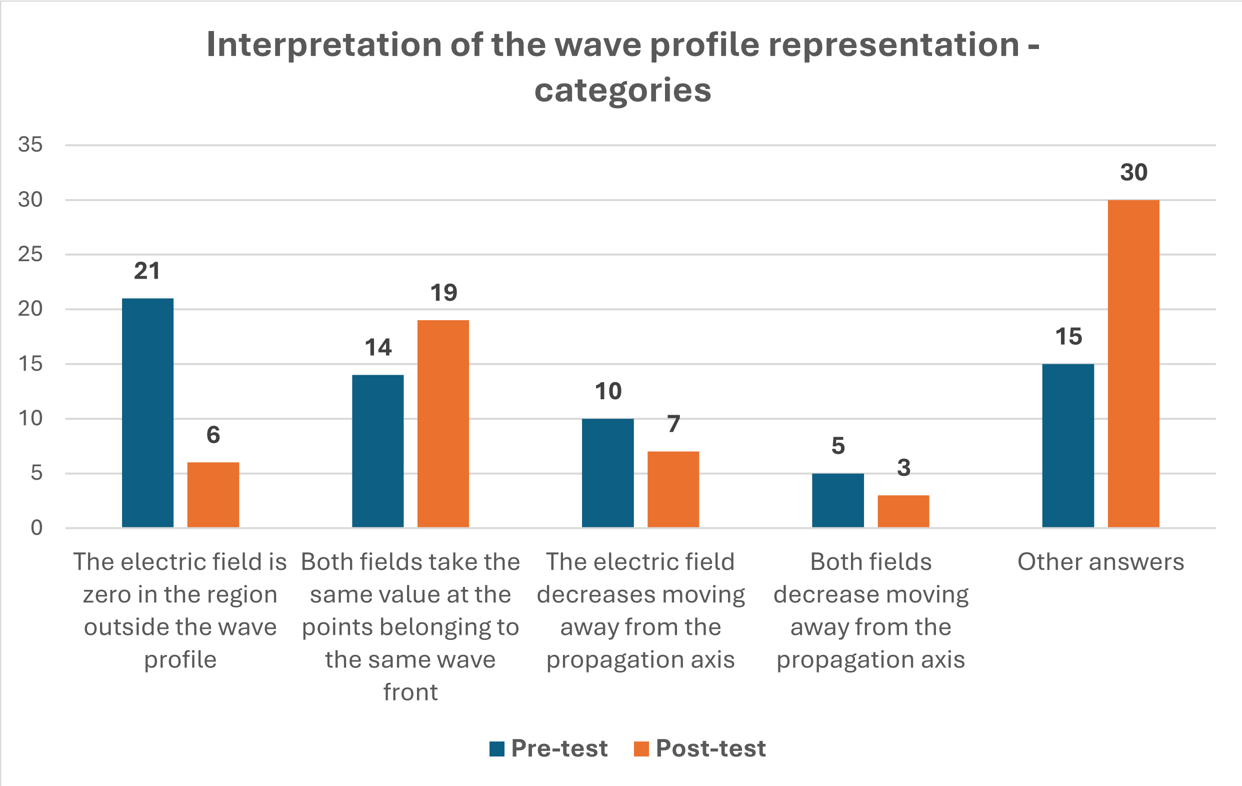


Figure 10.1: Categories emerging from the analysis of question n.1 (see the first version of the questionnaire in Paragraph 8.3.1) with comparison between the pre-test and post-test frequencies.

The results of the second question – shown in Figure 10.2 – are much more controversial. First of all, four categories emerge, ambiguously declining the meaning of the answer options: probably not all students were able to detect – even implicitly – the active role of the frame of reference in determining the motion of the source. The Bowker test shows a very significant change ($\chi^2[6] = 25.09, p < 0.01$) as the teaching treatment has succeeded in eliminating the category of independence between the temporal trend of fields and the motion of their source, that is characterized by a static conception of fields as a fixed spatial property in relation to the sources' presence. Despite this, the number of students who explicitly refer to specific variations in the fields detected – in relation to the frame of reference we are adopting – decreases (6 in the post-test vs 9 in the pre-test). Furthermore, the number of students who characterize the electric field generated by a charge in uniform motion as static, or place an equivalence between a single point charge in motion and a direct current increases (34 in the post-test vs 29 in the pre-test).

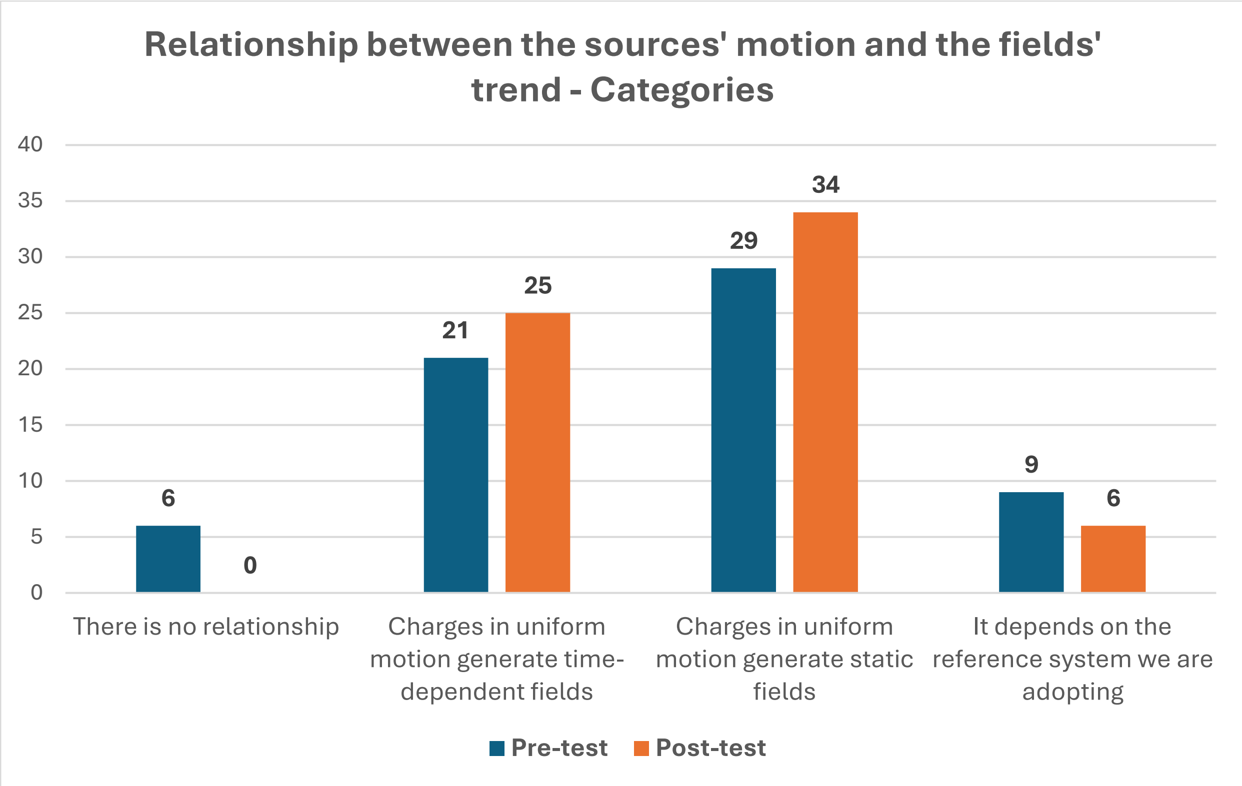


Figure 10.2: Categories emerging from the analysis of question n.2 (see the first version of the questionnaire in Paragraph 8.3.1) with comparison between the pre-test and post-test frequencies.

Figures 10.3 and 10.4 report the results of the third and fourth questions, which analyze the relationships of the propagation velocity of electromagnetic waves with different quantities: the fields' oscillation frequency, the relative observer-source velocity and the propagation medium. The Bowker test shows no significant differences for both question n.3 ($\chi^2[10] = 7.59, p > 0.05$) and question n.4 ($\chi^2[6] = 10.59, p > 0.05$), due to the drastic decrease in the "other answers" categories, which mainly included the students' responses who believed there was not enough information to answer the questions. A relevant difference emerge looking at the reasons provided in the identification of the relationships: they are almost always absent or inconsistent in the pre-test, while in the post-test – for both questions – the reference to the relation 9.8 often emerges. In this case, the difficulty index is lower in question 3 (18%, compared to 46% in question 4), although the improvement is similar in percentage terms. This may be due to the fact that a large majority of students (about 64%, that is very similar after treatment) identify a relationship of direct or inverse proportionality between velocity and frequency, often making implicit or explicit reference to the relationship $v = \lambda f$.

The reference to this relationship is in some cases present also in the pre-test answers, as the students know it from the study of mechanical waves. Two different mathematical interpretations of the same relationship emerge – direct or inverse proportionality – which generally do not specify which quantities have to be considered variable or constant.

Furthermore – in the answers to question n.4 – recurring references to the Doppler effect emerge, in particular in the category identifying an increasing relationship between the propagation velocity of electromagnetic waves and the sources' velocity, when it is approaching the observer. Also in this case, the references to the previous study of mechanical waves is evident.

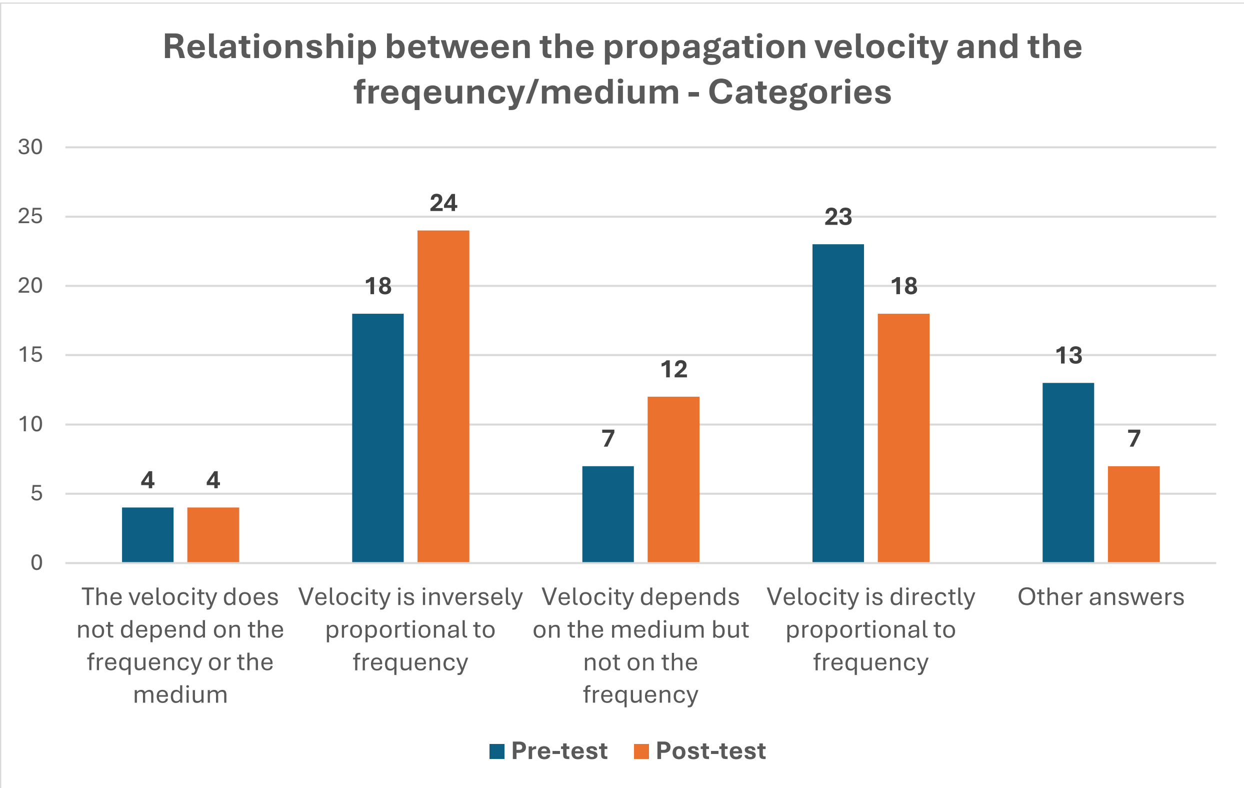


Figure 10.3: Categories emerging from the analysis of question n.3 (see the first version of the questionnaire in Paragraph 8.3.1) with comparison between the pre-test and post-test frequencies.

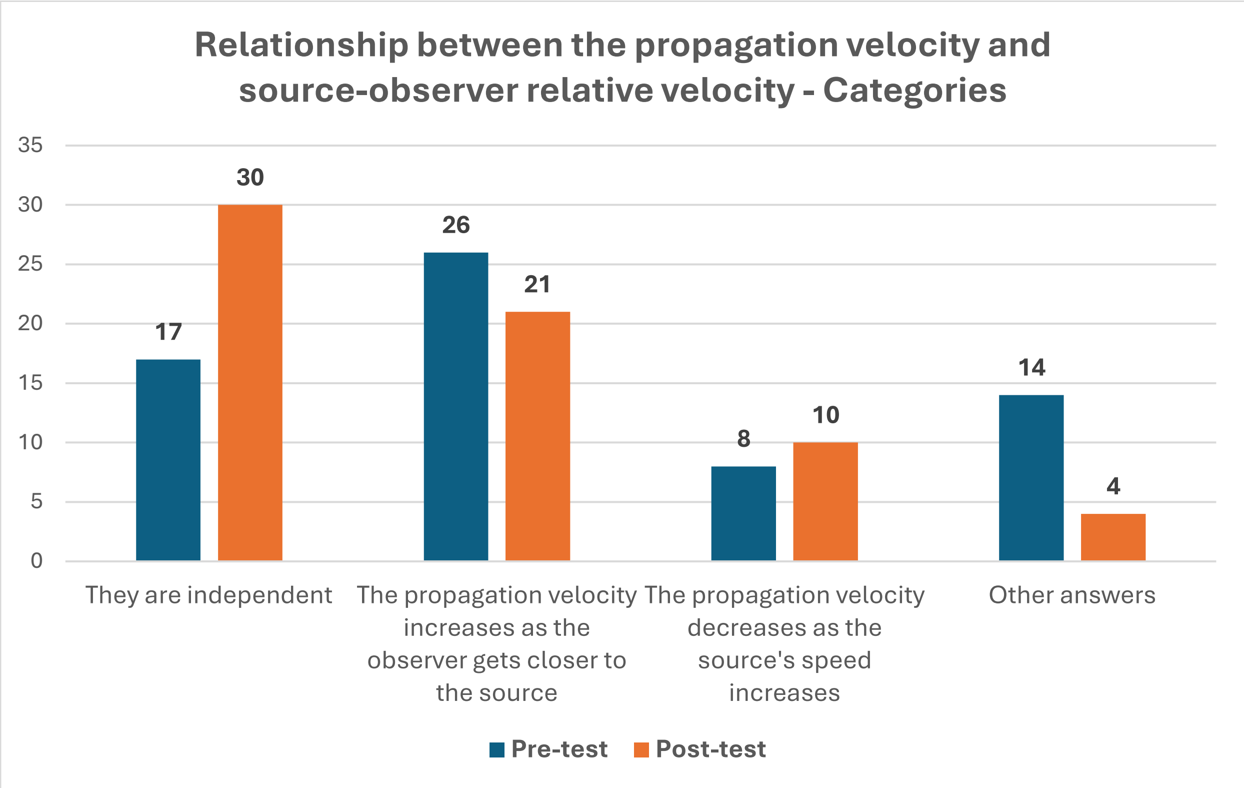


Figure 10.4: Categories emerging from the analysis of question n.4 (see the first version of the questionnaire in Paragraph 8.3.1) with comparison between the pre-test and post-test frequencies.

As in previous cases, also for question n.5 four categories emerge (see Figure 10.5), but this time the drastic decrease in the "other answers" makes the variation very significant ($\chi^2[6] = 18.89, p < 0.01$). The fact that two different parameters – wavelength and amplitude – are involved in the text means that (especially in the pre-test) the last category was dominated by sorting based on criteria – which took into account a dependence on both quantities – that were not easily decipherable when further explanations are not reported. Furthermore, in the hypothesis of non-linear trends, the students were asked a quantitative reasoning, taking into account the different quantities represented in the figure.

Although half of the students identify the (increasing) relationship between the amplitude of the electric field and the wave intensity – beyond its independence of the wavelength – nobody makes explicit the quadratic dependence on amplitude. Moreover, although the figure represented the fields' spatial trend by defining the values of the wavelengths in the three cases, most of the answers – in particular in the first and third categories – explicitly refer to frequency: this is surprising because, unlike the previous questions, the reference to the behavior of mechanical waves is absent. In this regard, a negative result consists in the increasing total number of students (19 in the post-test vs 10 in the pre-test) who report an inverse proportionality relationship between intensity and frequency, or the independence of the intensity from the amplitude of the electric field.

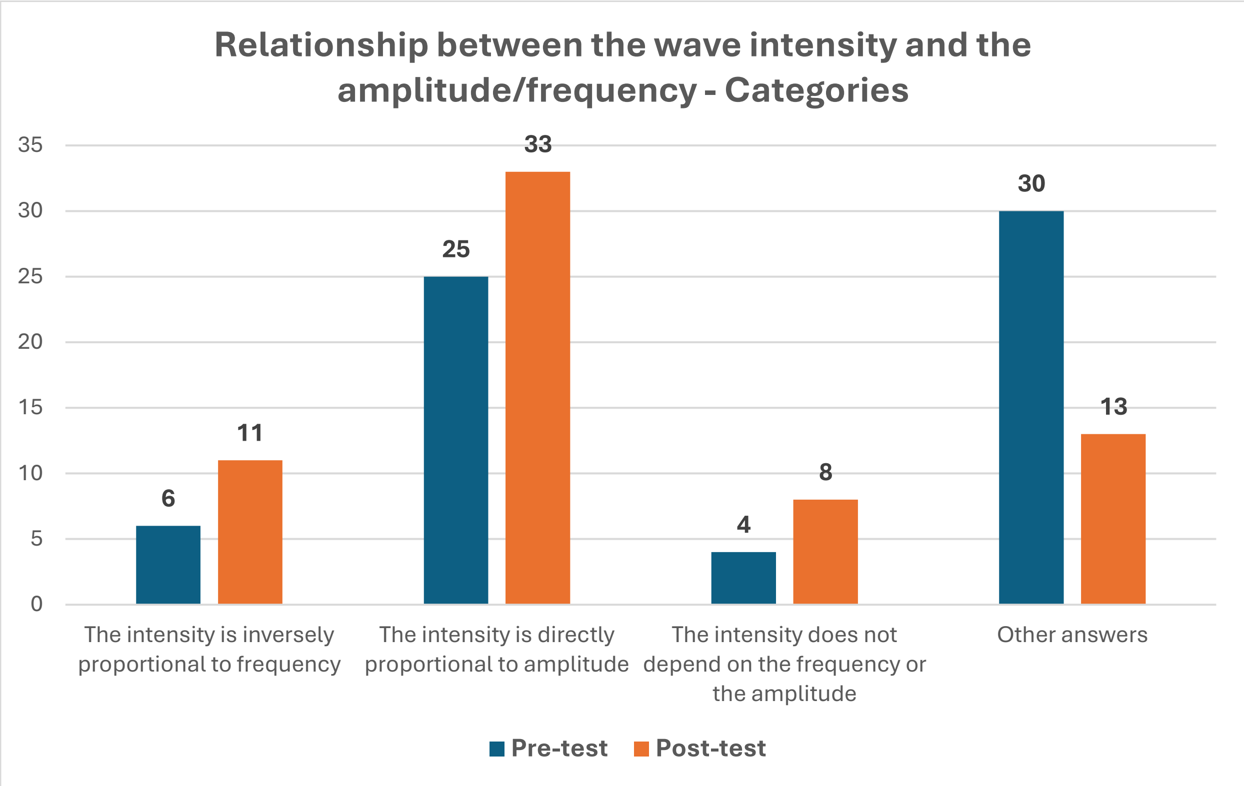


Figure 10.5: Categories emerging from the analysis of question n.5 (see the first version of the questionnaire in Paragraph 8.3.1) with comparison between the pre-test and post-test frequencies.

The last question – whose results are reported in Figure 10.6 – is peculiar as it shows an increasing number of categories compared to the answer options. Indeed – both before and after the teaching activities – the students who characterize the direction of signals' best reception as constant (i.e. the direction in which an antenna should be oriented) are divided into two groups: those who believe that this direction coincides with that of the electric field oscillation, and those who believe that this direction is indefinite (i.e. it cannot be determined a priori by the signals' characteristics, but it can be experimentally determined). Surprisingly, although no answer emerge that explicitly refer to the oscillation direction of the magnetic field as the direction of best reception, in the pre-test there are not answers mentioning to the Lorentz force as a mediator of the interaction (also for the categories who provide a correct answer). This motivation is made explicit in the post-test by almost all the students in the corresponding category: the difficulty index shows a very significant variation (8% in the pre-test vs 32% in the post-test).

The "other answers" category includes the answers of students who claim that other information is needed to answer the question – without further specifications – and those that, more or less explicitly, refer to the variation over time in the direction of best reception. However, they believe that this direction is not related to the signals' trend; rather, it would be determined by environmental or technological characteristics (not better specified) of the receiving apparatus. The drastic decrease in the size of this category contributes to the significance of the Bowker test ($\chi^2[6] = 14.91, p = 0.02$).

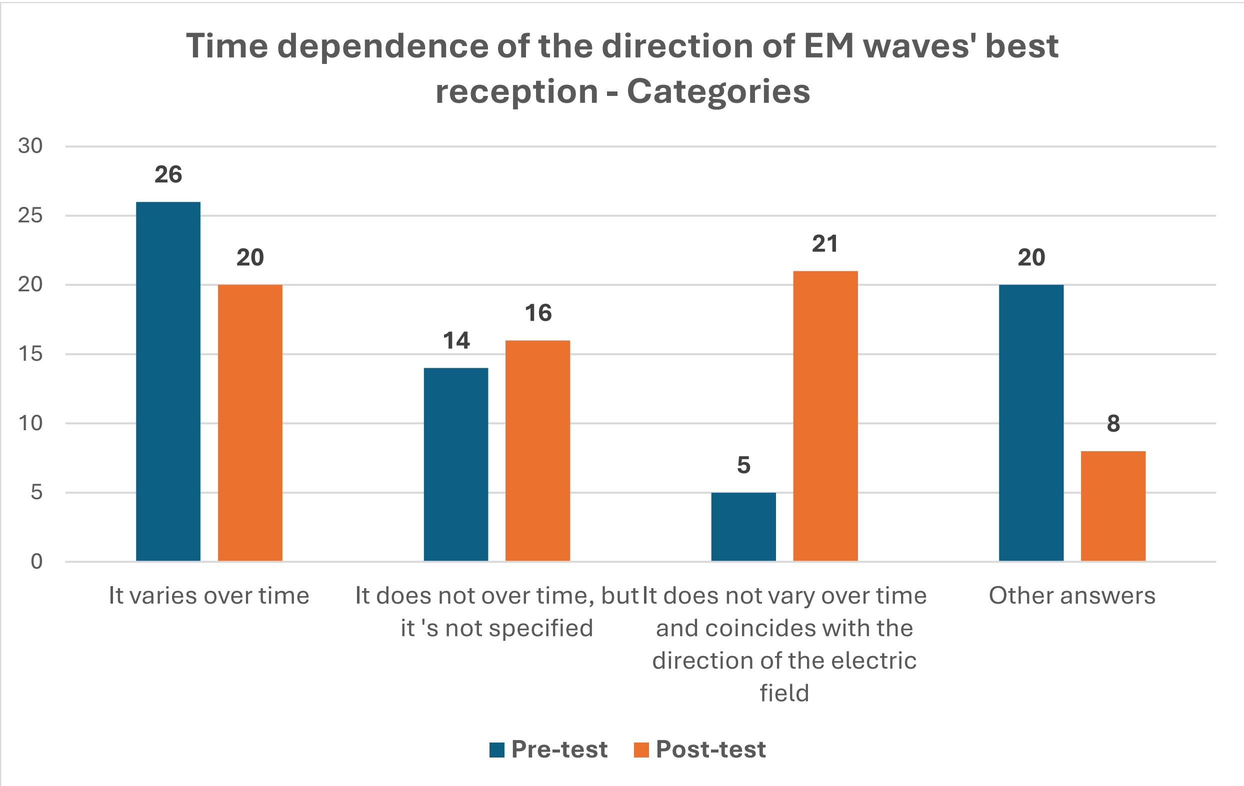


Figure 10.6: Categories emerging from the analysis of question n.6 (see the first version of the questionnaire in Paragraph 8.3.1) with comparison between the pre-test and post-test frequencies.

We conclude this paragraph with some remarks regarding the most significant answers we obtained in class through "stimulating questions", taking into account the most recurrent alternative conceptions: in Chapter 11 we will discuss how these remarks –

together with the results reported above – led to the modification in the second version of the learning path.

Students seem usually to be able to recognize – in several contexts – the difference between the fields' sources and those contributing to flux and circulation, thus demonstrating that they can distinguish between the concept of field and those of operators. On the other hand, it is more complicated to recognize the sources of electric and magnetic fields: for example, a direct current is usually considered a source of an electric field outside the conductive circuit; the difference between stationary and moving charges is recognized, but not between direct and time-varying currents; permanent magnets are considered to be (exclusive) sources of magnetic fields, even when they are in motion. Furthermore, the difficulty that emerged from the previous analysis regarding the active role played by the observer in determining the motion of the source – and, as a consequence, the values of the detected fields – is confirmed.

Although the students demonstrate they know how to apply Maxwell's equations to the case of a plane source, the transition from the analysis of the "transient phase" to the continuous production of electromagnetic waves is not immediate: indeed, most students do not seem to be able to determine the time interval necessary for the variations in the characteristics of the source to affect the variations in the field detected at a specific distance from the source. In particular, it is difficult for students to identify variations in the oscillation frequency (of the source and fields) and to take into account the direction of wave propagation in determining the time interval the phenomenon need to occur. This is less surprising if we consider that students often spontaneously assume the identity of the fields' and sources' frequencies, which is considered valid at every instant.

In analyzing the detection of electromagnetic waves through a circuit, students adopt different reasoning depending on the context we are considering: indeed, we can refer to the time variations of the magnetic flux enclosed by the circuit, or to the force exerted on the free charges in the circuit by the electric field. In both cases, the result is the generation of an induced current, but it is interesting to observe how two different mental models have been adopted – one at macroscopic level and the other at microscopic level – which seem to be alternative for students. Indeed, no one has spontaneously emphasized the consistency between the two points of view. This aspect is reflected in the difficulty in applying the reasoning to question n.2 of Table 9.1, when the circuit is represented as a simple rectilinear antenna; moreover, the answer requires non-trivial geometric reasoning and the analysis of the components of the (electric) field.

Finally, the difficulties in interpreting the formal representations of electromagnetic waves – in particular the spatial representation of the wave profile – clearly emerge from the difficulties in calculating the circulation (see question n.13 of Table 9.1). It confirms the alternative conceptions represented in Figure 10.1 also for points not belonging to the same wavefront, as well as detecting – through question n. 14 – the mental overlap that students pose between the representations as a function of space and time. Indeed, students do not seem able to correlate the value (sign) of the electric fields' circulation to the trend of the magnetic flux: on the contrary, they usually try to establish this relation referring to the same representation (see Figure 9.4).

10.2 Results of the second version of the learning path

In the following, each figure shows a bar graph illustrating the comparative results of the experimental group (N = 74) and the control group (N = 194), that are the operational categories which characterize all response profiles. They take into account the justification provided by students, as described in Chapter 8: at this stage, the selected option influences the analysis only when complete information is missing in the open answers, thus making the categorization more difficult. The results take into account all (and only) the answers of the students who completed the entire test, choosing at least one answer option for each question.

In Figure 10.7 the results of the first question show a non-significant difference ($\chi^2[4] = 4.26, p > 0.05$) between the two groups in the number of categories relating to the relationship between the values of the magnetic field and its circulation: the relative majority of students (50% in the experimental group vs 42.3% in the control group) recognize that circulation is a spatial property as well as a property of the field, that is, not all field sources provide a contribution to the calculation of circulation. A relevant difference between the two groups concerns the prevailing alternative conception: in the control group, students who consider circulation as a property determined by the contribution of all (and only) the fields' sources, show in most cases an overlap between the two concepts (field and operator) leading to consider them as uniform quantities in space (for example, to consider that the magnetic field generated by two wires crossed by opposite current is zero). On the other hand, the alternative conception of circulation as an exclusive property of the sources prevails: thus, it does not depend on the curve along which it is calculated, but it appears to rarely mate with the conception of a uniform operator (or field).

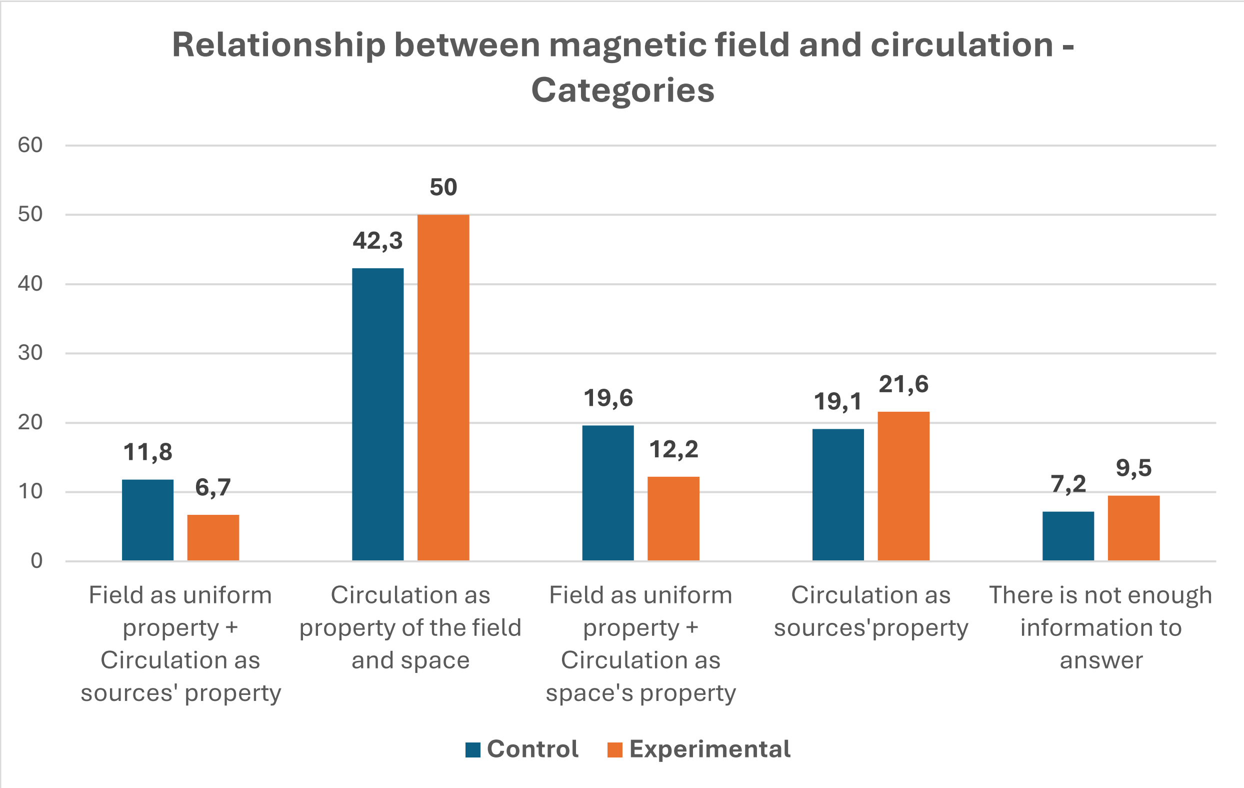


Figure 10.7: Categories emerging from the analysis of question n.1 (see the second version of the questionnaire) with comparison between the percentages of the control and experimental groups.

Figure 10.8 shows the results of the second question, which is the only one where the correct answer is prevalent in the control group (44.9%) compared to the experimental group (37.8%), even though the differences between the two distributions are not significant ($\chi^2[4] = 3.25, p > 0.05$). This category includes all the answers that referring to the magnetic flux as an agent of electromagnetic induction, even though not all of them explicitly refer to the temporal variations of this quantity as the cause of the induced current.

We can observe that all the alternative conceptions presented in the response options are prevalent in the experimental group, with the exception of the one only referring to the translational motion of the induced circuit as the cause of electromagnetic induction: thus, a slight prevalence in the experimental group emerge about the recognition of the symmetrical role of the objects' (magnet and circuit) motions. The "other answers" category includes combinations of cases different from those presented in the options, which do not present any motivation requiring the definition of different conceptual categories.

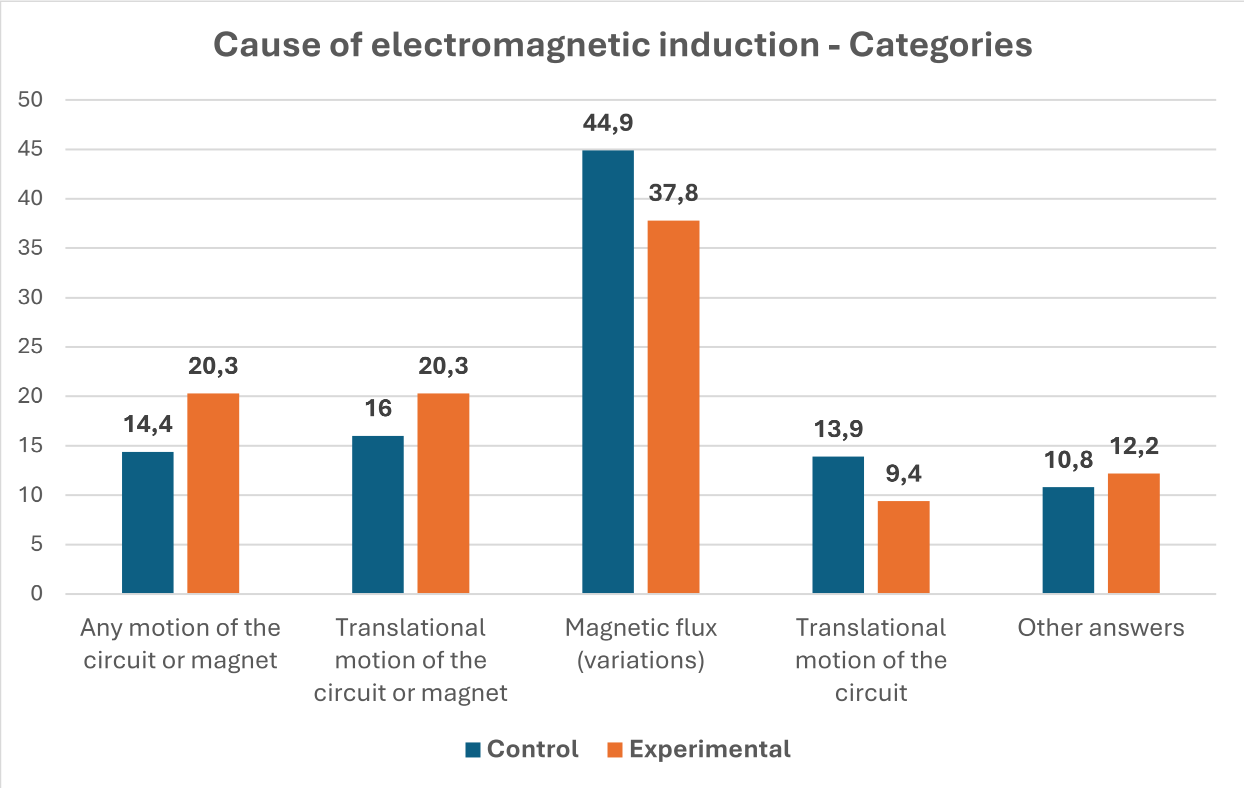


Figure 10.8: Categories emerging from the analysis of question n.2 (see the second version of the questionnaire) with comparison between the percentages of the control and experimental groups.

Also in the case of the relationship between the propagation velocity of the electromagnetic field and that of its source (see Figure 10.9 for the frequencies of the third question), we observe a non-significant difference between the two groups ($\chi^2[4] = 7.88, p > 0.05$). The prevailing category in the experimental group (40.5%) recognizes the independence between the two quantities, and that – in the case we are analyzing – the propagation velocity of the field is much greater (the numerical value or the expression 9.8 is often reported by students). On the contrary, in the control group the relative majority of students (35.6%) characterize the two quantities as automatically equivalent, since a cause-effect relationship between the motion of the source and the propagation of the field subsists. This identity is justified by different reasoning in the last category, where it is established as a contingent element in the specific case of conduction electrons. Limiting the comparison to these two categories – the second and the fourth – a consistent variation between the categories' size emerges, despite the fact that the chi-square test measures an overall insignificant change.

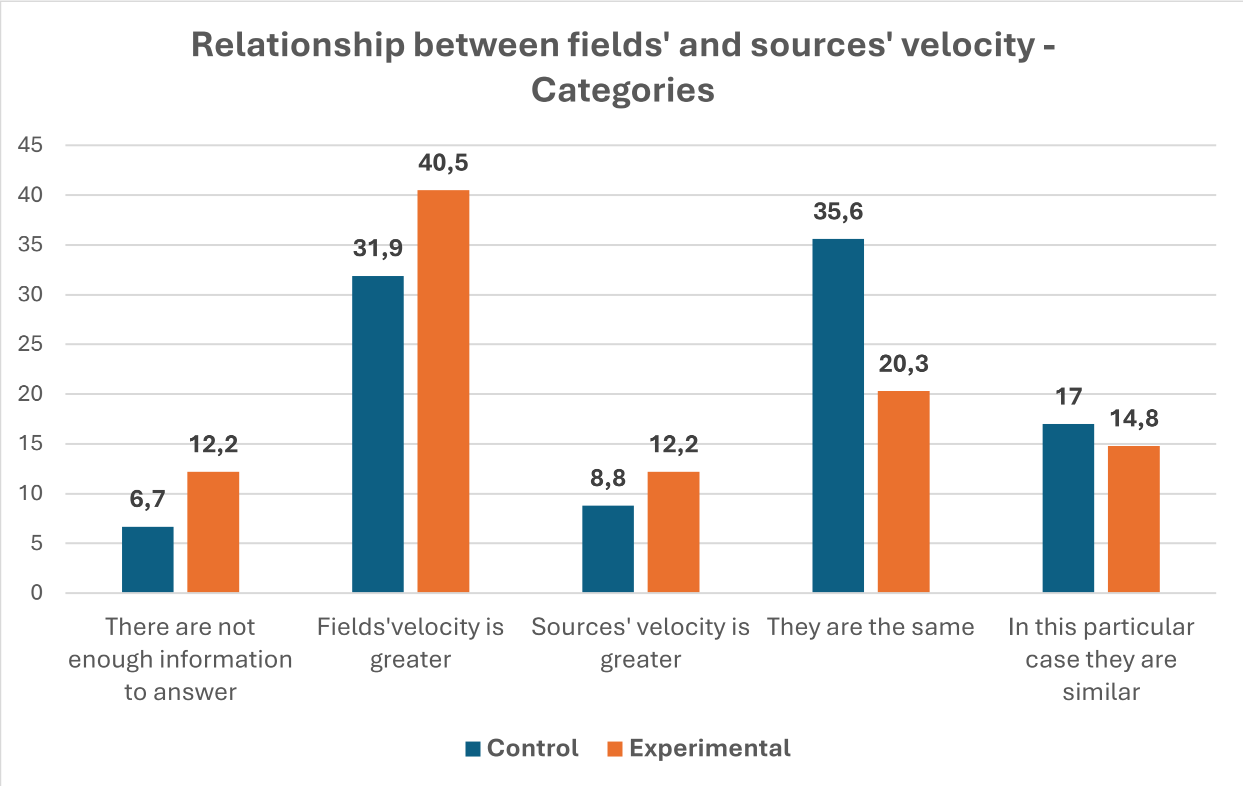


Figure 10.9: Categories emerging from the analysis of question n.3 (see the second version of the questionnaire) with comparison between the percentages of the control and experimental groups.

If the causal relationship is approached from a more general and qualitative point of view – i.e. how the trend of the electromagnetic field changes over time as a function of the motion of its source: see Figure 10.10 for the frequencies of the fourth question – we observe a very significant difference between the answers' distribution of the two groups ($\chi^2[4] = 20.42, p < 0.01$). The role of the observer (frame of reference) in determining this relationship is correctly highlighted by the relative majority of students in the experimental group (31.1%), while the vast majority of the control group considers charges in motion (i.e. any kind of motion) or only those in accelerated motion as sources of electric and magnetic fields varying over time (the frequencies are, respectively, 40.7% and 35.1%). We remark that the cumulative percentage of students who refer to the independence of the field characteristics from the motion of the source, or who provide answers in which this aspect is not made explicit (that are the first and last category) is much higher in the experimental group (18.9%) than in the control group (6.2%).

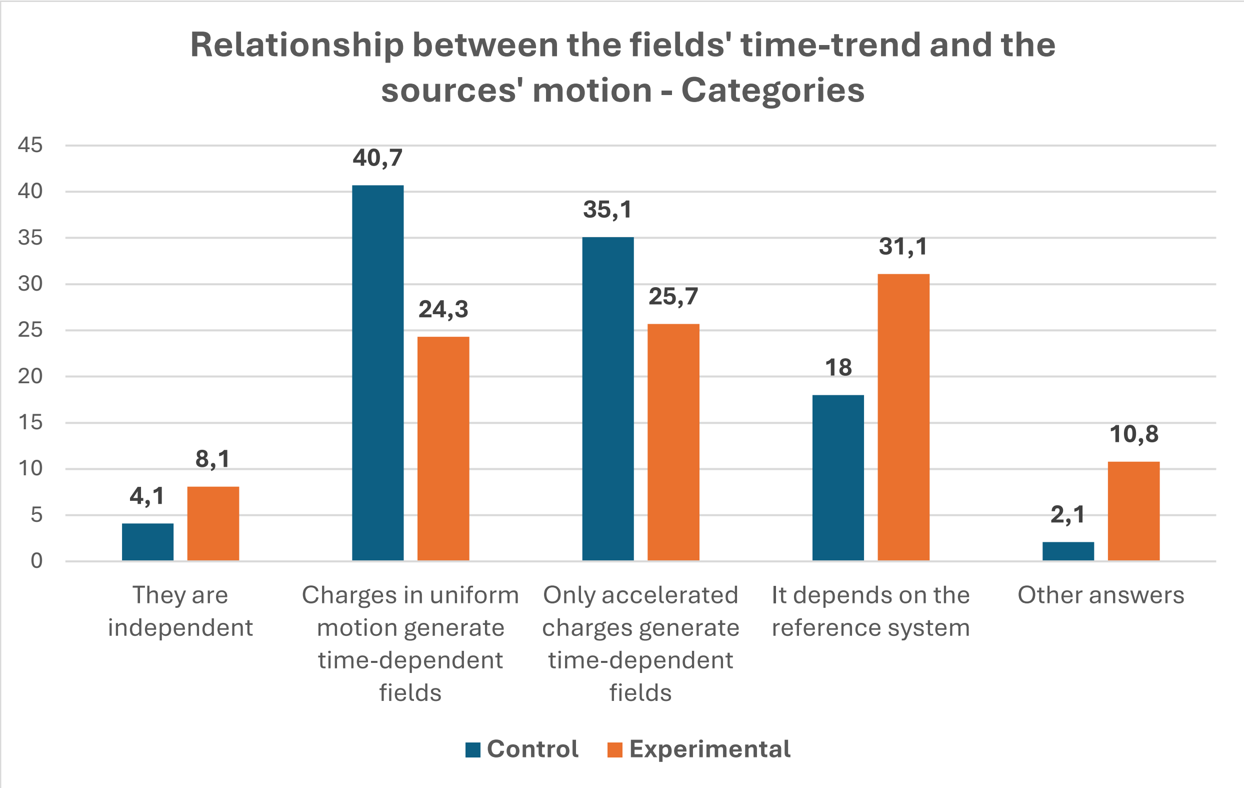


Figure 10.10: Categories emerging from the analysis of question n.4 (see the second version of the questionnaire) with comparison between the percentages of the control and experimental groups.

Figure 10.11 shows the results of the fifth question, which results as the most complicated for students in both groups. Nevertheless, the difference between the two distributions is statistically significant ($\chi^2[4] = 16.19, p < 0.01$) especially because of the first category, in which the majority of students in the control group fall (50.5%): they claim that all antennas receive (or they will not receive, in this specific configuration) the signal in the same way, as they belong to a plane that is orthogonal to the propagation direction of the electromagnetic wave: thus, no effect is produced by the interaction between the fields and the free charges in the conductors. This category is also prevalent in the experimental group (28.4%), even though the explicit reference to the propagation direction as that of signals' best reception is often absent. Even in the case where the direction of the electric field is characterized as the one of best reception (21.6% in the experimental group vs 16.5% in the control group), the explanation based on the Lorentz force is not explicit. Furthermore, the last two categories are characterized by a common explanation relating to the invariability of the direction of best reception, despite the fact that the non-simple application of geometric reasoning leads to different results regarding which is the brightest bulb.

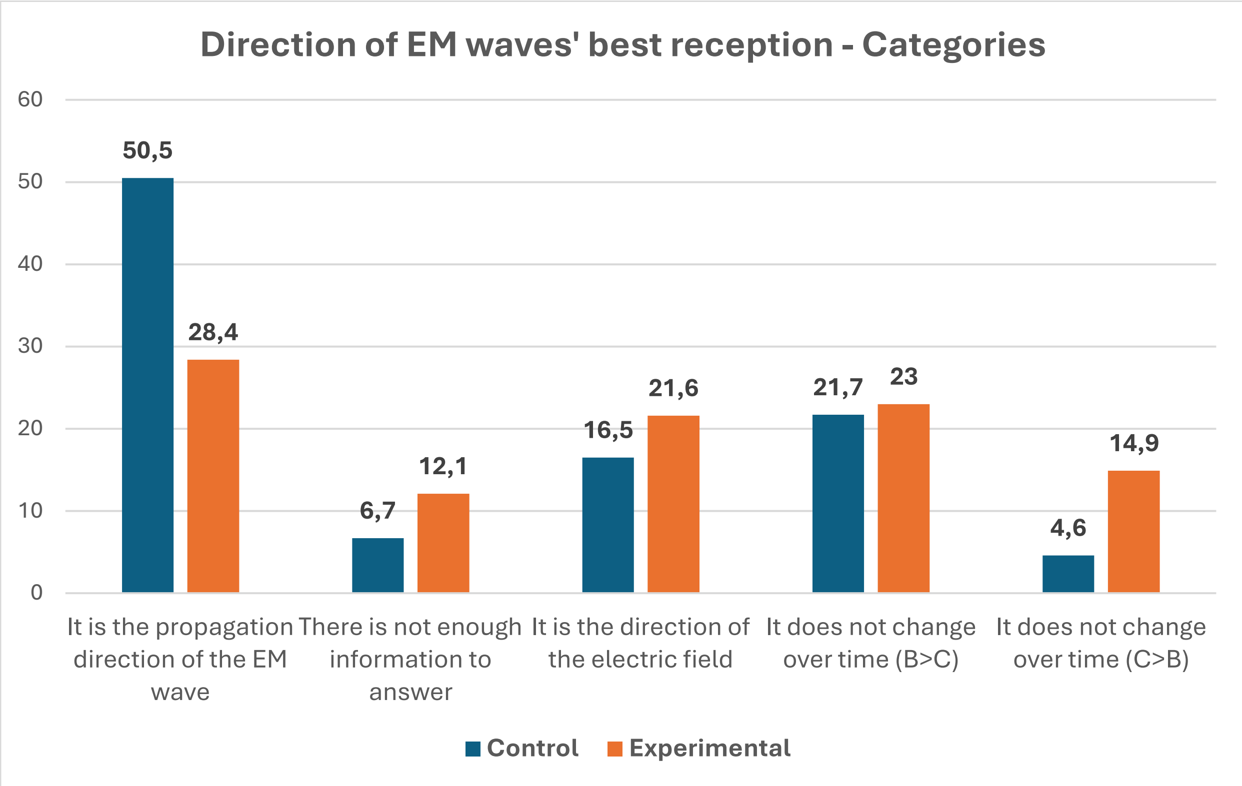


Figure 10.11: Categories emerging from the analysis of question n.5 (see the second version of the questionnaire) with comparison between the percentages of the control group and the experimental group.

Figure 10.12 shows the results of the sixth question, whose differences between groups are quite significant ($\chi^2[4] = 9.93, p = 0.04$). The percentage of students in the control group who identify a change in the fields as the frame of reference changes (19.6%) is compatible with the fourth category of question n.4, which analyzed a similar situation from the point of view of the motion of the source. This percentage is much lower in the experimental group (12.2%), as the relative majority of students correctly identify the change in both fields as the observer changes (39.2% in the experimental group vs 33.5% in the control group). Conversely, the fields' invariability is the second most populated category with a stable percentage (25.6% in the experimental group vs 26.8% in the control group). It is interesting to remark that the results are not symmetrical for the first two categories: in the control group, the share of students who recognize changes in the electric field is prevalent (16.5%, compared to 10.8% in the experimental group), while the situation is reversed for the magnetic field (3.6% in the control group vs 12.2% in the experimental group).

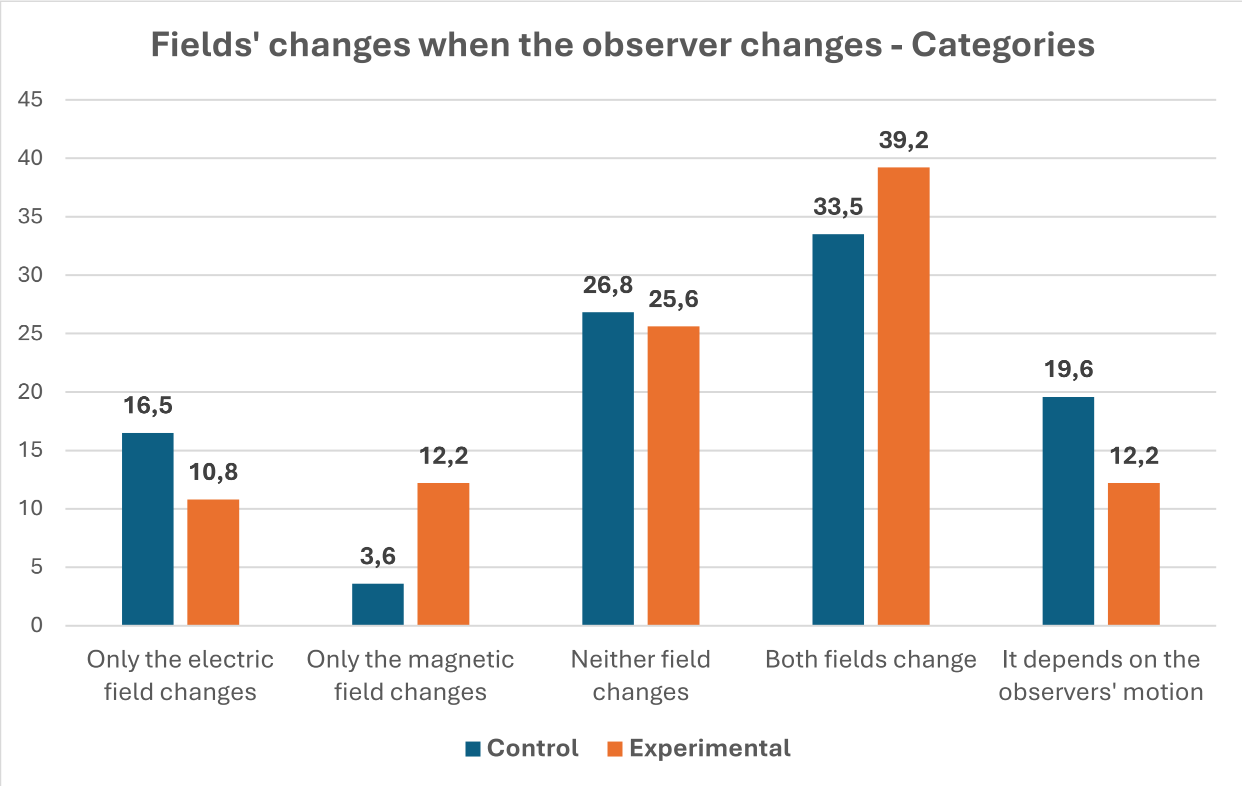


Figure 10.12: Categories emerging from the analysis of question n.6 (see the second version of the questionnaire) with comparison between the percentages of the control and experimental groups.

In the case of the seventh question (see Figure 10.13) the differences are not significant ($\chi^2[4] = 5.90, p > 0.05$). Although it investigates the relationship according to the propagation velocity of electromagnetic waves depends on two different quantities or entities (the fields' oscillation frequency and the propagation medium), the last three categories are basically characterized by responses focusing on a single quantity. Some answers falling into the last category (the propagation speed takes on its minimum value in vacuum) may be explained by the confusion between the time required for propagation – which all the options refer to – with the speed, which would require the reversal of the correct sorting. Moreover, the third and fourth categories seem to have a partially common root in the relationship $v = \lambda f$, which is interpreted by students (alternately) as a relationship of direct or inverse proportionality: nevertheless, such explicit reference is significantly more present in the control group. In any case, in both groups the relative majority of students (50% in the experimental group vs 45.4% in the control group) correctly identify the medium-dependent relationship and the velocity's independence from frequency, in some cases referring to the relation 9.8.

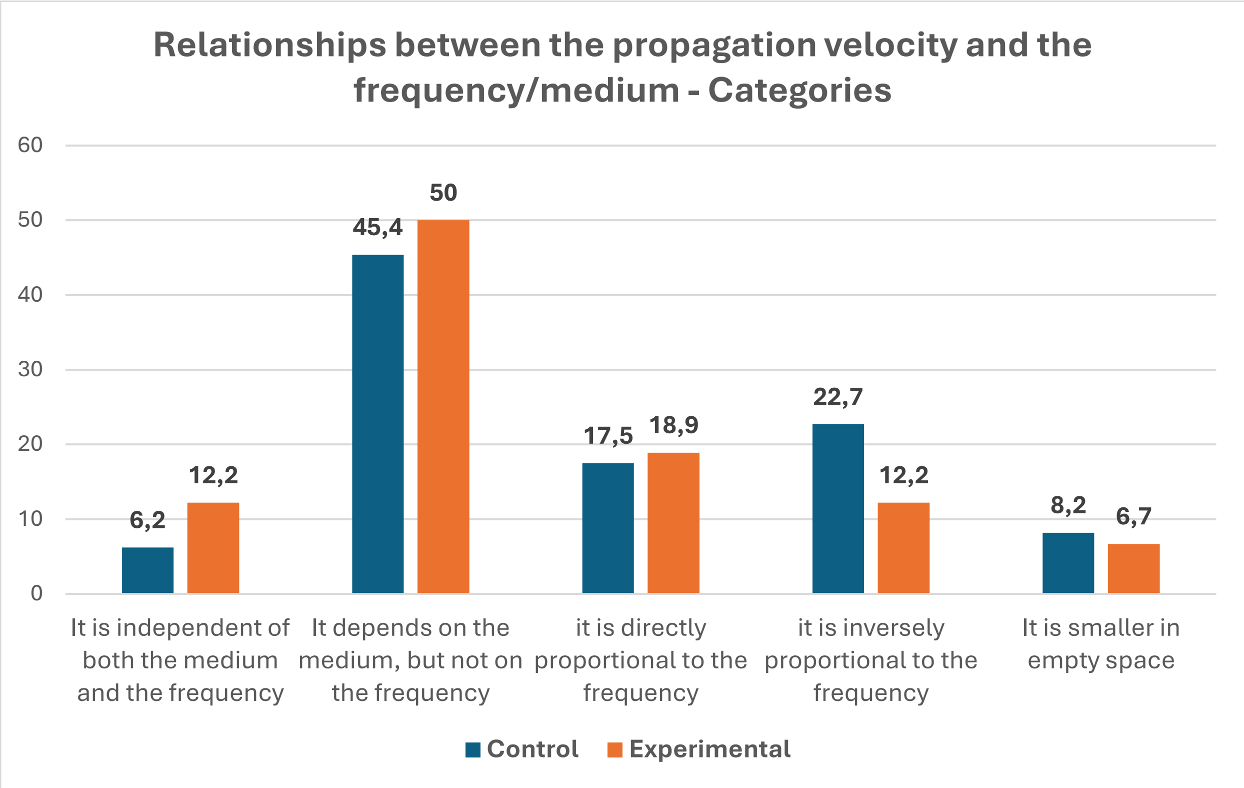


Figure 10.13: Categories emerging from the analysis of question n.7 (see the second version of the questionnaire) with comparison between the percentages of the control and experimental groups.

Figure 10.14 shows the results of the eighth question, which do not significantly differ between the two groups ($\chi^2[4] = 6.39, p > 0.05$). The category "other answers" refers to some sorting which are alternative with respect to those proposed in the options, which sometimes mention – in more or less explicit and consistent modalities – to the Doppler effect (e.g. v_2 should be greater than and v_1 and v_3) or the fact that in the third case (source in accelerated motion) the context provide a non-inertial system, so the velocity keeps constant or it is indeterminate. The percentage of students who correctly identify the independence of the propagation velocity from the frame of reference is very similar in the two groups (41.9% in the experimental group vs 40.7% in the control group), but the explanations provided are quite different. In the first case the reference to the relation 9.8 prevails (according to which the velocity does not depend on the fields, thus it also independent from the observer); in the second case the phenomenological reference to the independence of the speed of light from the frame of reference prevails, or it is taken as an implicit hypothesis without considering its incompatibility with the classical rule of velocities' composition.

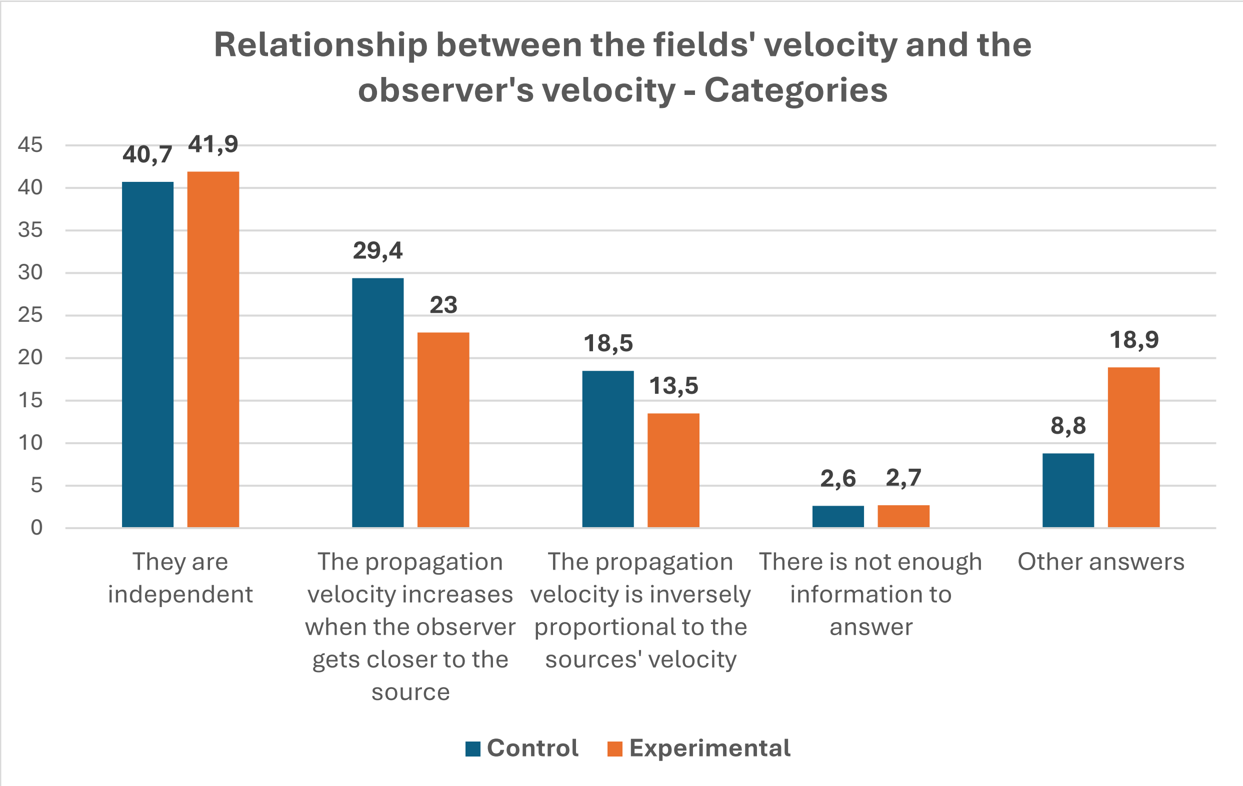


Figure 10.14: Categories emerging from the analysis of question n.8 (see the second version of the questionnaire) with comparison between the percentages of the control group and the experimental group.

For the ninth question (whose results are shown in Figure 10.15) the differences between the two distributions are very significant ($\chi^2[4] = 15.26, p < 0.01$). In particular, while in the experimental group the identification of the independence of the waves' intensity from the wavelength and its increasing relationship with the fields' amplitude prevails (43.2%), in the control group the prevailing category concerns the relationship of direct proportionality between intensity and frequency (42.8%). We found that the relationship between intensity and amplitude is often characterized as a direct proportionality, or as an unspecified increasing relationship. Furthermore, even though the representations are posed in terms of wavelength, the students' answers often refer to frequency. For this specific reason, we included in the first category students who characterize the relationship as an inverse proportionality, due to the confusion between the concepts of period and wavelength. Once again, the "other answers" category includes different kind of sorting from those proposed in the other options, without reporting explicit reasons for defining further categories. Finally, an interesting aspect is that the share of students who refer to the independence of intensity from both the fields' amplitude and the frequency (or wavelength) is much lower in the control group (2%) than in the experimental group (9.4%).

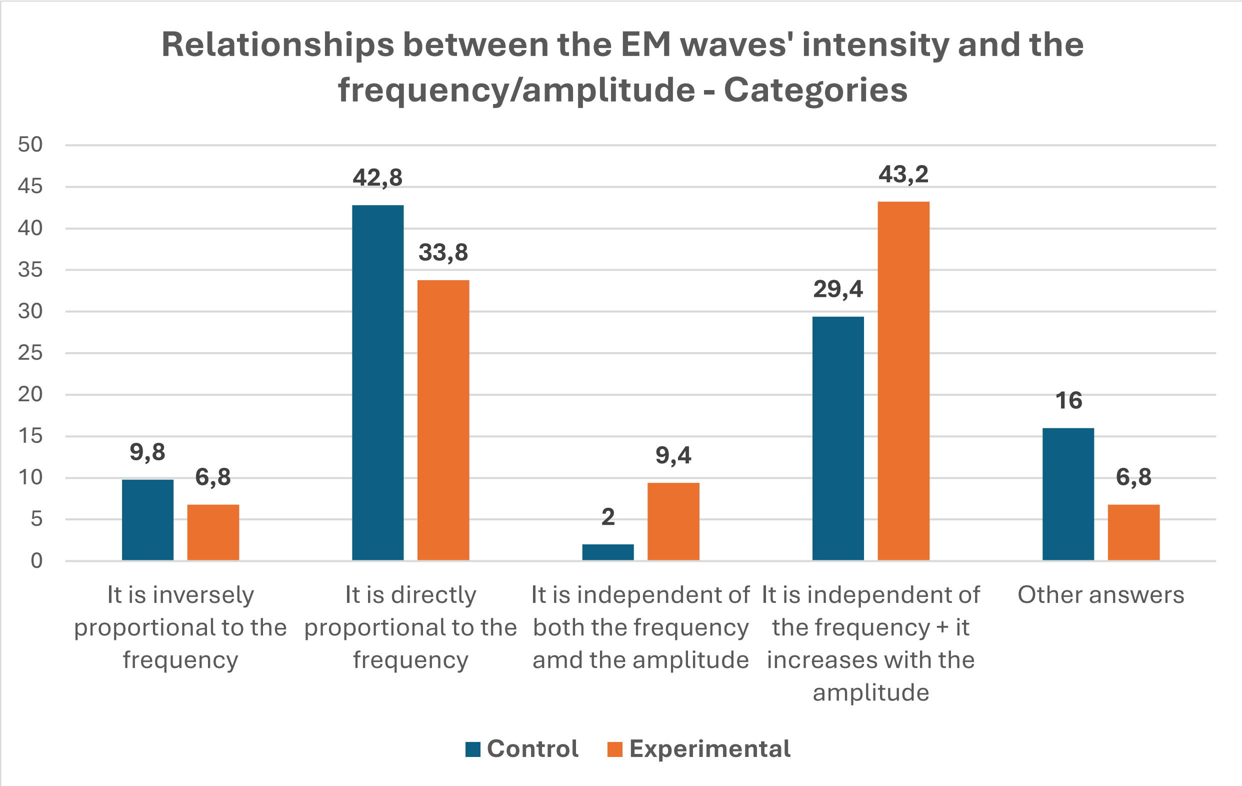


Figure 10.15: Categories emerging from the analysis of question n.9 (see the second version of the questionnaire) with comparison between the percentages of the control group and the experimental group.

Also for the last question concerning the representation of plane waves (whose results are reported in Figure 10.16) the differences are very significant ($\chi^2[4] = 34.78, p < 0.01$). Indeed, the share of students reporting decreasing fields moving away from the propagation axis is much larger in the experimental group (23.0%) than in the control group (2.6%), in which the "other answers" category prevails (48.4%). It includes alternative fields' sorting (e.g. $B_Q > B_P = B_R$; $B_P > B_Q > B_R$; $E_R > E_Q > E_P = 0$ etc.), which do not seem to be associated with other relevant conceptions. Moreover, they differ in types and numerosity between the two groups, not allowing the definition of further significant categories for our analysis.

The differences that emerged from the Chi-square test have to be attributed to the two categories just mentioned, rather than to the one in which students recognize that both fields take on the same values at all points we are considering. In this case the percentages are similar, although for the experimental group it is slightly higher (32.4% vs 27.3% in the control group). We remark that almost all the students in the experimental group – who belong to this category – explicitly mention the common membership of all the points in the same wavefront, in order to justify their answer. Finally, we observe that only a minority of students in the second category make it clear that this fields' sorting applies both to the instant in which the representation is provided (t = 0) and to the instant which the question refers to (t = T/2, i.e. half of the wave's period), while in most cases (also for the other categories) the reference to the temporal dimension is implicit.

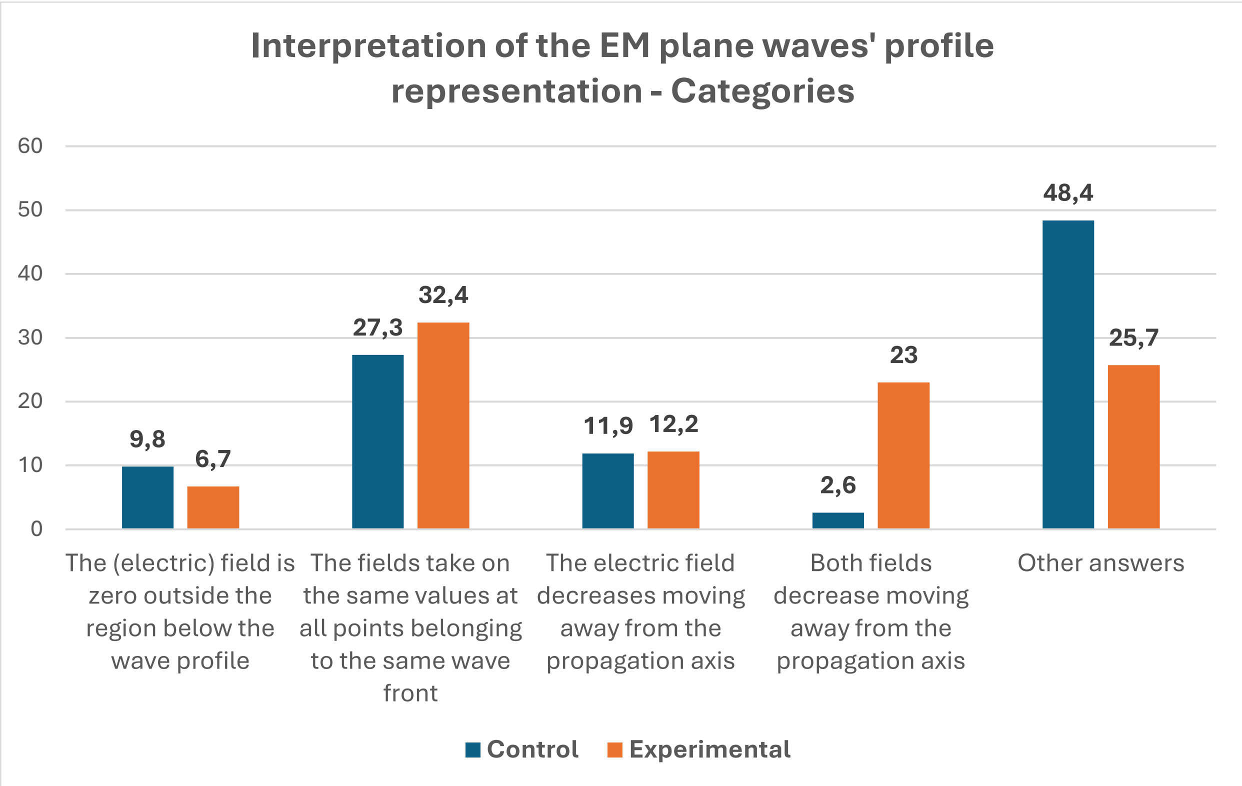


Figure 10.16: Categories emerging from the analysis of question n.10 (see the second version of the questionnaire) with comparison between the percentages of the control group and the experimental group.

We conclude this paragraph with some remarks regarding the most significant answers we obtained in class through "stimulating questions", and the most recurrent alternative conceptions in the experimental sample. Since the answers obtained from the questions we did not modified (compare Tables 9.1 and 9.2) are similar to those described in the previous paragraph (first version of the learning path), we will limit the discussion to the new outcomes.

As far as preliminary activities are concerned, students are able to recognize the action of an entity acting on the free charges in conductors, so as to charge the different parts of an overall neutral object. In order to experimentally characterize this entity as a vector quantity, students often suggest searching – through the sensor – for a point where the electric field generated by two sources is zero (in order to compare this experimental position with the expected one) or to operate with a single (symmetric) source whose field lines' distribution is already known, so that a comparison can be made at different points. When emphasizing the possibility of carrying out different measurements with different sources, some students suggest performing the operation described in paragraph 9.2 (i.e. verify the vector sum rule at any point by previously measuring the component fields), even though it is not very intuitive for them to establish which operation should be performed with the sensor to determine the direction of the electric field. On the other hand, they are able to set a simple experiment to determine the conservative nature of the electrostatic field by measuring its circulation, completely characterizing the variables involved.

Question n.8 (in Table 9.2) provides students with a further interpretative key for the fields' changes as the observer who detects them changes, as they show to recognize the invariance (from the classical point of view, between inertial frames of reference) of the force acting on a charge, together with its consequences from the point of view of the interactions produced. As a result, the majority of students recognize that a charge in motion is able to generate an electromagnetic field. In particular, for question n.10 the first option is selected by a small minority, even though the answers' discussion shows a prevailing conception that a magnet could not play a symmetrical role with respect to the charge (about the generation of a field which has both an electrical and a magnetic component varying over time).

Question n.14 shows better results than the previous one: in this view, it seems easier for students to recognize the effect of delay in propagation with respect to the fields' values at a point rather than their properties characterizing the variability over time (e.g. the oscillation frequency). Despite this, in question n.14 about half of the sample is equally divided between the first two answer options, highlighting the spread of alternative conceptions both concerning the independence of the field propagation from its source (once generated, the fields' values no longer depend on the characteristics of the source) and regarding the fields' absence when sources are not present (since it will no longer be in motion): Both these alternative ideas show a qualitative lack of understanding about the propagation mechanism illustrated by the example of the infinite conducting plane.

Finally, let's analyze the two "stimulating questions" concerning the signals' detection. In question n.17 we observed that option A is selected by very few students, while the others chose between the remaining options, with a slight prevalence of options B and D. Students who identify the propagation direction as that of best reception for electromagnetic waves, tend to provide explanations based on the fact that in this case the fields arrive “directly” arrive on the antenna. On the other hand, those who believe that this direction varies over time report justifications based on the variability of the signal (some students interpret the representation of the plane wave profile provided in this case in such a way that the fields turn out holding oscillation directions changing over time) or the fact that there is no reason to think that the reception direction must be unique and well-defined. These results are clearly comparable with those of the fifth question of the same questionnaire (see Figure 10.11).

A similar situation is observed for the interpretation of the phenomenon of resonance and its role in the signals’ detection (question n.18 in Table 9.2): also in this case the first option is selected by very few students – as the vast majority recognizes that it is not possible to act on the signals’ parameters – and the sample is (almost) equally divided among the remaining three options. Thus, the reference to the Doppler effect is still recurrent, as well as mentioning the impossibility of uniquely define an optimal modality for signals’ detection. Nevertheless, those who recognize that it is possible to act on the parameters of the receiving circuit often explicitly refer to the phenomenon of resonance, representing the possibility of maximizing the current induced in the circuit.

10.3 Results about undergraduate students’ conceptions

Tables 10.1-10.6 show the results of the categorization of each question for the sample of undergraduate students. We remark that the phenomenographic approach allows students to be grouped into descriptive categories, which does not imply that any student belongs to the same category when answering any question on that topic. Indeed, as Guisasola and colleagues (2004) observe, a student can implement reasoning characterizing another category as the context of the question varies. As a consequence, this analysis does not determine the response profile of individual participants, but rather aims to extract possible emerging categories from a sample of the population we are addressing.

Table 10.1: Categories (and subcategories) with relative frequencies for the sample of undergraduate students (N = 35) for question n.1 (first version of the questionnaire). The meaning of the categories is described in the last column.

Categories	Sub-categories	Frequencies	Explanation
A Identification of the meaning of wavefront	**A1** Fields take on the same values at any time	1	Both fields take on the same values in P, Q and R because they belong to the same plane, where the fields (by definition of the case we are dealing with) show uniform intensity + The sorting does not depend on the time instant it refers to.
	A2 The fields take on the same values after a period + The 3 points belong to the same wavefront	4	Both fields take on the same values in P, Q and R because they belong to the same plane, where the fields (by definition of the case we are dealing with) show uniform intensity + The representation of the wave is the same at the two instants we are considering.
	A3 The 3 points belong to the same wavefront	19	Both fields take on the same values in P, Q and R because they belong to the same plane, where the fields (by definition of the case we are dealing with) show uniform intensity + There is no reference to the temporal dimension of the phenomenon.
B Misconceptions deriving from the incorrect interpretation of the representation	**B1** The magnetic field takes on different values at the 3 points	6	The representation of the wave profile suggests the misconception that the field is zero at all points that do not coincide with the tails and/or tips of the vectors.
	B2 Sources' location on the propagation axis	1	The fields' values decrease as you move away from the

			source, which is located on the propagation axis.
C No or inconsistent explanations	**C1** Inconsistent explanations	3	References to the fields' oscillation in phase or to the intensity of the wave (*it also occur for 6 students belonging to other categories*).
	C2 No explanation	1	/

Table 10.2: Categories (and subcategories) with relative frequencies for the sample of undergraduate students (N=35) for question n.2 (first version of the questionnaire). The meaning of the categories is described in the last column.

Categories	**Sub-categories**	**Frequencies**	**Explanation**
A Accelerated charges are required to generate an electromagnetic wave	**A1** Accelerated charges are required to generate an electromagnetic wave in any frame of reference	6	/
	A2 An accelerated charge is necessary because those in uniform motion generate a static (magnetic) field	6	/
	A3 Combination of categories A1 and A2	5	/
B No accelerated charges is required to generate an electromagnetic wave	**B1** A charge in uniform motion can generate electromagnetic waves	2	A charge in uniform motion is enough to generate an electric and a magnetic field, possibly varying over time.
	B2 A stationary charge can be a source of electromagnetic waves	1	A stationary charge can generate electromagnetic waves by choosing a frame of reference such that the surface through which the field's flux is calculated results to be variable over time.
C No or inconsistent explanations	**C1** Inconsistent explanations	9	References to the motion of the frames of possibly or to possible relativistic effects.
	C2 No explanation	6	/

Table 10.3: Categories (and subcategories) with relative frequencies for the sample of undergraduate students (N = 35) for question n.3 (first version of the questionnaire). The meaning of the categories is described in the last column.

Categories	Sub-categories	Frequencies	Explanation
A The propagation velocity depends only on the medium	**A1** The propagation velocity depends only on the medium, and its maximum value occurs in vacuum	3	With reference to the formula descending from Maxwell's equations.
	A2 The velocity is independent of frequency, and its maximum value occurs in vacuum	13	/
	A3 The velocity depends only on the medium	6	/
B The propagation velocity is independent of the medium	**B1** The velocity is independent of both the medium and frequency	3	The propagation velocity of electromagnetic waves is a universal constant.
	B2 The velocity is directly proportional to frequency	4	Because $v = \lambda f$ (with a constant wavelength).
	B3 The velocity is inversely proportional to frequency	3	Because phase's delays accumulate (in relation to the refractive index).
C Identification of the duration of propagation with the wave's period		3	Students consider the time interval necessary for the wave to propagate between the two points, to be equivalent to the oscillation period.

Table 10.4: Categories (and subcategories) with relative frequencies for the sample of undergraduate students (N = 35) for question n.4 (first version of the questionnaire). The meaning of the categories is described in the last column.

Categories	Sub-categories	Frequencies	Explanation
A The speed of light is invariant for all observers		19	/
B Classical reasoning		3	The Galilean rule of velocity composition applies, according to which the propagation velocity depends on that of the

			observer and that of the source (in any case, they do not recognize the invariance of the speed of light).
C Inconsistent or no explanations	**C1** There is not enough information for a complete answer	3	/
	C2 Reasoning based on the Doppler effect	2	/
	C3 No explanation	8	/

Table 10.5: Categories (and subcategories) with relative frequencies for the sample of undergraduate students (N = 35) for question n.5 (first version of the questionnaire). The meaning of the categories is described in the last column.

Categories	Sub-categories	Frequencies	Explanation
A The waves' intensity depends only on the fields' amplitude	**A1** The intensity increases as the square of the fields' amplitude	11	/
	A2 The intensity is directly proportional to the fields' amplitude	4	/
	A3 The intensity depends on the fields' amplitude	6	Several explanations occur in this case: for example, some students believe that waves' intensity correspond to the time integral of the energy carried by the (electric) field.
	A4 The intensity depends on the propagation medium	1	/
B The waves' intensity depends on its frequency	**B1** The intensity is directly proportional to the frequency, and it increases with the fields' amplitude	6	/
	B2 The intensity is directly proportional to the frequency	3	The intensity depends only on the frequency, and the direct proportionality relationship is explained by means of the Planck's law or by analogy with mechanical waves.

	B3 The intensity is inversely proportional to the frequency	1	/
C No Explanation		3	/

Table 10.6: Categories (and subcategories) with relative frequencies for the sample of undergraduate students (N=35) for question n.6 (first version of the questionnaire). The meaning of the categories is described in the last column.

Categories	Sub-categories	Frequencies	Explanation
A In the direction of the electric field	**A1** In order to generate currents in the antenna	5	According to the Lorentz force.
	A2 Without further specifications	7	/
	A3 Inconsistent explanations	2	References to the fields' absorption or to the induction of "local, non-conduction" currents.
B In the direction of the wave propagation		10	So that the signal "directly" arrives on the antenna.
C Direction not determinable from the wave representation	**C1** In the direction of the source's oscillation	1	/
	C2 In the direction of the field interacting with the specific antenna	2	It depends on the modality the antenna works: some are built to interact with the wave in the oscillation direction of the electric field, while others refer to the magnetic field, and maybe others in alternative directions. Antennas are designed to convert, in some way, electromagnetic waves into electrical signals, but the specific modality depends exclusively on the antenna, while it does not depend on the signal.
D Inconsistent explanations	**D1** Inconsistent explanations	2	/

	D2 No explanation	6	/

Chapter 11

Discussion

Here we are going to discuss the teaching implications of the results illustrated in the previous chapter, which provide the evaluation about the effectiveness of the first version of the learning path together with the following modifications leading to the second version.

11.1 Discussion of the results of the first experimentation

The analysis of the results emerging from the questionnaire shows some significant changes after the implementing the learning path, while other aspects do not report significant differences. The former includes the relationship between the motion of the source and the fields' trend over time, the relationships of the waves' intensity with the fields' amplitude and their wavelength, the direction of signals' best reception and the interpretation of the wave profile representation.

In particular, the teaching activities seems to have determined a greater connection degree in students' reasoning between the source and the entity (field) that is generated, which is often absent in traditional teaching. Indeed, the textbooks' analysis revealed the missing link between Maxwell's equations and the relationships characterizing the propagation of the electromagnetic field: thus, it is relevant to derive the qualitative characteristics of the generated fields – as in the case of the point charge – and then the quantitative ones in a specific context, such as that of the infinite conducting plane (i.e. the transverse nature of the electromagnetic field and the expression of its propagation velocity). In this view, the analysis of the "transient phase" represents – despite the approximations which we referred to – an adequate context to provide a conceptual meaning to the fields' operators (flux and circulation) as entities determining the characteristics of the electromagnetic field propagation. Obviously, the transition to electromagnetic waves requires going beyond the analysis of the "transient phase", by considering a source whose properties change over time. We managed this by assuming the hypothesis that the fields' values generated in the space surrounding the source correspond – instant by instant – to those obtained through the "transient" method: this implies, for example, that the frequency of electromagnetic waves correspond to the oscillation frequency of its source (to simplify the analysis, we can consider it as a single charge).

The link between the electromagnetic waves propagation and the Maxwell's equations has also been useful for building the missing link between electrostatics and electrodynamics (see the discussion in Chapter 4), by fostering the development of mental models sharing conceptual referents between the two contexts. For example, the expression of the Lorentz force is useful for determining the interaction between fields and charges not only in the static case – in which students usually apply this law – but also when the fields change over time. This may explain the great difference we observed after the teaching activities regarding the model adopted to describe the electromagnetic waves' detection through a rectilinear antenna: beyond the geometric reasoning required to determine the negligibility of the magnetic field action, the main aspect consists in recognizing that the direction of best reception does not vary over time if the fields (responsible for the interaction with free charges in matter) keep a constant direction of oscillation.

Since determining the direction of signals' reception also requires geometric reasoning, it is important to compare it with the interpretation of formal representations of electromagnetic waves. In dealing with plane wave profiles (which expresses the fields' trend over space) we can deduce that the teaching treatment has been useful in counteracting some of the most widespread learning difficulties of conceptual nature – for example, the fields decrease as they move away from the propagation axis because it represents the location of the wave's sources – but additional (and difficult to be classified) difficulties may emerge due to the interpretation of the figure, even though they may be not related to conceptual aspects.

For this reason, we think it is useful to take into account the explanations provided by students who correctly identify the sorting of the fields' values at the points we considered. They often refer to the common membership in the same wavefront as the reason why the fields take on the same values: for this specific aspect, the learning path has been modified according to the description of Paragraph 9.2, by coupling the representation of the wave profile with that of the (plane) wavefront (namely, they are introduced together). Thus, students can gain a greater awareness of the meaning of wavelength as a distance between two consecutive wavefronts, and also they can identify wavefronts as set of points where the fields oscillate in phase (not always by assuming the maximum values, which explains the use of multiple representations derives).

On the other hand, using static representations does not allow to illustrate the propagation over time – at a fixed point of space – of electromagnetic waves: for this reason, the item of the questionnaire relating to the interpretation of the wave profile representation has been modified in the second version of the questionnaire, by referring the request to an instant in which the configuration of the electromagnetic field is changed: thus, it is possible to investigate the students' reasoning regarding the interaction between the spatial and temporal natures of the electromagnetic waves' propagation.

On the other hand, the aspects that do not show any significant improvement after treatment are related to the characteristics of the electromagnetic waves' propagation velocity, including its relationships with the fields' oscillation frequency, the speed of the source (with respect to the observer) and the propagation medium. Probably, this is due to the fact that these aspects are part of traditional teaching, although the analysis of textbooks reported in Chapter 6 has shown that information relating to such relationships is usually provided without adequate explanation. Indeed, the missing link between Maxwell's equations and the wave equation does not allow to provide justifications; despite this, students know that the maximum value of the propagation velocity occur in a vacuum, and that it is independent of the speed of the source. This information can be of a dual nature: it is often presented as a postulate of the theory of special relativity, thus representing a conceptual bridge towards teaching modern physics. On the other hand, some responses suggest that some students implicitly assume that the fields' propagation velocity is independent of that of the source generating it, probably due to the lack of references (in traditional teaching) to the modalities for generating electromagnetic waves.

Furthermore, the fact that the number of students who identify a functional relationship between the frequency of fields' oscillation and their propagation velocity is almost the same in the post-test, led us to introduce the discussion of this relationship $v = \lambda f$ into the learning path. In particular, in order for these three quantities to be considered as variables – so that the functional relationship can be studied – the case in which electromagnetic waves generated by different sources propagate in different media has been considered. Thus, it is possible to characterize the frequency as a quantity exclusively dependent on the source, while the wavelength (and, as a consequence, the velocity) will depend on the propagation medium. If we deal with the electromagnetic waves' propagation in vacuum by considering different sources, the previous relationship represents neither a direct nor an inverse proportionality, since two of the quantities considered are constant.

From the analysis of the answers provided to the "stimulating questions" during the implementation of the pathway, the need to deepen the role played by the observer in detecting electric and magnetic fields emerged. Indeed, we have previously identified clusters of students considering fields as fixed properties of space or being exclusively dependent on the sources' properties, even though some of them do not include the type of motion among these characteristics. In any case, the detected fields depend on the state of motion of the observer: the results suggest that, in order for students to recognize this aspect, stimulating questions should be asked to show the comparison between the different cases in which the same source – which is also the simplest, i.e. a point charge – is analyzed in different frames of reference.

Indeed, it is necessary to consider an aspect that may potentially confuse students: the propagation velocity of the electromagnetic field – unlike the values it takes on – does not depend on the relative velocity of the observer with respect to the source: these two variables are probably directly correlated according to the classical model adopted by the majority of students. For this reason, the ideal example of the field generated by an infinite conducting plane can help them conceptually characterize and distinguish the single variables, establishing relationships between them that are valid at every point in space and at every instant.

Using Maxwell's equations in this example also provides a key to interpret them in terms of relationships between the values of the electric and magnetic fields, rather than cause-effect relationships between the fields or their operators: the explicit recognition of the quantities appearing in the equations in order to describe the sources also contributes to this characterization, also showing how – even in the simplest case of a single point charge – it is not possible to uniquely determine the fields without considering the boundary conditions.

Finally, these results were also useful to obtain information about the items of the questionnaire, as – as explained in Chapter 8 – the validity and reliability of the questionnaire had not previously been tested on a large sample. In particular, the fact that in the answers about question concerning the interpretation of the wave profile, the reference to the temporal dimension was often absent – i.e. even students who recognize that the fields at the points belonging to the same wavefront take on the same value – do not seem to recognize the generality of this statement for any instantaneous configuration of the electromagnetic field. For this reason, in the second version of the questionnaire the time interval that separates the instant which the representation refers to and the one which the question refers to is no longer equal to an oscillation period of the electromagnetic wave, but to half of this quantity: in this way the fields' values (at the points we are considering) at these two instants will be different, and the students' reasoning will be able to make evident the mental models they adopt in the interpretation of the spatial and temporal dimensions of the electromagnetic field propagation.

Moreover, considering the variety of response profiles emerging with respect to the crucial question of the role of the observer, the new questionnaire contains several items aimed at investigating this aspect from different points of view: 1) the relationship between the fields' velocity and the speed of the source generating it; 2) the characteristics of variability over time of the electromagnetic field in relation to the motion of the source; 3) the variations in the fields detected as the sources' properties change.

11.2 Discussion of the results of the second experimentation

Also in this case, the significance of the differences between the two groups (experimental vs control) depends on the specific question. First of all, we observe that the analysis of the relationships between the electromagnetic field and its sources returns a composite picture: while the questions concerning the propagation velocity of the field and its relations with the velocities of the observer and the source do not present significant differences – as is the case with its relations with the propagation medium and the fields' oscillation frequency, confirming the results of the first version – these differences emerge for the variations of the field as the frame of reference varies, and even more for the relationship between the fields' time trend and the motion of the source. Considering the number of categories, we can state that the teaching treatment has been effective in highlighting both aspects concerning the generation of electromagnetic waves – in particular, why the sources are formed by accelerated charges, and how their motion determines the trend of the fields – and their detection – i.e. how the motion of the observer influences the values of the measured fields – despite the fact that there is still an asymmetry in the roles attributed to the electric and magnetic fields. Indeed, although the unification of the electromagnetic field has been discussed from different points of view, a stable share – between 20% and 25% of students in the two groups – still report that the motion of the source and that of the observer produce different effects on the two fields' component.

On the other hand, the recognition of the property according to which the entities involved in the description of the phenomenon – sources, fields and observers – move with independent velocities, present only partial differences between the experimental and the control group: it can be attributed to the fact that the propagation velocity of electromagnetic waves was introduced as a concept through the ideal example of the infinite conducting plane, where the fields are detected by a single observer, and

Maxwell's equations are analyzed according to the "transient phase" (namely, we did not consider a time interval during which the velocity of the charges varies). As a consequence, the fact that the propagation velocity of the field is a constant exclusively dependent on the medium is a result that is obtained at the end of the case analysis, and not by analyzing several cases in which the quantities involved vary. This may lead to a lack of advancement – compared to traditional teaching – of students' conceptual understanding of these relationships, even though a partial difference in students' reasoning emerges when looking at the size of some specific categories. Indeed, a clear reduction – in the experimental group – of students who do not distinguish between the velocities of the different entities (i.e. they consider them as equivalent), corresponds to a significant increase – in the same group – of students who characterize these as independent quantities, which cannot be determined from each other. As a consequence, although the analysis of different cases in which the velocities involved take on different values may help to recognize their mutual independence, the example we discussed has been successful in characterizing them as distinct quantities.

The other questions in which very significant differences between the two groups are measured concern the determination of the direction of electromagnetic waves' detection, the relationships of their intensity with the fields' amplitude and their oscillation frequency, and finally the interpretation of the plane wave profile representation. In the first case, the difference is probably due to the fact that traditional teaching relegates the Lorentz force as a mediator of the interaction between fields and free charges only to the static context: thus, students are not used to consider cases in which the force acting on a charge placed at a specific point can vary over time depending on the fields' trend, or according to the motion of the source that generating them. For this reason, a portion of the students in the control group who identify the direction of best reception as that of wave's propagation, still consider that this direction varies over time according to a periodic trend (following that of the fields). This can be explained by the fact that students do not own a conceptual reference to explain how signals are received and how it is possible to improve their reception, so they tend to rely on more naive (but intuitive) explanations, such as "the signal is received only by placing the (rectilinear) antenna in the direction in which the signal is propagating – this direction may be variable, depending on the position and on the time instant we are considering - because in this way the fields directly arrive on the antenna, and they are not diverted in any way". We also remark that – although question n.5 requires a quite complicated geometric reasoning – the students' answers in the different categories show reasoning taking into account the components of the fields (i.e. not only their directions).

The above reasoning represents a remarkable example of an alternative conception which may be generated by a specific interpretation of a representation (in this case the plane wave profile). This makes it even more relevant to study these interpretations in detail, and the differences between the two groups show how the coupling of two different types of representations – that of the wave profile and that of the wavefront – helps students – more than using of a single representation – to correctly recognize the propagation modalities of the electric and magnetic fields over space and time. In particular, among the students who recognize the common membership of the points represented to the same wavefront, in the experimental group there are many more participants generalizing the equality of the fields' values at any instant, probably due to the use in the learning path of representations reporting multiple wavefronts in different positions and instants. They are also able to recognize that the specific value that the fields take on depends on the time interval separating the representation of the wave profile from the instant which the question refers to, by identifying similarities in the periodic trend that characterizes both spatial and temporal propagation.

Also the relationships between the intensity and the other parameters of electromagnetic waves – i.e. (electric and magnetic) field's amplitude and wavelength or frequency – show significant differences that can be attributed to the teaching treatment, since the introduction of the intensity as a physical quantity was not carried out through the formal definition of the Poynting vector, but by providing a conceptual meaning and a reason to introduce a new variable according to those already defined. The reference to frequency as a quantity characterizing the different regions of the electromagnetic spectrum is also confirmed: to indicate that the intensity of an electromagnetic wave does not vary depending on the spectral region which it belongs to (even though the fields' amplitude is not changing), the students in the experimental group tend to refer to frequency, despite the representations of the wave profiles are reported in terms of the propagation direction by indicating the wavelength. It implies that these students recognize the dependence of the oscillation frequency on the characteristics of the source – the wavelength is also determined by the propagation medium – while the students of the control group tend – to a greater extent, in several questions – to consider equivalent descriptions (in terms of frequency or wavelength) because of the relationship $v = \lambda f$. Furthermore, we can deduce that mentioning the frequency does not depend on using multiple representations, since the characterization of wavelength as a distance between two consecutive wavefronts did not generate reasoning in which closer wavefronts – i.e. a greater number of wavefronts incident on the surface we are considering during that specific time interval – correspond to a greater amount of energy carried out by the electromagnetic wave.

However, the aspects concerning the preliminary activities we added in the second version of the learning path – in particular the description of electromagnetic induction phenomena in terms of cause-effect relationships, and the relationships between fields and operators – do not show significant differences between the experimental and control groups. These activities would need to be reviewed in order to represent an added value for the learning path, even though the main focus related to the characterization of flux and circulation as operators accounting for the correlation between electric and magnetic fields, takes a more complete meaning later through the description of the electromagnetic waves' propagation.

Another relevant aspect is the variation in the structure of the "stimulating questions" submitted to the control group, also through the introduction of answer options. These can help stimulate a peer-to-peer discussion – under the teacher's guide – that progressively refutes the most common alternative reasoning, even though this would require an improvement in the structure of the tutorials and a proven scheme for the discussion. In particular, the "stimulating questions" related to the preliminary activities were useful to introduce new concepts – such as flux and circulation – or to reintroduce other concepts – such as that of field and its sources – to the portion of students who did not previously deal with them in traditional teaching.

After that, the explicit reference to forces and their invariance – in classical mechanics – with respect to the frame of reference makes it easier for students to recognize the unification of the electromagnetic field, placing it in a coherent theoretical framework. This may also explain the improvements measured about considering the role of the observer with respect to the motion of the source and the propagation of the electromagnetic field it generates. As a result, students are usually able to recognize – to a greater extent than in the first experimentation – the equivalence between a charge and a magnet in motion in generating both components of the field, despite the asymmetries detected in their reasoning (see the discussion in the previous section).

The multiple representations were also effective in improving the rate of correct answers for question n.13 of Table 9.2, as the image of several consecutive wavefronts makes it easier to recognize that – at least in the case of plane waves – the angle that the propagation direction forms with the line connecting the (point) source with the position in which the fields are detected, does not affect the time interval needed for the effect (i.e. in this case the change in the measured oscillation frequency) to occur. On the contrary, when using representations with a single wavefront, we observed that a significant portion of students considers the propagation speed to be the component of the vector $\vec{c}$ along that direction (connecting the source and the point we are considering). In addition, the dynamism characterizing multiple representations also helps to increase the share of students who recognize the implications of the delay effect in the propagation of the electromagnetic field.

Regarding question n.17 of Table 9.2, most students – differently from the final questionnaire – chose options A and (especially) D, thus not identifying a direction of best reception over time: this confirms that in this case we are dealing with spontaneous conceptions, and not alternative ideas due to traditional teaching methods. Similarly, the answers to the next question show that – although they are sometimes aware of it – they do not recognize the need and role of resonance phenomena in the signals' reception, despite the fact that they recognize it is necessary to act in somehow on the receiving circuit. In this regard, most students do not seem to be able to recognize the differences in reception between a rectilinear and a circular antenna, that is they are not able to explain the differences between the two cases about the interaction of the fields with the free charges within a coherent framework: also in this case, a double description in microscopic or macroscopic terms is confirmed.

11.3 Discussion of the results for undergraduate students

Despite the limitations due to the small sample size, the results we obtained for undergraduate students provide a rather detailed picture of their conceptions, according to an ordered structure – consisting of macro-categories and sub-categories – which is not in correspondence with the answer options proposed in the questions. Thus, a possible change in the structure of reasoning is derived at a long distance from school education, before making new experiences about these topics in university courses.

Except the last question, in the other cases three macro-categories were derived, the last always grouping inconsistent answers – i.e. those that are difficult to interpret and cannot be grouped with other categories – and those who did not provide any explanation. In particular, regarding the interpretation of the wave profile representation, the inconsistent explanations refer to the oscillation in phase of the electric and magnetic fields or to the intensity of the wave (confusing it with that of the fields). On the other hand, the second category represents two alternative conceptions, relating to the cancellation of the fields at all points that are outside the region of space delimited by the wave profile, or to the decrease of their intensity moving away from the propagation axis represented in the figure. It is interesting to observe how these conceptions show different nature: while in the first case it is a formal conception exclusively relating to the representation itself, in the second case we can state that an alternative interpretation of representation implies the adoption of a specific conceptual model: although this subcategory includes only one student, it has been reported in the final table due to its relevance for teaching implications.

Indeed, category B1 includes students who believe that the (magnetic) field takes on different values at the points shown in the figure, being influenced by the vectors representing it. In particular, they believe that the fields are zero at points that do not coincide with either the tips or the tails of the vectors: this derives both from a misinterpretation of the vector representation itself – as it provides the value of the quantity considered at the point of application – and from the representation of the plane wave profile – which implies the symmetry of the electromagnetic field. On the other hand, category B2 contemplates an alternative reasoning that is not only formal, but rather conceptual: the fact that the only vectors represented in the figure have as application point those belonging to the propagation axis of the wave, leads students to think that these points represent the only ones which the fields are generated from: i.e. in this perspective, the propagation axis represents the location of the wave's sources.

This case is relevant because it represents a remarkable example of an alternative conception generated by a formal representation, as literature (Ambrose et al., 1999) and our previous results about high school students confirm. As a consequence, it is possible to discuss the chance of using representations of the plane wave profile in which the vectors (representing the electric and magnetic fields) are not reported, as this result would suggest this choice. However, the preference for one or the other type of representation should be specifically studied on a larger sample and with several questions focused on a common learning objective, in which students have to extract information from the different kinds of representations. It is worth noting that the ideal example discussed in the learning path – a field generated by an infinite conducting plane – takes into account the propagation of electromagnetic waves in relation to the position of the source: this aspect, often missing in traditional teaching, can help reduce this specific alternative conception.

In addition, the first category groups together the vast majority of students who recognize the common membership to the same wavefront of the points we are considering, which the equality of the values of the electric and magnetic fields derives from. Thus, reference to the wavefront as a conceptual referent is confirmed also for undergraduate students; nevertheless, in this case the analysis clearly brings out the distinction between the majority of students who do not add any other information beyond belonging to the same wavefront (categories A3) and those who premise that the statement is valid because the fields take on the same configuration after a time interval equal to the period of waves' oscillation (category A2). Finally, A1 category has also been added – despite being composed of a single student – as it corresponds to the most complete answer that can be provided. Indeed, due to plane symmetry, both fields verify the same sorting, regardless of the instant considered: this reasoning is particularly relevant, as it makes explicit the reference to the temporal dimension of the waves' propagation.

In the case of the second question, the dialogue between the students which it is based on makes the category C the most populated (15 students), and inconsistent explanations emerge referring to the motion of non-inertial frames of reference or to (supposed) relativistic effects that would influence the relationship between the motion of the source charges and the time trend of the electromagnetic fields detected by different observers. Category B groups together the misconceptions relating to the characterization of charges as sources of electromagnetic waves regardless of their state of motion: in particular, the reasoning on the non-static nature of the fields generated by a charge in uniform motion – without any reference to the observer who detects them – or even by a charge at rest, emerges.

In this second case, the only student belonging to category B2 reports a specific motivation to support his answer: he states that a stationary charge can generate electromagnetic waves in a frame of reference such that "the surface through which the (electric and magnetic) fields' flux are calculated is variable over time". Thus, this student is probably assuming that – according to Maxwell's equations – electric and magnetic fields with non-zero circulation will be generated, so that the propagation mechanism characterizing the dynamic case – according to which the variability over time of the electric component locally generates a magnetic component, and vice versa – will be activated according to an iterative process. It represents a very peculiar reasoning, but we decided to keep this subcategory to represent the response profile that does not identify any kind of link – causal or not – between the motion of the source and the trend over time of the electromagnetic field.

In addition, three subcategories have been defined for macro-category A relating to this question. They represent different justifications for the categorization of accelerated charges as the (only) sources of electromagnetic waves: while in the case of category A2 students believe that accelerated motion is necessary for the production of a time-dependent field, category A1 includes those who correctly identify the dependence of the source motion – and, as a consequence, of the temporal variability of the field – on the frame of reference. Finally, in category A3 those reasons are combined in an inconsistent modality, by means of causal links which do not occur between the motions of the physical entities we are considering.

Questions n. 3 and 4 investigate the relationships of the propagation velocity with other wave parameters, whose structure of categories is easier to interpret. Indeed, the majority of students identify the invariance of the propagation velocity for all (inertial) observers, even though they often refer to light in their explanations (probably due to the formulation of the special relativity postulate) rather than to the whole electromagnetic spectrum. In this case, the answers are quite uniform, while in the case of the relationships of the velocity with the propagation medium and the oscillation frequency, three subcategories have been identified for the same macro-category. In this case, all the students identify the dependence of this quantity on the medium, but in category A3 they do not specify either the possible dependence on the frequency, or the fact that the propagation velocity is maximum in vacuum. In category A2 these limitations do not occur, but only in category A1 students provide a satisfying justification by means of relation 9.8, being it derived from Maxwell's equations or from experimental evidences: these are usually based on the phenomenon of refraction and the variation of the optical path when switching between two different media.

A similar structure is obtained in both cases for macro-category B: in the fourth question it is characterized by classical reasoning, which does not recognize the invariance of the propagation velocity, by assuming the classical rule of velocities' composition (for the observer and the source). On the other hand, in the third question a tripartition of the second macro-category again occurs: in B1 the students assume the independence of the velocity from the propagation medium, as it is considered as a universal constant quantity. On the other hand, the categories B2 and B3 are similar in establishing a relationship between the propagation speed and the oscillation frequency: in both cases the students refer to the equation $v = \lambda f$, but in B2 it is interpreted as a relation of direct proportionality, while in B3 it is characterized as an inverse proportionality. While in the first case the interpretation seems to be based on considering the wavelength as a constant quantity, in the second case a qualitative reference (going beyond the relationship) also emerges: the students relate the increasing frequency with the accumulation of phase delays and – as a consequence – with the decreasing propagation speed. The reference to dispersion phenomena (according to which the refractive index is a function of frequency) appears clear, but it seems to be explicable in terms of a lexical or conceptual misunderstanding between the phase shift of one wave with respect to another, and the possible accumulation of delay in propagation.

Finally, there is a difference between the two questions, regarding the characterization of category C: while in the case of the fourth question it groups – similarly to the other cases – the answers provided without motivation or with inconsistent reasons (e.g. in this case some students refer to the Doppler effect), in the third question it represents a specific conceptual category. Indeed, three students consider the time interval necessary for the wave to propagate between the two points we are considering, to be equivalent to the oscillation period: this implies that – since the frequency is equal to the inverse of the period – a wave oscillating with greater frequency will propagate faster in space according to this model.

Regarding intensity (question n.5), four different groups are observed in macro-category A: these differences emerge both for the different kinds of relationships that students identify with the fields' amplitude, and also for conceptual aspects. Indeed, while in A1 the dependence on the square of the fields' amplitude is correctly identified, in A2 a relationship of direct proportionality is reported, probably due to the exclusive focus on the electric or magnetic field (see relation 9.12) rather than on the unified (wave) entity. In addition, A3 includes students who identify an (unspecified) dependence relationship, by reporting several explanations involving the definition of intensity: for instance, in one case it is considered equivalent to the time integral of the energy carried by the electric field.

Macro-category B includes students who identify a relationship between waves' intensity and fields' oscillation frequency, thus confirming the previous result showing that the reference to wavelength is not explicit, even in cases where the representation is provided in terms of this variable. In addition, the division into the three subcategories identifies the different types of relationships: in B1 the students report a further dependence on the fields' amplitude, while in B2 and B3 the reference to frequency is exclusive. The identification of the relations of direct (B2) and inverse (B3) proportionality actually seems to have a common nature, which can be explained alternatively in conceptual terms or through the interpretation of mathematical laws. Indeed, the Planck's law establishes a link of proportionality between the energy carried by a photon – which the students seem to identify with the intensity of the electromagnetic waves – and its frequency; on the other hand, an analogy with mechanical waves also emerges from the answers that can be summarized by a statement such as "a wave oscillating with higher frequency carries more energy".

Finally, the last question is characterized by the presence of four macro-categories, the last representing the inconsistent (or absent) answers. Category C groups together those who consider the direction of best signals' reception to be undetermined from its instantaneous representation. In this case, two clusters emerge: in C1 a student characterizes this direction as that of the oscillation motion of the source (imagined as a point charge), while in C2 it is indeterminate as it depends on the specific antenna and on the field (electric or magnetic) interacting with it, according to its operating mode. In other words, according to these reasonings, antennas are specially designed to convert – through unspecified processes – electromagnetic waves into an electrical signal, but the conversion methods depend exclusively on the antenna – in particular on the fact that some of them are built to interact with the wave in the direction of the electric field oscillation, others in that of the magnetic field, still others in any other possible direction – and not on the signal.

Once again, macro-category B confirms a result obtained with high school students, namely, that a significant share of the sample identifies the direction of waves' propagation with that of detection, by providing similar reasons. Similarly, no student reports the oscillation direction of the magnetic field as an answer, showing how this distractor can be removed in a possible new version of the questionnaire.

Finally, macro-category A groups together students who correctly identify the direction of the electric field oscillation as the one of its best reception through a rectilinear antenna. In A1 they provide an adequate justification in terms of the Lorentz force, also attributing a precise meaning to the lexical expression "best reception" in terms of maximizing the induced current in the antenna circuit, although there are no references to its variability over time and to the resonance phenomenon. On the other hand, in A2 students do not provide meaningful justifications, while in A3 inconsistent reasons relating to reception methods emerge, based on the fields' absorption or on the generation of "local" currents (other than those of conduction): these statements, however, are not (in our opinion) further classifiable from a conceptual point of view.

Overall, this discussion highlights that the differences about conceptual profiles and their sizes between the control groups – both for undergraduate and high school students – and the experimental ones show the overcoming of some learning difficulties after the teaching activities, beyond that any critical issues which to base a revision of the learning path on, also providing suggestions for a possible new version of the questionnaire to be validated.

Chapter 12

Conclusions

In this short concluding chapter we summarize the main results we obtained, which answer – at least, partially – the research questions defined in Chapter 8. They have been discussed in more detail in the previous chapter, also considering the results of the preliminary analysis' steps illustrated previously.

Regarding the generation phase of electromagnetic waves (RQ1), we observed that high school students – after experiencing the (second version of the) learning path – show greater awareness of the multiple reasons why accelerated charges can be considered as completely characterizing the set of their microscopic sources (i.e. causal link between the periodic motion of charges and the periodic fields' oscillation, energy emission in relation to the motion of the observer, quantities characterizing the waves' properties dependent on the source, etc.): from the answers to the first questions of the questionnaire – and looking at the comparison between its two versions – it is possible to understand how this improvement can be (at least, partially) attributed to the clarification of the mathematical definition and conceptual meaning of the integral operators of the vector fields (flux and circulation) for several reasons. Indeed, on the one hand, their conceptual meaning as quantities representing the characteristic constants of the fields allows us to clarify that the Gauss and Ampere laws (relations 9.2 and 9.4) do not contain all the fields' sources; on the other hand, the example of the infinite conducting plane shows that there is no causal link between the (electrical and magnetic) components of the field, but rather the application of Maxwell's integral equations allows determining the properties of the propagation of this new physical entity.

In this regard, the response to RQ2 showed the relevance of providing a new interpretative key to explain the characteristics of the propagation velocity. Indeed, although the textbooks' analysis and the comparison with the control group (or pre-test, in the case of the first experimentation) found that some of them were typically known to students, there was often a lack of some kind of motivation relating these properties with the theoretical context that had been previously presented to students. These relationships have been provided through the examples discussed above, allowing to characterize the propagation velocity as a quantity independent of the observer-source relative motion and of the fields' properties; dependence on the propagation medium also emerges, although our discussion is focused on the case of the vacuum. In addition, the step-by-step analysis of the fields generated by a specific source allows providing a more precise meaning to the propagation mechanism, also with reference to the "delay effect" (i.e. the time separating the changes in the field from the changes in its source), which is another innovative element for students. These clarifications also take place by exploiting the interpretation of representations such as the wave profile (in its spatial or temporal propagation) or the train of wavefronts, which (when they are coupled) produce beneficial effects on learning, avoiding the emergence of alternative conceptions generated by the same representations when the information (which they contain) is not made explicit. Nevertheless, the recognition of the fields' characteristics starting from the representations remains one of the most difficult tasks for students, even after the teaching activities.

The most significant improvement compared to traditional teaching is measured in the discussion of RQ3, as the textbooks' analysis had already highlighted how fields' detection through interaction with free charges in matter is a topic usually neglected. On the other hand, beyond providing a relevant application, this discussion completes the frame with respect to the generation and propagation of electromagnetic waves. Thus, it was quite easy to overcome some spontaneous conceptions generated by the lack of alternative mental models, especially regarding the determination of the direction of best signals' reception.

In this regard, one of the several similarities between the samples of undergraduate and high school students was measured, confirming our initial hypothesis. Similar results are also obtained with respect to the interpretation of representations, while undergraduate students seem (slightly) more aware – compared to students in the control group or in the pre-test – about the role of the observer and the implications of the relative motions between the physical entities involved in the description of the phenomenon. Nevertheless, in order to assess the presence of statistically significant differences between the two populations – which goes beyond our goals – we would need a larger sample representing the undergraduate students (whose size should be similar to the control group) and coming from several universities in different countries.

This represents one of the possible developments of this research, as well as others are possible to mitigate other limitations of the present study. In particular, tutorials and "stimulating questions" can be modified in order to be more structured, and more experimental activities can be inserted (thus modifying the duration of the learning path); however, this is quite complicated, considering the peculiarity of the topic we are addressing (quite advanced for the population of high school students) as well as representing a further logistical difficulty. On the other hand, considering a more immediate and easily achievable objective, it is possible to validate a test aiming at systematically searching the (undergraduate or high school) students' learning difficulties in this area. For this purpose, the information we obtained through the present study can be exploited, for example regarding the frequency of the different distractors (options), and several items related to the same learning objective can be included. This choice would transform the tool's nature from evaluative to diagnostic, thus adapting to the new purposes.

By thew way, the discussions we had with teachers showed us how it is difficult for them to identify the learning difficulties in this area, so we believe that this objective can also be fruitful for research, also considering these topics' relevance as a conceptual bridge between classical and modern physics. For this reason, small experimentations have been recently set as part of a teacher training course, which constitute another possible research development.

Finally, we want to point out some limitations of this research thesis. They can be classified as concerning the samples' selection, the implementations' methods of the teaching activities, and finally the testing phase. In fact, those selected are convenience samples involving schools whose average students' performance - measured through formal and informal surveys - is high. Furthermore, this section of the physics curriculum is peculiar as it is compulsory in Italian high schools where students attend a scientific curriculum, going beyond the simple illustration of Maxwell's equations. Thus, it would be necessary to expand the research with a more stratified sample, as well as to refine the structure of the tutorials and the stimulating questions submitted during the activities, as the subsequent class discussion did not follow a predetermined scheme. Furthermore, the testing methods would require a specific preliminary study, as the items' validity has not been previously tested on a control group, nor different items have been included for the various learning objectives: this represents another among the possible developments of this research.

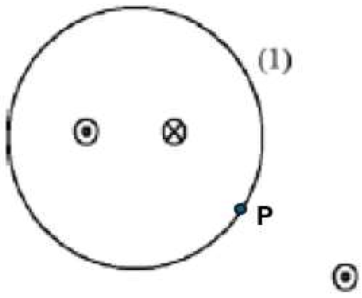
Acknowledgments

At the end of this thesis, I would like to thank all the people who made this research work possible. Among them, I want to mention *Prof. Kristina Zuza* and *Prof. Jenaro Guisasola* from the *University of the Basque Country* (*Spain*), where I spent three months of my research period abroad (Erasmus project for PhD students), with whom we developed a part of the questionnaire then submitted to the research samples, also laying the basis for our epistemological reconstruction of the theory we addressed. I thank *Prof. Ignacio Julio Idoyaga* from the *University of Buenos Aires* (*Argentina*) who made it possible to carry out a comparative study of Italian and Argentine school textbooks. I also thank *Prof. Maurizio Zani* and his research group (*Dr. Marco Costigliolo* and *Dr. Matteo Bozzi*) from the *Politecnico di Milano* (*Italy*) for having actively contributed to the sample selection and to the analysis of the results for the group of undergraduate students. I also thank *Prof. Guido Pegna* (former Professor at the *University of Cagliari, Italy*) and *Dr. Enzo Cortesi* for the fundamental contributions relating to the experimental devices used during the activities, and for bringing (in the case of Enzo Cortesi) to my attention the important theoretical work of educational reconstruction by *Dr. Francesco Dalla Valle* (former president of the Italian association of physics teachers). A heartfelt thanks also goes to all my colleagues (high school physics teachers) who made possible to implement our experimentation (and data collection) during curricular and extra-curricular activities. Finally, the last thanks is for *Lorenzo Santi* (former Professor at the *University of Udine, Italy*), who prematurely passed away in February 2025, who – together with *Prof. Marisa Michelini* – supervised my thesis work: I will try to inspire my career as a teacher-researcher to honor people with his human and professional value.

Appendix A

Original (Italian) version of the final questionnaire

<u>Question 1</u>



Nella figura precedente sono rappresentati tre fili percorsi da corrente ed ortogonali al piano del foglio. Il simbolo $\odot$ indica correnti che scorrono verso l’alto, mentre il simbolo $\otimes$ indica correnti che scorrono verso il basso. Sappiamo che un filo percorso da corrente genera un campo magnetico.

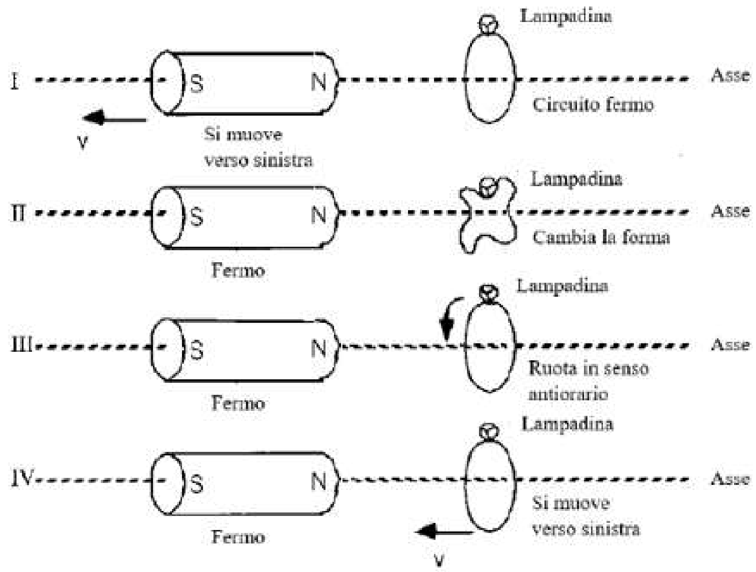
Quanto valgono la circuitazione del campo magnetico lungo la curva circolare (1) e il campo magnetico nel punto P ad essa appartenente?

A. Il campo magnetico è nullo, mentre la circuitazione è diversa da zero.
B. Il campo magnetico è diverso da zero, mentre la circuitazione è nulla.
C. Sia il campo magnetico che la sua circuitazione sono nulli.
D. Sia il campo magnetico che la sua circuitazione sono diversi da zero.
E. Non ci sono informazioni sufficienti per rispondere.

<u>Spiega brevemente il tuo ragionamento</u>.

<u>Question 2</u>

Le 4 figure seguenti rappresentano un magnete cilindrico e una lampadina connessa ad un filo conduttore a forma di circonferenza. Il piano in cui giace il filo è perpendicolare all’asse di riferimento rappresentato in figura. Gli stati del magnete e del circuito sono riportati in figura.



In quali delle figure la lampadina si illuminerà?

A. I, III, IV.
B. I, IV.
C. I, II, IV.
D. IV.
E. Nessuna delle precedenti.

Spiega brevemente il tuo ragionamento.

Question 3

Considera degli elettroni di conduzione che si muovono, sotto l'azione di un generatore, in un filo metallico. Essi genereranno un campo elettromagnetico.

Cosa possiamo dire sul rapporto tra la velocità con cui gli elettroni si muovono nel filo e la velocità di propagazione del campo da essi generato?

A. Non ci sono informazioni sufficienti per rispondere.
B. La velocità degli elettroni è molto minore di quella del campo.
C. La velocità del campo è molto minore di quella degli elettroni.
D. La velocità del campo coincide con quella degli elettroni.
E. Le velocità sono diverse, ma hanno lo stesso ordine di grandezza.

Spiega brevemente il tuo ragionamento.

Question 4

Quattro studenti discutono sugli stati di moto che una carica elettrica puntiforme deve assumere rispetto a un sistema di riferimento inerziale *S*, per generare un'onda elettromagnetica rilevabile in tutti i sistemi inerziali.

Lo studente *A* afferma: "Una carica elettrica puntiforme in stato di quiete in *S* è in grado di generare un'onda elettromagnetica".

Lo studente *B* replica: "Non sono d'accordo, perché una carica ferma non genera campi elettrici e magnetici variabili nel tempo. Abbiamo dunque bisogno che la carica si muova di moto uniforme".

Lo studente *C* replica: "Non sono d'accordo, perché anche in questo caso i campi generati sono costanti. Abbiamo dunque bisogno di una carica che si muova di moto accelerato".

Lo studente *D* replica: "Non sono d'accordo, perché anche una carica in moto uniforme è in grado di generare campi variabili nel tempo, ma non nel sistema di riferimento ad essa solidale. Per questo motivo abbiamo bisogno di una carica accelerata".

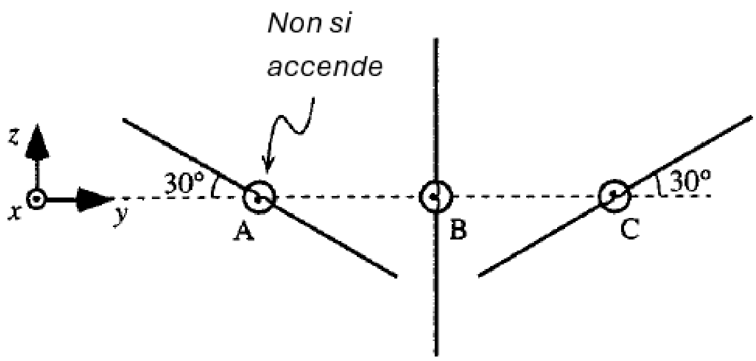
Con chi sei d'accordo?

A. Con lo studente A.
B. Con lo studente B.
C. Con lo studente C.
D. Con lo studente D.
E. Con nessuno degli studenti.

Spiega brevemente il tuo ragionamento.

Question 5

Nella figura successiva sono riportate 3 lampadine, collegate a fili diversi giacciono nel piano y-z. Un'onda elettromagnetica piana si sta propagando lungo l'asse x.



Sapendo che la lampadina A non si illumina, cosa possiamo dire sulla luminosità delle due lampadine B e C?

A. Nessuna delle lampadine si accende.
B. Non ci sono informazioni sufficienti per rispondere.
C. Entrambe le lampadine si accendono con la stessa luminosità.
D. Entrambe le lampadine si accendono, ma B avrà luminosità maggiore.
E. Entrambe le lampadine si accendono, ma C avrà luminosità maggiore.

Spiega brevemente il tuo ragionamento.

Question 6

Cosa succede ai campi elettrico e magnetico prodotti da una carica ferma quando vengono misurati da un osservatore in moto uniforme rispetto ad essa?

A. Cambia solo il campo elettrico.
B. Cambia solo il campo magnetico.
C. Non cambia nessuno dei due.
D. Cambiano entrambi.
E. Dipende dal moto dell'osservatore.

Spiega brevemente il tuo ragionamento.

Question 7

Tre campi elettrici, variabili nel tempo con andamento sinusoidale, sono generati in un punto O, con frequenze rispettive $f_2 > f_1 > f_3$. I primi due campi si propagano nello stesso mezzo dielettrico semplice, mentre il terzo si propaga nel vuoto. I tre campi si propagano nella stessa direzione, fino ad essere rilevati in un punto P.

In che ordine temporale essi arriveranno in P?

A. Arrivano insieme.
B. Il terzo campo arriva prima, poi gli altri due insieme.
C. Arrivano in ordine di frequenza.
D. Arrivano in ordine inverso di frequenza.
E. Il terzo segnale arriva per ultimo.

Spiega brevemente il tuo ragionamento.

Question 8

Un osservatore a riposo in un sistema di riferimento *S*, dotato di una radio portatile, riceve un segnale da una sorgente di onde radio. Considera i seguenti tre casi:

1. La sorgente è a riposo in *S*.
2. La sorgente si allontana dall'osservatore con moto uniforme rispetto a *S*.
3. La sorgente si avvicina all'osservatore con moto uniformemente accelerato rispetto a *S*.

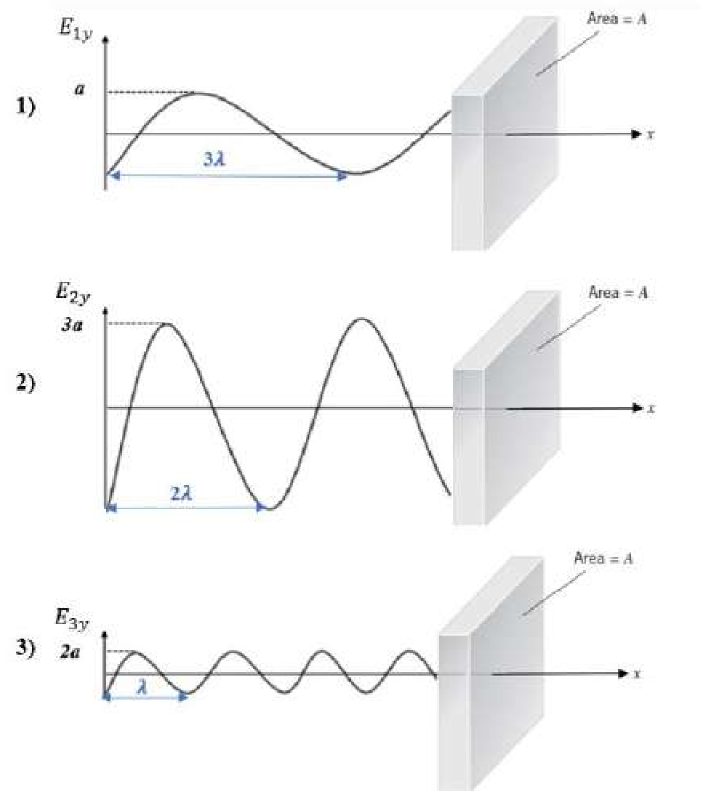
Detto v_i il modulo della velocità di propagazione del segnale radio misurato dall'osservatore, quale tra le seguenti alternative risulta verificata?

A. $v_1 = v_2 = v_3$.
B. $v_3 > v_2 > v_1$.
C. $v_1 > v_2 > v_3$.
D. Non ci sono informazioni sufficienti per rispondere.
E. Nessuna delle alternative precedenti.

Spiega brevemente il tuo ragionamento.

Question 9

Considera le 3 figure seguenti. Esse rappresentano le componenti nella direzione di propagazione di tre campi elettrici, che variano nel tempo con un andamento sinusoidale. Essi investono una superficie che giace nel piano y-z, di area. Le ampiezze dei tre campi sono *rispettivamente* a, $3a$ e $2a$, mentre le lunghezze d'onda sono *rispettivamente* 3λ, 2λ e λ.



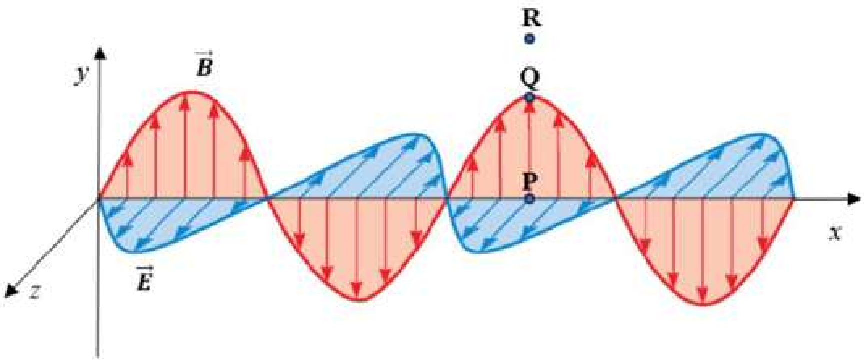
Detta I_i la quantità di energia che investe la superficie A in 1 secondo, quale tra le seguenti alternative risulta verificata?

A. $I_1 > I_2 > I_3$.
B. $I_3 > I_2 > I_1$.
C. $I_1 = I_2 = I_3$.
D. $I_2 > I_3 > I_1$.
E. Nessuna delle alternative precedenti.

Spiega brevemente il tuo ragionamento.

Question 10

In figura è illustrata la rappresentazione di un'onda elettromagnetica piana all'istante $t = 0$, con piano di simmetria parallelo al piano *y-z*. I punti P, Q e R hanno lo stesso valore dell'ascissa $x = x_0$.



Detto T il periodo dell'onda, quale delle seguenti opzioni rappresenta l'ordine corretto dell'intensità dei campi elettrico e magnetico nei punti considerati all'istante $t = \frac{T}{2}$?

A. $E_Q > E_P = E_R$ e $B_P = B_Q = B_R$.
B. $E_P = E_Q = E_R$ e $B_P = B_Q = B_R$.
C. $E_P > E_Q > E_R$ e $B_P = B_Q = B_R$.
D. $E_P > E_Q > E_R$ e $B_P > B_Q > B_R$.
E. Nessuna delle alternative precedenti.

<u>Spiega brevemente il tuo ragionamento</u>.

Appendix B

The measurement of the electric field circulation

In this Appendix we report some figures to illustrate the measurement of the electric field's circulation (similar considerations apply to the magnetic field).

Circulation of the Electric field with 2 spheres					
N°	l (m)	E (ua)	E l (ua)	e%	e.a.(ua)
1	0,050	-2,9	-0,15	5	0,01
2	0,050	-13,2	-0,66	5	0,03
3	0,050	-15,2	-0,76	5	0,04
4	0,050	-22	-1,10	5	0,06
5	0,050	-16,1	-0,81	5	0,04
6	0,042	-29	-1,22	5	0,06
7	0,050	-38,5	-1,93	5	0,10
8	0,050	-33,2	-1,66	5	0,08
9	0,050	-18,9	-0,95	5	0,05
10	0,050	2,2	0,11	5	0,01
11	0,050	50,8	2,54	5	0,13
12	0,050	44,3	2,22	5	0,11
13	0,050	33	1,65	5	0,08
14	0,050	18,9	0,95	5	0,05
15	0,050	-8,6	-0,43	5	0,02
16	0,050	24,8	1,24	5	0,06
17	0,050	19,4	0,97	5	0,05
18	0,042	-11,2	-0,47	5	0,02
19	0,050	21,7	1,09	5	0,05
Value of the circulation =			0,6	±	1,0
Sum of positive values =			10,8		
Sum of negative values =			-10,0		
Sum of absolute values =			20,7		
Circulation/Sum of absolute values =			3%		
Nota: The error about the electric field (3%) also includes the error about the angle, while for the length the absolute error is 1 mm and the relative error is 2%					

Figure B.1: Values we obtained by measuring the circulation of the electric field generated by two charged spheres.

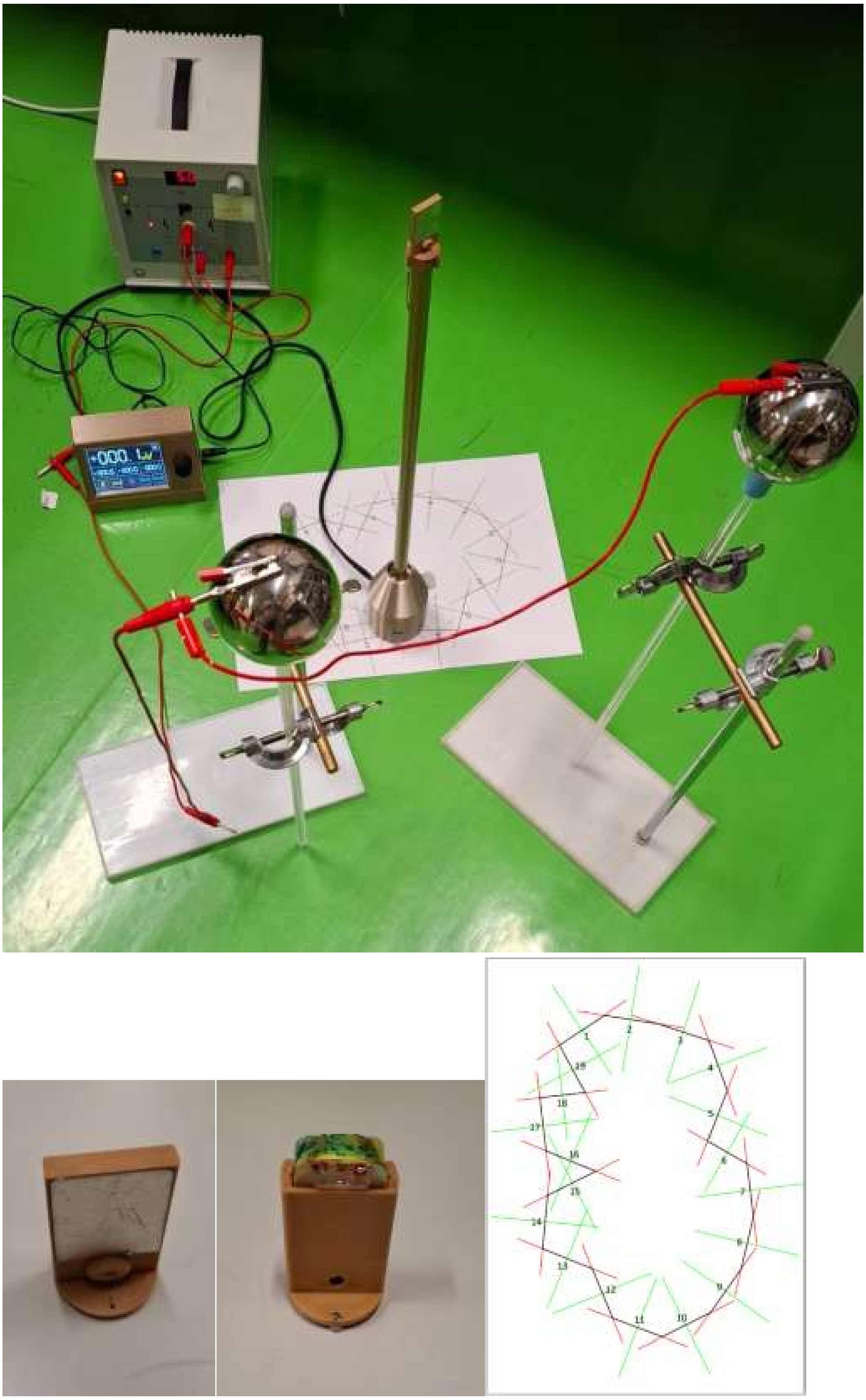


Figure B.2: Experimental device used to measure the circulation (on the top). Sensor heads for measuring the electric field (bottom left) and the magnetic field (bottom center). The closed line along which the circulation was measured: the red and green segments are only used to best orient the sensor (bottom right).

Appendix C

Measurement of the speed of light

We inserted in the learning path a discussion about the historical development on the measurement of the speed of light, which in our opinion has an intrinsic educational value.

Indeed, in ancient times light was often conceived as an entity that propagating instantly, or permeating all space (e.g. Aristotle). Other philosophers (such as Empedocles) believed that light, like all objects, should propagate with finite speed, but the debate was centered around metaphysical reasons. On the other hand, taking into account the value of the speed of light in a vacuum, it is easy to understand that simple terrestrial measurements cannot produce realistic estimates of this quantity[1].

This debate lasted until the first experimental estimates of the speed of light value in a vacuum were obtained, based on astronomical measurements. Figure C.1 shows the list of the main experiments providing estimates of this value. After Galileo's attempts that – which did not provide any quantitative estimates – the first astronomical measurement was made by Romer and Huygens in 1675: it was based on observations of the motion of Io – one of Jupiter's natural satellites – and the data available relating to its immersions in and its emergence from the shadow of sunlight visible from Earth. Indeed, knowing Kepler's laws about the motion of celestial bodies, a time delay emerged in the dives (or emergences) observed in the different periods of the year, depending on the time taken by the light to propagate in the considered stretch of the Earth's orbit. The final estimate differs by about 25% from the value we accept nowadays, but it still represents a fundamental result. A much more precise measurement was obtained by Bradley in 1729, based on the phenomenon of light aberration[2].

[1] Indeed, in a tenth of a second – which corresponds approximately to the sensitivity of an instrument measuring time intervals, taking into account the time needed for human reactions – light travels (in a vacuum) a distance equal to (about) the Earth's circumference.

[2] These measurements precede the theoretical estimate obtained by Maxwell in 1861 – in the essay "On Physical Lines of Force" – on the basis of which he hypothesized that light was made up of oscillating electromagnetic fields.

<1638	Galileo, covered lanterns	inconclusive
<1667	Accademia del Cimento, covered lanterns	inconclusive
1675	Rømer and Huygens, moons of Jupiter	220000
1729	James Bradley, aberration of light	301000
1849	Hippolyte Fizeau, toothed wheel	315000
1862	Léon Foucault, rotating mirror	298000±500
1907	Rosa and Dorsey, EM constants	299710±30
1926	Albert A. Michelson, rotating mirror	299796±4
1950	Essen and Gordon-Smith, cavity resonator	299792.5±3.0
1958	K.D. Froome, radio interferometry	299792.50±0.10
1972	Evenson *et al.*, laser interferometry	299792.4562±0.0011
1983	17th CGPM, definition of the metre	299792.458

Figure C.1: List of the main experiments carried out to determine the value of the speed of light propagating in a vacuum. The first column shows the year when the experiment was carried out for the first time, the second column shows the authors and the name of the experiment, and the third column shows the value obtained as a speed estimate (in km/s).

Estimates based on astronomical measurements have the limitation of providing the required value only in a vacuum, while the aim is to find a method allowing estimates for any propagation medium. To get the first terrestrial measurement, it was necessary to wait until the era of the industrial revolutions, in particular with the Fizeau experiment in 1849. In this case, a beam of light crosses a toothed wheel rotating with great speed, being transmitted only when it meets a throat of the wheel (that is the space between two successive teeth). Once transmitted, the beam travels several kilometres and is then reflected into an eyepiece, from which the light intensity can be observed. By adjusting the rotation frequency in order as to observe a minimum value for light intensity, the speed of light can be estimated, by taking into account that the time taken to travel the selected distance must be equal to that between one throat and the next tooth.

Mechanical measurements have produced increasingly precise estimates, but they have been abandoned since the end of the 19th century, leaving room for interferometric measurements used (until the 70s, when we got the value accepted nowadays). In the second version of our learning path, a measurement of the speed of light has been included, by exploiting the device illustrated in Figure C.2 (using the phase shift method).

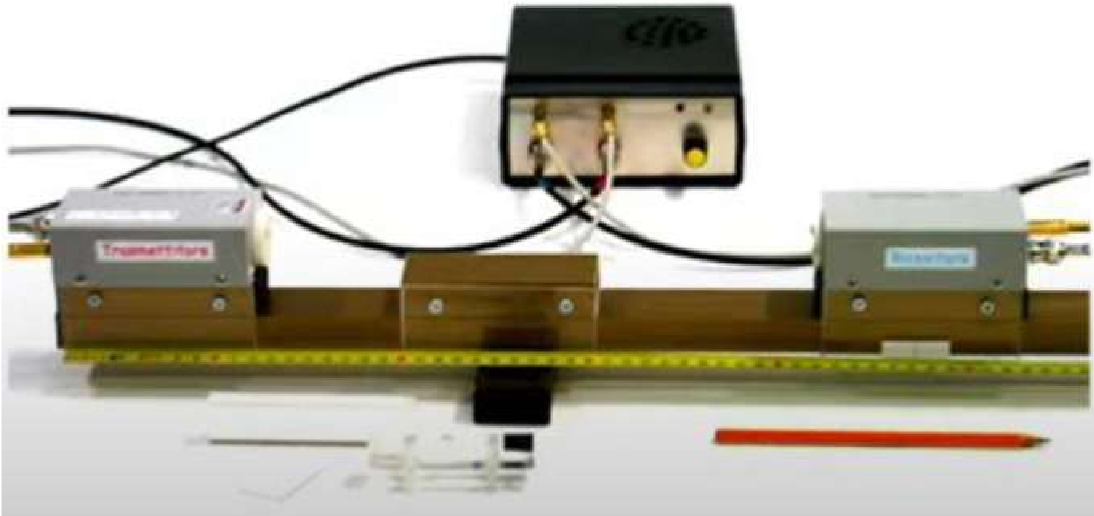


Figure C.2: Device we used for the measurement of the speed of light by means of the phase shift method, consisting of a control unit (on the top) generating a sinusoidal electrical signal, which is transmitted to a transmitter (consisting of a laser with driven pulses, on the left) and then reaching the receiver (consisting of a photodiode, on the right) in the form of light. It transforms it back into an electrical signal, which through another wire returns to the control unit, in order to be compared with the original signal.

The device consists of a control unit that generating a sinusoidal electrical signal, which is transmitted to a laser with driven pulses. This light beam travels a straight line on the optical bench, until it reaches a photodiode. It transforms it back into an electrical signal, which through another wire returns to the control unit, in order to be compared with the original signal. Indeed, by varying the position of the receiver on the optical bench, the distance travelled by the light pulse will vary: since the rest of the circuit is unchanged, the speed of light can be estimated as the ratio between the variation of the position of the receiver (ΔL) and the time interval corresponding to the delay effect ($\Delta\tau$, represented in Figure C.3).

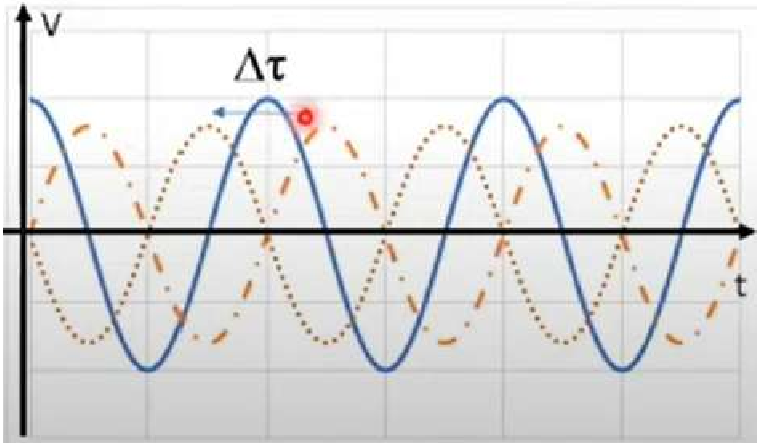


Figure C.3: Representation of the sinusoidal signal (voltage) generated by the control unit in Figure C.2 (in blue) and comparison with the signals propagating along the circuit for different distances between the transmitter and the receiver (in yellow, dashed line). The time interval $\Delta\tau$ corresponds to the delay between the two propagating signals we are taking into account. All signals oscillate in phase.

Thus, in order to estimate the speed of light we will use the relation[3]:

(9.18) $$\Delta\tau = \frac{\Delta L}{c} \rightarrow c = \frac{\Delta L}{T}$$

Where T represents the period of the signals considered. In fact, since the interval is very short, its measurement would require sophisticated instruments, not suitable for an experiment with educational purposes. For this reason, it can be shown that the result of the comparison between the two signals – reported in yellow colour in Figure C.3 – is made by means of an audio signal output: according to the indications of the apparatus' manufacturer, when $\Delta\tau = 0$ the output sound will have maximum intensity, while for $\Delta\tau = \frac{T}{4}$ it will have minimum intensity. This situation will repeat with a period T: thus, by varying the position of the receiver on the optical bench – in order to search for the minimum sound intensity – we may use the relation 9.18 to obtain an estimate of the speed of light (with its error)[4]. We remark that, from the teaching point of view,

[3] Actually, in this way we are estimating the value of the velocity in air, which is very similar to that obtained in a vacuum. The value obtained can be compared with the theoretical value reported in 9.8.

[4] Taking into account the previous considerations, the distance ΔL to be used in the relationship at the period T (i.e. the period of the signal emitted by the control unit) will be the distance between two non-consecutive positions of the receiver when the output corresponds to a minimum of sound intensity. The intensity of the output sound can be adjusted by rotating a knob: the best choice is to focus on the points corresponding to minimum intensity (silence), thus avoiding ear saturation. Since it is possible to show that these points are determined in the range of an area whose width is typically

it is important to observe that the numerator and denominator of this relationship do not coincide with the wavelength and period of the laser light, as students may easily conceptually overlap these relationships.

2-3 millimetres wide, the estimates we obtain differ from the expected value by a discrepancy of less than 1% (excellent result).

Appendix D

Deriving the wave equation

In this appendix we are going to derive the wave equation (and its general solution) starting from the Maxwell's equations – by considering a very simple case – through the application of mathematical methods suitable for the student population we are addressing.

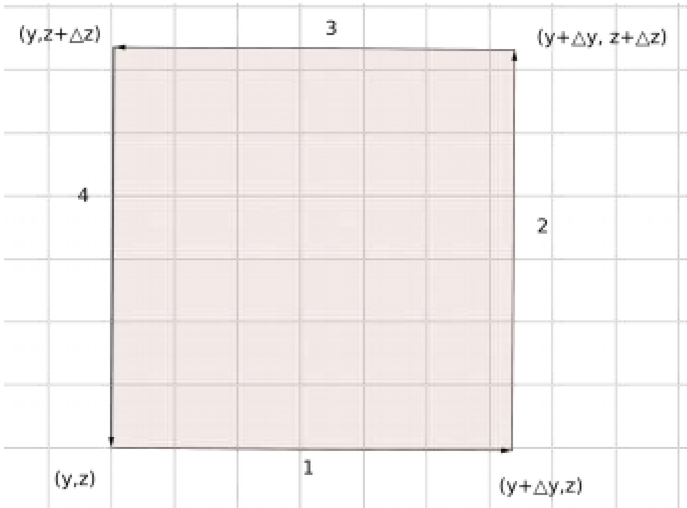


Figure D.1: Small square lying in the yz-plane.

Let's consider an electric field directed along the z-axis which is varying only in the direction of the y-axis, in the absence of charges and currents. By applying the definition of circulation to the small square represented in Figure D.1, we obtain[5]:

(D.1) $$\Gamma_E = \frac{\partial E_z}{\partial y}\Delta y \Delta z$$

As the magnetic field also is approximately constant at all points on the surface of the square, we have:

(D.2) $$\Phi_B = B_x \Delta y \Delta z$$

[5] Indeed, on sides 1 and 3 the tangent component of the field is zero. On side 2 the field is tangent and constant, taking on the value $E_z(y+\Delta y, t)$. On side 4 the field is still tangent, but it is oriented in the opposite direction, taking on the value $E_z(y, t)$. Thus, its circulation is given by the expression: $[E_z(y + \Delta y, t) - E_z(y, t)]\Delta z$. The result reported in D.1 is due to the relation: $E_z(y + \Delta y, t) - E_z(y, t) \approx \frac{\partial E_z}{\partial y}\Delta y$.

Finally, by applying relation 9.3 in this case we get:

(D.3) $$\frac{\partial E_z(y,t)}{\partial y} = \frac{\partial B_x(x,y,z,t)}{\partial t}$$

A similar reasoning applies to a square in the xy-plane, thus we derive:

(D.4) $$\frac{\partial B_y}{\partial t} = \frac{\partial B_z}{\partial t} = 0$$

That is, the magnetic field has no dynamic components[6]. Thus, we can state that the magnetic field has only a component along the x-direction, that is perpendicular to that of the electric field, and it only depends on the variables y and t.

A similar calculation can be applied to relation 9.4, thus obtaining:

(D.5) $$\frac{\partial B_x(y,t)}{\partial y} = -\varepsilon_0 \mu_0 \frac{\partial E_z(x,y,z,t)}{\partial t}$$

By differentiating equation D.3 with respect to the variable y, and by exchanging the order of derivation, we have:

(D.6) $$\frac{\partial^2 E_z(y,t)}{\partial y^2} = -\frac{\partial}{\partial t}\left(\frac{\partial B_x(y,t)}{\partial y}\right)$$

Finally, by using equation D.5 we have:

(D.7) $$\frac{\partial^2 E_z(y,t)}{\partial y^2} - \varepsilon_0 \mu_0 \frac{\partial^2 E_z(y,t)}{\partial t^2} = 0$$

That is the equation of a wave propagating in the direction of the y-axis with a velocity given by the relation 9.8. A similar reasoning also applies to the case of the magnetic field, which satisfies the same equation, thus propagating in the same direction and with the same velocity.

Thus, the elementary solution of this "wave equation" (for the electric field) is:

(D.8) $$E_z(y,t) = E_0 \cos\,(ky - \omega t)$$

From which the relation $c = \lambda f$ can easily be derived.

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[6] Any static components would be generated by stationary currents, which we assume do not exist.

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