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Heat Transfer Correlations for compressible flow in Micro Heat Exchangers

M. A. Coppola, G. Croce[1]
DIPIA, University of Udine
Udine, Via delle Scienze 206, 33100, Italy

E-mail: giulio.croce@uniud.it

Abstract. The paper discusses the definition of dimensionless parameters useful to define a local correlation for convective heat transfer in compressible, micro scale gaseous flows. A combination of static and stagnation temperatures is chosen, as it allows to weight the temperature change related to the heat transfer and that induced by conversion of internal energy into kinetic one. The correlation offers a purely convective local Nusselt number, i.e. correlating the heat flow rate with the local flow parameters and wall surface temperature. The correlation is validated through a series of numerical computations in both counter-current and co-current micro heat exchanger configurations. The numerical computations take into account rarefaction and conjugate heat transfer effects.

1. Introduction

Microscale heat transfer has been a popular topic in open literature for the last fifteen years, due to the ever increasing interest in micro scale equipment. It is thus well known that in microchannel flow a number of scaling effects, easily neglected at macroscale, must be taken into account [1,2]. To name a few, viscous dissipation work, due to the low Reynolds number, axial conduction in solid, due to the relative high thickness of the channel walls [3], roughness [4], due to manufacturing constraints, may play a significant role. If gases are concerned, as in the present work, compressibility, due to the large pressure drop required to drive the flow, and rarefaction, for the smaller sizes, induce further reasons to depart from standard macroscale behaviour [6,7]. Several papers deal with the evaluation of these effects in several exemplar or practical configuration, either via experimental analysis [7] or detailed numerical computation [7,8]; however, for a standard engineering design workflow, it would be much useful to have correlations that allow to take into account all of these effects in a simpler way, so that the information could be used in the necessarily simplified models required for the preliminary design of complex devices. This is the standard approach in macroscale, where the design of a heat exchanger relies (at least for standard geometries) on correlations for Nusselt number as a function of Reynolds number and a few geometrical parameters. Unfortunately, compressible flow regime excludes the possibility of fully developed flow, and thus these correlation needs to be defined on a local basis; furthermore, at higher Mach number regimes, the temperature changes are driven by two different phenomena: the heat transfer and the conversion of internal energy into kinetic energy, due to the flow acceleration. A straightforward extension of the uncompressible flow performance parameters

[1] To whom any correspondence should be addressed.

definitions, which neglect the latter effect, into this compressible regime, may lead to inconsistencies and unphysical behaviour.

Thus, the main aim of the present work is to look for a proper choice of dimensionless parameters that allows for a better understanding of the physics and the definition of a strictly local correlation for convective heat transfer in compressible gaseous flow in micro-channels. The correlation offers a purely convective Nusselt number, i.e. correlating the heat flow rate with the local flow and wall surface temperature.

This Nusselt number could be used in a variety of simplified modelling approaches, such as decoupled schemes or porous media modelling. The latter offers an interesting compromise between accuracy and efficiency, since, despite the high efficiency of current CFD tools, the detailed simulations of the whole device geometry (including tens, if not hundreds, of micro-channels) is not practical as standard design tool. Modelling of the whole device as a continuum porous media reduces the computational effort: the computational domain covers the whole device, and the single control includes both fluid and solid sections, and the wall effect is introduced via a porous resistance, as commonly done also in commercial CFD tools. This approach can be extended to the heat transfer case by defining, at each control volume, three different temperatures (hot side, cold side, wall) and solving the respective energy equations. Nusselt correlation defines the heat transfer, i.e. the source term in each energy equation due to the heat flux from the other components. A correlation is, thus, an essential step in order to extend this approach to the microscale compressible case: in fact, since it is based exclusively on local flow parameters, neglecting any global data (channel length, inlet and outlet temperature, pressure ratios), it is capable to reliably compute a local heat flux between fluid and solid under any possible flow arrangement.

The proposed parameter choice, and the consequent correlation, are here validated through a series of numerical computation in both counter-current and co-current heat exchanger configurations. The numerical computations are performed via a Navier Stokes code coupled with Maxwell first order boundary conditions, in order to explore the continuum and slip flow regime. We are concerned with the smaller scale of interest for MEMS applications, so we are restricted to lower Reynolds numbers and laminar flow. However, the heat exchanger configuration includes the conjugate heat transfer through the solid wall.

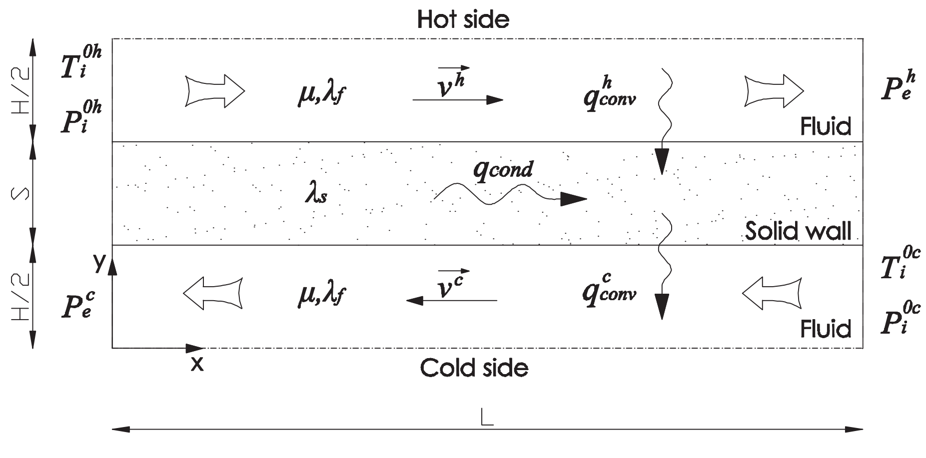

Figure 1. Sketch of computational domain

2. Geometry

The main aim of the present work is to identify the proper dimensionless parameters that define the heat transfer performance in compressible flow, where the temperature is not driven only by the heat transfer itself, but also by the cooling effect of the conversion from internal energy to kinetic one.

Thus, we consider a simple geometry, in order to focus on the main physical phenomena rather than the geometrical details. In particular, a plate heat exchanger, sketched in Fig.1, is considered: an array of two-dimensional plane microchannel separated by relatively thick solid walls. Despite the trivial geometry, most of the typical micro scale scaling effects are taken into account: the computational domain includes both the fluid and solid region, where velocities are set to zero and we solve only the energy equation, and the size of the channel falls within the slip flow regime. The plane channels have height H and length L, while the solid wall has thickness s. The elementary periodic cell reduces to one half of two adjacent channels and the solid wall of thickness between them, as depicted in Fig.1.
The standard parameter for the evaluation of the heat transfer performance is the local Nusselt number, defined as

$$Nu(x) = \frac{\alpha D_H}{\lambda_f} = 2\frac{q"H}{\lambda_f (T_{w,x} - T_{b,x})} \tag{1}$$

Where the subscript b refers to bulk, mass-averaged value over a cross section at location x, and w refers to the wall value at fluid-solid interface. For an incompressible flow, such definition is quite straightforward, since the temperature gradients (at least neglecting viscous dissipation) are exclusively a function of the heat transfer rate. In particular, in standard laminar, incompressible flow in a plane channel the Nusselt value is constant, except for the entrance region, and independent on either Re or fluid properties. On the other hand, in laminar, but compressible flow, the bulk temperature, and as a consequence the denominator in definition (1), is a function of two different mechanisms: the heat transfer at the boundaries, and the cooling due to acceleration in the core flow. Thus, in general, compressible bulk temperature at a given heat flux and wall temperature is lower than its uncompressible counterpart, leading to a lower Nusselt in a heated flow stream and a larger one in a cooled flow.

3. Numerical model

The viscous, compressible NS equations for 2D laminar flow are solved via a hybrid finite difference-finite volume method. The code, described in [9], follows standard numerical techniques and has already been widely applied and validated for the simulations of micro-flows [10].
We introduce the standard definition of stagnation state, defined by the pressure p_0 and temperature T_0 that the flow would attain after an adiabatic, isentropic arrest. The two are related by the local Mach number:

$$T^0 = T\left(1 + \frac{\gamma - 1}{2} Ma^2\right) \qquad p^0 = p\left(1 + \frac{\gamma - 1}{2} Ma^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{2}$$

The stagnation temperature gives a measure of the total energy content of the flow, and is thus usually more suited to energy balance considerations; in particular, the stagnation temperature is the temperature that could be measured in the inlet and exit plenums. The standard temperature T will be, from this point onward, referred to also as static temperature.
At the flow inlets we impose stagnation pressure $p^o{}_i$, stagnation temperature $T^o{}_i$ and the flow direction, assumed normal to the boundary, while static pressure p_e is imposed at the outlet. Symmetry is applied at the channels midlines ($y=0$, $y=s+H$).
The solid wall vertical faces at $x=0$ and $x=L$ are assumed adiabatic. The same gas flows on both the hot and cold sides, and for both channels we keep the same ratio β between inlet stagnation pressure and exit static pressure.
Viscosity μ and fluid and solid thermal conductivity λ_f and λ_s, are kept constant. Air (γ=1.4, Pr=0.7) and argon (γ=1.666, Pr=0.65) have been considered, in order to take into account possible influence of different gas thermos-physical properties. The fluid-solid interface, even if it is only an internal

boundary, needs special treatment due to the rarefaction effects, relevant at microscale. In particular, the Maxwell first order slip boundary condition is here used:

$$u_f - u_w = s_p l \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{3}{4} \frac{\mu}{\rho T} \frac{\partial T}{\partial x} \tag{3}$$

$$s_p = \frac{\sqrt{\pi}}{2} \frac{2-\sigma_v}{\sigma_v} \left(1 + 0.1366\sigma_v\right) \tag{4}$$

In the computations the accommodation coefficient σ_v is set equal to one (i.e., diffuse scattering of wall incident particles). The additional velocity derivative along the tangential direction x in eq.(3), although essential in capturing the behaviour of slip flow along curved walls, vanishes for straight walls, as in the present case. Fluid and solid side temperature on interface differ, due to the Smoluchowski temperature jump condition:

$$T_f - T_w = s_T l \frac{\partial T}{\partial y} \qquad s_T = \frac{2-\sigma_T}{\sigma_T} \frac{2\gamma}{\gamma+1} \frac{1}{\Pr} \tag{5}$$

where s_T is the temperature jump coefficient. We assume the temperature accommodation coefficient σ_T equal to one. Viscous effect magnitude is measured via Reynolds number, defined as

$$\mathrm{Re} = \frac{\overline{\rho u} H}{\mu} = \frac{\dot{m}'}{\mu} \tag{6}$$

where $\dot{m}'$ is mass flow per unit depth, ρ the density and u the average streamwise velocity. The full energy equation is solved at the fluid solid interface, thus taking into account the dissipation work due to the slip. Furthermore, if l is the molecule mean free path, we can define, as a measure of rarefaction relevance, a Knudsen number in terms of local, section averaged, values:

$$Kn = \frac{l}{H} = \frac{16}{5} \sqrt{\frac{\gamma}{2\pi}} \frac{\mathrm{Ma}}{\mathrm{Re}} \tag{7}$$

Finally, a relevant parameter is the axial conduction number M, defined by Maranzana et al [3] as the ratio between a representative conductive axial heat flux

$$q'_{cond} = \frac{\lambda_s s}{L} \left(T_{b,i} - T_{b,e}\right) \tag{8}$$

where the subscript b refers to the bulk temperature, evaluated at inlet and exit sections (i and e), and the total convective flux

$$q'_{conv} = \rho c_p H \overline{u} \left(T_{b,i} - T_{b,e}\right) \tag{9}$$

$$M = \frac{q'_{cond}}{q'_{conv}} = \frac{1}{\mathrm{Re}} \frac{1}{\Pr} \frac{\lambda_s}{\lambda_f} \frac{s}{L} \tag{10}$$

Two different aspect ratios L/H ranging, 20 and 100 are considered, with a relatively thick wall, so that $s/H=0.5$. Several computations were carried out, in order to explore a broad range of conditions, as summarized in table 1. Since we are dealing with compressible flow temperature and velocity fields are coupled: thus, the heat exchanger performances will be also a function of the operating temperatures.

Table 1. Main parameter ranges

Parameter		Min	Max
Conductivity ratio	Λ	10	100
Exit average Mach number	Ma_e	0.09	0.86
Pressure ratio	β	1.3	3.0
Temperature ratio	τ	0.035	0.318
Exit Knudsen number	Kn_e	0.006	0.009
Reynolds number	Re	18	100
Channel length	L/H	20	100
Axial number	M	0.05	0.2

All the parameters are set in dimensionless terms. In particular, any single run condition is defined by the pressure ratio $\beta = p^o{}_i - p_e$, which is chosen equal for hot and cold side, by the fluid/solid conductivity ratio $\Lambda = \lambda_f/\lambda_s$, by the length of the channel L (normalized over channel height H) and by representative temperature ratio

$$\tau = \frac{T_i^h - T_i^c}{T_i^c} \tag{11}$$

However, in order to give a little practical perspective, the condition here considered could be representative of a channel of 10μm of height, atmospheric pressure at the outlets and a cold side temperature of 283 K. Values of M suggest the presence of relevant conjugate heat transfer, as discussed in [3], Knudsen number involves small slip velocities at the wall, but well within the validity of Maxwell boundary conditions.

4. Result and correlation

The ratio behind the definition of the usual Nusselt number, eq.(1), is the one to one correlation between the heat flux and a temperature difference: if, on the contrary, such temperature difference is a function also of the flow acceleration, the definition loses some of his physical meaning (hence, the possible unphysical asymptotes and negative values). Thus, the choice of a proper temperature difference will be a key point in the search for a useful correlation.
Therefore, as a first step, we define the total (or stagnation) Nusselt number Nu^0 in terms of bulk stagnation temperature as follows:

$$Nu^0\left(x\right) = \frac{\alpha D_H}{\lambda_f} = 2\frac{q''H}{\lambda_f\left(T_{w,x} - T^0_{T\,b,x}\right)} \tag{12}$$

The total bulk temperature, in fact, is a measure of the total energy of the fluid and does not change with the conversion between kinetic and internal energy. This Nusselt number has to be correlated to local flow parameters, and such correlation can be sought using all the results available from the computations carried out to study the gas micro heat exchanger behaviour.
Due to the previously remarked strong influence of compressibility, a first step is to look at the influence of the local Mach number Ma. We will focus the analysis on the cooling stream, i.e. the hot fluid. Since the core flow is overcooled by the flow acceleration, if the convective heat transfer is low, and the Mach number high, we may end up with a flux from the wall to the fluid, despite a stagnation

temperature larger than wall temperature. This is clarified in Fig.3, which reports the temperature profile along transverse direction *y*, for such a case, at the hot fluid exit/cold fluid inlet. The difference in stagnation bulk temperature between the upper ($1<y<1.5$) hot stream and the lower ($0<y<0.5$) cold one is so small that the cooling due to the core flow acceleration is large enough to lower the static temperature of the hot stream actually below that of the cold one, and the lower channel fluid is cooled by the upper flow. Such condition yields a negative Nu^0.

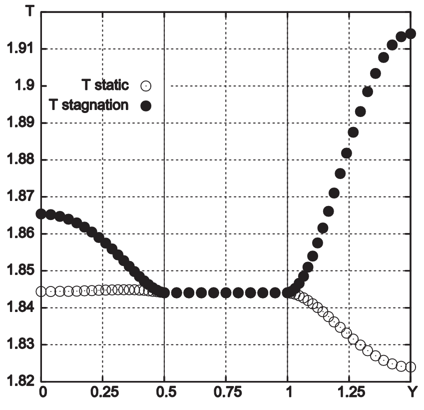

Figure 2. Temperature profiles normal to the wall, x=100, β=2.0, τ=0.18, Λ=100

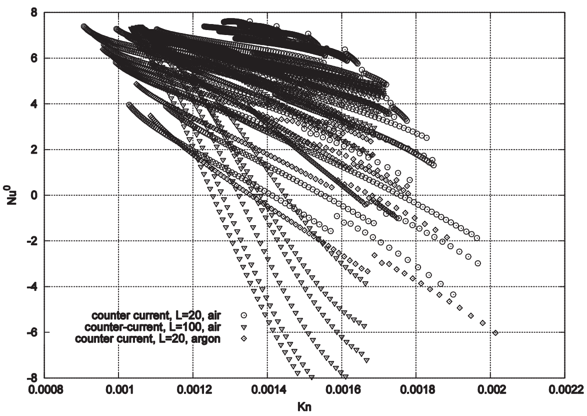

Figure 3. Stagnation Nusselt number plotted vs. the local Mach number, co-current configuration.

Fig.3-5 show the profile of the local stagnation Nusselt number number versus the local, cross section averaged Mach number or Knudsen for all of the performed computations. Since neither *Nu* and Nu^0 are constant along the channel, any single computation gives a full series of Nu^0-*Ma* or Nu^0-*Kn* couples, one for each streamwise location. Rarefaction do not seem to be the dominant effect, as can be deduced from Fig.3, where for the counter current case no clear pattern can be identified as a function of local Knudsen number.

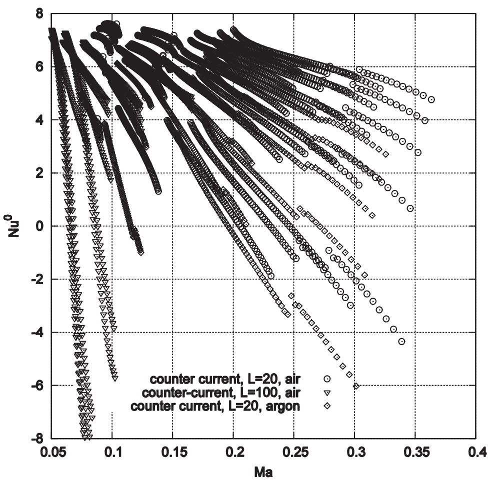

Figure 4. Stagnation Nusselt number plotted vs. the local Mach number, counter-current configuration.

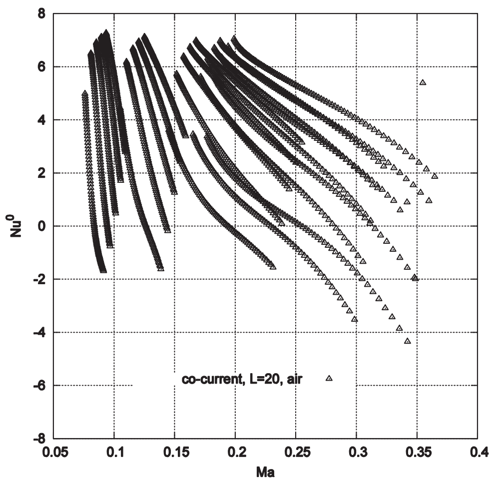

Figure 5. Stagnation Nusselt number plotted vs. the local Mach number, co-current configuration.

Figs. 4-5 show a clear decrease of Nu^0 with *Ma*, but we still have a large spread in the quantitative results: it is clear that we have to take in to account some other additional parameter. In particular, we found a remarkable influence of the temperature difference between the hot and cold stream, i.e. the temperature difference responsible for the convective heat transfer between the two streams. Thus, it is reasonable to look for a temperature parameter that normalize a convective related temperature difference with respect to the fraction of thermal energy converted to kinetic energy. In particular, we consider ($T_b^0 - T_b$) and ($T_b^0 - T_w$). The first term, as previously mentioned, expresses the amount of energy transported by the flow as kinetic energy and the second one is proportional to the total convective heat transferred in the process. Thus, we define a dimensionless temperature ratio parameter ϕ:

$$\phi = \frac{T_b^0 - T_w}{T_b^0 - T_b} \tag{13}$$

This parameter measures the ratio of the intensity of the two energy conversion modalities, and as such takes automatically into account the compressibility effects. A large value of ϕ implies that the temperature variation is essentially driven by the heat transfer, while at small value of ϕ the conversion in kinetic energy is dominant.

In Fig.5 we show all of the local results for Nusselt, except for the entrance region, as a function of the temperature ratio ϕ. It is immediately clear that we a much smaller spread than in Fig 2,3. In particular, fitting the data for air, an algebraic approximation may be defined as

$$Nu^0 = 7.54 - \frac{3\gamma}{2\phi^{\frac{6}{5}}} \tag{14}$$

Despite its simplicity, relations (14) offers a reasonable first guess for the Nusselt number on the basis of the local flow parameters, as demonstrated in Fig.6. In the positive range of Nusselt, i.e. in the most interesting region, the accuracy nearly everywhere within ±15%, and even in the negative range we have a similar approximation.

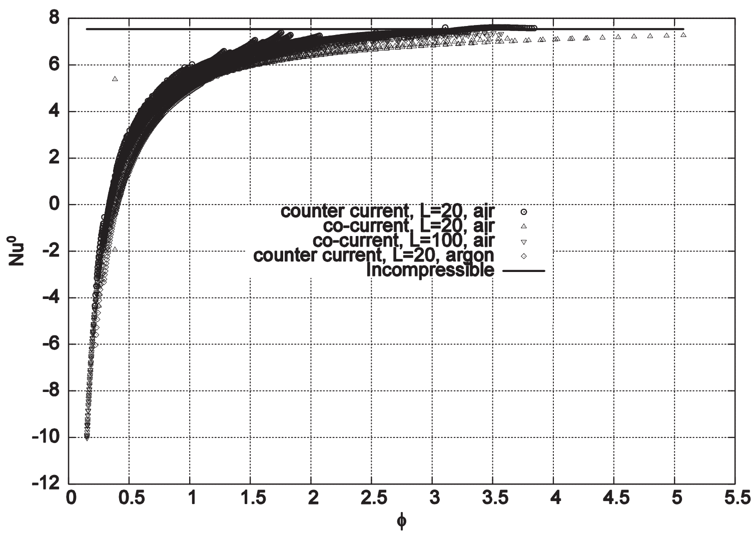

Figure 5. Stagnation Nusselt number plotted vs. temperature ratio ϕ

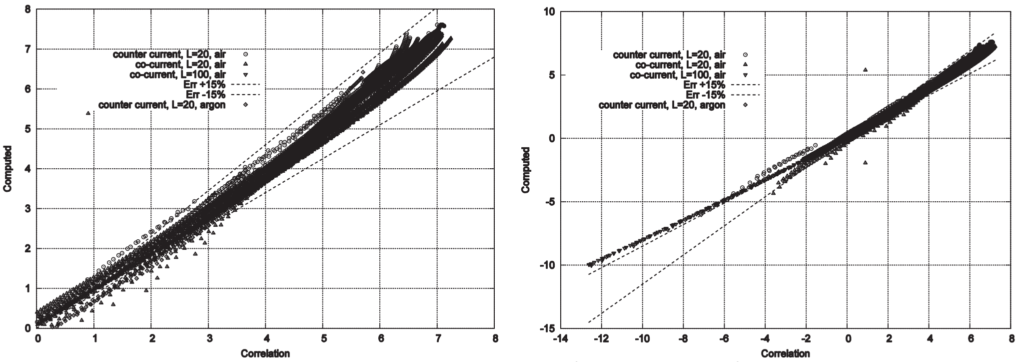

Figure 6. Correlation error: left, positive range of Nu^0, right, full Nu^0 field

5. Conclusions

The computational results for a wide range of operating parameter of a micro plate heat exchanger, in both co-current and counter current configurations, show that the compressibility effect significantly affects the heat transfer performances. In particular, this is due to the competition between the temperature gradients related to the convective heat transfer, and the temperature drop related to the conversion of internal energy into kinetic energy. This is confirmed by the fact that the local Nusselt number, based on the local bulk flow stagnation temperature, appears to be essentially a function of the ratio of the temperature difference between the stream stagnation and wall temperature versus the dynamic temperature (difference between stagnation and static one). It was thus possible to derive a correlation, based only on this dimensionless temperature ratio, which offer a reliable first guess of the local Nusselt value for the whole range of configurations. The proposed correlation can be useful in the framework of a porous media modelling of the heat exchanger: in fact, since it is based exclusively on local flow parameters, neglecting any global data (channel length, inlet and outlet temperature, pressure ratios), it is capable to reliably compute a local heat flux between fluid and solid under any possible flow arrangement. Furthermore, since we reduced to a dependence only on a single non-dimensional parameter, it ensures high computational efficiency.

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