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A TEACHING/LEARNING PATH ON THE CONCEPT OF MASS AND MASS-ENERGY RELATIONSHIP FOR UPPER SECONDARY SCHOOL

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X. Acknowledgments

Preface: General Outline

Because of the fundamental importance of mass concept and mass-energy relationship in physics, a content-specific educational research on them was carried out. Dealing appropriately with these concepts cannot be separated from modern physics education, which is enhanced by promoting science literacy. On the other hand, the recent outcomes from PISA and TIMSS international evaluations witness the necessity/importance of fostering innovation in science education. One of the most important problems for modern physics education is the internal coherence of teaching/learning pathways; for this reason the vertical perspective/approach was chosen. Thus, after having examinated paradigms, learning and educational theories, and conceptual reconstruction (chapter II), an educational research was designed according to the tenets of Design-based Research, adopting the Model of Educational Reconstruction (M.E.R.). Two teaching/learning paths were invented which could be tested in upper secondary school; it was planned to analyse students' conceptual change on the one hand and their learning pathways and reasoning profiles on the other. The research was aimed at answering the following research questions *a priori*, where "conceptualizing" means "gaining conceptual learning with understanding":

1) How do high-school students conceptualize the facets of mass in classical mechanics in ordinary and unconventional learning environments?
2) How do high-school students conceptualize the relationship between inertial mass and rest energy in ordinary and unconventional learning environments?
3) How and to what extent are the concept of inertial mass and mass-energy relationship utilized by upper-secondary students for interpreting real or modelled phenomenology?

The comprehensive research framework is explained in chapter III. A number of physics key conceptual nuclei were selected from consensus knowledge and educationally reconstructed for instruction (chapter IV) – according to the M.E.R. – and a critical selection of the educational literature on mass, energy, Special Relativity (SR) and mass-energy (chapter V) was provided. Two conceptual paths were built up using these ingredients, one dealing with mass in classical physics joint to mass in SR through a formal approach exploiting both four-vectors and reasoning by analogy, the other with the same module on classical mass joint to a phenomenological approach to mass in SR bringing to the relativistic idea of energy and to the new properties of mass as well (chapter VI). The set of (i) conceptual paths, (ii) prerequisites and scheduled duration of the activity, (iii) objectives and teaching/learning strategies, (iv) worked-out instructional materials furnishes the final teaching/learning paths. Chapter VII begins with an overview of the fourteen implemented formative experiments, whose samples were geographically widespread over Italy. Then the analysis of data from 6/14 experiments (two of them grouped in section VII.3) are described according the usual research format. Conclusions about the whole work and one consideration for further developments in the direction of Weak Equivalence Principle are in chapter VIII.

Introduction

The concept of mass is complex and crucial in physics; it produces a multiperspective vision both on the common sense ground and on the scientific one. As concerns common sense, the concepts of weight and the distinction/relation between mass and weight, rather than the concepts of mass itself, have been investigated by educational literature. On the disciplinary ground, each of the different historical theories elaborated its specific view of mass, starting from classical dynamics, in which it was discussed and separated into *"quantity of matter", inertial mass and gravitational mass*. The distinction between the latter two is extensively dealt with in physics education, although it is not considered as the focus of the "problem of mass" in physics.

In modern physics, namely Quantum Mechanics and Relativity, *the concept of mass was extended*. This contributed to its multifaceted character and did not allow to entirely understand its role within modern theories, the Standard Model above all. In fact, nature and origin of mass have not been fully understood yet: «there is no common opinion even among the experts what is the essence of this problem» (Okun, 2005). Nevertheless, the recent detection of a particle having proprieties matching those of Higgs Boson helped to enlighten that last issue (Okun, 1989 – 2005, 2010, 2012; Bowdery, 1996; Jammer, 2000; Kane, 2005; Roche, 2005; Wilczeck, 2006; Hecht, 2011; Silagadze, 2014).

Burniston Brown (1959) detected a growth in the lack of conceptual understanding of mass if a distinction between inertial and gravitational facets is made. This implies necessarily, in turn, misunderstanding on their proportionality, i.e. a lack of sharp acknowledgement of the Weak Equivalence Principle[1] (WEP).

Lehrman (1982) pointed out relevant critical issues on the concept of mass in high school textbooks instead, namely (i) confusion between weight and gravitational mass, (ii) belief that equal arm scales measure weight instead of gravitational mass, and (iii) *operational definition of inertial mass by* $m_i = F/a$ *without an independent generalized definition of force in dynamics*, so that a circularity problem emerges. The last issue is considered as a crucial problem for

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[1] Roughly speaking, the postulate by Einstein of equality between inertial and gravitational mass $m_i \equiv m_g$

the conceptual definitions of mass and force in classical physics by a wide literature (e.g. Maxwell, 1877; Jammer 1961, 2000; Goodinson & Luffman, 1985; Anderson, 1990; Roche, 2005; Hecht, 2006). The essence of the problem lies in the *status* of the concept of force in mechanics: it is not operationally definable in the general case, and the definition through the measure of a spring elongation (Bridgman, 1927) does not solve entirely the issue, since it is a special case.

An accurate survey by Doménech *et al.* (1993) showed the presence of a qualitative «pre-theoretical» conception of mass in a statistically significant sample of 16- to 18-years old pupils, with a marked teleological connotation – encouraged by the social view of science (Duschl, 1988) – instead of the scientific quantitative concept, in which mass should be *operationally* defined. For instance, most students identified mass with other quantities: volume or density on the one hand and weight on the other; in addition, the ontological conception of mass as amount of matter (*quantitas materiae*[2]) prevailed on inertial mass. These findings turned out to be due both to the belief that scientists describe objective reality straightforwardly both to «student bewilderment with the formal […] numerical reasoning used by scientists» (Doménech *et al.*, 1993).

Such a historic-epistemological and semantic perspective on the topic has not been considered anymore; thus it is necessary to take it into examination. By the way, the terms ‘conception’, ‘misconception’ and ‘concept’ have got a sharp meaning in science education research, that will be clarified in the next chapter.

Operational definitions are the only epistemologically unexceptionable ones, according to the approach that will be followed in the present thesis, i.e. Bridgman’s (1927). To define a physical quantity operationally means to identify the corresponding concept with the group of operations necessary to measure the quantity. Bridgman’s view was born in reply to the conceptual revolution due to the theory of Relativity, in order to avoid further redefinitions of physical quantities. From the educational standpoint, the development of an operational definition of a concept allows for a sound conceptual understanding (McDermott *et al.*, 1996). Favouring the formulation of operational definitions by students enhances functional understanding (Papadouris & Constantinou, 2014).

Mass is also *intimately related to energy in modern physics, owing to the well-known equivalence by Albert Einstein.* Therefore problems in learning with

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[2] «Quantity of matter» in Latin.

understanding mass have to be faced in parallel with those about energy. Unfortunately, *the operational approach is not feasible for energy in its comprehensive meaning*, as it will be presently explained.

The conception of mass as *quantitas materiae* proved to generate misconceptions concerning Einstein's mass-energy equivalence, viz. (i) mass is 'converted' into a generic 'energy' (the most frequent one); (ii) the equation $E = mc^2$ represents 'conversion' of mass into energy; (iii) mixing of energy conservation and mass conservation laws (Lehrman, 1982).

The educational problem involves some crucial content aspects which were also analysed in the historical evolution of the concept. They are:

- Mass, next to space and time, is a founding concept of physics (Jammer, 2000);
- Einstein (1905a) stated the equivalence between mass and energy;
- *Mass-energy and space-time are interrelated in General Relativity*;
- *The interactions of a particle with the surrounding "medium" or "field" add to its "bare mass"* m_0 – which turns out to be an abstraction – *a contribution* δm*, so that the particle's inertia is described by its effective mass* $m_{eff} = m_0 + \delta m$, which is the mass measured in real experiments. In particular, this contribution may be given by its interaction with the other components of a physical system observed as a whole.

The results of TIMSS (Trends in International Mathematics and Science Study) Advanced 2008 showed the most skilled Italian students are in the last position, and a more accurate analysis of these results displayed a lack of significant development of formal thinking (Euler, 2002). The results from PISA (Programme for International Student Assessment) and TIMSS appeared globally discouraging on the international ground at the beginning of the century, although in Italy the problem were more marked (INVALSI, 2013b). PISA measured the level of science literacy of Italian 15 years-old students and has been carried out every three years from 2000 to nowadays, while TIMSS is a quadrennial survey on learning gained by the students of fourth and eighth year of schooling about curriculum contents achieved through actual school practice. TIMSS Advanced was instead performed with the best students of high-school last year coming from ten countries in 2008.

All of these international evaluations *show the necessity to conceptualize physical quantities, of which mass is a fundamental one*. Its conceptual meaning cannot be fully grasped without considering it in modern physics, as briefly depicted above. *An innovative approach entailing teaching/learning* (t/l) *modern physics is thus necessary in order to enlighten the problem of mass.*

Modern physics at school is important and useful, independently of the specific topic, for it enhances *theoretical thinking building* by the formation of concepts and hypotheses without classical equivalents. More generally, *the interpretive and modelling methods of modern physics constitute a new cultural perspective and thus a new way of interpreting physical reality*. Therefore students should be provided with a complete picture of physics, including the 20th-century one, which is also the basis of modern technology whose spinoffs are present in their everyday life. Besides, modern physics is likely to be very involving because of its innovative and counterintuitive character, deriving also from its contribution to extend the domain of physics inquiry.

A *modern physics literacy* should thus be promoted, by developing concepts throughout their «supportive models» – in each of which every physical quantity / concept gains a different meaning (Doménech *et al.*, 1993). All afforded topics will become conceptually interrelated in this way, fostering long-term learning with understanding. This may be done in several ways, including the design of a «historical line», as suggested by Arons (1992). Defining and following a «historical line» means *carefully selecting only those fundamental concepts, from the involved theories, which are fruitful for students to understand the arguments and experiments concerning the intended topic.* For example, Huggins (1968) and Arons himself (1965) worked out a qualitative and phenomenological introduction to the ideas bringing in the theory of Relativity along that road.

The importance of an educational reconstruction of the cultural debate between the contrasting conceptions of space and time by Einstein and Minkowski was also demonstrated by Levrini (2004), who exploited that debate for endowing students with different perspectives and arguments on space-time in Special Relativity (SR). A research field has even been recently founded: history and philosophy of science and science teaching (HPS&ST). In that stream, Abiko (2005) and Giannetto (2009) maintained the cultural and educational importance of following a historical approach to the birth of SR. Generally speaking, this approach would allow building up a treatise of physics *inside the curriculum*, for supplying a consistent picture of physics, which is unitary. This internal consistency of any learning pathway is educationally more important than following the historical development of concepts, so the gaps should not be remarked too much. At the same time, students must be aware of classical physics boundaries. In particular, they should recognize and criticize the classical «implicit assumptions» by means of problem-solving activities aimed at knowledge reconstruction (Gil & Solbes, 1993).

In conclusion, the primary aim of the present work is to *achieve and enhance learning with understanding of the concept of mass extended to modern physics through a vertical approach, by exploiting a «historical line»* (Arons, 1992). Such an approach on this topic is lacking in educational research, as shown by the literature overview in chapter V. Furthermore, it may have interesting spinoffs on teaching. The major addressees of the present work are thus *researchers in physics education* and those *physics teachers especially interested in mass and SR education*. The choice of this specific topic is coherent with the new syllabus indications for the last year of Italian secondary school, Liceo specializing in scientific studies especially, in which modern physics is mandatory, with an emphasis on mass-energy relationship and its scientific and technological spinoffs.

I.1. Subject Matter Content: Mass, Energy, and the Equivalence

Mass is a fundamental physical quantity, necessary for developing classical dynamics, of which it was even defined «the key term» (Burniston Brown, 1959). Mass in Relativity is nothing but Newtonian inertial mass (Okun 1989 – 2010; Bergia & Franco, 2001; Fabri, 2005; Silagadze, 2014), taking new meanings within the novel paradigm (Kuhn, 1970). Jammer (2000) put mass in Relativity even at the same level of space-time, the relativistic "backstage" of every physical phenomenon. It is also worth remembering that Einstein mentioned *only mass-energy equivalence* among the consequences of the theory of Relativity during the Salzburg conference, «because it brings about a certain modification of the basic ideas of physics» (Einstein, 1909). From the particle physics standpoint, mass plays a founding role too, because if every particle were massless we would have a Universe of free particles travelling at the light speed without any interaction energy among them, and so no macroscopic object would exist. Mass endows matter with “something” characterizing it and determining its motion in the absence of electromagnetic fields, for example on a large scale[3]. Finally, *effective mass* is widely used in many areas of physics, chiefly Quantum Mechanics[4].

Mass has got a manifold *conceptual* character in classical physics. Isaac Newton (1642 – 1727) actually invented the scientific concept by introducing *quantitas materiae* – measurable through the product $\rho \cdot V$. At the same time, he worked out *inertial mass*, which measures the property of a body/system enabling it to oppose to a velocity variation when mechanic work is done on it. It is an intrinsic quantity of that body/system, namely the parameter in $F = ma$. Besides, inertial mass took up a *gravitational meaning* in Newton’s universal gravitation law, in which it gives rise to and undergoes gravitational interaction at the same time (active and passive gravitational mass respectively). This is a quantity stemming by interaction amongst objects, unlike inertial mass. Furthermore, Ernst Mach (1883) formulated an innovative operational definition of mass, in which it is measured by the inverse ratio of accelerations. This circumvents the notion of force, in order to avoid the circularity problem stated above.

In 1905 Einstein established a relationship between Newtonian inertial mass and the so-called ‘rest energy’ in the particular case of electromagnetic energy emission. Rest energy is given by the sum of all contributions to the energy of a body/particle but its kinetic macroscopic one. Till 1907 Einstein has been theoretically demonstrating mass-energy equivalence for a wider and wider range of phenomena. In Relativity mass becomes an *approximate* measure of inertia, with which it coincides only for speeds negligible with respect to light speed *in vacuo* c. In the general case, inertia is measured by total energy (Okun, 2010). This distinction is more marked for fast particles, namely those whose speed approaches c. An object’s mass is a *relativistic invariant*, i.e. it does not depend upon the inertial reference frame in which the object is observed, unlike energy, velocity and linear momentum. It measures an *inherent* property of the particle, just like charge, and thus is usually called *invariant mass* in particle physics. “Relativistic mass”, viz. a construct dependent on the speed of a body in a reference frame, is sometimes used as a proper physical quantity, although most of the scientific community considers it both useless and misleading in terms of teaching/learning

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[3] The physicist George Gamow considered gravity as the force that rules Universe.

[4] Although the first example of effective mass is given by classical hydrodynamic mass.

nowadays (e.g. Warren, 1976; Whitaker, 1976; Adler, 1987; Okun, 1989 – 2005, 2010).

In General Relativity *momentum-energy density*, energy being equivalent to mass, *is a major source of space-time geometry warping*: matter exerts an active action on space, differently from what occurred with Newtonian space.

Finally, *the inertial properties of an entity belonging to any physical system may also grow out of its dynamical interactions with the other components of the system itself and with the whole of the surrounding environment* – "medium" or "field" – *as well*. This brings to the concept of *effective mass* and to the *Higgs mechanism*, which are mentioned here since they lend themselves to further educational research. Roughly speaking, Higgs mechanism is the extension of the Standard Model (SM) ideated for endowing elementary particles and gauge bosons with mass, which was theoretically predicted to be null. Higgs mechanism thus enlightens the origin of elementary particles' and gauge bosons' masses as well as the hierarchy problem, i.e. the noteworthy fact that there are 11 magnitude orders between the smallest and the biggest value of these masses.

Energy is being considered because of its close link to mass in Relativity. The energy of a physical system may be defined as that quantity whose variation measures any change in the state of the system owing to interactions (Hecht, 2011). Actually, we are able to define only energy *variations* operationally. Therefore it is not possible to define energy in itself, because there is never solely an absolute energy value in a measurable process: energy is a quantity defined unless an arbitrary additive constant. Furthermore, energy is essentially characterized by its conservativeness and constancy: if all energy forms are added the result is constant over time in closed systems (Feynman, Leighton, & Sands, 1964).

Nowadays very different approaches are present in the literature on t/l energy (for a review see Heron, Michelini, Eylon, Lehavi, & Stefanel, 2014). The conceptual core of that dramatic debate stands in what energy is considered to be. Is it an entity existing in itself or rather a state property of physical systems? I have chosen the second interpretation, in agreement with the standpoint by Colonnese, Heron, Michelini, Santi, and Stefanel (2012), in which energy is thought as an abstract state quantity of a precise physical system, which is conserved but transforms when systems interact. Since there are a lot of energy forms not well-defined in physics, only four energy «types» have been considered in this approach: *kinetic*, *potential*, *internal*, and *associated to light*. The ideas of transfer and degradation of energy have been dropped, because they recall the conception of energy as quasi-material substance/entity. Further, degradation requires the additional concept of entropy for being properly understood.

On the *epistemological* and *methodological* ground, Mach's approach to inertial mass contained problematic aspects, for which he was criticized. In fact, for instance, his definition is not valid for any object on which a collision experiment in a laboratory may not be performed, like microscopic or astronomical objects (Jammer, 2000). Moreover, an actual measurement of the instantaneous accelerations will never be performed, because a finite time interval Δt has to be considered for any operational measure (Roche, 2005). Finally, a major objection by the "former" Jammer (1961): bodies' systems are never really isolated from the surrounding as they are supposed to be by Mach. As regards science progress, the 2012 discovery of a particle matching Higgs boson corroborated the current Standard Model tenets. Since SM is a quantum-relativistic theoretical framework for elementary particles and interactions, the fact that it was not invalidated entails SR and Quantum Mechanics are still valid too (*modus tollens*[5]).

I.2. A Call for Teaching/Learning Modern Physics

No actual insight into the problem of mass and obviously of mass-energy may be gained without considering it within modern physics. Furthermore, dealing with modern physics at high school cannot be overlooked for epistemological, cultural and educational reasons nowadays. The main reasons are the following.

First of all, *its theoretical frameworks* – made by "intermediate objects" – *are farther from the realm of phenomena* – made by "body objects" (Bellone, 2008) – *than classical physics frameworks* (Michelini, Santi, Ragazzon & Stefanel, 2008; Giliberti & Cavallini, 2010). Modern physics ideas are indeed not related to sensorial perception at all[6].

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[5] *Modus ponens* and *modus tollens* are two kinds of logical inferences. The first one asserts that if a proposition A and the implication $A \rightarrow B$ are true, then B is true too. The second one states instead that if $\bar{B}$ and $A \rightarrow B$ are true, then $\bar{A}$ is true too. They can be demonstrated by means of truth tables. *Modus tollens* is the logical basis for *testing a theory*: according to Popper's strict logical falsificationism, it is enough to show that a single consequence of certain basic hypotheses is false, to falsify the whole theory.

[6] "Objectification" is the cognitive act in which any group of sensory inputs, occurring with frequency high enough in common experience, is associated to the existence of a real "body object" (Lorentz, 1974). "Body objects" are located in space, and time is introduced to account for their perceived variations. Science as culture allows going *beyond sensorial perceptions* by building objects *not deriving from them,* organized

Moreover, affording quantum and relativistic mechanics just after classical mechanics allows showing students *what a theory is actually*, by comparing two different theories which are able to explain the same classes of phenomena under certain constraints. Of course, this requires additional time at school and so other topics have to be removed, but affording atomic structure or nuclei as well as introducing Relativity qualitatively and/or quantitatively is not impossible and very useful (Arons, 1992).

In addition, *the innovative interpretive and modelling methods of modern physics shape a new cultural perspective* (Michelini, Santi, Ragazzon & Stefanel, 2008), which made a strong impact on the literate climate of the early 20th century; so knowledge of Relativity and Quantum Mechanics is important for understanding the cultural production of that period (Arriassecq & Greca, 2012). In particular, modern physics allows interpreting physical reality since

a) It is the current theoretical framework paradigm for describing space and time, macroscopic "mechanics"[7] and gravitation (Relativity) and for interpreting the microscopic world at low speeds and matter physics (Quantum Mechanics). For instance, Relativity supplies an explanation for fascinating and exotic phenomena like black holes and the Big Bang, star evolution, particle creation and annihilation, identification of the new particles generated in a relativistic collision, and so on;
b) Even if it is always valid, it *focuses especially on* the 'extremes' of physics domain: short lengths (Quantum Mechanics), high speeds and strong gravitational fields (Special and General Relativity respectively); it derives its image of innovation and exoticism from this aspect too (Arriassecq & Greca, 2012).

Furthermore, modern physics is the basis of the whole modern technology, whose *relevant applications are somehow present in pupils' everyday life*. This fact is more marked for Quantum Mechanics, but, for example, Relativity underlies the working of GPS (Will, 2000; Ashby, 2003; Fabri, 2005; Will, 2006).

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in theories «in order to explain how "body objects" work» (Bellone, 2008): "intermediate objects". For example, the classical concept of force does not actually exist in itself, but it acquires meaning inside Newtonian mechanics: it is an "intermediate object". As for modern physics, a meaningful example is the concept of *particle*: it is simply a model, or a «symbol, by adopting which the laws of nature take on a particularly simple form» (Heisenberg, 1997). "Intermediate objects", together with the laws/rules for their behaviour and with consistent interpretations of the empiric reality, constitute a separate reality. Their reality *status* is variable, unlike body objects'. In particular *the concept of mass changes according to the different theories or models in which it is embedded* (Doménech *et al.,* 1993).

[7] Actually, the motion description is always *static* in Relativity, because of its 4-dimensional nature.

Eventually, modern physics is likely to be *very exciting and involving* for students, on the conceptual ground primarily, precisely because of its innovative and counterintuitive character (Shabajee & Postlethwaite, 2000; Meijer, 2005).

The most important feasible approaches to modern physics discussed in literature are (1) storytelling of the main results; (2) argumentation of crucial problems starting from their classical interpretation (semiclassical approach); (3) integration in classical physics. The debate is also on whether the crucial issues of modern physics are to be integrated or rather a complementary part of the curriculum and addressed to all citizens or only to the best students (Michelini, Santi, & Stefanel, 2015).

Another remarkable way of dealing with modern physics in secondary school is to enhance the *development of theoretical thinking* (Michelini, 2010a). In particular, modern physics allows building *novel concepts and new interpretive hypotheses*, i.e. concepts and hypotheses without a classical equivalent (Stefanel, 2007). I chose this latter way. This construction of abstract representations needs that learners form and elaborate mental imagery actively, i.e. intuitive pictures and images meant as set of symbols (Euler, 2002). I decided accordingly to favour *insight* in students, which is an inherently creative process fostering the development of higher mental functions (*ibid.*).

I.3. Science Literacy

The aim of the present paragraph is uniquely to introduce an important concept for updated education, which will be utilized hereinafter, i.e. *scientific literacy*. It is part of life skills, namely of mathematical and basic scientific and technological skills. Some education researchers have even identified science with the acquisition of a specific disciplinary literacy. «For Norris and Phillips (2003), to really understand science, as opposed to being knowledgeable about science topics, students need to know how to interpret, represent, and assess scientific claims, implying a foundational role for representational work» (Tytler, Prain *et al.*, 2013). For this reason the *representation construction* has been seriously taken into consideration in the work presented here.

Science literacy may be basically understood as *the use of science concepts and reasoning patterns for solving problems students experience in their everyday life.* The most recent (2012) definition of science literacy is far more extensive:

> the set of a person's scientific knowledge and the use of this knowledge to identify scientific questions, to acquire new knowledge, to explain scientific phenomena and to draw conclusions based on facts concerning science topics; the understanding of the distinctive features of science meant as a form of knowledge and inquiry typical of human beings; the awareness of how science and technology shape our material, intellectual and cultural environment, and the will of facing themes having a scientific value as well as science ideas, like a reflective citizen.

I.4. Key Educational Choices

One of the problems inherent to dealing with modern physics is *internal coherence*. In particular, De Vos and Pilot (2001) studied the problem and found out that the progressive update of topics tends to create a series of *ad hoc* thin juxtaposed 'layers' not actually connected by a logic thread, just like it happens with the chapters on acids and bases in a lot of general chemistry textbooks as knowledge increases. This points out the need of designing coherent teaching/learning paths.

It would be also desirable to build up a consistent treatise of any modern physics topic *across and integrated in the curriculum*, in order to give an image of physics as a coherent whole (Giliberti, 2002; Meijer 2005). To that end *vertical perspective* was chosen. It is the unitary reconstruction for instruction of the knowledge to be learnt throughout all the «supportive models» (Doménech *et al.*, 1993) of each concept, performed by logically connecting and/or merging the parts ('layers') traditionally taught at different ages of schooling. This approach is thus useful to enhance *didactical continuity* about one topic and let students become scientifically literate in modern physics, since all learned topics are conceptually interrelated. Verticality may be obtained by fostering «learning progressions» in the conceptual domain: strategies entailing the *progressive deepening of a single core science concept* for designing learning environments; content, instruction and assessment[8] are aligned to this aim. Researchers consider learning progressions as *modelled pathways of learning* occurring over empirically validated time periods.

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[8] It is specified that assessment is not part of the research work presented here.

In learning progressions conceptual learning is gained through intermediate steps from a «lower anchor», the current students' conceptions, to an «upper anchor», the expected learning level, set by national curriculum and informed by science education theory (Duschl, Schweingruber, & Shouse, 2007; Stevens, Delgado, & Krajcik, 2010; Duschl, Maeng, & Sezen, 2011; Neumann, Viering, Boone, & Fischer, 2013). This point will be expanded in III.4.1.

Therefore my intention was to design a teaching/learning path analogue to the Arons's «historical line» (1992), whose historical approach was consistent in its general lines with the one by Levrini (2004), Abiko (2005), Giannetto (2009), and Galili (2012). About modern physics, Gil and Solbes (1993) underline the importance of making *students aware of the limits of validity of classical physics* – which is beyond doubt the most studied paradigm in secondary school – by showing the deep conceptual, methodological and epistemological change in the passage to modern one. In particular, students should question the absolute character of space and time by a criticism of the «implicit assumptions of classical mechanics; implicit assumptions – because they are accepted as obvious, eluding any analysis – constitute one of the main difficulties in the development of science» (*ibid.*). *Grasping the relativity of space and time intervals for different observers* is the first step to understand four landmarks of Relativity – including mass-energy equivalence – according to these scholars. This aspect was stressed very much while building up the conceptual pathway. Moreover, Gil and Solbes claimed for a constructivist approach in which learners are engaged in problem-solving activities in order to re-construct knowledge, because «modern physics was constructed *against* the classical paradigm». At the same time it must be clear to pupils that *incommensurability in science is local* (physics is unitary), as witnessed by the Correspondence Principle, and so it is not educationally advisable to remark the gaps inside it excessively (Giliberti, 2002; Meijer 2005).

To sum up, the historical development from classical to contemporary physics *may* help for educational reconstruction, but the internal consistency of the learning path must be assured (Giliberti, 2002).

Background Issues on Scientific Learning and Conceptualization

Science education and didactics involve various aspects, among which *innovation for conceptual learning*, *educational and learning theories* (based on *philosophical paradigms*), and *education research paradigms*. This chapter presents these general issues, which I have deepened in order to obtain a complete picture of the discipline, although they are not directly involved in the focus of this PhD thesis.

First of all, PISA has highlighted the need of methodological *innovation* for fostering conceptual learning. Innovation necessarily concerns strategies and methods for developing learning environments too (TIMSS), it concerns technology, and finally subject matter contents. Since new contents are needed, this PhD thesis is committed to offer a contribution for clarifying the concept of mass in modern physics. In fact, school cannot overlook what is happening in the world outside nowadays; therefore organic and coherent treatises on fundamentals of modern physics are necessary, as asserted for instance by DeVos and Pilot (2001). The *reasons* for which innovation in instruction is needed, five of the Sjøberg's (2002) *suggestions for innovation*, the need of enhancing *social* skills for innovation, and *science innovation for society development* will be explained in II.1.

On the other hand, at the very beginning of any educational inquiry it is fundamental to set out which *overall framework* it will be based upon, because that framework will shape every phase of the work. Paradigms are indeed the philosophical worldviews establishing the 'lenses' through which everything is considered. Two fundamental contrasting paradigms on thinking and learning are *objectivism* – deriving from realism primarily – and the collection of theories under the name of *constructivism* – stemming from Kant's (1781) idealism. Paradigms will be illustrated from their philosophical genesis to their application in science education research in II.2, II.3, and II.4. The main Science Education Research (SER) strands are *post-positivism* and *interpretivism*[9].

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[9] It is specified that all previous and following names and technical terms will be resumed and clarified in the respective sections.

The prominent choice for my research work was cognitive constructivism and interpretivist paradigm, deriving from the former. At the same time, realism and therefore (post)-positivist paradigm is no doubt an indispensable standpoint for a trustworthy inquiry[10], as highlighted by Fischer (2013). The way out from the expounded dilemma has been the choice of a theoretical approach which aimed at developing strategies to systematically synthesizing different research methods or even paradigms towards a common project in social and educational sciences. It is called *'multi-method'* or 'mixed-method' strategy, and its most spread application, viz. multi-method measurement, is *triangulation* (Brewer & Hunter, 2006). That strategy will be explained in II.5.

Later on, the nature of learning and understanding from constructivist perspective will be extensively explained (II.6).

Finally, *content-oriented research theory* will be briefly defined (II.7), whose lines concerning *students' conceptions* and *conceptual profiles* (Driver *et al.*, 1994) as well as their *learning pathways* and *learning processes* (Niedderer *et al.*, 2007) were chosen for this work.

I selected *continuity in learning*, in particular *"conceptual reconstruction"*, as framework theory on learning processes. Continuity theory models learning as *personal involvement in a rational construction based on inquiry* for the acceptation of new conceptions/concepts. Learning occurs when concepts are *understandable*, *plausible* and may be exploited for *learning progressions* (Duschl, Schweingruber, & Shouse, 2007; Stevens, Delgado, & Krajcik, 2010; Duschl, Maeng, & Sezen, 2011; Neumann, Viering, Boone, & Fischer, 2013).

For the first time Posner, Strike, Hewson, and Gertzog (1982) focused on learning studied as a *rational process* rather than a mere absorption of a set of exact responses, verbal statements or behaviours. Their theory was founded on the union of rigorous epistemological studies and validated development psychology claims. The relevant following progresses in learning process studies – made through the researches on both conceptual change and conceptual reconstruction – have shown that learning processes may be observed and studied according to various

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[10] In this last respect, an unbiased standpoint which aroused my interest is the realism by Harré (1986), according to whom scientific knowledge is socially constructed and validated, but at the same time it is constrained by experimental tests. Experiments provide *indisputable* empirical information about the physical world, whose accumulation allows scientific progress. The experimental method in science is indeed an irreversible acquisition of humanity, from which unexpected answers will always come (Prigogine & Stengers, 1993; Driver *et al.*, 1994).

perspectives (e.g. Duit & Treagust, 2012; Tytler *et al.*, 2013), according to which a view of learning as rational individual process *is not enough* to account exhaustively for its complexity, as pointed out for instance by the socio-cultural approach by Vosniadou, Vamvakoussi, and Skopeliti (2008).

II.1. Innovation for Teaching/Learning Science

The worrying results from PISA and TIMSS led governments to make political choices at the international level in order to let students learn or improve life skills, *scientific literacy* in particular, through science education. Italian policy adapted to this new worldwide strategy.

Three major *reasons* related to these surveys for fostering educational innovation in Italy are the following. At the same time, however, it should be kept in mind that the specific PISA and TIMSS results, as well as the emerging geographical differences, are more meaningful than the general trends and mean values reported below.

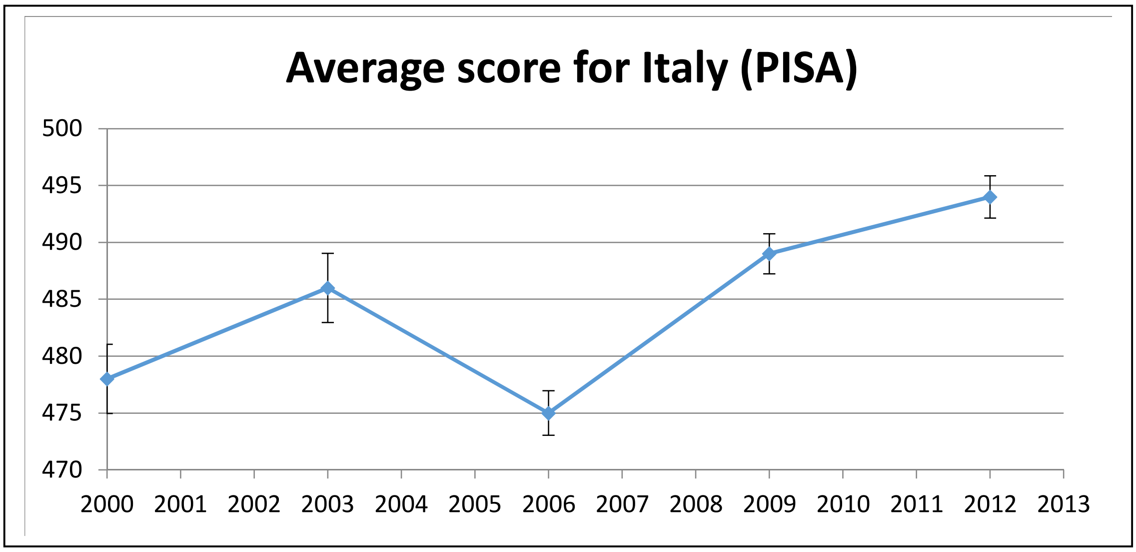

Figure II.1.Trend over time of the measured average science literacy level (with error bars) of Italian 15 y.o. students, the OCSE average being normalized at 500.

1) *Scientific literacy of Italian students is overlooked.* The scientific skills of Italian 15-years-old students were found to be on average significantly *under the mean value of OECD countries* by PISA. Furthermore, 18.7% of these pupils did not reach the lowest level of scientific literacy in 2012 (INVALSI, 2013b). Although the age of these students is 1 to 3 years below that of the ones

involved in my research, I have considered this datum as significant, for it supplies an early statistical indication on the average level of Italian high school students. Nevertheless, it is worth mentioning that a *sharp improvement was* in the period 2006 – 2012, especially in southern Italy regions, bringing Italian literacy score just slightly below the OECD average, namely (494 ± 4) points against (501 ± 1), as reported by INVALSI (2013a) and depicted in figure II.1. Italy reached the same level of France, Denmark and Norway.

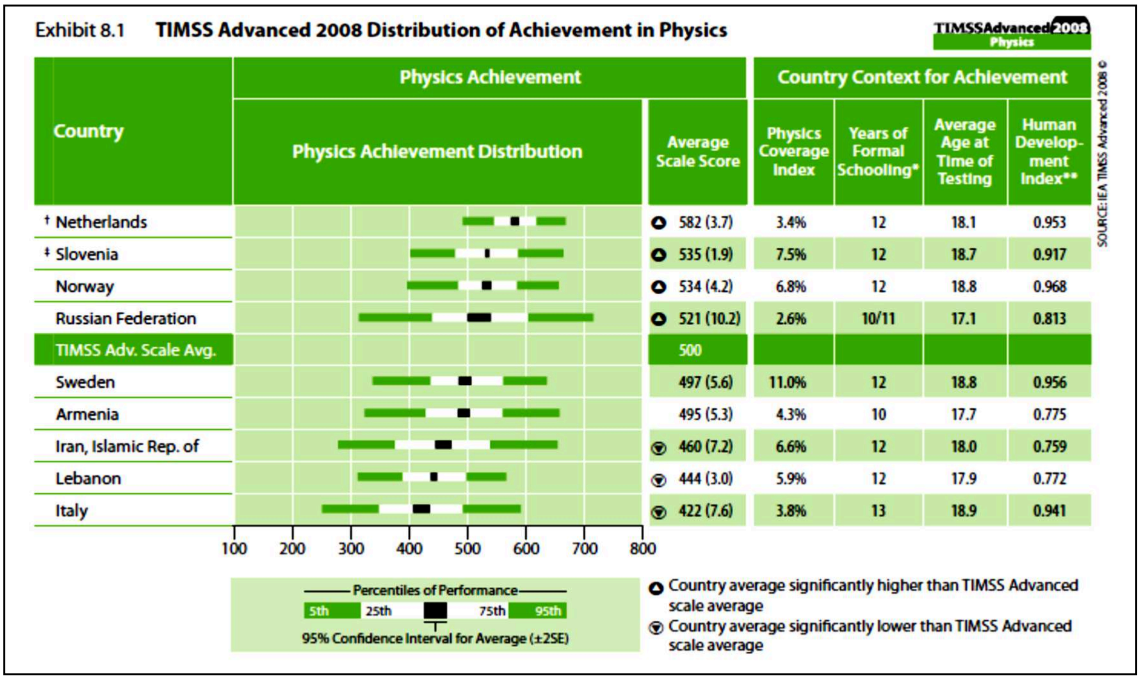

Exhibit 8.1 **TIMSS Advanced 2008 Distribution of Achievement in Physics**

Country	Physics Achievement		Country Context for Achievement			
	Physics Achievement Distribution	Average Scale Score	Physics Coverage Index	Years of Formal Schooling*	Average Age at Time of Testing	Human Develop-ment Index**
† Netherlands		▲ 582 (3.7)	3.4%	12	18.1	0.953
‡ Slovenia		▲ 535 (1.9)	7.5%	12	18.7	0.917
Norway		▲ 534 (4.2)	6.8%	12	18.8	0.968
Russian Federation		▲ 521 (10.2)	2.6%	10/11	17.1	0.813
TIMSS Adv. Scale Avg.		500				
Sweden		497 (5.6)	11.0%	12	18.8	0.956
Armenia		495 (5.3)	4.3%	10	17.7	0.775
Iran, Islamic Rep. of		▼ 460 (7.2)	6.6%	12	18.0	0.759
Lebanon		▼ 444 (3.0)	5.9%	12	17.9	0.772
Italy		▼ 422 (7.6)	3.8%	13	18.9	0.941

▲ Country average significantly higher than TIMSS Advanced scale average

▼ Country average significantly lower than TIMSS Advanced scale average

Figure II.2.Results from TIMSS Advanced 2008 with significant data for each country [27]. Noteworthy, Italy is in the last position. The Physics Coverage Index is the percentage of students selected for the survey from all students of the same age in the country.

2) The *scant mastery of science*, and *physics* in particular, of Italian students is further highlighted by the last position of Italy in TIMSS Advanced 2008, after Armenia, Iran and Lebanon, for which the Human Development Indexes, years of schooling and participants' average age are (far) lower than Italian ones. The findings are summarized in figure 2. As a consequence, Italian instructional policy needs to *promote excellence in physics at school*, which is reached and enhanced only by means of creativity and innovation (Michelini, 2011), and the latter are in turn fostered by learning life skills.
3) A rather strong *disaffection of students towards science subjects* has been demonstrated by several educational statistics data supplied by UNESCO, OECD and the EU. In particular, this decline of interest was highlighted by

three main European trends registered at the beginning of the 21st century by Sjøberg (2002):

- Reduction of the curricular choices in science at high school;
- Falling of recruitment in science and technology university studies, namely pure scientific disciplines as well as engineering;
- Weaker quality of works produced by newcomers in the universities.

Sjøberg (2002) identified eleven prominent tendencies in the political syllabus reforms of many European countries elaborated by education scholar commissions in order to face the situation depicted above, which are therefore suggestions for *innovation in instruction* at the same time.

Five of his suggestions were taken into consideration when the present work was undertaken:

1) Dealing with *non-specialist topics* at high-school level, i.e. avoiding a specialist treatment of the subject;
2) Broadening students' perspectives by highlighting the *cultural aspects of science*, in particular by exploiting meaningful historical facts and the epistemological debate about the intended concepts;
3) Favour students' *personal meaning-making* of the afforded scientific concepts inserted in appropriate contexts; according to the socio-cultural perspective (see below) this attribution of meaning may exclusively come from a cognitive exploration followed by an inter-subjective dialogue on what have just been explored, in which social and individual planes are matched;
4) Following a «historical line» (Arons, 1992; Levrini, 2004; Abiko, 2005, Giannetto, 2009; Galili, 2012);
5) The importance of manipulating digital media for a 21st century media literacy, according to the definition by New Media Consortium (2005): «the set of abilities and skills where aural, visual, and digital literacy overlap. These include the ability to understand the power of images and sounds, to recognize and use that power, to manipulate and transform digital media, to distribute them pervasively, and to easily adapt them to new forms».

Life skills have been said to foster innovation. Their development entails *social skills'* too, which are important for learning, since knowledge is built up by every comparison among possibilities (Galili, 2012) and reasoning is always generated by discussion (Vygotskij, 1931). So the social dimension of education was considered in two respects here: (a) the *dialogical nature of any meaning-making process* and (b) the *concept of «collective intelligence»* by Lévy (2010).

The theoretical framework on which the selected aspects are drawn on is being expounded.

According to the *socio-cultural perspective,* learning is an *internalization* process from social to individual plane: communication of ideas in a social context allows everyone to make personal sense of them (Vygotskij, 1978). Thus any meaning making process is essentially dialogic: «for [Mikhail Mikhailovic] Bakhtin (1895 – 1975), existence, language and thinking were essentially a dialogue» (Mortimer & Scott, 2003). This fact entails a view of scientific learning as learning of the *language worked out by the scientific community*, i.e. its ways of thinking and speaking. Socio-constructivists also claim that students should be made responsible of their own learning by building knowledge together through a *'co-construction' process* in which the teacher is merely a member of the research community. Besides, it is not possible to re-construct students' conceptions toward intended knowledge – and therefore *learn*, according to constructivist perspectives – without reconstructing their reasoning paths, and this may be exclusively achieved with socio-cultural support. Conceptual restructuring does not occur in the individual only indeed, but necessarily *inside a localized complex sociocultural setting* including specific tools and artifacts (Vosniadou, Vamvakoussi & Skopeliti, 2008). On the other hand, Lévy (2010) calls «collective intelligence» the process of inter-subjective expansion of individual mental skills by working on a problem within a social community. This will become the new method for producing knowledge in society. He asserts that it should be *developed in students* by school teaching. Students should know (i) how to solve a problem on their own – exploiting a broad background of already-known topics – as well as (ii) when turn to a larger community, pooling knowledge with the other members of the learning community towards a common goal.

All the previously expounded hints and ways for innovation are not actually restricted to the pedagogical and educational domain, but *concern the development of the entire society*, as shown by the 'Science with and for Society' section of Horizon 2020. Horizon 2020 is the biggest European programme for research and innovation in the period 2014-2020 and it is expected to achieve worldwide competitive research breakthroughs. 'Science with and for Society' programme aims at *uniting scientific innovation and society* by involving all citizens, in order to promote a shared responsibility and engagement in science research, taking into account ethical and gender equality issues too. So the whole of these actions favours *citizenship education*, which comes before and includes science education.

Figure II.3. The logo of the 'Science with and for Society' programme [9].

II.2 Philosophical Paradigms shaping Science Education

The set of philosophical paradigms useful for any educational theory, meant as theory of learning and understanding, may be thought as a *continuum* between two extremes: objectivism – in which reality is entirely external to the knower – and constructivism, in which reality is interior to the knower (Jonassen, 1992).

Objectivism is a theory of learning and understanding drawing on the assumption of the existence of an external reality independent of any observer and of human experience in general. This is a basically *realist* assertion. According to this view the true knowledge of reality may be approximately achieved or at least progressively approached. Moreover, reality has got a structure, which may be modelled by the knower by mirroring reality through symbolic representations; an approximate *matching* between knower's mental constructions and objective reality will be gained in this way. The meaning of reality is therefore independent of learner's mental processes (Jonassen, 1992). From objectivist standpoint, learning consists straightforwardly *in mapping conceptual referents on learners' mind*. Conceptual references are meant as the meanings attributed by the learner to objects and phenomena of external reality. The learner will approximately know true reality only if his/her mind acknowledges the models resembling reality.

Therefore students are taught *the* scientific explanation of real world, which is interpreted by teachers only: no personal construction of meaning is performed by students. It is assumed that *every student will be gaining the same learning with understanding* if the teacher's objectives are good enough in his own opinion (Jonassen, 1992; Vrasidas, 2000). According to the model of curriculum developed by Tyler (1949), namely an Input-Process-Output model with four basic steps, the instructional design deriving from objectivism shows a well-shaped structure. Each step ought to be strictly undertaken in the following order:

1) Identification of the aims (input);
2) Selection of useful learning experiences (process – 1st level);
3) Organization of the latter (process – 2nd level);
4) Assessment of learning outcomes (output).

At the other end of the above-mentioned *continuum* is radical *constructivism*. Constructivism is the most important philosophical paradigm for hard and soft sciences – namely social, psychological and therefore educational sciences – having emerged in the Twentieth Century. In constructivism the emphasis is shifted from the *object* of knowledge to the creative *processes* performed by the subject of knowledge. It was also defined as « rivoluzione pedagogica del XX secolo » (Bottani, 2002) and its roots are in the transcendental *idealism* by Vico and chiefly Kant. Some authoritative representatives of this broad stream are Kuhn, Von Glasersfeld, and Wittgenstein (science/logic philosophers), Heisenberg and Schrödinger (physicists), Bruner, Dewey, Piaget, and Vygotskij (socio-psychologists and educators). Reality, meant as actually unknown, is *fitted* by the knower's mental constructions, instead of being *matched* as in objectivism. It is human mental activity, scientific in particular, *which creates the structures of the external world* in order to *interpret* it, rather than merely *describing* pre-existing inherent structures, like from realist standpoint (Vrasidas, 2000). Therefore modelling acquires a new whole meaning in constructivism. On the other hand, constructivism does not necessarily assert that an objective reality external to knower's mind does not exist; it rather asserts that everyone builds its own interpretation of it, i.e. each knower builds its *own perspective on reality*. Eventually, the nature of the elements of any 'real' system is not considered meaningful, because thought as arbitrary, while their interrelationships are important (Jonassen, 1992; Watzlawick, 1992; Hruby & Roegiers, 2013).

Giambattista Vico (1668 – 1744) claimed that reality cannot be known by anyone but God, who created it, and thus a true and complete knowledge of natural world is impossible to achieve by men; *the only achievable knowledge is the one that the knower has been constructing*. Human beings cannot therefore gain access to any ontological truth (Jonassen, 1992; Watzlawick, 1992). In particular, in opposition with the claims of scientific rationalism, history is elevated to the rank of 'science' in *Scienza Nuova* (1744), for it deals with what *men actually made*, namely human civilization, while nature is only known by means of mathematics and metaphysics, but not constructed.

Immanuel Kant (1724 – 1804) argued in his *Critique of Pure Reason* (1781) that truth, the set of *noumena*, does exist, but it is not knowable by humans. He essentially claimed the existence of categories inherent to each knower for interpreting reality (for instance space, time[11], and causality); the knower, after experience of the external world, shapes what he/she experienced in it, i.e. *phenomena*, according to this knowledge *a priori*, generating thus knowledge *a posteriori* (Jonassen, 1992; Hruby & Roegiers, 2013).

Problems arise from *radical* constructivism when one has to define scientific progress, for there is no space for gradual approximation to truth or reality. For instance, according to Von Glasersfeld the only statements that can be made on reality on the bases of logics are about what it *is not* (Watzlawick, 1992).

Moreover, radical constructivism cannot explain intersubjectivity. I did not give up realism for these reasons too.

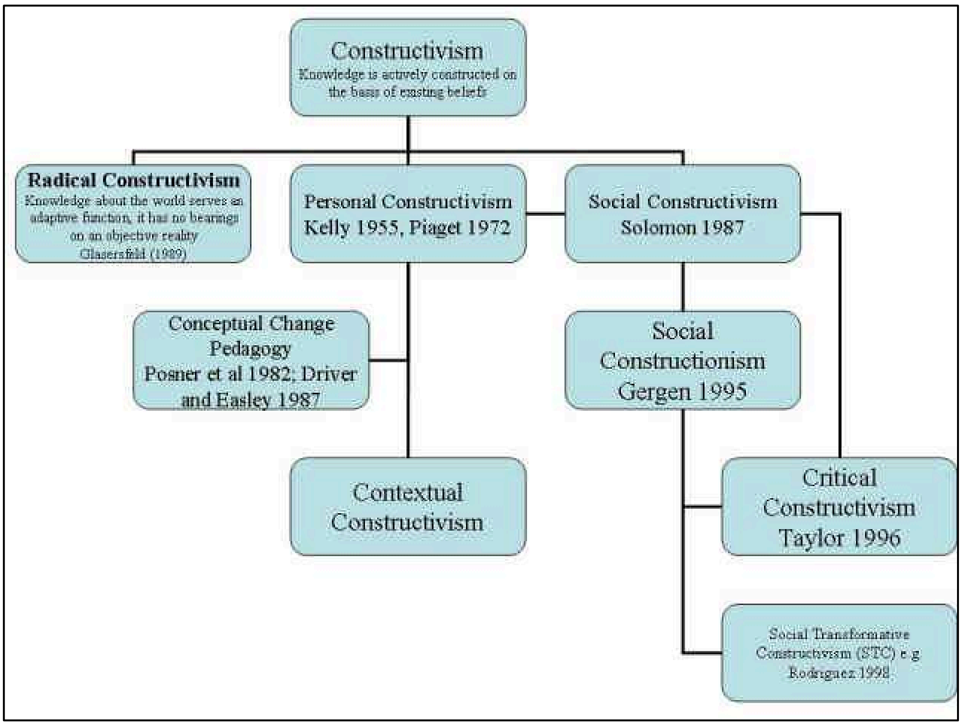

Figure II.4. Constructivism variants [44].

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[11] By the way, because of this peculiar role of space-time, Relativity may be considered a physical *meta-theory*: it is a theory of the framework in which all events occur and in which each other physics theory is necessary embedded (Bergia & Franco, 2001).

II.3 Learning in Constructivism

The passage from the philosophical constructivist view of learning to the educational theory and practice is problematic, as depicted in a seminal paper by Driver, Asoko, Leach, Mortimer, and Scott (1994). *Multiple perspectives* are to be taken into account as well as the fact that knowledge is never fixed or absolute. Learning is seen as nothing but *making meaning of experiences*, so «different individuals, with different prior knowledge, perspectives, histories, and values, will generate slightly different recollections of the "same" event» (Hruby & Roegiers, 2013).

Constructivism may be split into *individual* and *social* constructivism (Driver *et al.*, 1994), depending on whether knowledge is thought to be constructed in the individual mind or in the communities of practice/learning communities respectively. Jean Piaget (1896 – 1980), a very influential cognitive psychologist, anticipated constructivism by applying structuralism (deriving from Kantian ideas) to psychology. Piaget observed single cases and worked out a model for the *intellectual growth of a child*. He asserted that cognitive development occurs only when the child succeeds in coordinating inside his/her mind his/her personal interactions with the physical environment (Piaget, 1970). Piaget argued too that child's cognition develops through *four basic subsequent stages*, namely

- sensory-motor stage in infancy;
- pre-operational stage in early childhood, characterized by the developing of representations;
- concrete stage (developing of logical operations) in mid-childhood;
- formal stage (developing of operations on hypotheticals) in late childhood/pre-pubescence.

Each stage will be achieved only if a given set of skills, or *schemas*, are learned (Piaget, 1937). When the gained stage becomes inadequate for a large enough number of child's needs, a situation of disequilibrium generates, which is solved by the *equilibration* process, viz. adaptation to the new environmental conditions. This allows a child to gain the next stage. Equilibration may occur through either *assimilation* or *accommodation*. The former is the process of incoming knowledge re-structuration for matching it with the prior one; conversely, the latter is the re-structuration of prior knowledge for matching it with the incoming one, thus creating new knowledge (Jonassen, 1992; Driver *et al.*, 1994; Hruby & Roegiers, 2013). That is why the mere ‘addition’ of information by enrichment mechanisms is not enough for learning; on the contrary it has proved to be often the source of either internal inconsistencies in children’s conceptions or children’s conceptions not in agreement with scientific knowledge, called “misconception” (Vosniadou *et al.*, 2008). Accommodation is the type of restructuration useful to pass from everyday representations to scientific ones, which have proved to be significantly different under the ontological and epistemological respects (Driver *et al.*, 1994).

A worthwhile and meaningful definition of the individual understanding of any concept expressed by a word was provided by White and Gunstone (1992): it entails not only the *propositions* (verbal knowledge) but *also strings, images, episodes, intellectual and motor skills associated to that word*. «The richer this set, the better its separate elements are linked with each other, and the clearer each element is formulated, then the greater the understanding» (White & Gunstone, 1992). Nevertheless, this is not an exhaustive definition, since it does not specifies which elements of the set are more important than other. There is not a sharp answer to the last issue, for three reasons:

1) Conceptual understanding is actually a *continuum*, instead of a binary state ‘understood / not understood’;
2) Conceptual understanding is never complete;
3) Conceptual understanding is multidimensional: it not exhaustive to measure it on a linear scale composed of levels.

As a consequence, the measure of understanding is inherently subjective: it may be solely asserted that

- the ‘degree’ of the achieved concept understanding is an increasing function of the number of associate elements;
- understanding is also somehow function «of the mixture of different types of element and of the pattern of association that the person perceives among them» (White & Gunstone, 1992).

Anyway, learning with understanding is essentially a *meaning-making process* in constructivist perspective. Three ways for ascribing meaning have been individuated by White and Gunstone (1992), in which the generated learning with understanding will be *different for each person*:

1) The *insight*, sometimes called ‘Ah-ah!’ experience, in which new links are created or new propositions deduced by thinking individually about older knowledge without external hints;
2) The *incidental learning*, namely when a random occurrence – not planned for learning – forms a new episode that will enlighten new knowledge;
3) Meaning making *under the guidance of an authority*, typically under the tutoring of the teacher in a classroom or laboratory or informal learning setting. This point entails social aspects, thus it will be resumed later.

Even if it sounds strange, the understanding resulting from a lesson will be different for each pupil in the same class, because a different set of propositions, strings, episodes, images, and intellectual and motor skills is inside each student’s mind before any instructional activity. Therefore the latter will give rise to different learning gains.

However, this is not the end of the story, since *social interactions* have to be taken into account too.

Lev Semënovič Vygotskij (1896 – 1934), founder of the *sociocultural* tradition[12], argued that the cultural development a child, as well as concept formation, first occur between people, on the interpsychological plane, and *then* inside him, on the intrapsychological plane (Vygotskij, 1931), unlike Piaget’s claims. This happens by the internalization process mentioned in II.1, which is to be strictly considered as a *genetic law*: «Social relations or relations among people genetically underlie all higher functions and their relationships» (*ibid.*). Examples of ‘mental functions’ are memory, attention, and thinking. Thus higher mental functions have a social origin exclusively: they are never native, unlike elementary mental functions, according to Vygotskij (1978). He claimed essentially that elementary structures are psychological wholes of mainly biological origin, present in animals too, while higher functions are developed by using symbols, in particular those of language, and so they are characteristic of humans, being the last structures emerging in cultural development (*ibid*; Mortimer & Scott, 2003).

The followers of socio-cultural perspective tend to *consider science as a discourse built by the scientific community*. Therefore learning science is meant as an active meaning-making *dialogic process* towards Bakhtin’s social language of science (Holquist, 1981), which at the same time has to be consistent with the

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[12] For the sake of precision, Vygotskij’s stream had been named “socio-historical” at the beginning; James Wertsch proposed the current name.

results from experimental tests on hypotheses about nature. More precisely, learning is a process towards *school science* social language, which is different from language of real science since definite choices in the curriculum have already been made (Mortimer & Scott, 2003). Thus *learners have to be socialized into reasoning paths and practices of school science*, the latter being essentially a symbolic world (Bruner, 1985). Meaning should be built by learners during educational practice under the tutoring of an authority (usually the teacher) representing the orthodox science view. The teacher must support pupils in bridging from everyday representations to scientific ones. The teacher's main roles are (a) to introduce new hints and cultural tools for students, guiding them to make their own sense of these concepts, (b) to pinpoint how current activities are being considered by pupils for improving future design (Driver *et al.*, 1994). The teacher's roles, types of guidance and interactions with students may actually vary in a broad spectrum; compare for instance the work by Cacciamani and Giannandrea (2004), which will be referred to in VI.4.2. It should be clear from above that *learning by construction* is fostered in this context, namely enculturation, rather than learning by discovery. In fact, learning by discovery (insight) is associated to individual constructivism, while in social constructivism knowledge is constructed by socially sharing ideas through *talking* inside a community of practice consisting of different-skilled members. Besides it would be desirable if learners were socialized into a *critical view* on school science by reasoning on assumptions, boundaries and purposes of scientific knowledge. This would allow a meta-cognition on natural phenomena, which may be achieved exploiting history and philosophy of science.

Nowadays the most widespread standpoint is to join the two traditions, since *learning science entails social interactions but also individual meaning-making process of the new worldviews*. Socially constructed knowledge has to be necessarily matched with the prior one and re-worked inside any individual and, conversely, social processes are always born of cognitive inner processes.

II.4 SER Paradigms

Each philosophical paradigm translates into a specific SER paradigm that will be framing research on cognitive processes, from design research and stating research questions to data analysis and outcome report to the SER community. Treagust, Won and Duit (2014) wrote a seminal paper on SER paradigms that will be the main reference for this section. The two most important SER paradigms are *post-positivism* and *interpretivism.*

Positivist paradigms stem from naïve realism and are essentially meant as forms of logical empiricism here, i.e. «any approach that applies scientific method to the study of human action» (Schwandt, 2001), in which the sources of any trustworthy claim are sensorial experience and inferential or mathematical treatments. Positivist paradigms exploit logical argument and seek for generalizable results from empirical data, pinpointing some *variables* – 'social variables' hereafter – affecting social facts and educational phenomena. Positivists believe (i) in the *objective reality* of social phenomena viz. in their intrinsic independence from the particular perspective of the observer, (ii) in the existence of a *causal relationship* or at least *correlation* between social variables, (iii) in the power to determine and measure them and to find out their relations.

Post positivism is a weaker evolution of pure positivism prevailing with respect to it nowadays. Unlike their predecessors, post-positivists do not believe that collected data are immediately true and self-evident or that they are useful to draw generalized conclusions objectively, since every researcher is *inevitably conditioned* by its culture, personal values and beliefs. Post-positivist researchers and scholars try *to approach as close as possible to true assertions*[13] by means of systematic and comparative studies on overall collections of data, in order to get a rational explanation of a certain set of educational phenomena. Research outcomes are used for supplying worthwhile data to governments, school as institution, teachers and the research community itself, for instance in order to improve the syllabus or to contribute to teacher pre-service and in-service training. *Objectivity* of methods, *reliability* of instruments and results, *validity* of instruments and *significance* of results are necessary to reach trustworthiness and to develop

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[13] In contrast with pure positivists, truth is never entirely knowable for post-positivists, but it may be more and more approached to, according to Popper's view of science progress (1934).

evidence, i.e. a weak or strong support for whatever is being asserted (Fischer, 2013; Fischer, Boone, & Neumann, 2014).

Interpretivism emerged later instead, as a reaction to the narrowness of the (post)-positivist method, thought as too much detached from educational phenomena and not detailed enough in final reports because of a scant attention to the *context* in which anything occurs. In fact, interpretivists are interested in the *local meaning* of human experience inside a well-defined social, cultural, economic, political and ethnical context. They do not usually seek for a generalizable truth to be communicated to policy-makers or institutions, but for a *sensitive account of the situated knowledge construction by means of active experiences and relationships in social situations*. Thus social variables are complex and intimately interrelated, which implies in turn they are not linearly independent and therefore very difficult to measure. Moreover, an interpretivist researcher tends to be personally involved in the object of study and generally listens carefully and empathically to involved people, in order to understand their experience soundly. The influence of relativist ontology is apparent: *subjectivity* enters necessarily into play at various levels, namely in the interpretation of social interactions by people taking part into the research, in the interpretation of those people's cognitive experience by the researcher, and eventually in the interpretation of the expounded research outcomes by the science education community. Finally, there are criteria for trustworthiness in interpretive research too, which parallel those of post-positivist paradigm: *credibility*, *transferability*, *dependability*, and *confirmability*. The first one indicates *how well* the obtained results mirror the educational context under study: the elaboration process from raw data to conclusions has to be clearly and univocally expounded in order to achieve a good credibility. Transferability measures instead to what extent the research assumptions and findings may be extended to *other contexts*; this aspect will be estimable if the obtained data are extensively described. Dependability is an indicator of the extent to which the whole study *depends of the specific conditions* of the surveyed educational phenomenon, while confirmability measures the degree of *intersubjective agreement* on the obtained data and their supplied interpretation. The latter two are determined by the internal consistence degree of the study processes and of the research products respectively (Lincoln & Guba, 1985; Bradley, 1993; Zhang & Wildemuth, 2009).

II.5 Pragmatic Research

Traditionally, (post)-positivism and interpretivism have been considered as incommensurable, because based upon opposite assumptions, like objectivity *versus* subjectivity or realism *versus* constructivism respectively. However, less strict social inquiry stances, in which what actually guides inquirers in their methodological decision is taken into greater account, have developed up to now, evolving from the alluded *purist* stance (Lincoln & Guba, 1985) to the *a-paradigmatic* stance (Patton, 2002). These approaches are called 'pragmatic' (i) for their attention to make sense for social researchers to the needs encountered when putting the theoretic-epistemological assumption into practice, and (ii) for their taking care of the possible novel practitioners' interests as well (Greene, 2008). The extension to educational inquiry is straightforward, since it may be thought as a form of social inquiry, sharing with it most of methodology.

In the middle of the scale between purist and substantive theorists two worthwhile standpoints are:

- the *dialectic stance* (Greene & Caracelli, 1997; Maxwell & Loomis, 2003), according to which paradigms are separated indeed, but not untouchable, since they were worked out by a certain social community in a precise historical period; thus not only dialogue is possible between them, but it is likely to produce new insights;
- the *alternative paradigm stance* (Howe, 2003; Mertens, 2003; Johnson & Onwuegbuzie, 2004), which claims that a synthesis of different paradigms and a generation of new ones is possible, scientific realism[14] and American pragmatism being prominent examples; the new paradigms ought to enhance the mixing of methods too.

In other words, comparison, dialogue and even mixing of diverse paradigms are fruitful because they allow a form of *triangulation*. Mixed-method approach is usually composed by a social intervention (learning intervention in the present work) and its evaluation. *Triangulation* will be the purpose of mixed-method approach if research design involves *convergence*. Several other purposes, namely

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[14] Roughly speaking, scientific realism is the epistemological view according to which the propositions of the (constructed) best current scientific theories have *epistemic value*, i.e. provide an approximate *true* account of phenomena. Therefore no choices among theories describing the same phenomena are made for *pragmatic* reasons (Castellani, 2013).

complementarity, development, initiation, and expansion, were individuated by Greene (2008), each one relating to a different research design. At the beginning, data, investigator and methodological triangulation had been considered to be able to cancel out biases (Denzin, 1978), which would have implied the convergence on *the* objective description of the social (educational) phenomenon by one single proposition. Triangulation was thus being considered as the counterpart of the intersubjective agreement which warrants objectivity. Nevertheless, although convergence is sometimes found in triangulation, other two outcomes are more often found: inconsistency or even contradiction. In the former case the findings do not lead to the same conclusions, but to ones that may coexist; in the latter the findings lead to entirely incompatible statements. The way out is to consider not only the immediate data (1st level of evidence), as in the empiricist tradition of pure science, but also the specific research project from which they come as a whole (2nd level of evidence), and the overall knowledge of social world used for building the project and its evaluation (3rd level of evidence). So the social (educational) researcher has both to *explain the results*, which are never self-explaining, on the basis of the other two levels of evidence, both to *carefully report each source of evidence* from which he derives his explanation (Miles & Huberman, 1984; Mathison, 1988).

Two meaningful examples of this research approach, pragmatically merging qualitative and quantitative analysis:

1) Hohenshell and Hand (2006) used a mixed-method with a non-random sample, so that generalization is not allowed. Quantitative and qualitative methods were used *complementarily*, in the sense that the quantitative results accounted for science achievements and the qualitative findings served to enhance interpretation of the latter. Data and investigator triangulation were performed in qualitative analysis.
2) Prain and Waldrip (2006) carried out a study on multi-modal representations for learning primary science. Their mixed-methods approach included triangulation of different data sources to achieve convergence of results, according to the model by Denzin and Lincoln (1994).

II.6 Conceptual Reconstruction

To summarize, learning with understanding will occur from constructivist perspective if the learner reorganizes his/her knowledge structures actively. A *construction* of knowledge will be performed that way: new concepts will be generated. This will be clarified in the following.

First of all, it is important to separate *school science concepts* from *science cognitive conceptions*: the former are those ideas elaborated and currently accepted by the scientific community that have been selected for the science curriculum, the latter are the researcher-detected *mental models* which students elaborate individually or in group. *Having understood a concept means essentially to hold a mental model of it after instruction.*

However, what a concept actually is and how conceptual understanding is gained are open problems (Vosniadou, 2008b; Taber, 2011). For instruction, concepts may be conceived as «privileged linguistic markers through which conversations in the domain can productively proceed» (Tytler & Prain, 2013). That is to say, once a learner has been able to individually coordinate the various representations (signifiers) of each of a series of concepts (meanings), he/she may talk effectively (without further clarifications and in the minimum time) with other learners which gained similar personal insights.

Mental models are instead meant as entirely *interior representations* in cognitive sciences, by which a learner tries both (i) to make personal sense of the physical world, by explaining and predicting phenomena, both (ii) to understand discourse. Mental models supply to a learner information on the structure of physical systems and on their behaviour in his/her inner language, by means of analogy (Greca & Moreira, 2002). *Learning process may be associated to a sequence of different conceptions successively developed,* the scientific concept to be learnt being the *succession limit* (Niedderer *et al.*, 2007). Learning is characterized by a change in both mental *processes* and knowledge *structures*.

Nevertheless, conceptions are not directly detectable, so the researcher has to analyse their *external representations*, like speech, writing, paintings, and so on (Gilbert & Boulter, 1998). Eventually, conceptions may be considered either as parts of modelling of student mind by the researcher – built in order to find out pupils' paths of reasoning – or simply as "expressed ideas", i.e. what the researcher 'see' as external representations of mental models. In the former case a conception has to be *stable* over time, settings and students involved, whilst in the latter no general hypotheses are formulated on students' mind behaviour, thus the found pieces of knowledge have a *local value* only (Niedderer *et al.*, 2007).

In this research work conceptions are explored according to the second local meaning above, but in order to find students' reasoning ways, according to the first meaning above.

II.6.1 Conceptual Understanding and Learning in Science

The importance of scientific literacy was stated in the first chapter. Accordingly, learning with understanding a concept may be also pragmatically thought as the skills of knowing *how to use that concept in meaningful practical and everyday settings*, according to an empirical approach to scientific knowledge generation, whose situated and contextual nature is therefore enlightened. Conceptual learning with understanding needs *representation coordination* for solving specific problems and for developing explanations (Peirce, 1931 – '58; Wittgenstein, 1972; Tytler *et al.*, 2013).

In fact, learning essentially means *grasping the conceptual referents of real objects (world referents) through representations or signs*. These three elements are interrelated in Peirce's (1931 – '58) triadic model, usually depicted by a triangle like the one below. Learning a new concept is not limited indeed to grasping the concept (vertex 1), but also entails to acknowledge how the concept is represented (vertex 2) and the world phenomena which it refers to (vertex 3). On the other hand, it has been stated in section II.3 that learning science is a process of *enculturation into the scientific community discourse* from sociocultural perspective.

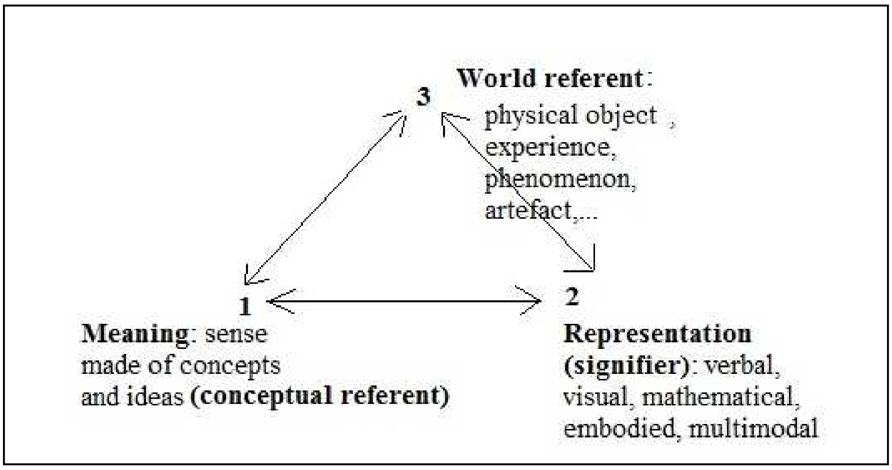

Figure II.5. Peirce's triangle for learning.

Furthermore, learning is not exactly the same than understanding: the former may be thought as *evolution of mental models* towards the intended knowledge, while the latter occurs by mere *generation of mental models* having some degree of agreement with the taught scientific concepts. Moreover, understanding entails cognitive transfer skills in similar contexts, whereas learning requires problem-solving skills in new contexts, also invented by the learner himself/herself (Tytler *et al.*, 2013).

All that is in sharp contrast with the verbal definition of concepts in most curriculums and textbooks.

Anyway, students have been found to use different mental models when engaged in reasoning about different versions of the same issue: research has shown the simultaneous existence of alternative mental models for a single topic in students' minds and even of a probability distribution for the activation of these «multiple models» (Bao & Redish, 2006). Furthermore, the external representations of mental models were found to be frequently activated in multiple modalities too. The importance of understanding how to *move within and between different modalities of representation* for science learning has been widely recognized since Nineties (e.g. Ogborn *et al.*, 1996; Ainsworth, 1999; Saul, 2004; Gilbert, 2005; Tytler *et al.*, 2013). For this reason, it is necessary that students develop the ability of handle with *multiple* and *multi-modal representations*.

- Multiple representations are important to deal with, since it is usual in science to refer to a *single concept* by depicting it in various modalities, like verbal, pictorial and numerical (Tytler *et al.*, 2013).
- Multi-modal denotes the *coordination and integration* among different ways of representing *complex statements*. Students proved much more interested in multi-modal representations when they could find a link between one of them and their personal model; therefore learning increased (Saul, 2004). Thus pupils essentially need a personal involvement in the knowledge-to-learn, as pointed out for instance by Michelini (2011).

In order to take into account these multiple perspectives, teaching and learning are more and more understood as *complex* processes calling for multi-method research (Brewer & Hunter, 2006). Eventually, students may also possess «hybrid models» (Greca & Moreira, 2002) deriving from *the merge of scientific concepts* and *naïve*[15] *or initial*[16] *conceptions*.

II.6.2 Early Conceptual Change and Conceptual Reconstruction

The broad SER strand named "Conceptual Change" stemmed from a parallelism between Kuhn's (1970) and Lakatos's (1970) views of the historical development of scientific theories and the changes in students' conceptions owing to the learning process. This model for learning, according to which a *revolutionary* change[17] occurs in learners' conceptions, came both from the older « belief that learning is the result of the interaction between what the student is taught and his current ideas or concepts » both from « Piaget's (1929, 1930) early studies of children's explanations of natural phenomena and his more recent studies of causality (Piaget, 1974) » (Posner *et al.*, 1982).

However, what has just been depicted is actually the *classical approach* to conceptual change: a description of *learning as an exchange of the students' initial conceptions for the scientifically correct concepts*. This is an outdated oversimplified description of learning, drawing on the alternative processes of assimilation and accommodation of a new conception into the learner's «conceptual ecology» (*ibid.*), by analogy with Piagetian theories[18]. To reach accommodation four conditions are to be fulfilled:

1) *Dissatisfaction* with the existing conceptions;
2) *Intelligibility* of the new concept;
3) *Plausibility* of the new concept;
4) Possibility of undertaking a *fruitful research program* using the new concept.

Noteworthy, this model of learning was first elaborated on the theoretical ground and then *it was applied to Special Relativity*. SR «was chosen because it has been commonly viewed as a prototype of a scientific revolution » (*ibid.*). By way of example, while the two postulates are overall rather intelligible, the whole theory, including radical changes in space and time concepts, is *not*. Therefore, a

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[15] This adjective refers to the case of children before any instruction. In this case the conceptions are usually called *pre-conceptions*.

[16] This term is used instead at the beginning of any teaching/learning path, at whatever level of instruction.

[17] A parallelism between the evolution of science theories and the ideas' change occurring in learning is also maintained by Galili (2012).

[18] It is worth specifying that the authors claim they do not rely on Piaget's theories in the original paper.

resistance to accept kinematical effects was detected, due to the reluctance to change the owned metaphysical beliefs and epistemological commitments on space and time. A real case study on the topic (Hewson, 1982) is worth mentioning, in which the strategy allowing the learner to overcome his metaphysical commitment[19] was showed to follow the four conditions above.

Nowadays the learning strategy called conceptual reconstruction consists in the *accommodation* of ideas deriving from real or imaginary phenomena into the learner's prior knowledge, preserving cognitive *continuity.* Generating accommodation in students as concerns the conception of mass conservation was my intention, in order to induce the opposite conception in students (according to $\Delta m = \Delta E_0 / c^2$). To that end it was necessary to stress anomalies and inconsistencies of the known theory, in this case Newtonian dynamics. Phenomenology-guided observation is able to suggest new hypotheses to learners (Michelini, 2005, 2010a), causing the well-known content restructuring process (Duit & Treagust, 2012; Duit *et al.*, 2012).

II.7 Content-oriented theory

The first part of the overall work consisted in *research design*, for which a *domain-specific approach to learning* was selected, which brought to the design of two topic-oriented t/l sequences (Méheut & Psillos, 2004; Vosniadou, Vamvakoussi, & Skopeliti, 2008) on mass and its relationship with energy.

In fact, the focus on content of SER community made Andersson and Wallin (2006) develop the concept of *content-oriented theory* – involving content-specific, nature of science, and general aspects (i.e. going beyond school science) – inspired by Lijnse's and Klaassen's «scenario» (2004). Content-specific or domain-specific theories have always come out from meaningful empirical or theoretical research results. Their outcomes are strictly valid for a specific topic only, but sometimes generalizable to a certain extent. Thus they may also supply SER community with useful «didactical structures» for design in similar topics (Lijnse, 2000; Cobb *et al.*, 2003). Content-oriented research's underlying assumptions are (Niedderer, 2006)

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[19] A mechanistic view, according to which objects have fixed and absolute properties (Hewson, 1982).

- Learning is *always* content-specific, simply because some content has necessarily to be learnt;
- There is a *limited* number of both alternative conceptions both learning pathways for every content area.

The attention was directed towards two related aspects of content-oriented research for the present work: *students' conceptions* and *conceptual profiles* (Driver *et al.*, 1994) on the one hand, and their *learning pathways* and *processes* (Niedderer *et al.*, 2007), as well as the process of *conceptual change*, on the other.

A strand of content-oriented research is *Design-based Research* (DBR) whose main characters are well summarized by Andersson and Wallin (2006). They assert that both U.S.A. and European design research are marked by

- Iteration process: the designed t/l sequence is tested, evaluated formatively, revised, and tested again for several times;
- The research work is meant for contributing to educational science, in various ways « e.g., by increasing understanding of conditions that favour learning of given topics in regular classrooms » (*ibid*);
- « Useful products » are obtained, i.e. guides for teachers and materials for students to put the work outcomes into practice;
- The researcher is regularly a designer, teacher, and teacher-trainer too;
- The research aims at improving school teaching directly.

Theoretical Framework of Research

Content-focused research, *design-based research* (DBR) in particular, and the *Model of Educational Reconstruction* (MER) are the pillars which my research draws on. The former was theoretically outlined in II.7; in particular, the inquiry into students' *learning pathways and processes* and the search for *students' conceptions* and *conceptual profiles*, along with *conceptual change*, were chosen as aspects of that research strand (Niedderer, 2006, 2010; Niedderer *et al.*, 2007), learning being meant as *evolution of ideas.*

The MER is a model methodology by Duit (2005, 2006) for conceptual reconstruction in which science content and educational issues are taken into consideration *equally*. A successful design of teaching/learning sequences is possible only in this way according to this perspective. «The model has been developed as a theoretical framework for studies as to whether it is worthwhile and possible to teach particular content areas of science» (Duit *et al.*, 2012). It is made of three components, as depicted in figure III.1. The first one is the *analysis of content structure*: clarification of *key concepts*, as well as *principles* and *processes*, in order to reconstruct the science content for instruction. Relevance of science in society and everyday life, views of the nature of science (*ibid.*) and possibly historical development of the topic (Duit, 2006) may be examined too. The research literature *on teaching and learning the intended science content* is then analysed (component 2). Finally, a learning environment is consequently designed (component 3) for implementing and evaluating the worked-out *teaching/learning sequence*. These three phases are intimately interrelated and were applied to my research, as it will be shown in III.2, III.3, and III.4 respectively.

Moreover, it is essential to state the aims (III.1) before undertaking the research work. Research must be also designed according to a SER paradigm (I chose pragmatic approach) and carried out according to a well-shaped style, the "experiment" in my case (III.5). After that, in DBR (Andersson &Wallin, 2006) a number of crucial conceptual aspects are to be tested through formative intervention modules, then formatively evaluated, consequently revised, and finally tested again in an iterative process of tuning (III.6). The entire t/l path may be experimented at this point. Eventually, a *qualitative* data analysis was performed, with the support of some statistical and educational parameters (III.7).

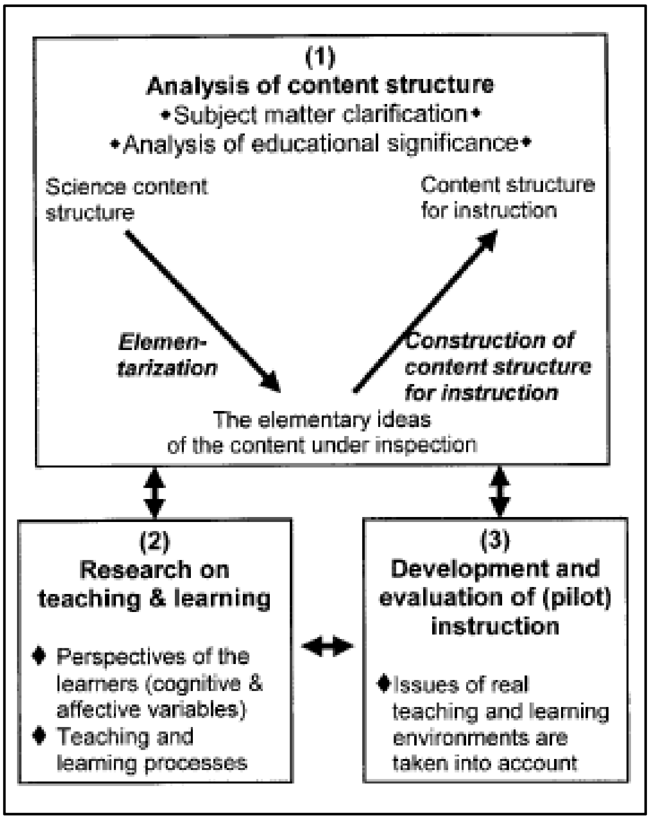

Figure III.1. Diagram of the three main components of M.E.R. (Duit, 2006).

III.1. Aims and Research Questions

Two general objectives were established to investigate into students' learning pathways and conceptions:

1) Analysis of conceptions embedded in a re-structured domain-specific framework theory[20] *after instruction*;
2) Analysis of reasoning *during* conceptual change (Niedderer *et al.*, 2007).

The first objective was pursued primarily by looking on the *sample conceptual change* (meant as variation between initial and final state) in terms of restructuring toward the scientific concepts (Vosniadou *et al.*, 2008), and secondarily by examining the students' reasoning paths after each formative intervention experiment for eliciting their *mastery of the understood concepts*.

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[20] This is precisely *learning* in the "coherence" approach to conceptual change process (Vosniadou, 2008b).

To achieve the second objective a search for the *sample's conceptions* was undertaken through tutorials administered *during the t/l process*[21].

The overall aims of my domain-specific research can be synthesized by three global *research questions* (RQs), in which "conceptualizing" means "gaining conceptual learning with understanding[22]":

1) How do high-school students conceptualize the facets of mass in classical mechanics in ordinary and unconventional learning environments?
2) How do high-school students conceptualize the relationship between inertial mass and rest energy in ordinary and unconventional learning environments?
3) How and to what extent are the concepts of inertial mass and mass-energy relationship utilized by upper-secondary students for interpreting real or modelled phenomenology?

Learning may be qualitatively probed through the external process of expression of a new idea *or* increase/decrease of the domain of an idea *or* link among several ideas so that a network may be developed (Givry, 2003). Besides, a hint for a quantitative measurement of the extent to which conceptualization has occurred may be supplied by learning gain (Aslanides & Savage, 2013).

III.2. **Educational Reconstruction**

The conceptual reconstruction was performed by individuating, progressively expanding and integrating the intended contents. The key conceptual elements of the new (relativistic) perspective have been dealt with in an innovative way. Mass and energy are of course the most important concepts under examination.

The former was first considered in its intrinsic meaning and cultural value. The role of mass in classical mechanics was indeed studied from original science texts for a historical reconstruction, more precisely for building a «historical line» (Arons, 1992), which involves only the fundamental contents for learning the ultimate target topic (mass-energy equivalence in this case), neglecting

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[21] Without forgetting affective variables and socio-cultural context in the data analysis discussion (Duit & Treagust, 2012).

[22] When the conceptions held after instruction proved consistent with school science, but neither reasoning patterns (or few uncorrelated ones) nor inclusion in a consistent theory-like framework were found in the analysis, a science concept was considered as *understood*, but not *learned* (Vosniadou *et al.*, 2008).

experiments or theoretical arguments not directly aimed at understanding it, being conscious that some modern physics contents must necessarily be neglected.

This first phase of conceptual «elementarization» brought to compare the fundamental ideas by Newton and Mach, allowing to reconstruct the concept of mass under the historical-epistemological standpoint. Classical mass conservation in physic-chemical processes was then considered.

Later on, mass in a new theoretical framework, i.e. Special Relativity, was conceptually clarified and explored. Relativistic mechanics four-vectors were accurately studied for founding the elements necessary to grasp mass at the formal level, in addition to the conceptual one. In fact, the modulus of momentum-energy four-vector furnishes mass in its relativistic meaning.

The roles of *energy* and its conservation in classical physics and SR were considered too, even though not with the same care as mass. A paper by Einstein (1935) suggested me the idea of deducing total relativistic energy expression by taking its classical limit and recognizing classical kinetic energy as a term of the series expansion. Then relativistic kinetic energy and rest energy can be defined, the former by the Correspondence Principle, the latter as zero-point energy: the energy owned by an object at rest. Energy concept is simpler in this paradigm than in classical physics. In Feynman's words (1964):

> Therefore we have a *new idea*: we do not have to know what things are made of inside; we cannot and need not identify, inside a particle, which of the energy is rest energy of the parts into which it is going to disintegrate. It is not convenient and often not possible to separate the total mc^2 [γmc^2 *in my perspective*, *Ed.*] energy of an object into rest energy of the inside pieces, kinetic energy of the pieces, and potential energy of the pieces; instead, we simply speak of the *total energy* of the particle. We "shift the origin" of energy by adding a constant $m_o c^2$ [mc^2, *Ed.*] to everything, and say that the total energy of a particle is the mass in motion times c^2 [$E = \gamma mc^2$, *Ed.*] and when the object is standing still, the energy is the mass at rest times c^2 [$E_0 = mc^2$, *Ed.*].

A t/l proposal based on this "four-vector approach" was developed and experimented; learning difficulties came out due to several reasons inherent to the proposal (expounded in VI.3.1).

So a more phenomenological approach was attempted for the relativistic part, in which *crucial experiments* were needed and which at the same time put a greater emphasis on *energy*. To that end, an educational experiment on the existence of an ultimate speed (Bertozzi, 1964) was put at the beginning of the new "energetic (phenomenological) approach". This path was more conceptual and less formal than the former. The phenomenological aspects included in both paths are Bertozzi's experiment, light reflection period for measuring time intervals, elastic relativistic particle collisions, β-decays and atomic mass defect, photon colliding against the walls of a mirrored box, emission/absorption of photons by an object in uniform motion/at rest, perfectly inelastic relativistic collision of identical particles with creation of a new particle at rest.

The following key concepts were organized in *founding nuclei* common to the two planned conceptual paths and other nuclei differing between one path and the other. The former nuclei are in the higher part of table III.1, the latter are split in the lower part. The ones relative to the classical[23] part of the t/l sequence are labelled by CN ("classical nucleus"), those concerning the relativistic part by RN ("relativistic nucleus"), and the overlapping ones by CRN.

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[23] It is specified that only macroscopic objects are considered in this part.

CN1) Newton's modern definition of mass: quantity of matter
a. The mass-density-volume vicious circle for the definition of mass $m = \rho \cdot V$
CN2) Inertial mass in Newton and Mach
a. Inertial mass as resistance to change motion state;
b. Motion state uniquely individuated by relative speed;
c. Inertial mass as conceptually separated from the tendency to persevere in the rectilinear uniform motion stated in Newton's first law;
d. The problematic operational definition of inertial mass in dynamics;
e. Mach's empirical approach to the definition of inertial mass;
f. A model of collision with impulsive forces;
g. Experimental measurement of mass by Mach, exploiting velocity variations in a collision;
CN3) Gravitational mass as evolution of inertial mass for celestial motions accounting for interactions between massive bodies
a. Third Kepler's law;
b. Weight as special case of gravitational interaction between macroscopic medium-sized objects and the Earth, 'near' to its surface;
c. Operational definition of gravitational mass in static conditions through weight, in particular exploiting a spring scales (Hooke's law);
CN4) Weak Equivalence Principle (WEP)
CN5) Matter and mass conservation under chemical-physical transformations; mass additivity

CRN1)The three laws of classical dynamics

RN1) The particle-like nature and behavior of matter and the mass-point model
RN2) Particle – wave duality of light;
RN3) The two postulates of Relativity
RN4) Time, space and space-time intervals
a. The operational definition of time and the methods for measuring it exploiting the second postulate
b. Quantitative measure of the back-and-forth time interval by a *light clock* of height h in two different inertial frames;
c. Proper time (the only invariant time in SR);
d. Dilation of time intervals and invariance of proper time;
e. Length contraction;
f. Space-time as a unity;
g. Invariance of four-interval;
RN5) Mass in SR is Newtonian inertial mass, which acquired a further meaning inside the new paradigm
RN6) $E_0 = mc^2$
RN7) Mass-energy-momentum relation
RN8) Correspondence Principle

RN9) Minkowski space-time a. Plot b. World lines of a particle c. World lines of light d. Pseudo-Euclidean metric RN10) Displacement four-vector analogue to classical displacement RN11) Four-velocity u^{μ} as the four-displacement divided by proper time RN12) Four-momentum defined as $p^{\mu} = mu^{\mu}$: mass times four-velocity RN13) Classical limit of the last component of four-momentum interpreted as *total relativistic energy* $E \equiv E_0 + K_{rel} = \gamma mc^2$ RN14) E_0 as the additive constant at null speed RN15) Relativistic total, kinetic and rest energy RN16) Relativistic momentum $\gamma m\vec{v} \cong m\vec{v} + O(v^3)$ RN17) Mass as norm of 4-momentum RN18) Energy and momentum as a unity in SR RN19) Physical systems owing energy show inertial properties, because of mass-energy equivalence: «Energy in all its forms behaves like matter» (Einstein & Infeld, 1938) RN20) The photon a. Photon momentum b. Compton effect	CRN2) Energy conservation law RN9') The difference between world at low and high speeds (speeds close enough to *c* to make relativistic effects significant at the intended level of approximation) RN10') (Classical) energy and work a. The conceptual and formal definition of work as force times displacement; b. The conceptual definition of kinetic energy as based on work; c. The mathematical expression of kinetic energy; d. The classical form work-energy theorem, resting on $F = ma$; e. Work-energy theorem in the form $\langle u \rangle \, \Delta p = \Delta K$; RN11') c as ultimate speed RN12') Relativistic momentum a. Elastic collisions; b. Collisions in center-of-momentum frame; c. The Correspondence Principle; d. *Invariance* of transversal quantities and of proper time; RN13') Relativistic kinetic energy a. Speed-dependent Lorentz's factor γ as characterizing relativistic dynamical quantities RN14') Nuclear transmutations a. Discovery of natural radioactivity by Pierre Curie; b. Instability of nuclei; c. Particle creation in the case of β-decay; d. Atomic masses approximate nuclear masses, to a level of accuracy of $5 \cdot 10^{-4}$; e. Empirical relation between the mass defect Δm in β-decay and the electron kinetic energy *E*, neglecting electron's mass and neutrino's energy; RN15') The relativistic idea of energy

	RN16')Loss of mass conservation law RN17')Idea of light as composed by elementary *quanta* a. Energy transmitted by means of discrete quantities; b. Photon's momentum and relationship with energy; c. Pressure of light; d. Photon as massless entity with energy and momentum e. Photon absorption RN18')Invariant mass of particle RN19')Mass non-additivity

Table III.1. Founding conceptual nuclei extracted for the educational reconstruction, sub-divided in key physical concepts.

The subject matter content discussion relative to the above nuclei constitutes the most meaningful deepening of my work. Therefore the whole chapter IV is committed to this.

III.3. **Learning Difficulties**

Several papers dealing with (i) conceptual learning of the selected physics content, (ii) students' conceptions detected by research, and (iii) indications for improving the *curriculum* either based on research or ensuring conceptual consistency are present in the literature. They were critically selected for this work and will be extensively explained in chapter V.

III.3.1. **Educational literature**

Weight is one of the first concepts a child learn at the intuitive level. However, its scientific meaning is difficult to understand for children, and both primary both lower secondary students find it difficult to separate it from mass, as well as understand mass in its proper meaning. A significant sample of upper secondary students were found to possess separate concepts of mass and weight, triggered by different words and associated to different quantities. In spite of its relevance in physics, the experimental data on teaching/learning mass are few: most studies are of descriptive or conceptual-theoretical character. It is worth mentioning the operational definition of mass by McDermott and colleagues (2001) and the gravitational approach by the PSSC (1995). Mass is the most important property of matter, so I also examined the chemistry education literature about matter conservation, whose by-products are learning difficulties on mass and weight conservation. Then I examined some literature on the problem of defining inertial mass operationally independently of force: it is well-known that Newton's II law by itself is not enough. The ways out are essentially to define acceleration or force operationally without the II law and then find mass value. Because of mass-energy equivalence, an examination of the different approaches to energy was necessary. Surveys on teaching/learning SR focus mainly on students' learning problems and critical point of curricula, but research on design and evaluation of t/l strategies are needed. Literature on mass-energy and "relativistic mass" was accurately searched for and a few works were found, mainly of historical or theoretical-conceptual character. Finally, I have selected three recent publications supplying t/l empirical data about the equivalence.

III.3.2. Idea explorations

In addition to the literature examination, I looked for students' expressed ideas on mass directly, its conservativity and the interplay among its facets in three planned formative experiments of the earlier research phase, namely the ones carried out in Udine (tutorial during 2011 summer school), Crotone (classroom intervention) and Cesena (classroom intervention).

III.4. Planning two Paths

Two teaching/learning paths were eventually designed drawing on the former two components of MER. More specifically, a module for teaching/learning classical mass was designed, then a relativistic formalized module was added, subsequently another relativistic module starting with phenomenology exploration and pointing to relativistic energy for characterizing mass in SR was worked out. I consulted original papers and books by Newton, Mach, Bridgman, and chiefly Einstein. In particular I took quotations from original Newton's (1687) and Mach's (1883) writings aimed at historical-epistemological analysis of inertial mass. I chose Mach's approach to inertial mass because it allowed to bypass the definition of force; then I consulted some worksheets by the physics education research unit of Udine (in collaboration with Naples [45]) for the measure of velocity variation ratio through online sensors. I also found a paper (Cohen, 1981) in which the historical path which brought Newton to Gravitational Law was competently reconstructed, which helped me to clarify the role of gravitational mass. Further, I read Feynman's well-known book (1964) accurately, as well as some University physics textbooks (e.g. Halliday, Resnick, & Krane, 1993; Halliday, Resnick, & Walker, 1995), and examined the t/l proposals already present in the literature. The path designed by Fabri (2005) inspired much of my work, as well as the conceptual path emerging from the books by Bergia and Franco (2001).

The overall designed pathway is composed of the conceptual pathway of the t/l sequence, the adopted t/l strategies, the exploited instructional materials and tools, and the carried-out learning activities. All that will be extensively described in chapter VI. A synthetic list is provided here, containing the three sections composing the two rationales: (1) + (2) and (1) + (3).

1) <u>Rationale (classical part)</u>: Operational definition of mass in *Principia* (1687): quantity of matter; inertial and then gravitational mass in Newtonian mechanics (Newton, 1687); problems of Newtonian mass; empirical and "relational" Mach's (1883) definition of inertial mass (criticism to Newton); experimental updated measure of Mach's mass ratio; mass conservation and additivity.

2) <u>Four-vector rationale</u>: The two postulates; definition of proper time and time interval dilation through the "light clock" thought experiment (TE from now on); space-time; length contraction; four-interval and its invariance; building of four-displacement through analysis of the world lines of a particle; building of four-momentum by analogy with classical momentum; series expansion of the temporal component of four-momentum in the Newtonian limit and inductive definition of this quantity as relativistic kinetic energy, apart from an additive constant; inference and interpretation of the mass-rest energy relationship: meaning of mass in SR. Relativistic momentum. Modulus of four-momentum and consequently relativistic mass-energy-momentum relationship. TE: "photon in a box".

3) Energetic (phenomenological) rationale: recall of introductory elements of classical dynamics and electrostatics; Bertozzi's experiment and crisis of Newtonian mechanics (classical kinetic energy expression in particular); the two postulates; classical duration invariance; definition of proper time and time interval dilation through the "light clock" TE; space-time; length contraction; four-interval and its invariance; relativistic momentum; relativistic kinetic energy; brief historical foreword on the discovery of natural radioactivity; phenomenological exploration of mass variation Δm and emitted energy E in some β-decays; empirical law of correlation $E - \Delta m$. Photon energy and momentum; TE on photon absorption. Meaning of mass in SR: equivalence to rest energy. Relativistic total energy: a *conserved* quantity; relation $E^2 - p^2c^2 = m^2c^4$ and definition of invariant mass; deduction of photon mass; mass non-conservation and mass non-additivity.

- T/L strategies: learning by inquiry; visualization. Writing to Learn strategy in science, POE and PEC strategies (RTEI), interactive/dialogic and interactive/authoritative discourse; «collective reasoning» technique.

- Materials and tools: thought experiments (TEs), real experiments (measure of mass by quasi-elastic collisions and measure of *c*), nuclide map, on-line applet.

- Activities: *hands-on* work with the nuclide map and on-line applets; demonstrations (i.e. "*ex-cathedra*" experiments, frequently used); real-time experiments (Sassi & Vicentini, 2008); analysis and structure reproduction of science texts (Halliday & Martin, 1993; Unsworth, 2001; Bazerman, 2009).

III.4.1. **Vertical Perspective**

Vertical perspective is useful to enhance *didactical continuity*, for it allows better facing vertical school transitions by avoiding overlapping, lacks of content, and/or repeated interruptions which frequently occur in school practice. By way of example, Fabri (2005) suggested to state Relativity Principle in its widest form at the beginning of the treatise of mechanics[24] at school (3rd year of high-school) in order to *resume* it two school years later instead of introducing it *ex novo*. He also

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[24] The Relativity Principle was first stated by Galileo, actually not limited to mechanical phenomena according to Fabri: *every type of phenomenon* is observed in Galileo's famous "esperimento del naviglio".

suggested to use an innovative approach to the SR postulates, in which they are not simply enunciated, but also made truthful, by exploiting the average level of technological knowledge of kids nowadays (for example about the existence and properties of electromagnetic waves).

Vertical pespective also includes *contextualized learning*, which exploits *inquiry-based learning* (*IBL) problematic approach*, useful for enhancing the student's personal active involvement in any studied topic. This is a necessary condition for effective learning (Bednar, Cunningham, Duffy & Perry, 1991; Merrill, 1992; Michelini, 2011). A gap was highlighted between the traditional abstract way of teaching science and learners' interests, perspectives and needs. This traditional simplistic teaching approach endowed and still endows science with an «image of authority» (Sjøberg, 2002): it is perceived as too difficult or not relevant for everyday needs. *Simplification* and *concreteness* are therefore necessary in science education.

The challenge of the present work has been to use the vertical approach for building up a t/l pathway which could *adapt the physics content of interest to the cognitive structures and affective needs* of students aged from 16 to 19. Education research interest on relativistic "dynamics" seems to be small indeed, as it will be stated in IV.3 and IV.4, particularly as regards those topics directly involved in deducing and making meaning of mass-energy equivalence.

III.4.2. **Thought Experiments**

It has been repeatedly stated the importance of developing formal theoretical thinking. It may be built and developed by means of *thought experiments* (TEs from now on), of which an extensive use was done in the present work. The usefulness of TEs for scientific inquiry was firstly taken into account by the physicist and science philosopher Thomas Samuel Kuhn in 1964, when he wrote an essay in which he argued that thought experiments teach a scientist «something on his own concepts and the world at the same time». About his/her conceptual apparatus, because they make evident inconsistencies to him/her which were implicit in his/her own previous way of thinking. TEs also increase the knowledge of a scientist about the *world*, although they cannot supply new experimental data, because they let him/her access to information already present (coming from the past real experiments) but again «in the periphery of scientific consciousness» (Kuhn, 1964). The TE allows to put it in the foreground and may generate a reconceptualization analogue to the one occurring during the scientific revolutions (Kuhn, 1970).

Of course, students will not invent a whole TE by themselves. However, they will be able to question their initial conceptions (and perhaps invent new hypotheses too) by "running" an already-known TE, guided by the teacher or by themselves. Three main roles of TEs for physics education were enucleated indeed (Park *et al.*, 2001):

- ***Manifesting existing knowledge***: examining a problem in modelled imaginary experiments allows a better understanding of *background knowledge*, standing «in the periphery of scientific consciousness» (Kuhn, 1964);
- ***Falsifying existing knowledge***: interior contradictions in the experimental background knowledge can be highlighted through a consistent application of it (e. g. Newton's bucket, Schrödinger's cat);
- ***Inventing new knowledge***: new hypotheses can be formulated in order to solve the contradiction above; they are to be compared with actual experimental data (e.g. Galileo's hypothesis about free fall in vacuum). It is also possible to draw *unexpected conclusions* from background knowledge.

According to Gilbert and Reiner (2000), thought experiments are composed by six basic elements:

i. ***Problem or hypothesis.*** For instance: "If the photon is a massless particle, what about its mass when it is a component of a physical system?"

ii. ***Creation of an imaginary world*** of entities linked by mathematical laws;

iii. ***Design*** of the TE;

iv. ***'Running'*** by the experimenter (usually the teacher);

v. ***Production of a new outcome*** with the laws of logic;

vi. ***Conclusions*** drawn from the outcome.

The presence of *Thought Simulations* (TSs), incomplete version of TEs, has been found in some common textbooks (*ibid.*). A TE begins with the statement of a *problem* or a *hypothesis*, goes on with the design and then "running" of the experiment, "observation", and *conclusions taken from results*, according with the pattern above. TSs always begin with the statement of the conclusion, then sometimes melt design, running and/or observation, and eventually claims the results only (sometimes by repeating the initial assertion). The characteristic elements of an experiment are lacking that way, and thus students cannot perform an effective cognitive reconstruction.

III.5. Research Design and Methodology

Research design is different between interpretivism and post-positivism: it frequently evolves or even emerges in progress for the former, while all aspects have to be accurately designed before undertaking the research for the latter. The former tends to utilize qualitative data analysis, while the latter quantitative analysis. Qualitative researchers design case studies, phenomenological studies, and narratives; quantitative researches design experiments (e.g. Ryoo & Linn, 2012), or large-scale surveys (Velayutham *et al.*, 2011) with variable control method. *Pragmatic approach* was taken for following out the present research, in which an epistemological choice is not made *a priori* and the two main SER paradigms, namely post-positivism and interpretivism, are complementarily mixed up for the actual researcher's needs, even if the implicit background is post-positivism according to Treagust, Won, and Duit (2014).

In order to accomplish my research objectives (III.1) I chose the "experiment" as research style (Cohen *et al.,* 2007). *Empirical* research has been undertaken, which mirrored the laboratory one. Of course, it must be kept in mind that empirical research in itself can never prove anything (Popper, 1934). «The results have to be discussed and interpreted, but conclusions are tentative and open to revision» (Fischer *et al.*, 2014).

Three lines of action were followed (see Cohen *et al.,* 2007):

- *Analysis of students' reasoning paths* about the proposed t/l path, by looking at coherence and structure;
- *Comparison of initial and final states under controlled conditions*, by means of pre and post-tests;
- *Establishing statistically significant correlations.*

III.6. Design-based Research and formative intervention modules

Fourteen formative experiments were carried out with very different modalities for collecting data on sample of students spread all over Italy. After the focalization of some crucial points to test, nine classroom and five University interventions were implemented. The sample size was at least of twenty, which is the minimum mandatory number according to Fischer (2013). A tuning phase made by five experiments for setting out strategies and methods evolved into a more stable and structured experimenting phase.

The research project implementation met several *limitations*. The available time (3 years) was enough to set out two t/l pathways, but not for finding samples large enough for a proper study of conceptual change process nor for analysing all available data. Further, quantitative analysis could not be undertaken: most activities took place in ordinary classes, with a too small sample size. Finally, most formative intervention modules lasted few days, so that long-term learning with understanding was not analysed. Six of the fourteen carried-out experiments were carefully analysed in this thesis, selected according to different geographical positions and different cognitive skills.

III.7. Data Analysis Methods

Qualitative data analysis supported by statistical and educational parameter calculation was chosen for the research reported in this thesis. More precisely, *qualitative analysis of content* was used[25] (Krippendorf, 1989; Mayring, 2000, 2004; Zhang & Wildemuth, 2009). It is an evolution of the outdated quantitative content analysis, which aims at analyzing texts or other communicative materials in order to find out *regularities* and to account for data contents in a synthetic way: «Then, during data analysis, the researchers immerse themselves in the data and allow themes to emerge from the data. The purpose of this approach usually is to validate or extend a conceptual framework or theory» (Zhang & Wildemuth, 2009). *Conceptual categories were then induced from data* through a «criterion of selection» (Mayring, 2004). The induced categories did not come out directly from raw data, but they also draw on a basis of both previous

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[25] This analysis method may be considered as mixed-method approach put into practice. Zhang and Wildemuth (2009) assert it is feasible when the researcher works in an interpretative paradigm.

research findings and theoretically-driven research questions *a priori*. This is important, since it allows the findings to go beyond the contextual activity performed. Nevertheless, some categories were unavoidably *a posteriori*. A researcher should carefully avoid to draw either ideology-driven conclusions, i.e. not stemming from data, or conclusions not accounting for data variety.

The categories were *operationally defined*: for each question of the test, a set of quotations from students' answers was individuated and grouped when they were considered to have the same conceptual content. Each set of answers defined a category, in the same ways as operational definition in physics. More specifically, conceptual elements were at first enucleated, then they were recognized inside each answer and combined for building more than three mutually exclusive categories. However, in some cases the final categories were not mutually exclusive, because of the nature of the correspondent question.

To sum up, the categories were outlined *before* each formative experiment on the basis of the RQs and then refined or changed *after* it by reading the replies, according to the tenets of qualitative content analysis. In addition, the *category relative frequencies / percentages* were calculated for eliciting their global discrete distribution (Cohen *et al.*, 2007) over the sample, useful for conceptual understanding evaluation. When possible, pre- and post-distributions *were compared* in order to seek for conceptual change.

Profiles are usually useful for looking at the coherence of the categories over the answers to different questions. At first I based on the literature in which 5 conceptual profiles, indicating the «levels of physical representation» (Gorodetsky, Hoz, & Vinner, 1986; Doménech, Casasús, Doménech, & Buñol, 1993), were mentioned. «Conceptual profile» means here the owned set of (possibly contrasting) parallel conceptions actually used in different domain-specific contexts (Driver *et al.*, 1994), which translates into different *answering styles*. In fact, each student may display a relational conception of mass in some answers and functional or ontological in others. In many cases a conceptual profile will prevail on the others. Later on, I passed to the 3 phenomenografic profiles by Fazio and colleagues (Fazio, Battaglia, & Di Paola, 2013; Pizzolato, Fazio, Mineo, & Adorno, 2014), since they allowed a more precise determination of the real students' profiles, which are never well-shaped. In the first case (levels of representation) I associated one profile to each student, exception for few cases, while in the second case (phenomenografic profiles) I found the percentages of each profile in every student and built a bubble plot showing the incidence of each of the three profiles in the sample. The plot is a triangle whose vertexes represent a profile each; the nearer the bubbles are to a vertex, the more relevant the correspondent profile is. It was built by a simple algorithm.

This «directed content analysis» (Mayring, 2004) allows building up empirical models of the student reasoning paths under certain constraints and those models bring to *context-specific statements* about student learning. Roughly speaking, these statements may be extended to *rules* if a *randomized sample* is taken, i.e. a sample which can be assumed as including «cases with all relevant attributes as in the population» (Mayring, 2007), essentially because no selection was done *a priori*.

The *central limit theorem* is necessary for going further. Simply put, that theorem asserts that whatever the distribution of a large enough number of independent random variables be, their sum will be distributed approximately as a standard Normal variable (Ross, 2008). Applied to scientific measurements, in physics as well as in biology or soft sciences, this entails that the limit distribution of the measured values of a single quantity will be a Normal distribution, if the measure is affected by many small equally-important random errors (Taylor, 2000). This means for our purposes that *the size of the randomized sample has to be greater than a "high enough" threshold* (Cohen *et al.,* 2007; Ross, 2008) and that *each measured value* (e.g. the score of each student) *has to be considered as affected by the sum of a lot of small random errors*. Inferences might be drawn on the entire population under these conditions: the sample would be statistically representative[26] of the universe under study, which are the Italian students attending from 3rd to last year of Liceo specializing in scientific studies, Liceo specializing in classical studies, and technical college in this case.

Unfortunately, ***our samples were too small-sized for a serious statistical analysis***. So *I limited myself to calculate some statistical* (frequency of the categories, correlation coefficient, phi-coefficient; Cohen *et al.*, 2007) and *educational* (students' score, learning gain) *parameters* (Aslanides & Savage, 2013) which could provide hints for corroborating and analysing the qualitative analysis results.

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[26] Actually, another hypothesis has to be introduced for representativeness: the population distribution is Normal too. It is a very likely assumption, which is regularly made in social sciences.

Finally, a better parameter than sample size for significance is *effect size*: it allows discriminating whether the research results are significant because of a calculated statistical parameter, for instance correlation coefficient, or because of the sample size itself (Cohen *et al.*, 2007).

Educational Reconstruction of Subject Matter Content

The physics content of interest for the present research is expounded in this chapter. It was selected so that it could be 'elementarized' and then used for building up the physics content *for instruction*, by joining the extracted conceptual nuclei to form a conceptual path which makes sense for students at the target level of schooling, according to the M.E.R. (Duit, 2005, 2006; Duit *et al.*, 2012). The result of the next step – namely the « content structure for instruction » – will be extensively displayed by the conceptual pathway itself in the VI chapter. I shall *recall* and *highlight* the intended physics content for the present domain-specific research – taken out from the current consensus knowledge – in the present chapter. The classical facets of mass were deepened as well as the conceptual evolution of inertial mass to modern physics. More specifically, quantity of matter (Newton), inertial mass in Newton and Mach, gravitational mass (Newton), and mass in Special Relativity were picked up. Further modern physics contents – meaningful for learning with understanding mass in its widest meaning – were not developed: (i) mass in General Relativity, and (ii) the Higgs mechanism. The inertial properties of a composite system deriving from dynamical interactions (iii) were considered by the "photon in a box" TE, and Weak Equivalence Principle (WEP) was finally given a little try. The work is thus suitable to be completed.

IV.1. **Space and Time**

'Space' and 'time' are words often used in common as well as philosophical language with diverse meanings and they might be examined under many respects. They constitute the background for any physical phenomenon and passed historically from being the well-known Kantian *a priori* categories for shaping the external reality to the geometrized and therefore idealized, although measurable at the same time, framework in which everything occur, according to Classical Physics as well as Special Relativity (SR). They are primitive concepts in physics and therefore undefinable in their widest meaning. Nevertheless, they are *operationally* identified by the group of operations performed for measuring *time* *and space intervals*. This is a definition entirely different from that one exploiting the concepts' *properties*, as in Newton's *Principia Mathematica* (1687).

Two ideas are necessary for characterizing a geometrized space (Motz & Weaver, 1991): *distance* (which accounts for the intuitive concept of "extension") and *direction* (which accounts for "orientation"). Space interval, i.e. the distance between two points in space, is measurable and therefore operationally definable. Namely, it is the length of the segment connecting the two points along a *straight line*, and it is invariant in Euclidean geometry and consequently in Newtonian mechanics. Moreover, modern physics established an *indetermination* on the simultaneous knowledge of length and linear momentum of the same physical system, owing to the Heisenberg's uncertainty relation $\Delta x\ \Delta p \geq \hbar/2$. Here $\Delta x, \Delta p$ are the intrinsic uncertainties on the respective values of the quantities: their product is not allowed to be less than $\hbar/2$. So, while in classical physics the uncertainty product of simultaneous measurements of x, p could be in line of principle reduced without limit, here the impossibility of going below $\hbar/2$ is intrinsic to the nature of physical systems[27] (Krane, 1987). On the other hand, if De Broglie equation $p = h/\lambda$ and the relativistic relationship $E^2 = p^2c^2 + m^2c^4$ are combined, it turns out that higher and higher energies are needed for more and more precise determinations of distance. The lower limit set by Heisenberg's principle is Planck length: $l_p \sim 10^{-35}$ m, under which the concept itself of distance has no meaning.

Direction is also necessary, because of the three-dimensional nature of space: three straight lines mutually orthogonal are required in order to represent every straight line in physical space as a linear combination of independent components. Thus vector quantities are needed for accounting motion in space, next to scalar ones. The concept of angle is eventually needed as a measure of the rotation for changing direction, in order to define the latter operationally (Feynman *et al.*, 1964; Motz & Weaver, 1991).

Time interval is operationally defined as the physical quantity measured by a *periodic* phenomenon, namely the clock, whose period supplies the time unit value, whatever its internal mechanism is. For everyday phenomena the pendulum may be a good clock, its period being of the order of 1 s. Newton believed that «true» absolute time was *duration*, not achievable by clocks utilized in practice –

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[27] The classical limit $\Delta x\ \Delta p \geq 0$ for Heisenberg's principle may be obtained by taking $h \rightarrow 0$.

pendulums *in primis* – which measured the relative (apparent) time instead. He believed apparent time was only a «sensitive and external (accurate or approximate) measure of duration by means of motion» (1687). However, measurements of smaller time are needed nowadays; to that end *electronic oscillators* may be used, in which the periodic phenomenon is the oscillation of electrical current visualized by an oscilloscope. Electronic oscillation periods are about 10^{-12} s. More accurate time measures are allowed by *atomic clocks*, as the periods of atomic vibrations are approximately 10^{-15} s. Their time measurement is very useful because of the *independence of external physical conditions and constraints*: their working depends exclusively on the photon absorption frequency of the atoms (typically ^{133}Cs or ^{57}Fe). Time resolution has been further increased up to 10^{-24} s, which is both the lifetime of "resonances"[28] both the time taken by light to cross the proton diameter (Feynman *et al.*, 1964).

Eventually, the *time in which a substantial physical state change occur in any physical system* is related to the *uncertainty in the energy of that state* by $\Delta E\ \Delta t \geq \hbar/2$ in non-relativistic quantum mechanics[29]. Planck time is currently thought as the smallest existing time interval, given by the time taken by light to cross Planck length: $t_p \sim 10^{-43}$ s.

Once a set of clocks has been defined, they have to be *synchronized* in order to provide the same measurements in different locations. In classical physics this is a trivial operation: all the clocks to synchronize are brought close to each other and their inner mechanisms are set to beat all at the same rate. Then they are brought back to their original position. Clocks at *relative rest* are synchronized that way. Every clock will also beat exactly the same time of any other clock *in relative uniform rectilinear motion*, according to classical mechanics. In the popularized (and common sense) conception, there is a universal absolute time. Nonetheless, what is actually done for measuring a duration (time interval) is observing and counting *simultaneities*, which is different than comparing the elapsed time interval with the unit time, like in distance measurements. The very idea of temporal succession comes ultimately from simultaneity.

Relativity of simultaneity is the basis for the Einstein's conceptual reformulation of time in physics. Interestingly, it does not invalidate the causality

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[28] A class of unstable extremely short-lived particles not directly observable.

[29] The energy-time Heisenberg's principle has the same mathematical form of the momentum-length uncertainty principle, but its theoretical origin is quite different (Griffiths, 1995).

principle. Time in SR is somehow similar to spatial coordinates, even if it is never on the same ground of them: space-time is a unity of two different things that does not imply they are indistinguishable or merged, although deeply related[30] (Taylor & Wheeler, 1965; D'Inverno, 1992; Bergia & Franco, 2001; Ferraro, 2007).

For *synchronizing clocks* an electromagnetic wave is used in Relativity, since it is the only wave whose propagation speed is invariant and therefore objective, assuring a scientific observer-free procedure (please refer to the next paragraph for further clarification). n clocks are placed aligned and equally spaced, with a distance d between them: a light signal is emitted at the instant $t = 0$, at the location of the first clock. The instant at which the wave reaches the nth clock must then be

$$t = (n-1)\, d \,/\, c \ \ (n = 2, 3 \ldots).$$

Once n clocks at relative rest have been synchronized, it may be examined what happens when one clock *moves steadily* with respect to another. It turns out in SR that time intervals between the same couple of events vary with the inertial frame, as it will be seen in IV.5.1.

IV.2. **Inertial frames**

An *event* is defined as a physical fact happening in a small region of space and for a brief time interval. « Mathematically, we idealize this concept to become a point in space and an instant in time. Everything that happens in the universe is an event or collection of events » (D'Inverno, 1992).

A Cartesian *coordinate system*, in which the geometry of space is Euclidean[31], has to be considered to assign a unique spatial position to each event.

It must be underlined that a *reference frame* does not coincide with a coordinate system, i.e. a tern of orthogonal Cartesian axes. It should be rather thought as a *real physical space endlessly extended, in which any physical quantity may be actually measured in any point and at any instant*, in contrast with the geometrized abstract space introduced in the previous paragraph. A reference frame includes one or more coordinate systems at relative rest.

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[30] The reader will be shown that the same holds for momentum and energy.

[31] The only geometry known in Newton's time.

However, a frame of reference has strictly a kinematical meaning, because only relative motions are described, without referring to masses or forces. A dynamical account of motion implies differentiating *inertial reference frames* (IFs) from *non-inertial frames* instead, by defining as inertial those in which Newton's laws are valid[32]. A timescale has to be associated to every frame in order to do this. The division into inertial and non-inertial frames is ultimately due to the state of rest or uniform motion of the former with respect to the *absolute space* in Newtonian physics (Ferraro, 2007).

The first law of Newtonian dynamics, or Principle of Inertia, is *problematic* in its original wording (1687):

> Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.

In fact, how may one perform an experiment to test this claim, if the described situation is not real? It is not possible indeed to remove the action on a body of any other object completely. In the first law a commonly used model (free body) is considered, but it neither is real nor can it be considered as a thought experiment. In fact, a point mass[33] satisfying the conditions of the Principle of Inertia ought to be completely alone in the Universe, while it would necessarily interact with an observer in a thought experiment. In our Universe a completely isolated object does not exist.

In addition, the physical situation depicted in the first law of dynamics (Galileo's principle of inertia) is *not* a special case of the second law, as it is sometimes believed and written in textbooks. In fact, in the principle of inertia a *tendency to persevere in a "natural" state* of rectilinear uniform motion (or rest if $v = 0$) is considered when no forces act on the body, this tendency being just called 'inertia' in a strict sense. In the second law, *the resistance to change the body's motion state* is considered when the resultant of the external forces acting upon the body is null ($\sum \vec{F} = 0$). This resistance, expressed by mass, is not *a priori* coincident with 'inertia' in the sense of the first law (Coelho, 2007). The

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[32] Unfortunately, a circularity problem is generated by this outdated definition: if an inertial frame is *defined* as a frame in which the center of mass of a physical system always moves at uniform velocity, with respect to what will it be moving uniformly? To the inertial frame itself.

[33] "Point mass" or "mass-point" is the well-known model in which an object has finite mass but negligible volume and thus is imagined as without internal structure. The term "particle" – more appropriate in Special Relativity – will be interchangeably used too.

conceptual identity between the two ideas[34] is implicitly assumed, but it should be tested by an experiment, which is easily workable in the second case but never in the first one. Therefore

1) Newton's first law is conceptually separated from the second;
2) 'Inertia' in its original sense is not necessarily related to mass, unless that implicit assumption is made;
3) Mass comes into play only in the second law.

The way out has been to consider the first law as *an assertion which postulates the «motion of reference» in Newtonian theory*, i.e. uniform rectilinear motion. The second law indicates instead how to quantify the degree of *deviation* from that kind of motion (Coelho, 2007). Eventually, the third law states that Newtonian forces are simultaneously exerted *on* and *by* each body/point mass belonging to a system, thus highlighting the interactive nature of Newtonian forces[35]. In other terms, the first law has become the assumption of the existence of at least one inertial reference frame *a priori* (DiSalle, 2009). Infinite IFs have to exist then: all those moving with rectilinear uniform motion with respect to it.

A typical example of IF is the real or imaginary *laboratory* ('LabIF' from now on) in which these measures are performed. A particular attention will be paid to the measurements of time and space intervals (durations and lengths respectively). Moreover, in *each* IF the measures may be thought to be performed by infinite[36] 'observers' endowed with clocks and rules. Alternatively, an IF may be thought an infinitely extended network of clocks connected by rules (Taylor & Wheeler, 1965). Therefore it should be reminded that different observers do not necessarily imply different IF: this occurs *only when the observers are in relative motion*. It has been found instead that «In effect, the students treat observers at rest relative to one another as being in different reference frames» (Scherr, Shaffer, & Vokos 2002). For deepening this point also refer to another similar paper by Scherr, Shaffer and Vokos (2001). The observers will determine the values of all mechanical quantities by drawing on distance and duration measurements, if they also know the mass of the physical system under study. Observers associated each one to a *different IF* are being considered now. Measure each observer a precise

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[34] Or at least, the fact that this resistance stems from the inherent tendency to persevere in uniform rectilinear motion.

[35] By way of counterexample, fictitious forces do not give raise to any reaction.

[36] The ideal observers need to be infinite ($\aleph_1$) because they are in one to one correspondence with the points of the 3D space.

quantity and write his/her result on a notebook, observers will name ‘invariant’ every quantity which has the same value in all their notebooks. *An invariant physical quantity is therefore a quantity whose operational measure is the same for every inertial frame* (Fabri, 2005). That value may be thought objective or, going a little further, *real*, in a way.

Eventually, *simultaneity is invariant* in classical mechanics: any instant is equal in every couple of IFs. So when an event A occurs at the same instant of another event B in one inertial frame, i.e. $t_A = t_B$, then $t'_A = t'_B = t''_A = t''_B$ and so on: if an event occurs at a specific instant, this is valid for every classical IF. The instant of this event is thus *objectively* determined. This is expressed by the second equation in the first system below.

A particle moving at instantaneous speed *u’* in the frame *K* is considered, *K* being in uniform rectilinear motion with respect to LabIF at velocity *v* with the same direction of *u’* (figure IV.1). The formula for the speed *u* measured in LabIF may be easily derived through Galilean transformation of coordinates.

$$\begin{cases} x = x' + vt \\ t = t' \end{cases} \Rightarrow \begin{cases} \Delta x = \Delta x' + v\Delta t \\ \Delta t = \Delta t' \end{cases} \Rightarrow \frac{\Delta x}{\Delta t} = \frac{\Delta x'}{\Delta t'} + v$$

If the limit $\Delta t \to 0$ is taken, the last equation will turn into the special case of Galilean transformation of velocities describing this physical situation. It should be noticed that *time interval invariance* ($\Delta t = \Delta t'$), which stems from *simultaneity invariance* ($t = t'$), is a sufficient condition for the scalar classical addition law $u = u' + v$.

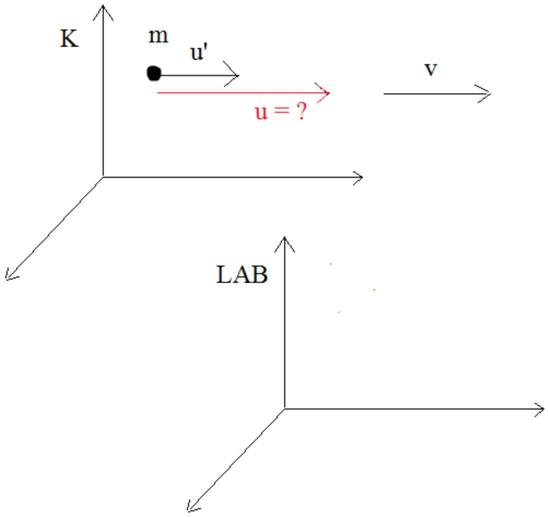

Figure IV.1. Relative speeds in classical mechanics.

IV.3. Mass in Classical Physics

Next to space and time, another fundamental ingredient is needed for building up all mechanical quantities: *mass*, a multifaceted quantity. In everyday knowledge it is usually identified with the amount of matter, considered in turn as a primitive concept. Noteworthy, it is also its first historical definition in the proper scientific sense.

At the same time, mass is a quantitative measure of inertia, the opposition exerted by a physical system to motion state (velocity) variation. Inertial mass is difficult to define because it is related to acceleration and force by Newton's second law only. Nevertheless, it is possible to perform a prior kinematical measure of acceleration based on experimental observation, without Newton's laws. Then there are two alternatives:

i. Determine mass relative to a sample mass by either relative acceleration or final speed ratio (exploiting linear momentum conservation) and eventually define force by $F = ma$,
ii. Determine force by examining the dynamics of a phenomenon *different* from the one for acceleration and then define mass operationally by $m_i = F/a$.

The third facet involves the mutual macroscopic[37] action of each massive body on every other massive one, namely *gravitational* interaction. Inertia actually stems from the interactions of each body with all the others in the Universe (*Mach's principle*) and this intimate relation between inertia and gravitation is well stated by WEP at the formal level: $m_i = m_g$ for all bodies.

IV.3.1. *Quantitas materiae*

Isaac Newton is to be considered the inventor of the modern concept of mass (Jammer, 1961, 2000; Cohen, 1981; Okun, 1989 – 2010). Mass is first of all «*quantitas materiae*» in Newtonian physics, as defined in the opening paragraph of *Philosophiae Naturalis Principia Mathematica* (1687):

> The quantity of matter is the measurement of the latter obtained from the product of density and volume. […] Air of double density, in a double space, is quadruple

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[37] Gravitational interaction is negligible at the microscopic level with respect to the other ones.

> in quantity […] And the *norma* of all bodies, which be differently condensed for any cause, is identical […]. I will mean hereafter, everywhere, this quantity under the name of body or mass.

This «pre-theoretical» ontological definition (Doménech *et al.*, 1993) rests upon the idea that in all bodies *something* is present which is always measurable in terms of a not uniquely defined concept, *matter*, whose meaning is thought 'primitive' and therefore uncritically accepted, although a primitive concept is necessarily undefined at the same time[38]. Moreover, the quantitative definition contains a vicious circle, because mass is indirectly measured through density (primitive quantity) and volume, but it does not specify *how to define and measure density independently of mass*. One equation is trivially not enough for two variables, assuming volume has been measured on the side. Relative densities might be considered, but there is a problem anyhow: the density ratios are defined by Archimede's law, which is ultimately based on the assumption $m = \rho V$ as well. In fact, for measuring the density ρ_x of an object it is possible to measure 'directly' its volume V_x, plunge it into a fluid (typically water), wait for static equilibrium, measure the plunged volume V_w and apply the well-known equation

$$\rho_x V_x\, g = \rho_w V_w\, g \;\Rightarrow\; \rho_x = (V_w/V_x)\rho_w$$

where ρ_w is the density of water.

The vicious circle is not actually removed, since this argument rests upon the conventional definition of weight $W = m\,g = \rho\, Vg$.

The physicist and philosopher of science Ernst Mach pointed out that criticality in his influential book *The Science of Mechanics in Its Historical-Critical Development* (1883).

> As for the concept of mass, we observe that Newton's formulation is unfortunate. He says that mass is the *quantity of matter* in a body measured by the product of its bulk by density. The vicious circle is prominent. In fact density cannot be defined but as mass per unit volume. Newton acknowledged that a quantitative property determining motion is inherent in all bodies and it is different from weight […] but he did not succeed in stating this knowledge correctly.

Newton invented *inertial mass* as well, but he never sharply differentiated it from *quantitas materiae* (e.g. Okun, 1989, 2005).

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[38] Just like space and time (compare III.1).

IV.3.2. **Inertial mass**

Inertial mass may be defined as the measure of the property of a body/system/mass-point enabling it to resist to a velocity variation when mechanical work is done on it. It has been considered as a quantity *intrinsic* to a body/system/mass-point in the present work, even if it gains a 'relational' meaning when it changes into gravitational mass. It is intimately related to force or momentum variation rate in the fundamental equations of classical or relativistic dynamics respectively.

Hereinafter the classical case only will be considered, in which it is not possible to define dynamical mass and force at once with a sole law, namely $\vec{F} = m\vec{a}$, just like static mass and density in III.3.1. Likewise, an *independent definition* either of force or of acceleration is needed to avoid considering force as a primitive quantity and consequently defining «an *ignotum per ignotius*» (Jammer, 2000).

The well-known difficulties in force and mass definitions have been recently recognized to stem from the *circularity in the first law of dynamics* (Coelho, 2012a). Since the principle of inertia presupposes acceleration, but the inverse is not true, it is possible and convenient to *define acceleration independently of Newton's laws,* drawing on experimental observations. Acceleration may thus be the first in the sequence of the three concepts involved in the fundamental laws of dynamics. Once it has been operationally defined, two ways are feasible:

I) To define mass operationally by means of accelerations or velocity variations (Mach, 1883; Weyl, 1927) and then find force through $\vec{F} = m\vec{a}$, which would become a definition of *force* (acceleration-mass-force chain);

II) To define force independently of acceleration at first, through an «extra phenomenon» (Coelho, 2012a), and then to find mass through $m_i = F/a$. So Newton's second law would turn out to be a definition of *mass* (acceleration-force-mass chain). Unfortunately, to define dynamic forces in such a way is not possible, unless to make some special assumptions, as Bridgman (1927) and Arons (1965, 1992) did.

The mentioned approaches will be described in the following.

1) *Bridgman's solution* (1927) has been to take into consideration the *static* force measured by elastic deformation and to extend it to *dynamic* systems. The concept of mass in disentangled by the one of force by this approach. More

precisely, a spring with a hanging weight in an isolated laboratory without gravitational field (empty space) is considered as example. Hooke's law $\vec{W} = -k\ \Delta\vec{l}$ relates the spring elongation to weight in static conditions. In a situation of "static motion" – e.g. a body launched by a compressed spring – the static force exerted by the released spring is expressed by Hooke's law. *It is assumed that this force be the same appearing in the II law of dynamics*: a *dynamic* force. In equations

$$k\ \Delta l = F_{static} = F_{dyn} = ma\ \Rightarrow \boxed{m = \frac{k\ \Delta l}{a}}$$

Thus force may be operationally defined by directly measuring the elongation of a dynamometer – for instance – or more generally the deformation of an elastic body. So the specific situation *of setting in motion* (in which $F = ma$ is strictly valid) is first considered, then the definition of inertial mass is generalized by induction. Finally, it is reminded that the equation above is valid for the spring scales too, thus a correction would be needed.

In Bridgman's words (1927):

> Suffice it to say that we are eventually able to give to each rigid material body a numerical tag characteristic of the body such that the product of this number and the acceleration it receives under the action of any given force applied to it by a spring balance is numerically equal to the force, the force being defined, except for a correction, in terms of the deformation of the balance, exactly as it was in the static case.

2) Another prominent operative definition of mass, used in few textbooks nowadays, was elaborated *by Mach* (1883). He proposed a definition based on measures of accelerations exploiting a meaningful result (below) derived from Newton's second and third laws of mechanics, in which he put m_A as mass unit.

$$\frac{m_B}{m_A} = -\frac{a_A}{a_B}$$

The minus sign is owing to the opposite directions of the accelerations. In order to better understand Mach's conception the case of *identical bodies* is considered at first, in which equal and opposite accelerations are expected to be mutually transmitted in central collisions, because of a symmetry principle. As for different bodies, it is assumed that if the mutually transmitted accelerations of A and B are $+a_A$ e $-a_B$ respectively, the mass of B will be

$-a_A/a_B$ times the mass of A. If the latter is taken as mass unit it will be thus asserted that the *mass of a generic body be equal to the opposite of the acceleration impressed to the unit mass body divided by the acceleration received by it in a central collision.* This «concept of mass does not stem from any theory. It entails only the precise assessment, designation and definition of a fact» (Mach, 1883).

3) An equivalent definition is due to Hermann Weyl (1927), who considers a perfectly inelastic collision between two objects in which the final product is at rest (Jammer, 2000). Momentum conservation is written, for a perfectly inelastic collision (owing to mass conservation law),

$$m_A u_A + m_B u_B = (m_A + m_B) u_{A+B}.$$

The Weyl's «metrical measurement» of mass is obtained by putting $u_{A+B} = 0$: $m_B/m_A = -u_A/u_B$. That is nothing but Mach's equation with a null final speed for each object.

4) Eventually, a Newtonian operational definition of mass without logic circularities and not limited to static cases is actually possible (Arons, 1992), but it is not *general*. It has to be induced indeed from a necessarily small number of cases, i.e. the ones feasible in a terrestrial laboratory. Moreover, the other two objections raised against Mach's solution – that an instantaneous measure of acceleration cannot be operationally performed and that Mach's definition holds only for a dynamically isolated couple of objects[39] – are valid for this case too. Eventually, a further prominent Jammer's (1961, 2000) objection to Mach's approach is that it brings to *different results in reciprocally accelerated reference systems*, because the relative accelerations add up. Nonetheless, the definition by Mach is good for inertial reference frames, and this is enough for the present purposes.

The mentioned procedure (Arons, 1992) consists of the following steps:

a) a *scale of forces* is defined by measuring the accelerations in m/s^2 undergone by a test body connected with a spring, setting the force values on

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[39] Jammer (1961) pointed out that two-body systems gravitationally independent (or better, negligibly gravitationally interacting) are very difficult to find in nature. Perhaps a two-star system might be an example, but they are very far away from the Earth and their motion is difficult to study. Thus what is the usefulness of such an operative definition, if it cannot be put into practice?

the dynamometer as numerically equal to acceleration for that body ($F = a$), so that the hypothesis of spring linear behavior[40] is not necessary;

b) Once the dynamometer has been calibrated, another object is taken and accelerations are measured in correspondence of the *same force values* as before; of course, different numbers will be found;

c) If the procedure is iterated, a plot of force (in arbitrary units) against acceleration (in m/s^2) will show a set of straight lines passing through the origin; the test body will correspond to the line at 45°.

d) Therefore an experimental law will be discovered and the lines' *slopes* will be numbers inherently associated to each body; they may be called *inertial masses* of those bodies.

There is an overall issue to solve too: acceleration is *caused by* a force in classical dynamics, thus it ought to conceptually presuppose force, even though implicitly. This is in sharp contrast with all of the expounded approaches. The assertion is true, however, exclusively within the conceptual framework based on the law of inertia (Coelho, 2012b). It seems therefore necessary to go beyond ("outside" from) that framework for an effective educational reconstruction.

To summarize, *the science content was reconstructed in the following way for the t/l sequence, according to Mach's approach (1883):* (i) *velocity variation ratio in an elastic collision, which is also acceleration ratio, was calculated and* (ii) *mass was found in units of a sample mass. Force* (iii) could have been also found, but it is unnecessary for our purposes. So *the chain acceleration-mass-(force) was chosen, but without considering* $\vec{F} = m\vec{a}$ *as definition of force.*

Inertial mass invariance is *assumed* in Newtonian mechanics, based on the macroscopic experimental results at low speeds just described (collisions typically). In fact, *it turns out from those experiments that* m/m_s *is independent of the initial uniform speed of the body*, m_s being the sample mass.

A final objection may be raised, however: every operational definition entails macroscopic medium-sized objects to be taken. What about the mass of the Sun or an atom or an elementary particle? A more comprehensive definition will be given by SR, but for now the solution is to consider *theoretical definitions*, because they are based on generalized laws (Jammer, 2000).

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[40] Namely, the hypothesis of linear proportionality between exerted force and elongation of an elastic body (Bridgman, 1927) is *unnecessary*. In my opinion the hypothesis is *implicitly done* in PSSC instead (1995).

IV.3.3. **Gravitational mass and WEP**

Inertial mass takes on a new significance within the Newton's *universal gravitation law*, whose historical relevance was stated, among other scholars, by Cohen (1981). He pointed out Newton's discovery was the best of the Scientific Revolution, because the scientist incorporated the major physical phenomena occurring in the observable Universe *into a single law*. So celestial physics and physics on Earth must be the same. Three ancient problems were solved at once: (i) the physical meaning of Kepler's laws; (ii) the origin of tides; (iii) the independence of free-fall motion of bodies' weight. Cohen (1981) argued that «Newton had achieved Kepler's goal of developing a physics based on causes».

In brief[41], *inertial masses take on a gravitational meaning*, namely a specific meaning with reference to a universal *reciprocal action*, beyond their inherent inertial proprieties. Whilst inertial mass «determines the inertial behavior of particles or bodies», gravitational mass «determines the gravitational behavior of matter» (Jammer, 2000).

Here it is worth mentioning *Mach's principle*, which stated essentially that the inertia of a body[42] stems from the masses of all other bodies in the Universe: inertia is asserted to have a gravitational origin, for it depends of the configuration and entity of all masses. The formulation of this principle is due to Einstein, who attempted to synthesize in it both the type of research carried out by Mach, aimed at establishing the interdependence of phenomena, both Mach's view of scientific knowledge as the simplest description of relations between elements.

The experimental fact that $m_i = m_g$ is valid for all bodies, independently by their internal structure and chemical composition, was elevated to the epistemological *status* of a principle by Einstein. It was called "Weak Equivalence Principle" (WEP). The acknowledgment of this fact paved the way historically to General Relativity. The II law of dynamics for a free-falling body in the Earth's gravitational field[43] may be written indeed

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[41] A detailed exposition of the physical content referred to here is in VI.2.1. The subject matter is not reported here because it concerns how gravitational mass concept is re-constructed throughout the path, differently from inertial mass and *quantitas materiae*, the former and the latter being respectively almost entirely and partially considered as prerequisites.

[42] Inertia is deeply related to mass both in classical mechanics both in SR.

[43] Free fall occurs close enough to the Earth's surface to consider g as a constant at the intended level of approximation.

$$m_i a = m_g g \leftrightarrow a = \frac{m_g}{m_i} g$$

Therefore (i) m_g/m_i is constant for all bodies *if and only if* (ii) all of them fall with the same acceleration. The second experimental fact was acknowledged by Galileo (Okun, 2005), while the first by Newton, who observed that inertial mass «is revealed by the weight of each body. Through very accurate experiments on pendulums I found that it is proportional to weight, as I will later show» (1687).

The period of a simple pendulum (approximated to a harmonic oscillator for small oscillation angles) is given by

$$T = 2\pi \sqrt{\frac{m_i}{m_g} \frac{l}{g}}$$

Two pendulums-balls having different structure, size and shape and made by different chemical substances may be used: if the mass ratio is dependent on one of these characteristics of the body, different periods will be found for the different balls. Newton measured the period ratio between pendulums loaded with gold and lead, observing that they oscillated «together forwards and backwards, for a long time, with equal vibrations» (1687). The period ratio was judged to be equal to 1, with an accuracy of 0.1%. After that, the experiment was performed with silver, glass, sand, common salt and wheat, but the results did not change. Newton then argued the proportionality between inertial and gravitational mass.

The same units are worth using for the two quantities, because they have the same physical dimensions. Thus one is finally left with the equality

$$m_i = m_g \ .$$

There is actually only *one* mass in physics (Okun, 1989 – 2010; Doménech *et al.,* 1993; Will, 2006; Bergia, 2009). The most recent experimental tests of WEP are well summarized by Will (2006).

IV.4. **Energy and Work**

Unlike mass, *energy* varies according to the IF in which it is measured. It is impossible to define energy in an operational way, but the concept is *founding* for all of physics and science, because of its conservativeness.

In fact, energy is an abstract physical quantity characterized by *conservation* throughout every transformation, according to the approach by Bridgman (1927), Feynman, Leighton, and Sands (1964), and Arons (1989, 1992), amongst others. Scientists trust in energy conservation to such a degree to consider it as a *principle*, i.e. something which is assumed *before* formulating theoretical and empirical laws, and to be held when experiments invalidate some of the former. This happened historically in the study of β-decay and the consequent hypothesis of neutrino by Fermi for keeping energy and momentum conservation.

At the same time, energy manifests itself in diverse ways, which vary over time, i.e. *energy changes over time at the phenomenological level.* Its *conservativeness* in natural phenomena derives ultimately from being a constant in equations of motion. Namely, some *constant functions* depending only on initial conditions will be necessarily obtained if equations of motion are deduced. These constant functions are energy, linear momentum and angular momentum for mechanical systems (Bridgman, 1927).

It is worth mentioning here the two major different ways for changing physical systems' internal energy: *heat* and *work*. The former occurs at the microscopic level, the latter at the macroscopic level. Unlike what is frequently stated, heat is *not* an energy form, but rather a quantity of ancient origin representing the *energy exchange between the particles of a system and the surroundings* in the kinetic (mechanistic) model (Atkins 1984; Arons, 1989; Tarsitani & Vicentini, 1991; Arons, 1992; Alonso & Finn 1995). This mechanical energy of the particles is interpreted by the macroscopic *temperature* of the whole system. When there is a difference between the mechanical energies of the particles belonging to different systems an energy exchange occur (Loria & Michelini, 1976). Arons (1999) proposes this approach too, but he associates «transfer» to heat, thus considering energy as an entity travelling in space on a carrier rather than associated to a precise system under determinate conditions. *Work* in its widest meaning may be thought instead as a way for macroscopically restructuring physical systems by means of an interaction. A way for teaching (i) work (PSSC,

1995) and (ii) its relationship with kinetic energy in classical physics (work-energy theorem) is being afforded.

Consider a body modelled by a mass-point moving through uniform and constant force fields. Each force ($\vec{F}$) application produces a shift $\Delta\vec{x}$ of that mass-point and work is generally defined by the scalar product $L = \vec{F} \cdot \Delta\vec{x}$. If $\vec{F}$ and $\Delta\vec{x}$ are parallel, the mechanical work done by that force on the body is simply given by $F\ \Delta x$. This is the only case of interest here. Work value is numerically equal to the mass-point *kinetic energy* variation, i.e. the variation of the energy form expressing the *motion state of the mass-point.* This is work-energy theorem in a qualitative form. Now formalization may be introduced.

If the second law of classical dynamics $F = ma = m\,\Delta v/\Delta t$ is inserted in the definition of work, the work-energy theorem mathematical expression will follow, which is valid *exclusively for a mass-point*, since the resultant work is obtained by calculations over the boundary of the physical system (Bridgman, 1941, p. 30; Arons, 1992).

$$L = F\Delta x = m\,\Delta v\,\frac{\Delta x}{\Delta t} = m\left(v_f - v_i\right) v_m \Rightarrow$$

$$L = m\left(v_f - v_i\right)\frac{\left(v_f + v_i\right)}{2} = \frac{1}{2}m\left(v^2{}_f - v^2{}_i\right)$$

$$L = \frac{1}{2}mv^2{}_f - \frac{1}{2}mv^2{}_i = \Delta K$$

In fact, displacement divided by elapsed time is the definition of average speed, which can also be written as the average of the initial and final speeds in the case of constant acceleration. This result can be also derived at a more advanced level by means of differential calculus:

$$F = ma = m\frac{dv}{dx}\frac{dx}{dt} = mv\frac{dv}{dx}$$

$$L = \int F\,dx = \int mv\,dx\frac{dv}{dx} = m\int v\,dv$$

$$L = m\int_{v_i}^{v_f} v\,dv = \frac{1}{2}mv_f{}^2 - \frac{1}{2}mv_i{}^2 = \mathit{\Delta}K$$

However, if an *extensive body* or, equivalently, *a particle system* is considered (real case), work-energy theorem will not be strictly valid (Arons, 1992). The following equations will hold instead:

1. $(\sum F_{ext})\Delta s_{CM} = \Delta\left(\frac{1}{2}\,M{v_{CM}}^2\right) = \Delta K_{CM}$. This one comes out from the second cardinal equation of dynamics $\sum F_{ext} = Ma_{CM}$, on whose left member is the resultant external[44] force, while on the right member the total mass and the centre-of-mass acceleration are. The left member of the first equation *is not work*, because it is not referred to the application point of the external resultant (the "boundary" to which the sum of force has to be applied, see above).
2. $W = (\sum F_{ext})\Delta s = \Delta K_{CM} + \Delta U + \Delta E_{int}$, Δs being the shift of the application point. This means that the *work* done by the external resultant is numerically equal to the algebraic sum of different energy form variations. It comes from total energy conservation, but it is not work-energy theorem.

Now focus will be shifted to a particular potential energy: the electrostatic one. Be a charged test particle[45] immersed in an electrostatic field. Since the field is conservative, it is possible to define a physical quantity $U(r)$, function of the spatial position, such that the work for bringing the test charge[46] from a starting point A to an arrival point B be equal to the difference between the values of the function in A and B, regardless of the path: $L_{AB} = U_e(A) - U_e(B) = -\Delta U_e$. Exploiting work-energy theorem one obtains the well-known result

$$\Delta K = -\Delta U_e.$$

Therefore kinetic energy increases when the particle pass from a state of electrical potential energy U_A in A to another state of potential energy $U_B < U_A$ in B: the particle is accelerated. On the contrary, when $U_B > U_A$ kinetic energy decreases: the particle is slowed down. The analogy with the case of gravitational field is full. Kinetic energy depends counterintuitively on charge but not on mass in this context, which makes the electron-volt very practical in calculations involving particles (Halliday, Resnick & Walker, 1995).

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[44] The resultant internal force is of course null, since it is given by a sum of action-reaction couples.

[45] A particle whose charge q_0 be small enough to interact negligibly with the surrounding electrical field at the intended level of approximation.

[46] The term "charge" is used for "charged particle" here. This substitution is widely used in technical language, just like "mass" for "massive particle". However, from the educational standpoint it is not advisable to exchange an object for one of its properties (Arons, 1992).

If the electrostatic *potential* $V \equiv U/q$ is introduced it will be possible to deal with potential energy for unit charge and the equation above will change into $\Delta K = -q_0 \Delta V$. This result is utilized to increase the kinetic energy of a charged particle beam in *electrostatic accelerators*. A constant potential difference is kept, so that by crossing it the particles reach very high speeds because of a strong electric field. In the Van de Graff electrostatic accelerator a potential difference ranging from 0.1 to 10 MV is reached (Giacomelli, 2002), thus accelerating charged particles potentially up to 10 MeV.

IV.5. Relativity: Basics and Mass

The rationale of the t/l pathway will serve as a guideline for the theoretical topics dealt with in this paragraph. The reader is therefore sent to VI.3 for deepening each of them.

Relativity is a theory of principle[47] built on two fundamental postulates: the Relativity Principle (RP) and the invariance of light speed *in vacuo*.

1) *Theoretical RP*: every physical law has the same expression in all IFs, included electromagnetic and optical laws, unlike classical RP[48]. *Experimental RP*: any kind of experiment carried out in the same conditions in different IFs gives the same outcomes.
2) *Invariance of c*: the value for light speed *in vacuo* is neither dependent of the specific IF in which it is measured nor of the propagation direction.

IV.5.1. Space-time, proper time and four-interval

The time interval between two events is different if measured in different IFs: *durations in motion are measured as longer*. This kinematic effect is therefore called "time dilation" or, more properly, "duration dilation". It is quantitatively expressed by

$$\boxed{\Delta t' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\Delta t \iff \Delta t' = \gamma \Delta t}$$

Δt being the time interval in a frame, Δt' the interval measured in another frame, and *v* the relative speed of the frames. The deduction of this result is an integral part of the worked-out conceptual path, so it will be expounded in VI.3.1.

This effect is sometimes referred to by the sentence "travelling clocks slow down", although time itself, rather than the instrument, is slowed down. The Lorentz's factor is a nonlinear function of relative speed always greater than 1 indeed:

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[47] The use of universal principles for building up an entire theory, whose validity may be experimentally tested only through its observable statements, was an historical novelty: models had been formulated for explaining observed behaviours until then. Einstein (1919) separated the theories of principles (like his own) from the constructive theories. The two postulates and WEP were not properties *a posteriori* following from a theory, but assumptions *a priori* for building it (Sanchez Ron, 1985; Arriassecq & Greca, 2012)

[48] Although Fabri (2005) would not agree on this distinction (see III.4 and V.4 for his approach).

$$\gamma(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \geq 1$$

Notice that the formal relationship between two time intervals above *cannot leave the travelled distance apart*, since the Lorentz's factor depends on the relative speed, deriving in turn from the spatial coordinate variation Δx. The instant and the spatial position in which something happens are therefore unified in Relativity: there is not a flowing absolute Newtonian time next to three-dimensional space, but a *space-time made by events*. The mathematician Hermann Minkowski invented a geometrical representation of spacetime, by putting space (usually x-coordinate) against time (namely ct), as depicted in figures 2 and 3. The motion of each point mass is described by a continuous curve called "worldline" of the point mass. In figure 2 four worldlines of differently moving particles have been drawn. The red worldline is the trajectory of a body at rest; the orange one is the closed trajectory of a particle which moves at constant speed, inverts its motion at a certain instant and still moves at constant speed; the white one represents a closed path too, but travelled at non-uniform velocity[49]; the yellow one is eventually the path of an object moving at *c*, typically the photon.

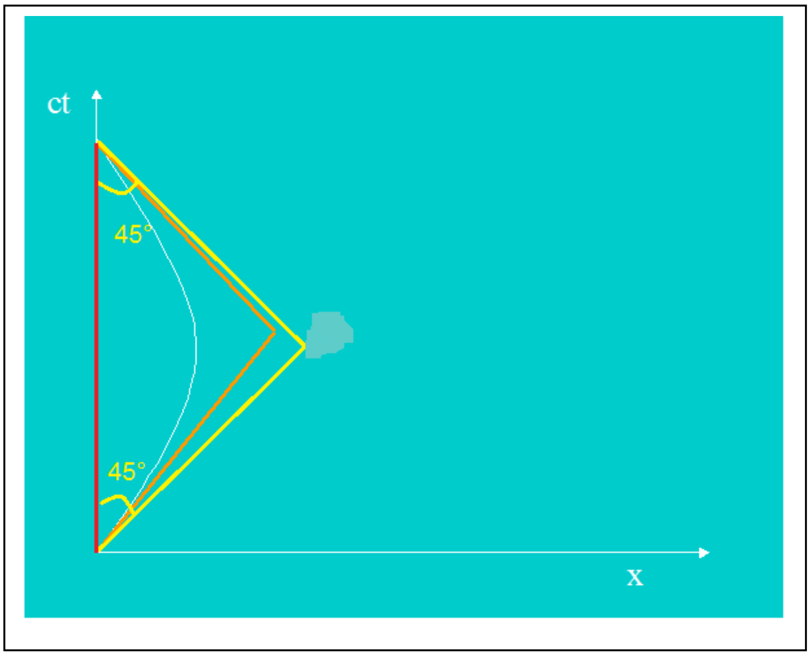

Figure IV.2. Painting of Minkowski spacetime.

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[49] In fact, mass-points moving at constant speed are represented by straight lines in the Minkowski diagram, since $\beta = u/c = 1/c(dx/dt) = dx/d(ct) = 1/m \Rightarrow m = c/u$, m being the slope of the worldline.

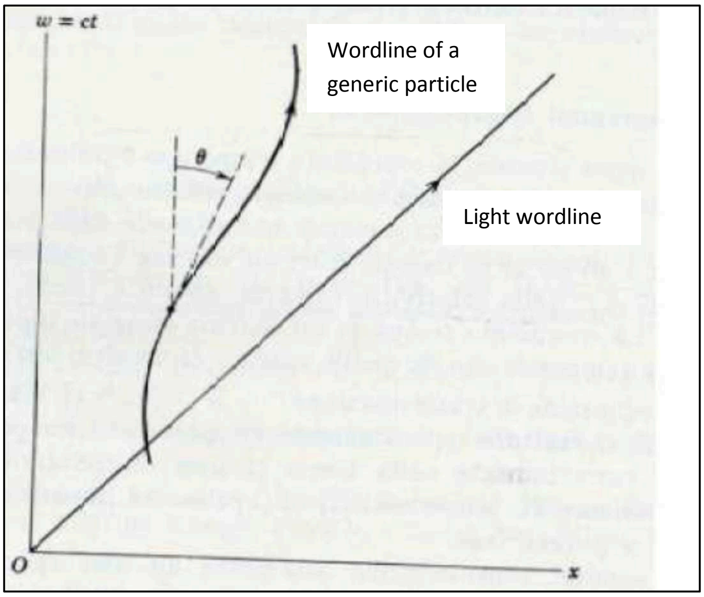

Figure IV.3. Painting of Minkowski spacetime.

Each event is in one to one correspondence with the set (ct, x, y, z), where t is the 'coordinate time'[50]. The *time interval between two events occurring in the same position* (in this case the emission and reception of light occurring in the same spatial point) is called *proper time interval* $\Delta\tau$.

There is a subtle conceptual difference between *proper time* interval $\Delta\tau$ and *coordinate time interval measured in the proper frame* $\Delta\bar{t}$. It is theoretically shown in General Relativity (GR) by the relationship

$$\Delta\tau = \sqrt{g_{00}}\,\Delta\bar{t} \cong \sqrt{1 + \frac{2\Phi}{c^2}}\,\Delta\bar{t}$$

Φ being the gravitational potential. In SR the Minkowskian metric component is $g_{00} = 1$: proper time is equal to the time measured at rest with respect to proper frame. A good approximation for the component of the metric tensor in the *weak gravitational field* limit ($\Phi/c^2 \ll 1$), matching Newtonian gravity, is $g_{00} \cong 1$ too. However, when the gravitational potential cannot be neglected in the proper frame «the relation between proper time (the clock rate) and coordinate time result to be altered by the presence of a gravitational potential» (Ferraro, 2007). Roughly speaking, gravity slows down the clocks in proper frame with respect to coordinate time in that frame.

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[50] *ct* is of course used for dimensional reasons. The name 'coordinate time' is due to the new meaning that time assumes in Relativity, *like if it were* a fourth spatial coordinate, in contrast with 'proper time'.

The pseudo-Euclidean metric of SR is obtained by subtracting the sum of the squared Cartesian spatial coordinates from the squared temporal one. The "minus" sign is due to the fact that geometry of spacetime is not Euclidean, but Minkowskian. So the invariant squared *space-time interval* is in differential form

$$ds^2 \equiv c^2 d\tau^2 = c^2 dt^2 - (dx^2 + dy^2 + dz^2) = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

In technical language it is said that two events are separated by a "time-like" interval when $ds^2 > 0$, so that the time component of displacement four-vector $(cdt, d\vec{l})$ is greater than its space component, namely $cdt > dl$. Thus the events are separated in every IF by a space interval smaller than the one crossed by light in the time interval between them and one may condition the other. Conversely, if the interval is "space-like" no causality relationship is possible between the two events, since $cdt < dl$ $(ds^2 > 0)$. When $cdt = dl$ the two events may be connected only by a light ray and therefore the interval is called "light-like" $(ds^2 = 0)$.

The interval expression may be also deduced by the following formal argument, based on the two postulates and space isotropy. It was taken from D'Inverno (1992) and requires some mastery of differential calculus. Light propagation observed in two different IFs K and K' is considered. Consider the spherical light wave propagation in K': in the time interval $\Delta t'$ the wave must be covering the generic spatial distance $\sqrt{\Delta x'^2 + \Delta y'^2 + \Delta z'^2}$ from the emission point in every direction. As a consequence $(c\,\Delta t')^2 = (\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 \Rightarrow$ $(c\,\Delta t')^2 - (\Delta x')^2 - (\Delta y')^2 - (\Delta z')^2 = 0$. The difference of squares in the left member will be called *I'* ('interval'). Then $I' = 0$ for light paths in K'.

The same phenomenon, described by the same physical laws, must be happening in K, because of the RP. Moreover, the light wavefront must be propagating at *c* because of the second postulate. Thus, using a reasoning analogue to the one above $I \equiv (c\,\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 = 0 \rightarrow I = 0 \Leftrightarrow I' = 0$. *If the interval I is null in an IF, it will be in all the others too.*

Now consider the space interval between two generic points infinitesimally close each other (x, y, z) and $(x + dx, y + dy, z + dz)$. Light will take an infinitesimal time dt for travelling from one point to another. Therefore the space-time interval between two point separated by an infinitesimal in spacetime will have the form $ds^2 \equiv (c\,dt)^2 - (dx)^2 - (dy)^2 - (dz)^2$. The conclusion above thus translates into $ds^2 = 0 \Leftrightarrow ds'^2 = 0$. Since the two interval *are infinitesimal of the same order*, we shall find[51] $ds'^2 = a\ ds^2$. The standard configuration of the two IF is considered, in which the x-axes of the two coordinate systems coincide and have the direction of their relative speed. Because of space isotropy, nothing has to change if the axes of the two IFs are reversed.

Figure IV.4a. Standard configuration of two general frames (D'Inverno, 1992).

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Figure IV.4b. Frames of figure 4a with reversed axes (D'Inverno, 1992).

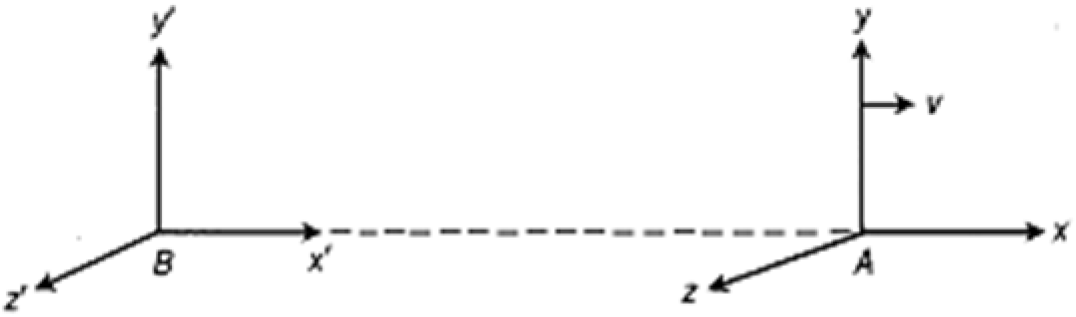

Figure IV.4c. The configuration of figure 4b seen by B (D'Inverno, 1992).

The configuration seen by the observer B in K' (figure 4c) is perfectly symmetrical with respect to the initial one (figure 4a) and becomes identical if A

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[51] This crucial passage could not have been used with the finite differences.

and B are exchanged, as depicted below. So one may also write $ds^2 = a\, ds'^2$, with the *same multiplicative constant*. Consequently

$$ds^2 = a\, ds'^2 = a^2\, ds^2 \quad \Rightarrow \quad a^2 = 1 \quad \Rightarrow \quad a = \pm 1$$

The sign is positive because the two IFs have to coincide for null relative speed:

$$ds'^2 \xrightarrow[v \to 0]{} ds^2$$

In conclusion, $\boxed{ds^2 = ds'^2}$ *for every pair of inertial frames*. The proof is valid for IFs in standard configuration only, but it is enough for the present purposes.

Eventually, it is remarked that relativistic "dynamics" is not actually the study of the causes (interactions) determining certain effects (momentum variations) afterwards, like in the Newtonian case. Space is unified with time, so there is not "the" global flowing time, but only a local one. In short and simplifying, it can be asserted that *there is one flow of time per wordline*, i.e. the classical linear causal ordering occurs only for the events belonging to each single worldline (Dieks, 1988, 2006; Valente 2013).

IV.5.2. Mass in SR

Mass and its relationship with rest-energy $E_0 = mc^2$ are the *core* of the present work at the content level. Newtonian inertial mass, equal to gravitational one because of WEP, takes on a new meaning in SR. At the same time, it is related to energy and linear momentum, or, more precisely, to *total* energy by $E = \gamma mc^2$, which is the first component of momentum-energy vector (four-momentum). The *equivalence between mass and rest energy* means that a system at rest in the laboratory with energy E_0 is endowed with a total mass E_0/c^2 and it behaves accordingly under the inertial respect. In the general case, a collision with total energy E in the center-of-momentum frame means that a mass $m \leq E/(\gamma c^2)$ is at disposal for the creation of new particles, provided that relativistic momentum is conserved. Conversely, an energy up to Mc^2 (in the center-of-momentum frame) may be released from a system of total mass M whose center-of-momentum be at rest, because of the conversion of rest energy in other energy forms.

Rest energy may be considered as energy "internal" to the system too, although not limited to the thermodynamic meaning of the phrase, because *all energy forms but the macroscopic kinetic one* contribute to rest energy, not only those to which a temperature value may be associated. So atomic and nuclear binding energies have a mass equivalent, since they are beyond doubt “internal”.

The relative decreases of mass due to energy emission are briefly calculated in four phenomena using the equivalence.

1) ***Chemical reaction.*** An energy release $\Delta E = 68$ kcal $= 2.85 * 10^5$ J occurs when a molecule of water is formed from its atoms. The corresponding mass variation according to Einstein’s relation is

$$\Delta M = \frac{\Delta E}{c^2} = 3.1 * 10^{-10} kg$$

1 mole of H_2O may be taken as quantity of reagent matter, to which corresponds a mass $M = 0{,}018$ kg. So

$$\frac{\Delta M}{M} = 1.7 * 10^{-8}$$

This is a very small relative mass variation, not detectable by any existing instrument. The one used in most common chemical reactions is even smaller. Therefore mass may be considered as a conserved quantity to a rather high level of accuracy: Lavoisier’s law is *approximately valid* in SR.

2) ***Fusion.*** The power transferred by the waves emitted by the Sun is $\Delta E = 4 * 10^{26}$ W, which corresponds to a mass decrease (per second)

$$\Delta M_\odot = \frac{\Delta E_\odot}{c^2} = 4.4 * 10^{-9} kg$$

If it is compared with the Sun mass, an abundantly negligible relative loss comes out:

$$\frac{\Delta M_\odot}{M_\odot} \cong 2.2 * 10^{-21}$$

3) ***Uranium fission.*** Consider one of the possible fission reactions of Uranium:

$$n + {}^{238}U \rightarrow {}^{143}Ba + {}^{93}Kr + 3n$$

There is a loss of energy $\Delta E \approx 10^{13}$ J for 1 mole of Uranium-238, which corresponds to the relative mass decrease

$$\frac{\Delta M}{M} = 7.6 * 10^{-4}$$

The percent variation is much higher than in the other cases. Therefore *fission is a good candidate for making mass non-conservation evident to students.*

4) ***Nuclear β-decay.*** The decay of Carbon-14 is considered, for it is widely used in archaeological dating: ${}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^- + \bar{\nu}_e$. The energy released in a single decay is $\Delta E = 0.1565$ MeV. Therefore $\Delta M = \Delta E/c^2 = 2.78 * 10^{-31}$kg. The atomic mass of Carbon-14 is 14.003242 a.m.u.[52] and so

$$\frac{\Delta M}{M} = 1.2 * 10^{-5}$$

This is a result close to the one above. The effect is observable, so *β-decays are also worth considering to illustrate a phenomenology which shows a correlation between mass variation and released energy.*

Mass in GR. In GR *momentum-energy density* is a physical quantity represented by some components of the tensor $T_{\mu\nu}$, which roughly stands for the mass-energy content of the Universe and is therefore the source of space-time geometry warping. The tensor $G_{\mu\nu}$ represents instead the modified geometry, which determinates the motion of matter. The two components are linked by the Einstein's field equations below, stating this reciprocal action of matter on geometry: «Space acts on matter, telling it how to move. In turn, matter reacts back on space, telling it how to curve» (Misner *et al.*, 1970).

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \, .$$

IV.6. **"Relativistic Mass"**

A quantity called "relativistic mass" appeared and still appears in several treatises of relativity, its expression being

$$m_r = \frac{m_0}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Using the latter, one can rewrite relativistic momentum in the form $p = m_r u$. Is it enough to justify the use of the expression above for representing a *proper physical quantity*? Namely, a speed-dependent quantity expressing quantitatively inertia in the wide relativistic sense? Taylor and Wheeler

---

[52] The conventional unit for atomic mass is a.m.u.: $1\,\text{a.m.u.} \equiv 1/12\, m\,({}^{12}_{6}C)$ = 1.66*10^{-27}kg. Atomic masses are approximately equal to nuclear ones, because $m_e \cong 10^{-4}\text{a.m.u.} \ll m_p \cong m_n \cong 1\,\text{a.m.u.}$; no distinction will be made in this context.

(1965) underlined that while "relativistic mass" is the first component of relativistic four-momentum, invariant mass is the scalar magnitude of the latter.

On the other hand, why should m be interpreted as the inertial classical mass in the following master equation, and consequently in the whole relativistic dynamics?

$$E^2 - p^2c^2 = m^2c^4.$$

"Relativistic mass" is utilized and considered as a proper physical quantity by several physicists and physics education scholars as well, but more and more considered as misleading by most of the scientific and SER community, as well as by me. Actually, there are reasons both to consider mass in continuity with Newtonian mechanics both to interpret it as a completely new quantity[53] in a new paradigm (Kuhn, 1970); «the expression $\frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}$ is best suited for THE mass of a moving body», wrote Tolman (1934). Moreover, from the mathematical point of view the two quantities are completely symmetric: classical mass can be generalized to two quantities with different tensorial characters[54] (Bickerstaff & Patsakos, 1995). One strong objection to relativistic mass is that

$$m_r = E/c^2$$

varies with the reference frame, thus it should not define a physical quantity representing an intrinsic property of a real particle or body. In spite of this, Richard Feynman (1964) claimed the importance of "relativistic mass", conceptually separating it from "rest mass":

> Newton's Second Law, which we have expressed by the equation
>
> $$F = d(mv)/dt,$$
>
> was stated with the tacit assumption that m is a constant, but we now know that this is not true, and that the mass of a body increases with velocity. In Einstein's corrected formula m has the value
>
> $$m = \frac{m_0}{\sqrt{1 - v^2/c^2}},$$
>
> where the "rest mass"m_0 represents the mass of a body that is not moving.

---

[53] The philosopher of science Feyerabend (1965) considered mass in SR as a «*relation*, involving relative velocities, between an object and a coordinate system», instead of a «*property* of the object itself».

[54] Similarly, one can generalize classical time either to proper time or to the first component of (ct, x, y, z).

IV.7. The Photon and its Properties

The photon was taken into account for the educational reconstruction since it is an *ideal relativistic object*. It will be examined only under the phenomenological respect, for it is the most useful for didactics.

The electron emission from metals hit by high-frequency invisible light – UV light typically – is named *photoelectric effect*. The kinetic energy (K.E.) of those electrons may be measured while light frequency varies, by arresting them through a positive potential difference, according to the well-known equation $-e\Delta V = \Delta E = 0 - E_i \Rightarrow E_i = e\Delta V$. The first historical observation of the effect was performed by Becquerel in 1839, who measured an electrical current produced by the action of visible light on chemical reacting liquids. In the end of the Nineteenth Century (1887) Hertz identified then in the ultraviolet light the causes of several electrical discharges. His line of research was followed by Hallwachs, Stoletow, Righi, Elster and Geitel, who collected a large amount of data in the period 1888 – 1897 (Robotti, Giuliani & Galdabini, 1992). Lénard observed the ionization of gases by ultra-violet light in 1900 and carried out an accurate experimental work that lead him to discovery the *Lénard effect* in 1902.

In 1905 Einstein hypothesized that *light-matter interaction occurred through discrete energy units directly proportional to light frequency*, starting from a completely different theoretical study – the study of "black body" – according to the known Planck's equation $E = h\nu$. A black body is defined as a body absorbing and emitting waves in the whole electromagnetic spectrum; it may be depicted as a black cavity with a little hole, through which radiation goes in and is then absorbed and re-emitted by the cavity walls. The atoms of these walls may be modelled as oscillators. In 1900 Max Planck had hypothesized that the *oscillator energy were quantized*, the energy quantum being a universal constant (h) times each oscillator frequency. Differently, in 1905 Einstein supposed that the *emitted radiation energy* were quantized, the quantum being proportional to the light frequency. This was an entirely new heuristic assumption, which allowed him to explain the photoelectric effect and other phenomena. In particular, this hypothesis brought him to propose the following well-known equation for calculating the *maximum* photoelectron K.E.:

$$\boxed{K_{max} = h\nu - L_e}.$$

$L_e = h\nu_0$ is called “removal work” from the metal. It is the lowest energy to be supplied for making one electron break the binding with an atom of the material, viz. a threshold energy which obviously matches the threshold frequency. K_{max} is therefore the highest possible residual energy brought by the photoelectrons after ionization.

To summarize, Einstein described light as composed by indivisible entities, *light quanta* or *photons*, always moving at the ultimate speed. These entities carry discrete amounts of energy and may be emitted and absorbed only as individuals: *electromagnetic radiation interacts discretely with matter*, by “packets”.

Millikan found the experimental plot below in 1916, which confirmed Einstein’s hypothesis.

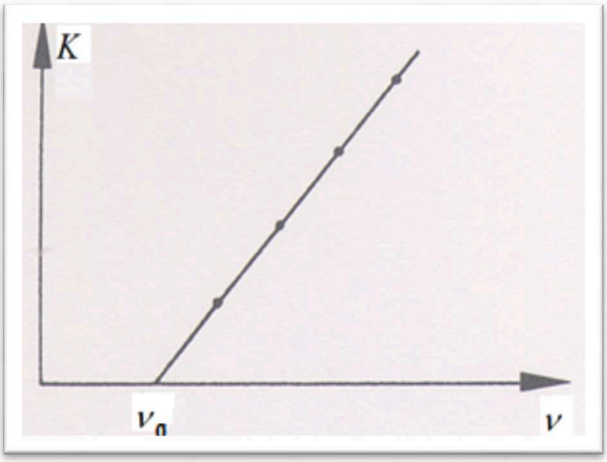

Figure IV.5. The historical experimental trend of emitted-electron kinetic energy against light frequency.

My intention was *not* to follow the historical path for this section of the conceptual pathway. I wanted to show instead the experimental trend at first, in order to begin with a phenomenological exploration, and then arriving to state Einstein’s hypothesis. In the graph below it is striking that:

- Photoelectrons are emitted only if incident light frequency stands over a minimum value, depending on the used material. The existence of this *threshold frequency* ν_0 seems quite strange: it is not clear why when UV light turns into red or IR one electrons are not emitted anymore.
- *Photoelectron kinetic energy is linearly increasing with light frequency*. This is strange as well: these physical quantities are not thought to be related in classical physics.

- Kinetic energy is *completely independent* of light intensity. Very curious indeed: if light is a wave and transmits power continuously, why will the variation of transmitted power per unit area not make electron K.E. vary?

These observations are well explained by the light quantum hypothesis.

Anyway, physical quantities associated to photons *besides energy do exist too*. In the end of Nineteenth Century Maxwell speculated on light, making the assumption that it exerted a thrust on matter, on the basis of his understanding of it as electromagnetic radiation. He foresaw a light-matter *momentum exchange*, the supposed expression for light's momentum being $p = E/c$, with E incident light energy. Lebedev (1901), Nichols and Hull (1903) ran several experiments for testing this hypothesis, using a torsion balance made by two mirrors and a fine quartz thread. A light beam was directed first on one mirror and then on the other, and the mobile equipment was observed to *rotate slightly*: a pressure *was* actually exerted by light. The rotation angles were measured. By the way, stars stand at their size because of this small effect.

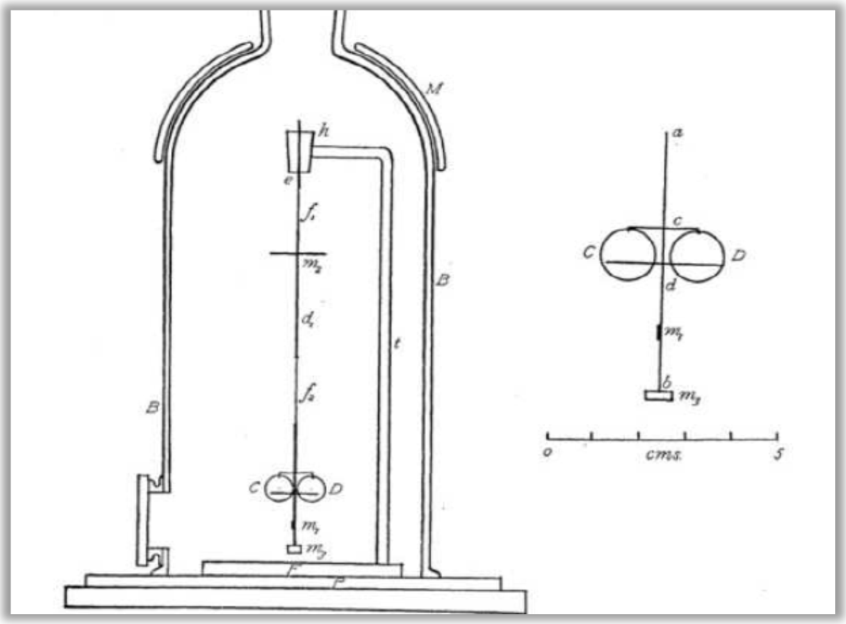

Figure IV.6 (Nichols & Hull, 1903). The Nichols radiometer for the measure of radiation pressure: a torsion balance made by two small silvered glass mirrors (enlarged on the right) suspended by a fine quartz fibre. The air pressure inside the enclosure was set to 16 mmHg, at which the influence of the air on the phenomenon was found to be negligible.

Those experiments displayed with diverse degrees of accuracy that

1. The module *F* of the force (impulse per unit time) exerted on mirror by light is directly proportional to the power *P* (energy per unit time) of the beam.

2. The proportionality constant is approximately equal to 2/c for a reflection, within the experimental errors. Thus

$$F = P\left(\frac{2}{c}\right) \qquad \text{[Empirical law]}$$

$$F = \frac{\Delta p}{\Delta t} = P\left(\frac{2}{c}\right) = \frac{E}{\Delta t}\left(\frac{2}{c}\right) \rightarrow \Delta p = 2p \text{ [reflection]} = \frac{2E}{c} \rightarrow p = E/c$$

The final quantity is the *linear momentum to be associated to incident light*, *E* being its energy, in agreement with Maxwell's hypothesis.

This result has necessarily to be extended to every light quantum in the beam, and then it may be interpreted as the equation relating energy and momentum of a photon:

$$\boxed{E = pc}.$$

Every entity moving at c has null mass according to SR theory: $m = 0$ is not forbidden for a single entity, like it is in classical mechanics. This value is intimately related to the ultimate speed, but not necessarily to photons (Bergia & Franco, 2001). On the other hand, it may be demonstrated that *a system of two photons has not necessarily null mass*, but a value in the range

$$0 \leq m_{1,2} \leq h(\nu_1 + \nu_2)/c^2$$

depending on the angle between the photon directions (Gabovich & Gabovich, 2007).

An experimental test of Coulomb's law provided the upper limit for photon's mass: $m_{photon} < 10^{-14}$ eV/c^2 (Williams, Faller, & Hill, 1971). More updated tests provided $m_{photon} < 10^{-18}$ eV/c^2 (Amsler, Doser, Antonelli *et al.*, 2008). For comparison, $m_e = 511$ KeV/c^2 and $m_\nu \sim 1$ eV/c^2.

Review of critically selected Educational Literature

Mass is a concept not yet extensively explored in physics education research literature (PER literature). Few papers are available which illustrate outcomes of t/l research on the concept of mass in SR. There are more publications for mass in classical physics, but not so much anyway.

In most of the surveys reported in this chapter the involved students' age is *under* or *above* the age span of the ones to whom the designed t/l proposal is addressed. As for the former case, it is possible to individuate the *formation of conceptions* (meant as inner mental models) at an earlier age, which will be likely present in secondary students too. As for the latter case, the assumption is made that domain-specific learning problems emerging at higher instruction level will be especially significant for high school students.

First of all, it is important to conceptually separate *weight* from *mass* for didactical purposes, since children have the spontaneous conception of weight from their first infancy (paragraph 1.1). On the contrary, they usually find it difficult to learn the concept of mass and to understand the scientific meaning of weight: gravitational interaction near the surface of the Earth. In addition, children and teenagers tend to overlap the two *terms*. However, this does not necessary implies that high school students overlap the two *conceptions*. On the contrary, it was shown that they own the distinct concepts at the intuitive level, but triggered by two separated words, each not univocally corresponding to the scientific meaning. This allows to *discriminate* between the owned intuitive concepts (for educational research) and to *separate* them (for teaching).

Mass is the most important quantitative property of matter, although not the only one. Total energy and mass are both conserved separately in classical processes, while total energy (including rest energy, equivalent to mass) is conserved in SR. A second important research objective has been therefore to study how (classical) mass conservation in chemical-physics transformations is conceptualized by students (paragraph 1.2).

Furthermore, inertial mass has to be operationally defined independently of force by exploiting the laws of dynamics (paragraph 1.3). The operative definition of mass by Mc Dermott, Shaffer and the Physics Education Group at the University of Washington (2001)[55] as well as the gravitational approach to mass by PSSC (1995) are considered as the most important (the most frequently cited) ones in the international literature on topic.

Mass is interrelated with energy in Special Relativity. So the latter concept is to be soundly understood by students, who usually find it difficult, especially as regards its conservativeness. Students should also focus on the two most important ways for internal energy change: heat and work. At the same time, other processes in which energy is varied, like nuclear transmutations and electromagnetic wave emission/absorption, are not to be neglected. Finally, it will not be easy to match mass and energy on the conceptual ground (section V.2).

In educational research on teaching/learning SR the design and evaluation of teaching strategies is underdeveloped, unlike research on students' *learning problems* on the one hand and on *critical points of traditional curricula* on the other hand. Moreover, the developed teaching strategies are evaluated merely in terms of either motivation they foster in students or conceptual change activation. In particular, relativistic "dynamics" is the theoretical framework in which mass-energy relationship is inserted. Nevertheless, the conceptual aspects to consider are essentially the same of kinematics, because (i) relativistic dynamics requires kinematics and, differently from classical dynamics, (ii) it cannot be a causal study of the effects of forces / fields on particles/systems, since spacetime is four-dimensional (section V.3). Finally, "relativistic mass" proved to cause a lot of *misunderstandings* and *learning difficulties* (section V.4).

V.1. **Mass**

V.1.1. **Weight and Mass**

The first spontaneous idea gradually developed by small children, i.e. from the age of five, is *weight* in its operational facet through what Piaget would have called a series of «stages». In particular, children develop two major naïve conceptions: (a) weight as "force" exerted by the objects on human muscles[57] provoking effort; (b) at an older age, weight as amount of matter (Piaget, 1930). Anyway, the mental image being formed in children is never the school science

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[55] See also McDermott, Shaffer, and Somers (1994) and McDermott (1996).

[57] Weight has got the translated meaning of 'heaviness' in this case.

one, i.e. the gravitational interaction near to the Earth's surface (Galili & Bar, 1997). For this reason the *scientific concept* of weight is difficult for children; moreover high-school or even university students usually hold *hybrid models* of weight resulting from the attempt of putting their spontaneous mental image and the taught knowledge together. This causes misconceptions (Galili, 2001; Vosniadou *et al.*, 2008). Therefore Galili (2001) has called for an operational definition of weight distinct from that the gravitational one in instruction, asserting that this shift would be able to improve students' understanding. He individuated three possible operational definitions, the latter of which is especially interesting:

> (3) Weight of the body is the force, which acts downwards and causes spontaneous falling. Numerically, weight is given by the product mg*, with g*—the acceleration of a free fall, as it is measured in a particular frame of reference.

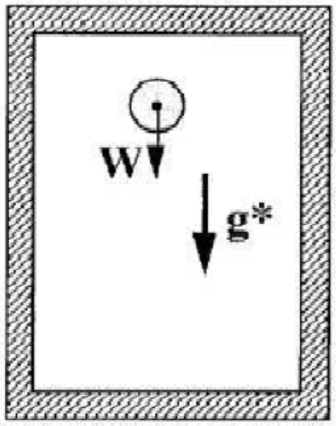

Figure V.1. Weight depicted as a force acting "downwards" in the direction of g* (Galili, 2001).

Using this definition at the beginning of high school it is possible indeed to match it to a special case of the gravitational interaction, which will be dealt with later. This has been my intention while designing the t/l pathway[58].

The concept of *mass* has also proved difficult for primary students, as recently shown, for instance, by Cheeseman, McDonough, and Ferguson (2014) and therefore in most cases is incorrectly understood at basic science instruction levels, as it is easily seen in school practice. This fact has been recently confirmed by a study on the contribution of computer simulations to the conceptual learning of weight and mass by a sample of Portuguese 12 – 13 years old students (Sarabando, Cravino, & Soares, 2014). The lack in students' learning/understanding is also doubtless due to the presence of alternative

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[58] The only problem of this definition is the use of the word «downwards» in an absolute sense. It has to be replaced with «towards the centre of the Earth».

conceptions in *teachers themselves*. For example, serious misconceptions were detected in 267 science and physics pre-service teachers about the key concepts of inertia and gravity as well as on gravitational acceleration, gravitational force and weight (Gönen, 2008). The same holds for weightlessness, of which 9 pre-service teachers were found to own incorrect scientific explanation in an analogue study (Tural, Akdeniz & Alev, 2010).

Although the problem of understanding and/or learning mass and weight has been recognized to be basic in physics education, it has been almost always studied either in *descriptive* ways (e.g. Galili, 1993; Morrison, 1999) or on the *conceptual-theoretical ground*, seeking for consistent (e.g. Hecht, 2006, 2011) and/or didactically proper definitions, as in the case above (Galili, 2001). Experimental data on t/l are almost entirely missing. Two noteworthy exceptions are the above-mentioned research by Sarabando, Cravino, and Soares (2014) and the following older but important one.

Mullet and Gervais (1990) carried out three combined cognitive experimentations in five French schools and showed that the selected statistically representative groups of high school students, aged from 13 to 15, owned the *same intuitive concept* for the terms 'mass' and 'weight': that of *weight*, function of density, volume, and gravitation[59]; in symbols $W = f(\rho, V, gravitation)$. However, the pupils also owned a separate *intuitive concept of mass*, function of density and volume, activated by the phrase 'quantity of matter': in symbols $m = f(\rho, V)$. So, unlike previous Piagetian findings (1930), weight is neither cognitively confused with mass, as previously asserted by Halbwachs and Bovet (1980), and by Bovet and Halbwachs (1983) too, nor better mastered than mass, as claimed by Gomez (1983). *Both concepts* are present in students' minds, and

(i) mass differs from weight in the absence of any reference to *gravitation*;

(ii) paradoxically, mass is exclusively activated by the term 'amount of matter' or 'quantity of matter', while weight by 'mass' or 'weight' equally.

Supposing to express the combination of the cognitive contributions from the involved physical quantities through an equation, one can synthetically write $'quantity\,of\,matter' = m = \rho + V \neq \rho + V + g = W = 'mass' = 'weight'$ (Mullet & Gervais, 1990).

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[59] Gravitation is spatially contextualized on the Earth, on the Moon, and in outer space in the tasks of the survey.

V.1.2. **Matter conservation**

Mass is the fundamental quantity characterizing matter in physics and chemistry; nevertheless matter is not necessarily associated to mass. Matter may be indeed characterized by energy without mass in modern physics. Nevertheless, massless particles like photons, and thus the electromagnetic field, or gluons[60], as well as the gravitational field, are incorrectly *not* thought as *matter* in a number of physics textbooks. Further, in the same books almost-massless neutrinos are not considered as matter in a clean-cut way (Okun, 2005).

Conservation of matter is a basic assumption of physics and a *core concept* of chemistry, whose reactions do not preserve the substance, differently from physical ones. Matter is a primitive concept in chemistry and in physics. It is thought to be made by atoms and molecules in chemistry, the basic unit being the atom: it is fundamental for a chemist to focus on the conservation of these units while molecules interact and change. A reorganization of atoms occurs, but their identity is preserved (Gomez *et al.*, 1995; Krnel, Watson, & Glažar, 1998). This is essentially Lavoisier's principle, in which he stated that elements are invariant in chemical transformations (1789). Children from three to eight years old have even proved to know that matter remains the same when objects are destroyed (Smith *et al.*, 1985). More generally, it may be asserted that the whole of science is characterized by searching for something which is conserved in transformations, be it mass, energy, momentum, or matter.

No specific studies have been found on mass conservation learning/understanding. However, the notions of mass and weight conservations are *secondarily* considered within several chemistry education papers reporting surveys on students' difficulties in understanding *matter conservation* in physical-chemical reactions (Stavy & Stachel, 1985; Stavy, 1990, 1991; Doménech *et al.* 1993; Gomez *et al.*, 1995; Krnel, Watson, & Glažar, 1998).

A child's understanding of matter conservation was traditionally thought to be gained in the «concrete» stage of his/her cognitive development (refer to II.3), as a special case of the construction of *conceptual invariants* needed to interact with the external world. The notion of matter conservation may be either (i) based on sensorial perceptions, or (ii) directly inferred from observation, as well as (iii) indirectly inferred from other concepts through formal reasoning, for instance in

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[60] The vector gauge bosons for the strong nuclear interaction.

chemical transformations like combustions and solutions (Piaget, 1970; Piaget and Inhelder, 1974). However, Stavy and Stachel (1985) found that matter (weight in this case) conservation was used only in a few administered tasks by 7 – 11 years old children. This implies the levels of Piaget are not actually the best description of children's intellectual development. The third way above for understanding matter conservation, called «conservations beyond observations» (Gomez, Pozo, & Sanz, 1995), is very important for school science, because most conservations dealt with in *curricula* are of this type, i.e. they entail non-observable properties. These kinds of conservation need thus to be *constructed on the conceptual ground* rather than being founded on perceptions (*ibid.*).

It is worth adding that mass even proved to be identified with matter, particles and bodies at the same time in its «ontological level of physical representation» by pupils aged 16 – 18 (Doménech *et al.*, 1993).

A research by Gomez, Pozo and Sanz (1995) addressing the conceptions on matter conservation of University chemistry and psychology students, as well as secondary[61] ones, showed that matter conservation is far more easily understood when a *change of state* is examined with respect to a *chemical reaction. Solutions* stand in the middle: matter conservation is correctly understood if considered in familiar (everyday) contexts, but incorrectly understood in specific chemical contexts, just because they tend to be interpreted as changes of state or as chemical reactions respectively. This research outcome indicates that matter conservation tends to be better understood in physical reactions than chemical ones, for the following reasons:

- Physical reactions are felt closer to everyday experienced phenomena;
- In the former no substance change occur, while in the latter different substances are mixed up: «the interactive use of reactions makes this content the most difficult, as it asks students to use concepts of interaction that are opposite to the linear causal reasoning they usually employ» (*ibid.*).

More generally, the involved chemistry students showed a soundly consistent reasoning pattern, but the conceptions of the other students were also interestingly not uncorrelated. So *mastering matter conservation is not too much difficult*, and it may be included as a topic of the rationale to be designed.

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[61] These students were from 12 to 17 years old.

V.1.3. **The problem of defining inertial mass in dynamics**

As for inertial mass, I focused on the definition by Mach (compare IV.3.2) since it is an *operational* one and it *circumvents the problem of dynamically defining force independently of mass*. In the first and fourth chapters it was claimed indeed that an operational generalized definition of mass in dynamics by means of Newton's second law is not achievable, due to a vicious circle, often found in high-school textbooks (e.g. Paracchini & Righi, 1994; PSSC, 1995).

Actually, as stated in IV.3.2, there is a feasible Newtonian operational definition of mass in dynamics, obtained by empirically establishing a scale of the forces exerted on bodies initially at rest accelerated by a pulled spring to which they are connected. This approach does not require the hypothesis of a linear behavior of the spring; however it is not valid in the most general case. It is due primarily to Arons (1965, 1992) and *stems exclusively from physics experiments*, rather than coming from theoretical hypotheses or *a priori* conventions, even if it involves the concept of force, unlike Mach's approach.

Many other consistent definitions of mass are of course possible. My intention is to explore both one similar to the above, not exploiting $F = ma$ – namely Goodinson and Luffman's (1985) – both another using Newton's second law as the *dynamic definition of force*, extended to relativistic speeds, viz. Brehme's approach (1985). As far as I know, these outdated approaches are among the most relevant ones for didactical purposes, which is confirmed by the fact that they are the only ones present in the bibliography STCSE (Duit, 2009).

Goodinson and Luffman (1985) suggest to accelerate a sample body S and a body B sequentially, by hanging on them a floating "weight" (W). The respective acceleration a_s and a are measured. After that, the operation is repeated for a series of different "weights", allowing to plot a against a_s. A linear relation like $a = ka_s + c$ will be obtained. In order to study the dependence of the two constants from *the particular table* or *experimental setting*, another table is then used. A straight line of equation $a = ka_s + d$ will be obtained again, with *the same slope*: k proves to be related to inherent properties of S and B. Trying with different bodies in place of B, straight lines with different slopes will be found, each experimental k_i being in one-to-one correspondence to each body. At this point it is asserted that $m_i \equiv 1/k_i$: inertial mass is thus operationally defined and its additivity follows from the following experimental property of the slopes: $1/k_i + 1/k_j = 1/k_{i+j}$. However, the order relation built up between the masses is based upon the fact that an object *is perceived to be «heavier than» another* (p. 41). So what about inertial mass of objects in a spaceship? Or on a horizontal table? This definition appears to relate two domains which *should be separated* for educational purposes: the gravitational and inertial one.

In the ideal case of an infinitely smooth table: $c \to 0$ and $d \to 0$. This is nothing but Mach's definition with $m_s = 1$. Moreover, this kind of definition solves the problem of measuring the *same mass* for the same object in reciprocally accelerated IFs, as the reciprocal acceleration is absorbed by the additive constant, which vanishes in the no-friction case (Jammer, 2000, p.19 – 20).

Alternately, in Brehme's approach (1985) mass is defined by *momentum conservation* – for every speed and then specializing to low speeds – and then force is determined by the second law of dynamics[62]. The last is thought as *definition of force* in a strict sense, the only feasible one in relativistic mechanics. This pathway is essentially equivalent to Mach's under the theoretic-conceptual and educational respects.

V.2. **Energy**

Generally speaking, students typically show serious problems in understanding energy, its conservation especially (Duit & Häußler, 1994). In particular, introducing the energy concept by force and work[63] has proved to bring students to a limited conception of energy (Lehrman, 1973; Duit, 1985; Papadouris & Constantinou, 2011). Furthermore, some students' alternative conceptions of energy, present after instruction too, either mirror their everyday conceptions or depict energy as a kind of fuel (Doménech *et al.*, 2007). Other representations of energies have been found, including (i) a quantity "stored" in physical systems and (ii) something like a fluid (Watts, 1983). After instruction, pupils regularly (Duit, 1986; Arzi, 1988):

- Do not learn the basic aspect of the concepts (see below);
- Do not use the proper language when they have to explain processes in which energy is important;

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[62] The form of this law depends upon the chosen theory: $F = dp/dt$ in the relativistic case, $F = ma$ in Newtonian dynamics.

[63] This t/l practice is still rather diffused, as well as the one in which only abstract energy calculations are made (Nordine *et al.*, 2011).

- Are not able to apply the taught energy concept appropriately to real everyday situations.

As anticipated in I.1, in the literature there are various approaches to t/l energy. To begin with, there is the socio-constructivist approach for primary school by Rizaki and Kokkotas (2009), exploiting history and philosophy of science.

Another proposal has been to introduce and discuss *energy forms* in an extensive and coherent way at every school levels: energy forms are considered as an intermediate conceptual step useful to bring students to the correct thermodynamic view (Hobson, 2004; EIA, 2009). This approach is motivated by the social role of energy, especially as concerns the language used by media. In particular, Hobson (2004) argues that all physical processes may be described in terms of energy transformation, i.e. passage from some energy forms to other ones. According to him, this interpretation of processes is interesting when the latter might be analyzed in terms of Newtonian forces, and unavoidable in every other case. Finally, he supports the use of *energy flow diagrams*, because they depict energy transformations «visually and approximately quantitatively» (*ibid.*), and they also allow to consider energy in relation with the environment.

This brings to the third approach, which highlights the importance of identifying energy fluxes and utilizing the concept of *energy carrier*, as in the Karlsruhe Physics Course by Hermann (2000).

Alternately, energy may be conceptually introduced and considered from the standpoint of the II law of thermodynamics (Ogborn, 1990), as in the “Energy and Change” project (Boohan & Ogborn, 1996; Stylianidou & Ogborn, 1999). Stylianidou and Ogborn (1999) also suggest to replace the term “transformation” with “transfer”, according to the National Curriculum for England and Wales, since energy should be considered as something flowing from one system to another; the kind of transfer is more relevant rather than the energy form in their opinion.

According to Duit (2014), energy is conserved in closed systems, transforms and transfers: «conservation amidst change» occurs. He considers *conservation*, *transformation*, *degradation* and *transfer* as the four basic interrelated aspects characterizing the concept, and thus allowing students to gain learning with understanding. These ideas endow students with tools for handling with the (social) issues of *energy supply*, which is in turn important for scientific literacy. Duit (2014) also maintains that it is mandatory to consider degradation too for conceptually understanding energy. "Degradation" means here that work cannot be done by transferring the same amount of energy that was previously transferred if a change in the thermal state of a system has occurred. In a nutshell, «Energy is conserved however its utility value unavoidable decreases» (*ibid.*), in agreement with the II law of thermodynamics. Finally, Duit asserts it would be fruitful for understanding conservation if students mentally visualized energy *like* an indestructible substance flowing from one system to another, transferred by different energy carriers, rather than generically imagining the more common 'forms of energy'.

Neumann and colleagues (2013) developed a learning progression on energy based on the four core concepts individuated by Duit. This learning progression goes from a lower anchor in which students think of energy in terms of sources and forms to an upper anchor in which they have fully understood the four concepts. The intended sequence of conceptions is the following: form and sources, then transfer, degradation, and finally conservation. Each conception is supposed to be learnt through four stages of increasing complexity: *facts* (the basic ideas), *mappings* (representation of the concept by physics measures), *relations* (links among the different aspects of the concepts and related measures) and *concepts* (abstract concept grasping). Neumann and colleagues (2013) apply it to energy forms, for which (i) facts are forms; (ii) a mapping is speed measure for kinetic energy, for instance; (iii) relations are the links between physical measures and the respective forms of energy (for example, height for gravitational potential energy); (iv) concepts is the final achievement that « energy is a somewhat abstract quantity that is assigned to different forms based on observed measures » (*ibid.*).

Unfortunately, Duit's and Neumann's approaches presume a *materialist view* of energy – namely the conception of energy as a substance-like entity flowing from one system to another, like water – which is not good under both the scientific and educational respect. In fact, it fosters the idea that energy is *localized* in space, which in turn makes people think that it needs a propagation medium. Moreover, entropy should be considered too if one wanted to deal with energy degradation appropriately. So I decided to *drop the concepts of transfer and* *degradation* as referred to energy, according to the approach by Colonnese, Heron, Michelini, Santi, and Stefanel (2012) explained below.

An improvement in long-term understanding of energy conservation/ transformation interplay was detected when the following conceptual sequence had been used (Nordine *et al.*, 2011):

(i) *energy types* are introduced, each one characterized by an «indicator»;
(ii) it is observed that if an energy type increases, at least another will systematically decrease and vice versa: this brings to the qualitative idea that energy is *transferred* and *transformed*;
(iii) it is then noticed that the decrease of every type of energy is *numerically* equal to the sum of the increases of all the other forms: this brings to the idea of *conservation.*

Since only energy variations may be operationally defined, thought experiments including them were designed for the t/l path (Doménech *et al.*, 2007). Eventually, students should be shown that energy variations may take place not only through work or heat, but also by means of other processes (Duit, 2014) – such as electromagnetic radiation absorption/emission or nuclear transmutations – so that their phenomenology of reference becomes wider.

After examination of the different kinds of educational approach to energy, I selected the one by Colonnese, Heron, Michelini, Santi, and Stefanel (2012), because it is closer to the orthodox physical interpretation of energy, and also since it is a research-based vertical coherent approach. Energy is conceptualized as an *abstract state quantity* of a precise physical system in this approach, which *is conserved but transforms when systems interact*. Energy makes no physical sense without the systems owing it. In this approach the term "energy forms" is considered as misleading, for they are nothing but the different phenomenological manifestation of a single quantity. Since chemical, mechanical, nuclear, gravitational, radiant, thermal, motion, sound, electrical, solar, wind energy are melted in popular treatises (compare EIA, 2009), but they are not precisely defined in physics and not at all in thermodynamics (Millar, 2005), only four forms of energy are used in this approach, called "energy types": *kinetic*, *potential*, *internal* energy, and the energy *associated to light*. In fact, all the others forms are combinations of these "types". *Transformation* of energy is considered as a valid concept for education, since it spans from everyday interactions to processes related to large-scale energy supply. Energy is however never created nor lost, so strictly speaking there are no sources of energy, but rather of each "type" of energy. These points were deepened with primary[65] and secondary students. As for upper-secondary students, each type of energy was converted into the internal energy of a physical system (aluminium cylinder), different physical parameters being varied for identifying the types' expression, and the two ways for changing internal energy according to the first law of thermodynamics, namely "heat" and work – were explored. The work-energy theorem was not used for introducing energy.

In conclusion, my focus is on *qualitative approach* to energy conservation, even though without losing the quantitative aspects inherent to the physical concept. Energy types are not seen at first as numerical values for drawing up balances between the initial and final state of a physical transformation: quantification should come later.

Eventually, a big challenge for the purposes of the present research was how to *combine energy with mass* in an effective way for learning, since the former concept is abstract, viz. far from concrete experience and immediate intuition, whilst mass is semantically related to the amount of "stuff", muscular effort, inertia, heaviness, and weight in common sense knowledge. Besides, energy is formally defined unless an additive constant, while mass is not.

V.3. Special Relativity

Teaching and learning Special Relativity has not been extensively explored in PER so far, as recently pointed out by Selçuk (2011) and Levrini (2014) among other researchers. However, a number of papers have been published on the topic, focusing mainly on (i) students' learning difficulties and (ii) criticalities in traditional *curricula* (e.g. Hewson, 1982; Villani & Pacca 1987; Hewson & Thorley, 1989; Villani & Arruda 1998; Scherr, Shaffer, & Vokos, 2001; Scherr, Shaffer, & Vokos, 2002). These research strands, the first one in particular, are well-established, a varieties of both *diagnostic* and *explicative* results having been obtained in approximately the last thirty years[66], together with guidelines for

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[65] As for primary students «some partial results were also obtained about the deep-rooted conceptions on "energy loss", reshaping students' attitude from an energy disappearance conception to an energy transformation way to look at phenomena, associating energy to systems, but also, in few cases, growing up a primordial idea of energy conservation» (Colonnese, Heron, Michelini, Santi, & Stefanel, 2012).

[66] From the publication of the founding paper by Strike, Hewson, and Gertzog (1982), to be precise.

improving instruction. Nevertheless, since research on science education is still in its pre-paradigmatic phase (Kuhn, 1970; Fischer, 2013), not all of the educational theories are yet complete and sound. In the case of SR, research on *design and evaluation of teaching strategies* is lacking. Few innovative proposals have been formulated indeed, while most of the experimented pathways are based either on the old-fashioned *algebraic approach* by Resnick (1968) or on the *geometrical approach* by Taylor and Wheeler (1965), which is more innovative but has already been extensively experimented. Moreover, these teaching strategies have been worked out exclusively either for motivating students or for activating conceptual change. Historically and epistemologically founded teaching materials are needed, with the final aim to «promote physics as a culture» (Levrini, 2014), according to Galili's approach (2012), contributing thus to promote scientific literacy in its updated sense.

Several recent works have been carried out on the basics and/or kinematics, among which it is worth mentioning the ones by De Hosson, Kermen, and Parizot (2010); Arriassecq and Greca (2010); Dimitriadi and Halkia (2012); Velentzas and Halkia (2012).

In particular, the paper by De Hosson, Kermen, and Parizot (2010) has provided meaningful results on the role of *reference frames* and the *relativity of simultaneity*. The survey involved 94 prospective chemistry and physics teachers.

It turned out that:

- Mathematical tools like the Lorentz transformations and spacetime diagrams are not didactically effective, in agreement with the findings by Scherr, Shaffer, and Vokos (2001);
- The definition of inertial frames as a set of intelligent observers in different spatial positions at relative rest (Scherr *et al.*, 2002) is not effective too;
- Motion somehow "contaminate" events, in the sense that if the relative observer speed is mentioned, in most answers the location of the observer is not separated from the other variables into play;
- Some students do not differentiate between the instant in which an event occur and the one in which it is perceived by an observer;
- Relativity of simultaneity is thought to occur between observers of the same reference frame in some cases;

- Students tend to use classical patterns when solving relativistic kinematic problems;
- The classical concepts of event and inertial frame are seriously misunderstood even in classical theory in some cases: a scientifically correct conception of space and time is lacking in these students, which is the most prominent obstacle for grasping relativistic concepts.

The students involved in this survey need to exchange their epistemological beliefs («metaphysical commitments» according to Hewson, 1982) that phenomena occur in space while a universal time is flowing for a view of physical reality as made by *events in spacetime*.

At the science content level, all of the examined t/l sequences deal with either the *two postulates, relativity of simultaneity*, *time dilation* and *length contraction* (in agreement with algebraic approach, which in its original form also includes Lorentz transformations) or *events*, *invariance* of *proper time*, *unity of spacetime*, and *Minkowski diagrams* (in agreement with geometric approach).

The two postulates are important because relativity is a theory of principle, unlike previous ones. It is important for learning to make meaning of everything is theoretically assumed, since neither it comes out *ex nihilo* nor it stems from experiments solely. In Einstein's words (1950): «The theoretical idea […] does not arise apart from and independent of experience; nor can it be derived from experience by a purely logical procedure. It is produced by a creative act. Once a theoretical idea has been acquired, one does well to hold fast to it until it leads to an untenable conclusion».

Relativistic dynamics have been much neglected instead, in particular those parts directly involved in deducing and making meaning of mass-energy equivalence. Exceptions are Relativistic Concept Inventory (RCI; Aslanides & Savage, 2013) and attempts to deduce momentum (Peters, 1986) and energy (Sonego & Pin, 2005) expressions. Nevertheless, all these research outcomes are significant for the present research. Even if the geometric-kinematic contents have been given preferences in PER so far indeed, the conceptual aspects to be considered for the author's purposes are *essentially the same*. In fact, the topics of interest in the present work *include* the ones above; what has been added is substantially summarized by the relationship among total energy, mass and momentum $E^2 - p^2c^2 = m^2c^4$. It is a “static” equation, because of the inherently four-dimensional nature of relativistic motions (Einstein & Infeld, 1938).

V.4. **Mass-energy and “Relativistic Mass”**

An interesting proposal for defining mass in the special-relativistic case is the previously anticipated one by Brehme (1985). It rests on momentum conservation, which is equivalent to assert that *if we want momentum to be conserved, then mass must be defined in a certain way.* A relativistic perfectly inelastic collision is considered, in which a sample body S of mass m_s is steadily moving at speed v_s and will hit a target A initially at rest[68]. Be m the mass of the latter. The collision product will be a body of mass M and speed V. The four-dimensional formalism is used: momentum conservation equation may be split into a three-vector equation with the spatial components and a scalar one with the temporal ones. As for the collision being examined, this brings to

$$\begin{cases} \gamma m_s \vec{v_s} = \Gamma M \vec{V} \\ \gamma m_s c + mc = \Gamma M c \end{cases} \Rightarrow \boxed{m = m_s \gamma \left(\frac{v_s}{V} - 1\right) \xrightarrow[v/c \to 0]{} m = m_s \left(\frac{v_s}{V} - 1\right)}$$

A definition of mass in terms of a sample mass is thus obtained, both for the relativistic case both for the low-speed limit. In the latter case it is nothing but a variation of Weyl’s definition set out in IV.3.2, and it may be obtained from Mach’s definition by assuming a null initial speed for A.

This is of course *a model*:

- the objects’ speeds are assumed to be constant before and after the collision;
- the forces in the collision are assumed to be impulsive, i.e. lasting for a very short time[69].

However, the first requirement is not satisfied in practice, due to friction (in macroscopic collisions). So an experimental issue is: *when* is it more appropriate to measure the speeds, how long before and after the “instant” of collision t_{coll}? The values looked for would be in principle $v_s(t_{coll} - dt)$ and $V(t_{coll} + dt)$. If the speed measurements are performed with an experimental apparatus of sampling frequency f_s, rough discrete estimates are $v_s(t_{coll} - \Delta t_s), V(t_{coll} + \Delta t_s)$

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[68] This particular case is considered for making calculations easier, but in principle the procedure may be exploited in every collision.

[69] Typically of the order of the millisecond; anyway much shorter of the timescale in which any effect of the other forces is observed.

respectively, where $\Delta t_s \equiv 1 / f_s$. Better estimates are $v_s(t_{coll}), V(t_{coll})$, provided by a linear interpolation of the functions $v_s(t)$ and $V(t)$ respectively.

Most literature on mass-energy and the quantity commonly called "relativistic mass" is of *historical* or *theoretical-conceptual* character. The two facets are sometimes mixed up, as in the majority of papers by Okun (1989 – 2012). He has been much involved in struggling against the diffusion of "relativistic mass", which he considers as a «pedagogical virus», as well as against the logically consequent spread of mass-energy relation in the form $E = mc^2$, which has become a dangerous «element of mass culture» in his opinion (1989). The equation written in this form fosters indeed the idea that mass *is* energy, while it is actually «*a direct measure*[70] of the energy contained in a body» (Einstein, 1905a). Mass *represents* energy (Einstein & Infeld, 1938), since they are different quantities. In order to stress this point, Okun prefers to indicate rest energy with E_0 and write $E_0 = mc^2$, while Einstein's equivalence should be expressed by the total energy expression $E = \gamma mc^2$ in his opinion. In fact inertia, i.e. resistance to speed variation, is not expressed only by mass in Relativity, but it also depends of speed itself (Einstein & Infeld, 1938).

Dib (2013) made a recent noteworthy attempt to deduce mass-energy relationship for a mechanical mass-spring system according to Einstein's historical thought experiment (1905a). More specifically, Dib tried to parallel Einstein's lines of reasoning by showing that a system of three blocks connected by compressed springs (figure V.2a) owns a potential energy E_p *equivalent to a system additional mass*. In other terms, it was deduced that an additional mass $\Delta m = E_p/c^2$ is necessary to be consistent with Lorentz transformations of velocities. So (i) a simple mechanical system and (ii) relativistic velocity addition law were used in place of Einstein's (i) emission of electromagnetic radiation and (ii) the two postulates[72], in order to simplify the argument for didactical purposes. The *core idea is however the same*: to examine a system in two different energy states from two different IFs, the first at rest with respect to the center-of-momentum system (CM), the other in uniform rectilinear motion with respect to CM. Dib deduced first $E_p = 2mc^2(\gamma - 1)$ from relativistic energy conservation

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[70] Italics added.

[72] Curiously, the second postulate was not explicitly mentioned in the historical paper by Einstein (*ibid.*), perhaps because he meant it as included in the Maxwell equations (which is actually not true: the constancy of *c* in electromagnetism is different from its relativistic invariance).

in the rest frame. He then passed to the other frame, in which he showed that Galilean velocity transformations bring to an identity, if applied to classical momentum conservation. Nonetheless, *the same trick in the relativistic case leads to a contradiction.* The way out was to suppose that a term Δm had to be added to the initial total mass in momentum conservation

$$(M + 2m + \Delta m)\,\Gamma V = M\Gamma V + m\gamma_1 v_1 + m\gamma_2 v_2$$

Γ being the Lorentz factor associated to the relative speed V (figures V.2b, V.2d). If Lorentz transformations are used, we are left with $\Delta m = 2m(\gamma - 1) = E_p/c^2$. The argument is conceptually rather simple, but *it needs Lorentz transformations*, requiring thus to put too much emphasis (at least in my opinion) on mathematical formalization instead of the physical meaning behind the argument.

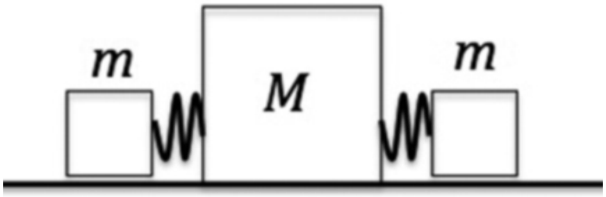

Figure V.2a. The mechanical system used in Dib's TE (2013): two blocks of mass *m* connected by two compressed spring to a central greater block of mass *M*. This is the initial configuration in the rest frame.

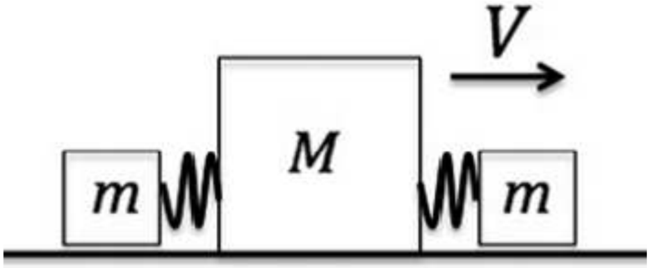

Figure V.2b. The system in the initial configuration seen in the moving frame (Dib, 2013).

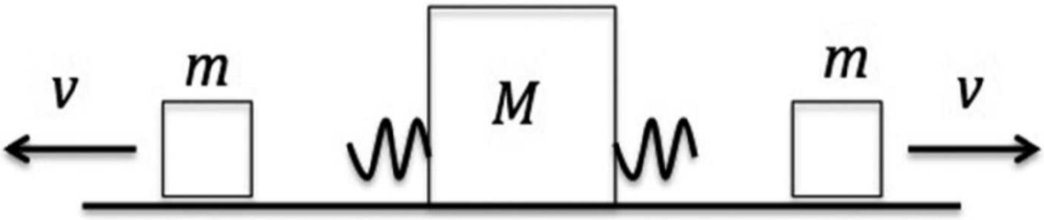

Figure V.2c. The system in the final configuration seen in the rest frame (Dib, 2013).

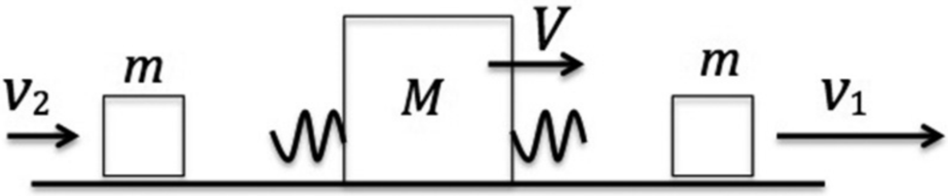

Figure V.2d. The system in the final configuration seen in the moving frame (Dib, 2013).

Elio Fabri (2005) is one of the few Italian physicists who engaged seriously in designing a t/l sequence on Special and General Relativity for high school. He worked out a thought experiment by tracing Einstein's one too, but without Lorentz transformations, so that it was interesting for the present purposes. He also claimed the didactical importance of avoiding "relativistic mass", which complicates easy notions. Moreover, "relativistic mass" is frequently used inside a contradictory framework and thus misconceptions are easily generated.

It seems useless to say that I share this viewpoint entirely.

On the contrary, Ireson (1998) claimed that mass is not an invariant quantity in relativistic mechanics. I have already asserted that several scientists and practitioners believed and still believe in a speed-dependent mass, among whom Kuhn, Feynman (1964), Born (1976), D'Inverno (1992), Jammer (2000), Rindler (2001), Penrose (2005). On the other side, Oas (2005a) pointed out that "relativistic mass" is not consistent with the geometrical approach to SR, and that it should be abandoned because SR is more and more recognized to be a *geometrical theory* essentially. Finally, Silagadze (2014) asked himself if this dispute makes any sense, and he worked out a positive answer, since modern physics is not currently taught worthy of the times, and it should be. He strongly claims indeed that current education ought to draw on the basic principles of special relativity and quantum mechanics, from which Newtonian theory should be derived as a special case, stressing the limitations of Newton's concepts. So he considers the introduction of "relativistic mass" useless and out-of-date, both because it hides the real importance of classical mass in the whole of modern physics, and since in modern physics (namely, modern field theory) the principle of least action is used, in which "relativistic mass" is unnecessary. Moreover, linear momentum is introduced by Noether theorem in modern physics, so there is no actual need to bring it back to the classical form by means of "relativistic mass". For these reasons, the latter is said to «hinder understanding of modern physics».

Eventually, very few PER papers containing *t/l empirical results on mass-energy equivalence* have been found in the literature. Three of them are reported here by way of example, all coming from surveys performed on university students:

1) a study on the understanding achieved by first-year engineering students visiting an exhibition in a science museum, as part of a t/l sequence (Guisasola *et al.*, 2009);
2) an analysis of difficulties in understanding the concepts of time, length, mass, and density, as well the relationships between them, faced by some pre-service teachers (Selçuk, 2011);
3) the findings from RCI, administered to 53 first-year physics students (Aslanides & Savage, 2013).

The following results came out respectively from these inquiries, as concerns *mass in SR* and *mass-energy relation*:

1) (Guisasola *et al.*, 2009) 31% of the involved students answered by stating mass-energy equivalence and describing the CERN simulation[74], but they *did not explain* what happens at CERN; 31% distinguished qualitatively between mass at rest and mass in motion at relativistic speeds; 38% argued that when speed reaches values close to *c*, it cannot be further increased. The second and third type of answers stem from the *conception of "relativistic mass.*
2) (Selçuk, 2011) About 66 – 67% of the «introductory group» understood mass in SR as an invariant quantity, but the remaining 33% considered it as *a speed-dependent quantity*; this is attributed by Selçuk to misconceptions formed during high school.
3) (Aslanides & Savage, 2013) The last question of RCI is about mass-energy equivalence, and concerned photon mass too. This makes it particularly interesting for the present research, thus the text is reported below:

> Consider a closed box, containing an equal amount of matter and antimatter. The total mass of this box and its contents is initially M. The matter and antimatter are then allowed to annihilate inside the box, turning into photons in the process. What is the total mass of the box and its contents after the annihilation? (a) Greater than M; (b) Equal to M; (c) Less than M.

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[74] It had been watched during the exhibition.

The question has got (i) negative normalized learning gain $g = -0.1$; (ii) null discrimination of understanding, (iii) a strong negative correlation with the previous question, probing causality invariance (invariance of the order of two events separated by a time-like interval). These results mean that: (i) the fraction of the correct answers *decreased* after instruction; (ii) all of the students obtained exactly the same percentage of correct answers in the post-test; (iii) the students that answered correctly to the question on mass-energy were very likely to have taken the wrong answers on causality and vice versa. Thus causality invariance seems to mislead students when answering on mass-energy equivalence. So the question proved unsuitable for quantifying students' learning according to Aslanides and Savage, who proposed to *drop it from the test*. However, if there are no other reasons, the negative value of g and the zero discrimination are not necessarily due to one bad question. *They might be due to wrong instructional tools*. The instructional strategies described in the paper seem in line with the most recent hints from educational research: lectures in a studio space to encourage interactions, discussion in small groups, and visual approach to relativistic effects by exploiting laboratory with Real Time Relativity software. However, the evaluation of the latter (Savage, Searle, & McCalman, 2007) showed some problems as for students' program handling and involvement. Truthfully, it must be added that the simulation was about relativistic effects, not matter-antimatter collision, photon mass and mass-energy equivalence.

The Teaching/Learning Pathway

VI.1. Research Operationalization: Writing specific RQs

The extracted conceptual nuclei listed in III.2 have been considered, together with the intended pieces of consensus knowledge (IV chapter), in order both to gain the «content structure for instruction» (Duit *et al.*, 2012) both to design research. This content was then reconstructed by considering the teaching/learning knots of interest, described in chapter V. A brief recall of each knot concerning both the classical part and the relativistic part of the t/l sequences is being provided, associated to the way worked out for overcoming it.

VI.1.1. Mass in Classical Physics

- Primary students[75] find the concept of mass and the scientific concept of weight difficult to learn; for this reason I inserted the gravitational interaction into the t/l pathway in such a way to join it with the operational definition of weight, which should be introduced at an earlier stage of instruction, according to Galili's suggestion (2001).
- The following "cognitive equation" is valid: $'quantity\ of\ matter' = m = \rho + V \neq \rho + V + g = W = 'mass' =' weight'$, in which the words in quotation marks are the exact word/phrase activating in students the corresponding concept, indicated by the quantity's symbol (m or W); the other quantities are conceptually "summed" for cognitively triggering either weight or mass (Mullet & Gervais, 1990).
- So (i) the owned intuitive conception may be *discriminated* by probing the presence of any reference to *gravity* (in educational research) and (ii) the science concepts can be also *separated* at the verbal level, by mentioning gravity explicitly (in teaching practice). Moreover, asking students to compare between 'mass' and 'quantity of matter' is useful to explore whether the pattern above or the ontological definition of mass (*quantitas materiae*) is rooted in their minds.

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[75] These difficulties will likely emerge at a more advanced level of education too, if they are not carefully dealt with by the teacher.

- Matter is not necessarily associated to mass in physics, although this one-to-one correspondence is often present in textbooks, so language was accurately selected[76].
- Gomez, Pozo, and Sanz (1995) found that matter conservation, and therefore mass conservation (in this context), tends to be better understood when a *change of state* is examined with respect to a *chemical reaction.* This occurs because, in general, physical reactions are perceived closer to everyday phenomena, and substance does not change, while in chemical ones different substances interact *all at once*, in contrast with the students' usual linear causal reasoning.
- An operational dynamical definition of classical mass in the general case *neither exists nor is achievable*. I chose the definition by Mach (1883) after having examined several alternatives with their strengths and weaknesses. This definition was improved in order to exploit on-line measurements of the speed variations. It is noted that it is merely a *possible* choice and not the one and the only. It is based upon the most general definition of force as *interaction*, but definitions resting on force as macroscopic cause of deformation – according to Bridgman's approach (1927) – are equally worthwhile for didactics. By the way, it is reminded that the circularity problem of interest is closely related to the vicious circle inside the first law of dynamics in its traditional form.
- Finally, several *t/l strategies exploiting history* and *philosophy of* science, lacking in educational research, have been developed and implementedin this work (see VI.3.2). The most important of them was the section on *classical mass* of the t/l pathway. The students examined excerpts from science texts through «interactive/dialogic approach» at the beginning (Mortimer & Scott, 2003). They reconstructed then the chain of arguments which brought Newton to attribute gravitational meaning to mass by a simplified treatise under the interactive tutor's guidance («interactive/authoritative approach»). They were finally illustrated Cavendish's experiment to account for the value of *G*.

Some of the knots above gave rise to *specific research questions* (*RQs*). They are listed below in the same order of the correspondent points, "C" standing for "classical". For example, RQ5C is the fifth RQ relative to the classical part. Other RQs were added *a posteriori* in order both to complete mass conceptual

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[76] Only *mass* is named in the path: neither reference is made to matter non-conservation nor to the (spread) idea that photons are pure energy (Fabri, 2005).

exploration at the content level both to probe if and how guided qualitative and quantitative phenomena exploration fosters conceptualization of inertial mass.

RQ1C) How and in which contexts do students relate to and use the word "mass"?

RQ2C) Which concept was actually activated through students' reflections and revisions of the classical aspects of mass, according to the research outcomes by Mullet and Gervais (1990)? Weight or mass?

RQ3C) How do students link the terms "mass" and "weight"? Do they associate the words "quantity of matter" and "mass" (*ibid.*)? If so, how do they motivate these associations?

RQ4C) Are students aware of the vicious circle in the Newtonian definition of mass as *quantitas materiae*? If so, how do they express it?

RQ5C) Do students acknowledge conservation and additivity of classical mass after the proper formative intervention module?

RQ6C) How do students conceptualize mass conservation law in the six most important typologies of chemical-physics transformations, namely spacetime translations, deformations, breakings, changes of state, solutions, and redox (IPS Group, 1967)?

RQ7C) Do students consider chemical transformations different from physical ones? If so, do they find mass conservation in chemical transformations more difficult to explain than in physical transformations? Are chemical transformations described using a linear reasoning?

RQ8C) How do students conceptualize inertial mass? Are they aware of the problems in its definition?

RQ9C) How do students conceptualize gravitational mass? Are they aware that it stems from inertial mass and in which way?

RQ10C) What role(s) do students attribute to gravitational mass in universal gravitation law? How do they express it?

RQ11C) How do students relate the facets of classical mass among them?

RQ12C) Do students recognize the linear relationship between speed variation ratios and number of system components in cart collisions? If so, how do they justify it?

RQ13C) How, if so, do students relate the measured speed variation ratio to inertial mass ratio in Mach's definition?

VI.1.2. **Mass in Special Relativity**

Learning knots and choices for the educational path (*relativistic part*):

- The concept of energy is misunderstood by most students, in particular its conservation. If energy is introduced using force and work, in particular as the "ability of doing work", its understanding will likely be limited (Lehrman, 1973; Duit, 1985; Papadouris & Constantinou, 2011). The same is valid if it is considered as an abstract accounting quantity (Nordine *et al.*, 2011): it is not enough for an effective understanding, for quantification ought to come at a later stage (compare V.2).
- *Conservation* and *transformation* are the basic aspects for achieving a sound understanding of energy (Colonnese *et al.*, 2012; Duit, 2012; Neumann *et al.*, 2013). They are examined in Bertozzi's experiment (1964), which involved the motion and stopping of electrons in the relativistic regime, and also let students focus upon heat and work as ways for internal energy change. For this reason the experiment[77] was proposed in the second t/l pathway. Total energy conservation occurs in the calorimetric measure associated to it, the system being composed by the electrostatic accelerator, the electron beam, and the stopper. In this instance, energy transforms from *electrostatic potential energy* of accelerator and electrons[78] to *kinetic energy* of the free electron beam, and finally to *internal energy* of stopper, the total amount remaining constant. In fact, considering the whole system as split in these three separate sub-systems, they *interact*[79] and each sub-system decrease in energy is compensated by the increase of the subsequent one in a chain[80].

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[77] More specifically, in 1964 William Bertozzi carried out an experiment in which electrons emitted by a cathode were accelerated to high speeds – namely approximating light speed *in vacuo* at 1% of accuracy – in the wake of Kaufmann's measurements of the electron charge/mass ratio for determining the supposed functional dependence of mass on speed. The relativistic speeds were achieved through an electrostatic accelerator (Van de Graaf) and a LINAC for getting higher energies in the last two runs. Unlike Kaufmann's and many other similar experiments, Bertozzi's one at MIT had clear educational aims.

[78] Noteworthy, potential energy was considered as belonging to the whole system, for it makes no sense to consider it for isolated objects (Heron *et al.*, 2010). Some scholars claim that this statement is valid for kinetic energy too, as well as for any other form of energy, the latter being *a property of a system* instead of its components (Doménech *et al.*, 2007; Duit, 2012).

[79] The concept of energy transformation may be qualitatively associated to the state/configuration changes made by any interaction between them (Colonnese *et al.,* 2012).

[80] Of course, this would be strictly valid only if energy "losses" owing to the real experiment implementation were neglected.

- The task of considering processes different from heat and work – stated in section V.2 – was achieved by a *photon absorption* thought experiment inspired by Fabri (2005) and by the energetic analysis of *β-decays*.
- The mass-energy matching problems were solved (i) by highlighting the new *abstract* meaning that mass takes on in SR, in which it is operationally defined by an equation involving other directly measured quantities and (ii) by defining the additive constant – i.e. zero-point energy – as rest energy.
- Owing to the problems reported by De Hosson, Kermen, and Parizot (2010), the crucial role of *inertial frames* and *relativity of simultaneity* has been taken into consideration in the present proposal. The part of t/l path including those contents was proposed during a winter school for students attending the fourth and fifth year of upper secondary school. Namely, the following contents were tested:
  - Relative motions;
  - The role of observers in inertial frames (Scherr *et al.*, 2002);
  - Emission/perception of signals by intelligent observers (*ibid.*);
  - Relativity of simultaneity (*ibid.*);
  - Time dilation and length contraction;

    (Lorentz's transformations were carefully avoided).
- Okun and others warn against $E = mc^2$, which has become a dangerous «element of mass culture» (Okun, 1989): for this reason the equation above has never been written, but $E_0 = mc^2$ and $E = \gamma mc^2$ have been exclusively used in their proper different contexts. The former is indeed the real mass-energy equivalence, stating a direct relationship between mass and rest energy. The second is the equivalence in its widest sense, supplied by the relativistic expression for (total) energy (compare V.4).
- Eventually, since relativistic mass has proved to cause many *misunderstandings* and *learning difficulties* (V.4), due to the emergence of several theoretical-conceptual inconsistencies, it has never been mentioned in the designed t/l pathway, designed so as to such a "quantity" cannot be conceived.

Specific RQs ("R" standing for "relativistic"):

RQ1R) Do students recognize the role of c in Relativity, as for invariance and ultimate speed character? How do they express their final conceptions?

RQ2R) Do students acknowledge that simultaneity is not invariant in SR? If so, how do they express the grasped concept?

RQ3R) Are time dilation and length contraction effects understood by students, or described as "distortion of perception" effect, or not achieved at all? In the first and second cases, how do students characterize them?

RQ4R) Do cognitive transfer about time dilation occur in students?

RQ5R) Do students understand and/or learn the meaning of dynamical quantities in a new paradigm (SR)? In which ways do they reason about them?

RQ6R) How do (skilled) students interpret the conceptual extension from mass in its classical meaning to mass as rest energy in the relativistic context?

RQ7R) How do students interpret and justify phenomena in which mass is not conserved?

RQ8R) How, if so, do students correlate a phenomenology in which mass is not conserved (i.e. β-decay) to mass-energy equivalence?

RQ9R) Do students acknowledge that mass is *neither* conserved *nor* additive in SR? In which ways do they express these physical claims?

RQ10R) Do students discriminate between mass non-conservation and mass invariance in SR?

Some RQs concerning thought experiments may also be asked:

RQ11R) Are thought experiments effective in relativistic context?

RQ12R) Do they generate accommodation or assimilation?

RQ13R) Suppose a negative learning outcome is detected. Does it stem from a thought *simulation* instead of a thought experiment (Gilbert & Reiner, 2000)?

VI.2. **Classical Mass**

As anticipated in III.4, two t/l paths have been worked out, one being compounded by the classical part and the formal relativistic one (four-vector approach), while the other by the classical part and the relativistic part with phenomenological approach arriving to relativistic energy (energetic approach). The whole path may be subdivided in independent modules which were experimented in my research work.

The first common part was designed with the aim of affording the theoretical concepts concerning mass in classical physics through a *historic-*

epistemological analysis of science texts. On the other hand, theory was integrated with a *real experiment*.

VI.2.1. **Classical part of the Conceptual Pathway**

This section of the conceptual pathway was tested at two different depth levels in different formative intervention modules; namely the path sub-section in VI.2.1.2 was not implemented at the earlier research stage and the ones in VI.2.1.1 and VI.2.1.3 were more deepened at the final stage than at the earlier one.

VI.2.1.1. *Newtonian mass*

As stated in IV.3.3, Newton acknowledged that mass is different from weight. He also supplied an operative definition of the former through the latter, as a consequence of its accurate experiments with two pendulums. Gravitational mass can be easily determined by a dynamometer in this way. It is also recalled that Newton was the first to establish WEP experimentally, although Galileo had discovered the pendulum isochronism law (Motz & Weaver, 1991; Jammer, 2000). Newton invented *inertial mass*, but he never sharply differentiated it from *quantitas materiae*; in fact inertia, and not mass, is the main subject of the excerpts from *Principia* inserted in the t/l pathway.

Inertial mass takes on *a new significance within Universal Gravitation Law*, as it was discovered by Newton (Cohen, 1981). The argument for deducing the latter begins with the assertion that a lot of celestial bodies move on elliptical orbits, a special case of which are *circular* orbits, the only ones being considered in this context. The motion of a planet with inertial mass m_i around a star with inertial mass $M_i \gg m_i$ is being studied, more precisely the origin of the *centripetal force* contrasting the inertial motion of the planet. In the case of uniform circular motion the deduction is simple. Three laws are being assumed:

1. The second law of dynamics $F = m_i a$ in scalar form;
2. The third law of dynamics $\vec{F}_{12} = -\vec{F}_{21}$in vector form;
3. The third Kepler's law $(r^3 / T^2 = k)$ extended to any stellar system.

The centripetal acceleration of the planet is

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 \left(\frac{r^3}{T^2} \right) \frac{1}{r^2}$$

into which the III Kepler's law may be plugged, for obtaining the centripetal force $F_c = m_i\, a_c$ that is the force exerted *by the star on the planet*:

$$F_c = m_i \frac{v^2}{r} = 4\pi^2 \frac{k\ m_i}{r^2}$$

The magnitude of this force has an intrinsic component, given by the direct proportionality to the planet's mass, and a geometrical one, given by the inverse-square proportionality to the distance between the planet and the star.

Because of the third Newton's law the planet must exert a reaction on the star by attracting it too, even though far more slightly. The magnitude of the force exerted *by the planet on the star* must therefore be

$$F_c = \frac{4\pi^2 k' M_i}{r^2}$$

This implies the general interaction expression contains the mass product:

$$F_g = \frac{4\pi^2 k'' M_i m_i}{r^2} = G \frac{M_i m_i}{r^2}, \quad G = 4\pi^2 k''$$

This reciprocal attraction was historically called 'gravitational' force F_g, always attractive, as masses are always positive. The law above was extended to all bodies in the Universe[81]: in Feynman's words, Newton «proposed that this was a *universal* force – *that everything pulls everything else*» (Feynman *et al.*, 1964). The universal gravitational constant was determined by Cavendish in 1794. From the administered worksheets: «G was found through the accurate measure of the (weak) gravitational force between two couples of lead spheres of different volume fixed at the ends of a torsion balance (Cavendish's experiment, 1794). A direct measurement of the rotation angle allowed to find the force magnitude; since the sphere masses and their distances (length of the balance arm) were known, it was possible to find the gravitational constant with high accuracy, by reversing the law above: $G = F_g\, r^2/(M\, m)$.»

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[81] It was a very significant conceptual step in the physics of the 17th century.

The genesis of the Gravitation Law is also educationally reconstructed through further indirect and direct quotations from *Principia* and then compared to Coulomb's electrostatic force law, in order to draw a *formal analogy* between gravitational masses and electrical charges. Later the *definition* and *estimate* of the former are separately considered: the exact theoretical definition may be obtained from the Universal Gravitation Law, while an estimate is provided by the ratio

$$m_g \equiv \frac{W}{g}$$

where weight is measured by *static scales*, for instance a spring balance. This comes from the specialization to the bodies "close enough" to the Earth's surface by introducing the intensity of gravitational terrestrial field $g = G\ M\ /r^2$, constant for "small enough" relative variations of the distance from the Earth's centre.

Exploiting Hooke's law, the static equation above turns into the following, which includes a structural parameter of the spring (k), a directly observable quantity (compression/elongation) and a tabulated physics constant (g):

$$\boxed{m_g \equiv \frac{k\ \Delta l}{g} \text{(static conditions)}}$$

Finally, the Newtonian concept of mass is being *destroyed* under the *quantitas materiae* and inertial respects. To begin with, Mach's criticism against mass as amount of matter (see IV.3.1) concluded the first part, pointing out the density-mass-volume vicious circle.

VI.2.1.2. *An up-to-date implementation of Mach's solution*

The force-mass-acceleration vicious circle in $F = m\ a$ has been extensively dealt with in IV.3.2 and V.1.3. A possible solution was provided by Ernst Mach through his empiricist operative definition of inertial mass that did not need introducing the critical notion of force, because it was exclusively based on measuring the accelerations owing to the mutual interactions occurring in an elastic collision; the minus sign is due to the opposite directions of the outgoing accelerations:

$$\frac{m_B}{m_A} = -\frac{a_A}{a_B} \Leftrightarrow m_B = -\frac{a_A}{a_B}\ m_A$$

Since the collision time intervals $\mathit{\Delta} t_{coll}$ are supposed to be equal, if $m_1$is taken as the unit mass, viz. the mass of a sample object – the empty cart in the run experiment (figures VI.1 and VI.2) – then the mass m_2 of any other object – the mass of the other empty/charged cart in the experiment – is measured by the simple equation

$$\boxed{m_2 = -\frac{a_1}{a_2} = -\frac{\Delta v_1}{\Delta v_2}}$$

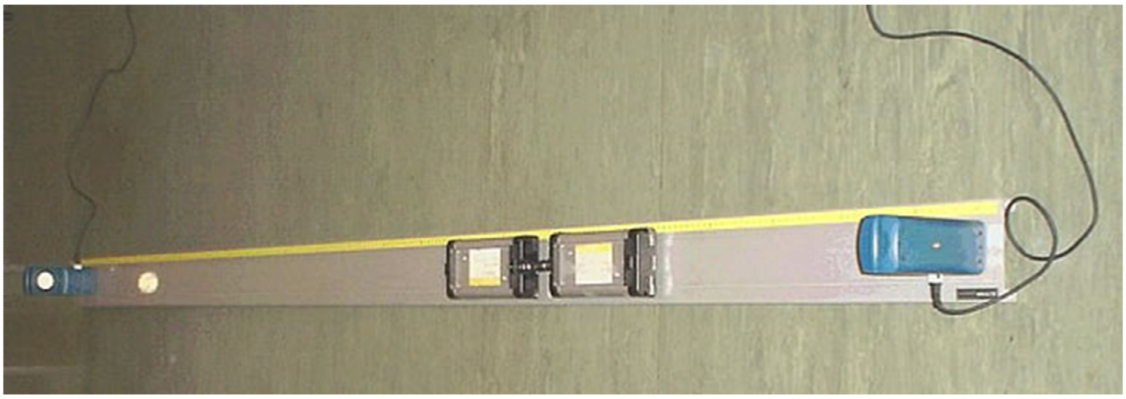

Figure VI.1. An educational real experiment: the kinematic study of quasi-elastic one-dimensional collisions between macroscopic carts (in the centre of the image) running on a horizontal rail. The experiment is performed with the help of real-time motion sensors (the blue devices at the ends of the rail). It is called Real-Time Experiment and Images (RTEI; Sassi & Vicentini, 2008) for the important role played by visualization. It is recalled that real collisions are *quasi-elastic*, since the kinetic energy of the carts is never exactly conserved, because of friction.

Figure VI.2. The colliding carts may be uncharged (empty, on the left) or one may be charged with up to three metal bars (on the right).

Mach’s definition thus becomes a conceptual referent. It is valid in any reference frame in uniform linear motion with respect to the laboratory, since accelerations are invariant for Galilean transformations in classical mechanics, and thus speed variations are too. The numerical value of an object’s inertial mass is thus *univocally determined* in inertial frames.

VI.2.1.3. *Mass conservation and additivity*

Eventually, since mass *is conserved* in classical physics, its measure cannot vary if a massive object undergoes the six most important types of chemical-physical transformations: (i) space-time translations, (ii) deformations, (iii) breakings, (iv) phase transitions, (v) chemical solutions and (vi) redox (IPS Group, 1967). Mass turns out to be an *additive* scalar quantity, because of its invariance for breakings. A comprehensive definition of additive quantity is then supplied with examples. Finally, the attention is focused on the logical interrelation between conservation and additivity of mass, the second with reference to the collision experiment.

VI.3. **Mass in Special Relativity**

VI.3.1. **Four-vector Approach: Conceptual Pathway**

The first worked-out relativistic conceptual pathway consisted essentially of *building up relativistic kinematical and dynamical four-vector quantities structurally similar to the three-vector mechanical classical quantities*. The aim was to re-construct the concept of mass in SR educationally through (i) a focus on invariance and invariant quantities, in particular the Minkowski norm invariance, and (ii) the introduction of the momentum-energy four-vector, whose Minkowski norm just gives mass: $E^2 - p^2c^2 = m^2c^4$.

At the beginning the two fundamental postulates of Relativity are stated:

3) *Theoretical RP*: every physical law has the same expression in all IFs, included electromagnetic and optical laws, unlike[82] classical RP. *Experimental RP*: any kind of experiment carried out in the same conditions in different IFs gives the same outcomes.
4) *Invariance of c*: the value for light speed *in vacuo* is neither dependent of the specific IF in which it is measured nor of the propagation direction.

Time measurements are then considered. A *light clock* of height *h* is considered to that end, in which the periodic phenomenon for the operational definition of time interval (see IV.1) is the back-and-forth path of a light ray reflected by a mirror S, the time unit being $2h/c$.

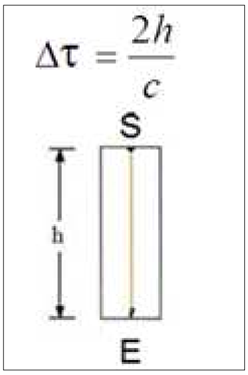

Figure VI.3. Sketch of a light clock with height *h* and period *2h/c*, the latter being chosen as the unit time.

Being A and B two observers in different IFs endowed with two *identical light clocks*, suppose A is standing in a train moving at uniform speed *v* compared

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[82] Although Fabri (2005) would not agree on this distinction.

with the station and B is standing still on the station's platform. The latter will be seeing its light clock as static and the identical clock of A as travelling at *v*, so a horizontal distance Δx will be covered in a time interval Δt. First of all, it is important to remind that *only the observer B is undertaking the whole analysis.* The light of the train clock goes through a longer trajectory than the one in the other, since hypotenuse is longer than vertical cathetus in the right triangle seen by B (figure VI.4). He calculates the length of the path forward by exploiting the Pythagorean Theorem:

$$\frac{c\Delta t}{2} = \sqrt{h^2 + \left(\frac{\Delta x}{2}\right)^2}$$

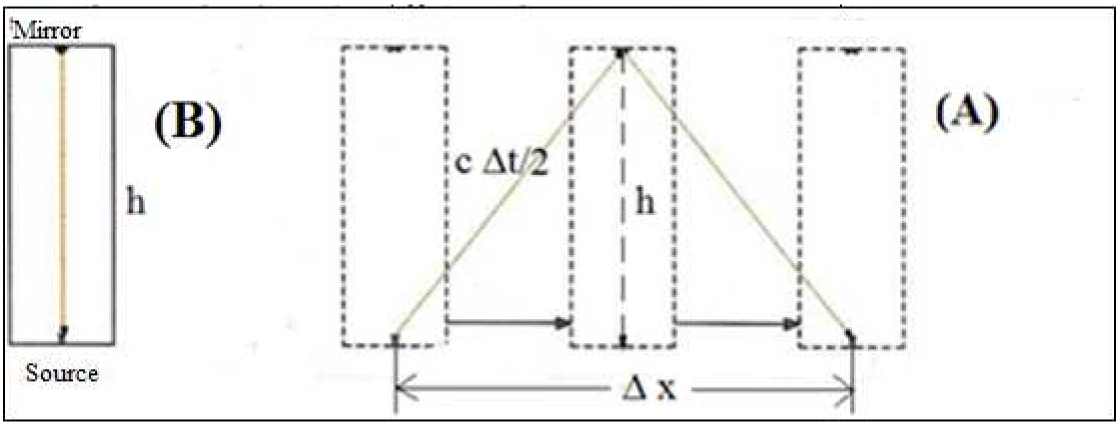

Figure VI.4. Sketch of one beat of the light clocks of the observer B (left) and A (right) *both seen by B, at rest compared to the platform.*

On the other hand, the observer B must find $2h/c$ for the back-and-forth path of his identical clock. The *time interval between two events occurring in the same position* (in this case the emission and reception of light, occurring in the same spatial point) is called *proper time interval* Δτ. It is important to define proper time in terms of events in a space-time structure instead of phenomena related to one object in the rest frame (Levrini, 2014). For instance, the time for one-way path of an horizontal light ray (i.e. moving in the direction of motion) is often thought to be a proper time because the ray is at rest with respect to the observer and the train, but it is *not*, for there is a spatial distance between the departure and arrival events (Levrini & diSessa, 2008). Since any transversal length is invariant[83], B is allowed to write

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[83] In fact, the motion along x-axis has no relativistic effects in any direction orthogonal to it. The *transverse components of a physical quantity* are always invariant.

$$\frac{c\Delta t}{2} = \sqrt{h^2 + \left(\frac{\Delta x}{2}\right)^2} = \sqrt{\left(\frac{c\ \Delta\tau}{2}\right)^2 + \left(\frac{\Delta x}{2}\right)^2}$$

$$(c\Delta t)^2 = (c\Delta\tau)^2 + (\Delta x)^2 \qquad (1)$$

$$\frac{\Delta x}{\Delta t} \equiv v \Rightarrow (c\Delta t)^2 = (c\Delta\tau)^2 + v^2(\Delta t)^2$$

$$(\Delta\tau)^2 = \frac{c^2 - v^2}{c^2}(\Delta t)^2 = \left[1 - \frac{v^2}{c^2}\right](\Delta t)^2$$

$$\boxed{\Delta t = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\Delta\tau \iff \Delta t = \gamma\Delta\tau} \quad (2)$$

The reader is sent back to section IV.5.1 for further theoretical details.

The *length contraction* effect is also *qualitatively* deduced from time dilation, even if it is not strictly necessary for going ahead; for this reason it has been dropped in the second conceptual pathway. Consider the observer B measuring the length of a *horizontal* light-clock by means of the time for one back-and-forth light path. B sees that light takes a (proper) time $\Delta\tau$ for coming back to the point E in its static clock (picture VI.5, left); therefore the double length of the clock will be $2l_0 = c\Delta\tau$, where l_0 is called *proper length.* B also sees the identical clock owned by A travelling at *v* in the train, and he will measure the same light speed, but a different back-and-forth time, for two reasons: the motion of the second clock (classical effect) and time dilation (purely relativistic effect). One can qualitatively assert that, in this case, since the elapsed time interval measured by B is shorter – the unit time being greater because of time dilation – and light is assumed invariant, the *space between the clock walls will have to contract.* Thus B will find the other clock shorter than his (picture VI.5, right).

Figure VI.5. Sketches of the horizontal identical light clocks of the observer B (left) and A (right) *both seen by B, at rest compared to the platform.*

It is also possible to formally deduce the expression for length contraction in a simple way, which was inserted in the path as an additional sub-section and tested in only one formative experiment (2012 winter school, compare VII.6), because the path was becoming too formal. The observer B will write for the clock at rest $l_0 = \frac{1}{2} c\Delta\tau$ and for the moving one $\Delta t = \gamma\Delta\tau$, Δt being the back-and-forth time. Thus $l_0 = \frac{1}{2} c\, \Delta t/\gamma$. For finding Δt, let us consider the back and the forth path times separately: since the clock is moving, it can be written $c\Delta t_F = l + v\Delta t_F \Rightarrow \Delta t_F = l/(c - v)$; $c\Delta t_B = l - v\Delta t_B \Rightarrow \Delta t_B = l/(c + v)$. $\Delta t = \Delta t_F + \Delta t_B = 2lc/(c^2 - v^2)$. By plugging the last equation in the expression for l_0, one finds $l_0 = \frac{1}{\gamma} c^2/(c^2 - v^2)\, l = \gamma\, l \Rightarrow \boxed{l = l_0/\gamma}$ (without Lorentz's transformations).

It follows from equation (1) that $(c\, \Delta\tau)^2 = (c\, \Delta t)^2 - (\Delta x)^2$. The right member of the latter has duration and length inside as measured by an observer in *any IF*, but the same proper time – univocally defined – is in the left member in every case. Therefore time and space intervals have different values in different IFs, but *proper time interval must be a relativistic invariant,* as well as the squared *difference* of the right member of (1). The same is true for the *space-time interval* Δs, defined by $(\Delta s)^2 \equiv (c\, \Delta\tau)^2 = (c\, \Delta t)^2 - (\Delta x)^2$. This supplies the basis for showing the space-time interval in its complete form, extending to the other spatial dimensions (compare IV.5.1):

$$(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2.$$

Then the *displacement four-vector* $(\Delta x, \Delta y, \Delta z, c\Delta t)$ is introduced as extension of the classical position vector, by analogy. Its modulus is deduced to be the space-time interval, also said “four-interval” or “line element” of Minkowski geometry, which in differential form is

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2.$$

Alternately to the time dilation calculus, the formal way resting on the two postulates and space isotropy may be followed, which was expounded in IV.5.1.

Four-velocity is then defined as the four-displacement divided by proper time (the only invariant time in SR) in the limit $\Delta\tau \to 0$, that is the derivative of position 4-vector (x, y, z, ct):

$$u^\mu \equiv \left(\frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau}, c\frac{dt}{d\tau}\right) = \left(\gamma v_x, \gamma v_y, \gamma v_z, \gamma c\right).$$

The result was obtained by exploiting time dilation formula in differential form: $dt = \gamma\, d\tau$.

Four-momentum is given by $p^{\mu} = mu^{\mu} = (m\gamma v_x, m\gamma v_y, m\gamma v_z, m\,\gamma c)$, m being the Newtonian inertial mass. If the classical limit of the last component is taken[84], the following result will be obtained:

$$\boxed{\gamma mc^2 \underset{\frac{v}{c}\to 0}{=} mc^2 + \frac{1}{2}mv^2 + O(v^4)}$$

As the second term is the classical kinetic energy expression, the relativistic correspondent may be *inductively* assumed to be $K_{rel} \equiv (\gamma - 1)\, mc^2$, neglecting fourth or higher order terms, by exploiting the Correspondence Principle, since the zero-order term must have the same form for high speeds too: it does not depend functionally on speed. Therefore, the *temporal component of momentum-energy vector may be interpreted as total relativistic energy* $E \equiv E_0 + K_{rel} = \gamma mc^2$, where E_0 is the additive constant at null speed. Thus we are left with $E_0 = mc^2$, which is the actual innovative physical result.

Relativistic momentum is then found, extending the correspondence above to the first three components: $\gamma m\vec{v} \cong m\vec{v} + O(v^3)$ for low speeds. Thus we are left with $\vec{p} = \gamma m\vec{v}\,, E = \gamma mc^2$. By analogy with 4-displacement, the norm of 4-momentum will be given by the squared last component *minus* the square modulus of the vector formed by the other components:

$$(\gamma mc)^2 - \gamma^2 m^2 v^2 = \frac{1}{\frac{c^2 - v^2}{c^2}} m^2\left(c^2 - v^2\right) = m^2 c^2.$$

If a simple substitution is made in the left member, we are just left with the master equation anticipated above, which indicates that energy and momentum are *a unity* in SR, exactly like space and time.

$$\left(\frac{E}{c}\right)^2 - p^2 = m^2 c^2$$

This part of conceptual pathway was inspired by Eddington (1920), Einstein (1935), Taylor and Wheeler (1965).

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[84] For the sake of precision the multiplicative factor *c* is added, for dimensional reasons.

A *thought experiment* was then designed and 'run' in order to support the validity of mass-energy relation.

A photon is bouncing between two mirrors in a box, while the box (of total mass M) is being thrust by an external force F at a fixed acceleration a along the axis of propagation of the photon. The relativistic momentum expression $p = E/c$ for the photon (ultrarelativistic limit) is used for dealing with the interaction photon-electrons of the atoms in the walls. It was modelled by a Compton scattering. Taking into account that the reflections on the forward and backward mirror occur at different times, due to the finite propagation velocity of the photon, the impulses transmitted to the box by the photon in the reflections are unbalanced, because of the different velocity of the box at those instants. The running of the experiment through mathematical calculations gives the following relation between the rate of impulse variation and the external acceleration:

$$\frac{\delta I}{\delta t} = -\frac{E_\gamma}{c^2}\, a.$$

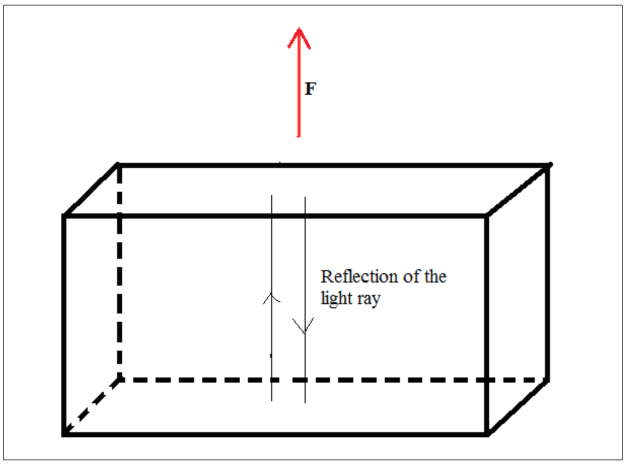

Figure VI.6. The box undergoing a constant force with one photon inside.

The sign is due to the fact that the impulse by the photon contrasts the system acceleration. Therefore the net result is a *very slight* backward thrust on the box that requires an *additional external thrust* to keep acceleration constant, the force variation being expressed by $\Delta F = \left(E_\gamma/c^2\right) a$, where E_γ is the initial photon energy. Calculations are provided in appendix 5. The *semi-classical* framework of the experiment is worth underlining. In fact (a) studying the system motion using force and acceleration is not a relativistic way of reasoning; (b) the classical relationships between photon "force", impulse and linear momentum were considered; (c) the classical kinetic energy and momentum were put into the equation, because the acceleration is meant to be too faint to increase speed until relativistic values.

In conclusion, *the inertial properties of a physical system observed as a whole are described by a dynamical parameter different from the mass sum:* $M + E_\gamma/c^2$ in this case. Mass is not additive indeed. So the parameter describing the additional inertia of the entire system with respect to the empty box turns out to be E_γ/c^2. It is an additional inertial mass δm according to Newton's law, valid since the box is not relativistic. This is consistent with Einstein's equivalence.

This conceptual pathway utilizes the *specialized metaphor*, but it traces a too much formal and forced way: nothing justifies the strong analogy drawn between three-vectors and four-vectors. Further, this conceptual pathway does not ensure the four-vector norm invariance *a priori* (Sonego & Pin, 2005). It is scientifically correct, but it risks to be not intuitively grasped by students. Moreover, the logical need for passing from postulates to time measurements is not clear. Eventually, there are problems with the thought experiment: the box kinetic energy is summed to the photon one, which is *very much smaller*, in the initial energy conservation equation. This is mathematically correct, but makes little sense in physics. It is analogue to the sum of the kinetic energy of a box containing gas and the energy of a single gas particle. This thought experiment may be therefore used for introducing a qualitative and approximate quantitative idea of the fact that, roughly speaking, *energy has mass*.

For all the reasons above another relativistic conceptual pathway was designed.

VI.3.2. Energetic Phenomenological Approach: Conceptual Pathway

Mass-energy relationship was achieved from the *energetic side* in the second conceptual pathway, due to four basic reasons.

First, conceptualizing mass through energy is a demanding challenge, but it is also *more logical and interesting* than starting from mass[85] and then extending to energy just for stating the relationship. Secondly, energy is *more fundamental than mass* and it is a *quantity extremely significant in the whole of science*. Since «everything that has mass has energy, yet not everything that has energy has mass», it is more appropriate to conceptually define mass drawing on energy in Relativity (Hecht, 2011). Likewise, I think it is better to arrive to mass from energy than following the converse path. Thirdly, energy is a *transphenomenological quantity*, which «has been identified as one of a small number of disciplinary core ideas for science learning» (Papadouris & Constantinou, 2014). Therefore learning energy fosters the unitary view necessary for *innovation* in t/l modern physics (compare II.1), of which Einstein's relationship is a major part. The same cannot be asserted for mass, for it is present in a smaller number of physics branches. Fourthly, learning issues on energy have been studied *far more extensively and in depth* than those on mass, so that the research outcomes may be considered as better established.

VI.3.2.1. *Bertozzi's educational experiment*

I consider the experiment by William Bertozzi (1964) as a good starting point involving energy and its properties (compare VI.1.2).

A recall of introductory elements of classical dynamics and electrostatics is provided to that end, by work and kinetic energy definitions, then derivation of work-energy theorem in its classical form and definition of electron-volt[86]. The equation $\Delta K = \pm\, e\, \Delta V$ is finally obtained by combining work-energy theorem with electrical work expression for an elementary charge. If the particle is initially

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[85] The actual objective of the whole work at the content level.

[86] At this point the electron-Volt (eV) is defined as energy unit for the charged particles. 1 eV is said to be the *kinetic energy gained by an elementary charge when passing through* 1 V, since electrical charge is assumed to be quantized, the quantum being e = 1.6 * 10^{-19} C. From $\Delta K = -q_0 \Delta V$ it follows 1 eV = 1.6* 10^{-19} J. The eV is then extended to be the measure unit for all particles and forms of energy.

at rest that equation will turn into[87]

$$e\,\Delta V = \frac{1}{2}m\mathrm{v}^2 \Rightarrow \mathrm{v}^2 = \frac{2e}{m}\,\Delta V \tag{3}$$

Thus classical mechanics predicts a *theoretical limitless speed increase with potential difference increase* according to the power law $v^2 \propto \Delta V$.

In 1964 William Bertozzi tested this prediction for high-speed accelerated charged particles. He used electrons instead of protons because $m_e \sim \frac{1}{2000}\, m_p$: high speeds are far more rapidly reached by increasing the voltage, as follows from equation (3). The experimental apparatus is depicted in figure VI.7.

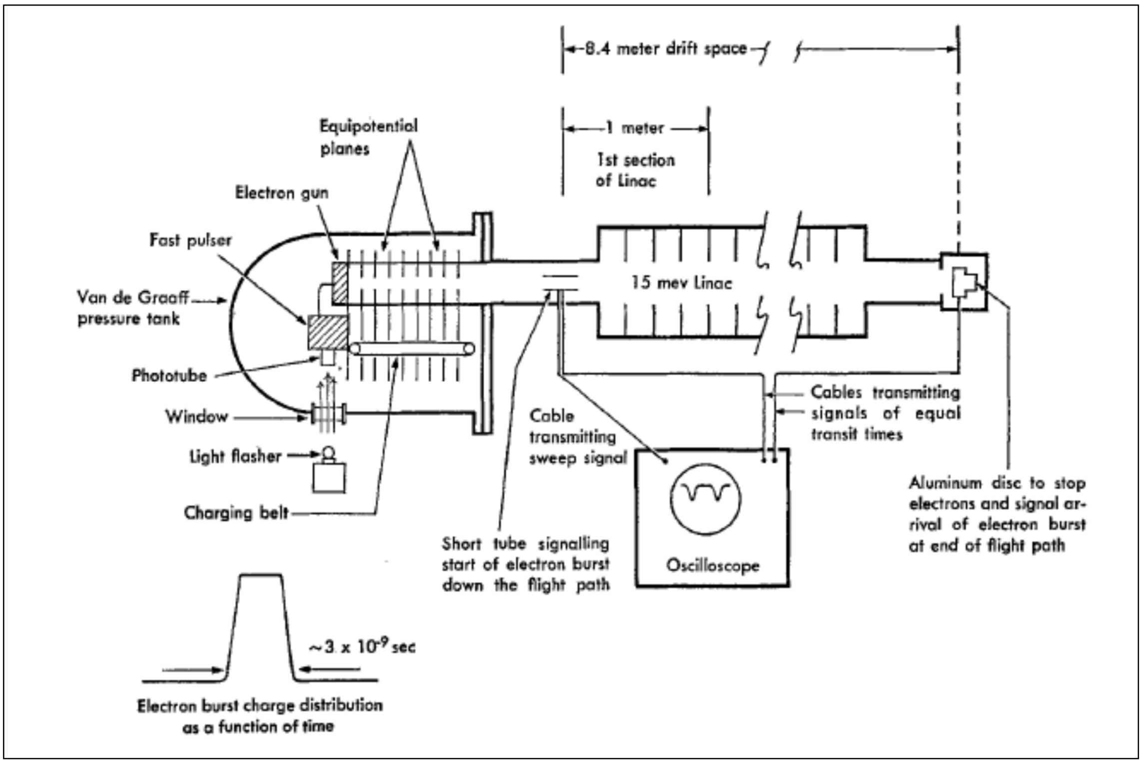

Figure VI.7. Experimental apparatus by W. Bertozzi for electron acceleration (1964). The path of the particles is in red; it is composed by three parts (from left to right): acceleration by Van der Graff; flight at constant speed until the capacitor tracing the instant of particle crossing (*departure*); free flight until the aluminium disk tracing the instant of *arrival.*

The *educational value* of the experiment is multi-faceted. First of all, even though a lot similar experiments were in Sixties, it was designed to highlight the existence of an ultimate speed in Nature as straightforwardly as possible in a visible way; for this reason the plot in figure VI.8 was shown during almost all the corresponding video, belonging to the PSSC (1995) educational video series. The

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[87] The modulus of the particle charge has been taken in this passage.

experiment shows the divergence between the predicted and observed velocity increase when the accelerating voltage is in turn increased, thus highlighting the *limitations of classical physics*, which is important for fostering modern physics literacy (Gil & Solbes, 1993). Likewise, the logical exclusion of the classical expression for kinetic energy follows from the calorimetric measure in the experiment, which in turn brings to the invalidation of $F = ma$ and of the entire classical dynamics consequently (*modus tollens*). So, if the present path were followed, a student could also learn hypothesis revision when an experimental test gives a negative outcome. It is thus a way to show hypothesis falsification and an occasion for speaking about theory falsification, which is not ordinarily dealt with in school curricular programs. Furthermore, the final experimental result is *local*, which allows to discuss the problem of generalization and of induction. Last but not least, the four basic properties of energy – conservation, transformation, transfer and degradation – are displayed in the experiment in several ways (compare VI.1.2).

In that experiment the mean speed is ‘directly’ measured through the time of flight (TOF) for a fixed distance: 8.4 m. TOF for different voltages was measured by an oscilloscope revealing *departure* and *arrival*, i.e. the instants when the particles cross the capacitor and hit the aluminium disk respectively. In the last two runs (d, e in figure VI.9) speed is not constant, since a linear accelerator (LINAC) rather than electrostatic accelerator was used. The collected data are *surprising* from the classical point of view.

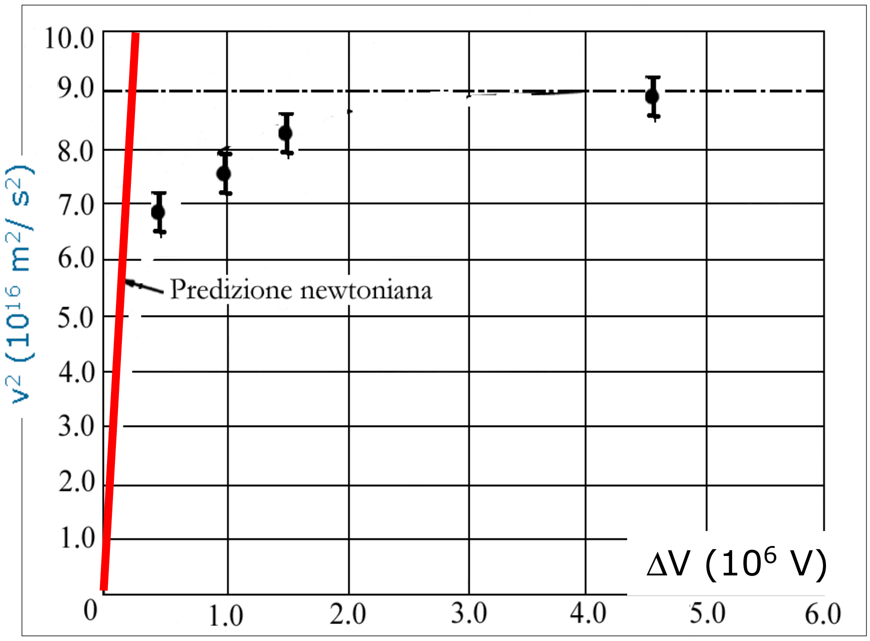
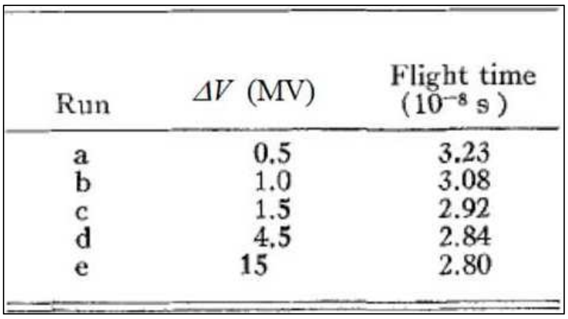


Figure VI.8. . Measures by Bertozzi, but the fifth one, in which the motion is not uniform at all. The expected classical trend is represented by the red line. A prominent divergence between the predicted and the observed trend is present, which increases with the potential difference.

When the potential difference is raised, the squared speed gets closer and closer to the value of about $9 \cdot 10^{16}$ m^2/s^2 never reaching it: a (horizontal) *asymptotic trend* was found out, while Newtonian mechanics predicted a linear trend. It may be noticed that the value of this ultimate speed approximates the one of light speed *in vacuo* very well. Because the classical prevision is based upon three hypotheses, they are to be examined to decide which one(s) bring(s) to conflict with experimental data:

1) The expression for electrical work is valid: $W = e\,\Delta V$;
2) Work-energy theorem is valid: $W = \Delta K$;
3) The expression for kinetic energy is valid: $K = \frac{1}{2} m\, v^2$.



Run	ΔV (MV)	Flight time (10^{-8} s)
a	0.5	3.23
b	1.0	3.08
c	1.5	2.92
d	4.5	2.84
e	15	2.80

Figure VI.9. TOF in 10^{-8} s units for each run (Bertozzi, 1964).

Energy conservation principle is of course assumed. The expression for electrical work may be considered as valid, since it has been deduced by arguments inside electrostatics only (not dynamics), exploiting electric field's conservativity. It should be inquired instead if the work $e\Delta V$ done on a particle actually corresponds to its kinetic energy variation.

Bertozzi carried out a *calorimetric measure* for answering this question. If the target's mass *M* and the specific heat of aluminum *c* are known, the measure of its internal energy variation will be obtained by means of the temperature variation detected by a thermocouple. The first law of thermodynamics and the fundamental law of calorimetry are expressed by $\Delta E_{int}^{(target)} = Q - \overline{W}$ and $Q = M\, c\, \Delta T$ respectively. Since the target, an aluminum disk, stands macroscopically still, the undergone external work $\overline{W}$ is null; thus $\Delta E_{int}^{(target)} = Q = M\, c\, \Delta T$.

It is reminded that the external work $\overline{W}$ is different from the electrical work W done by the Van de Graff on the electrons, involved in the work-energy theorem.

A further measurement displays that *the increase of the disk internal energy is numerically equal to the electrical work* for each run in the experiment: $\Delta E_{int}^{(target)} = e\,\Delta V = W$ (4). The first hypothesis has thus been corroborated.

Besides, the energy of the travelling particles is *necessarily kinetic*, for electrons have no internal structure and are free: if there are not losses, kinetic energy must vary exactly of the same amount of target's internal one, because of energy conservation: $\Delta E_{int}^{(target)} = \Delta K$ (5). The total energy of the system 'beam + target' is conserved, changing from entirely kinetic to entirely internal one.

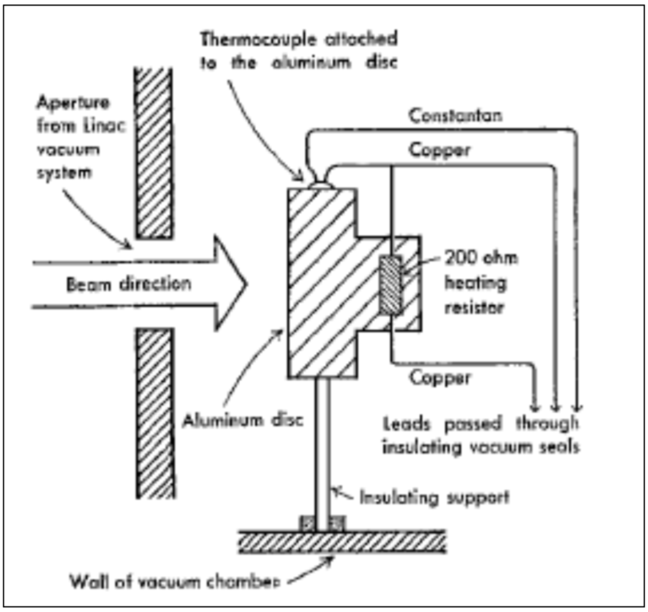


Figure VI.10. Scheme of the apparatus for the calorimetric measure (Bertozzi, 1964). The beam colliding with the aluminium target is depicted on the left, while the thermocouple with a resistor

Eventually, if (4) is put together with (5) it will be found that *work-energy theorem is valid at relativistic speeds too*: $\Delta E_{int}^{(target)} = W = \Delta K$.

As a consequence, the third hypothesis must be not valid.

$$\boxed{W = e\,\Delta V = \Delta K \neq \Delta\left(\frac{1}{2}mv^2\right)} \quad (6)$$

According to *methodological falsificationism* the accumulation of experimental results like the above invalidates Newtonian mechanics. In fact work-energy theorem in its classical form follows from the founding law $F = ma$ and the definition of work, as displayed in IV.4.

If the classical work-energy theorem is invalidated, it will also at least one of its assumptions, namely $F = ma$, the definition of work being considered valid in any theory. But classical mechanics cannot stand without the classical form of the II law (Motz & Weaver, 1991; Bergia & Franco, 2001).

To sum up, *an ultimate speed for the accelerated electrons does exist and turns out to be the light speed in vacuo c.* Therefore the well-known classical kinetic energy expression cannot be valid (although work-energy theorem is): the correct formula must be significantly different at *relativistic speeds*, i.e. comparable to *c*. It is not clear yet, however,

(a) whether *c* be the ultimate speed in *every* inertial reference frame;
(b) whether, assumed that *c* is the limit, it keeps the *same numerical value* in every inertial frame. For instance, sound speed in a medium is fixed, independently of the source motion relative to an observer, but it has not the same value in different inertial frames.

VI.3.2.2. *The two postulates and invalidation of classical duration invariance*

The Relativity Principle has to be considered in order to answer the question (a) of the previous paragraph. The first statement of this principle dates back to Galileo and his famous "esperimento del naviglio". Summarizing it, the physics which is valid in any laboratory on the ground describes any phenomenon observed in a ship moving steady compared with the ground with the utmost care. In general, *physics in steadily moving IFs is exactly the same than in the Lab IF*[88]*. This is not trivial at all.* For instance, in the (instantaneous) IF joint to the observed stars, galaxies and the Moon, the same physical laws are valid and experiments of any kind carried out in the same conditions supply the same outcomes. So scientists may use Earth LabIF physics for describing every phenomenon observed in space. Some examples (Fabri, 2005):

1. Any experiment in *airplanes* always gives the same result than on the ground, if the motion is stationary, no matter how much high is the speed; the same holds for high-speed trains;
2. *Space probes* may be considered IF when they travel far enough from not-negligible masses; Earth LabIF physics does work inside them. For instance, a mass-spring experiment's results are in agreement with Hooke's law, as can be witnessed by spacemen;

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[88] The reader is reminded that IF stands for 'inertial frame' and LabIF for 'inertial frame of the laboratory'.

3. *Gravity acceleration on the Moon* is independent of any physical quantity of a falling body, mass in particular, exactly like on Earth;
4. A model based on Earth physics has been built for explaining the observed *evolution of fast moving stars*; this model explains astronomical observations to a high degree of accuracy[89].
5. The same assumption may be made for *galaxies*, which move much faster than stars, at speeds comparable to light's when are very distant from Earth. In particular, each element's spectrum is perfectly equal to the terrestrial one, taking Doppler redshift into account.

Question (a) may be answered now: c is the ultimate speed in every IF[90], because of the Relativity Principle (RP). It is the *cosmic speed limit*.

In order to answer question (b) a simple reasoning is useful instead: be Bertozzi's (real) experiment run in a spaceship moving at speed v with respect to the Earth, keeping the Van der Graff's axis oriented in the motion direction. If an electron were accelerated up to $c - \varepsilon$ in the spaceship ($\varepsilon < v$, small at pleasure), it would move at $c - \varepsilon + v > c$ with respect to the Earth, c being the ultimate speed at the same time. Therefore *if c is the speed limit, then it will have the same value in every IF, i.e. it has to be invariant*. In fact, since ε is to be considered infinitely small we would have anyhow electrons accelerated beyond c as far as the spaceship speed is small, in contradiction with the experimental trend. This is impossible from both the physical and logical point of view.

The two founding *postulates* of Relativity are stated at this point of the t/l sequence. They entail that a unique "preferential" frame[91] in which light travels at c does not exist: all inertial frames are equivalent. When passing from an IF to another c and their relative velocity cannot be added up: it is forbidden to write $c \pm v$, nor a speed $c - \varepsilon$ may be added. In conclusion, the Galilean transformations for velocities have proved incorrect. Since classical velocity transformations are

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[89] It should be remarked that stars are known to move faster than 100 km/s compared with the Earth.

[90] According to the reconstruction by Silvio Bergia, this was an Einstein's *hypothesis* stemming from a thought experiment *previous to* the formulation of the second postulate. Namely, if an observer might travel at the same speed of a monochromatic plane light wave, he/she would observe a stationary sinusoidal profile. It is, however, in sharp contrast both with Maxwell equations both with every known electromagnetic experimental outcome. Therefore no observer may reach c, and so it must be the ultimate speed for every observer and consequently in his/her inertial frame. The experiment by Bertozzi thus furnishes an experimental corroboration of Einstein's hypothesis. However, this line of reasoning has been considered too much complex for students, above all because it requires mastery of electromagnetism.

[91] Namely, the historical 'ether'.

invalidated by Bertozzi's experiment, so are simultaneity and time interval invariance (from which they stem), because of *modus tollens*.

A thought experiment allows working out in which way durations in different IFs are related. In other words: since classical time duration invariance has been invalidated, one may ask in which way durations transform.

VI.3.2.3. *Light-clock, time interval dilation and space-time*

A *light clock* of height h is considered to that end, in which the periodic phenomenon (refer to IV.1) is the back-and-forth path of a light ray reflected by a mirror S, the time unit being $2h/c$.

This section of conceptual pathway is identical to the one worked out in the previous conceptual pathway. The reader is therefore sent back to VI.3.1, except for length contraction, which has been dropped.

VI.3.2.4. *Relativistic linear momentum*

It was shown by Bertozzi's experiment that *work-energy theorem is valid in the form of a relation between physical quantities*, but $K \propto u^2$ is not valid anymore (equation 6). On the other hand, the second law of dynamics in the form $\vec{F} = \Delta\vec{p}/\Delta t$ may be kept, because any interaction (*cause*) necessarily produces the variation rate (*effect*) of a quantity involving both state of motion and inertia, since the uniform motion state has been broken (D'Inverno, 1992; Coelho, 2007). The resistance of an object to variations in its state of motion is inertia, measured by mass in classical dynamics. The latter will be not enough for quantifying this resistance in relativity, since it will depend on $\gamma(u)$ too. Thus force may reasonably be proportional to the variation rate of a function $\vec{p} = \vec{p}(\vec{u}, m)$ of velocity and mass, such that $\vec{F} \propto \Delta\vec{p}/\Delta t$. In scalar form, if the function $p = p(u, m)$ is not made explicit and considering a unitary proportionality coefficient, it will be found that (consider that u_m is the mean speed *by definition*, independently of the theory once LabIF is established):

$$F = \frac{\Delta p}{\Delta t} \Rightarrow L = F\Delta x = \frac{\Delta p}{\Delta t}\Delta x = \Delta p \frac{\Delta x}{\Delta t} = \Delta p\, u_m$$

$$L = \Delta p\, u_m = \Delta K$$

Instantaneous speed is taken into account instead in the differential form of the work-energy theorem $dK = u\, dp$. *Since kinetic energy formula has to change, linear momentum expression has too.* To that end, we *hypothesize* the existence of the quantity p above, which be *conserved* at any speed value in the absence of external interactions, just like classical momentum.

Considering two identical particles *elastically* colliding, their motion may be studied without knowing the details of the interaction by means of momentum and kinetic energy conservation principles, which are *assumed* in this path[92]. The study of the elastic collisions was inspired by PSSC (1995), while the whole teaching meditative constructive TE (Gilbert & Reiner, 2000) by Feynman (1964), Taylor and Wheeler (1965), and Fabri (2005).

In the centre-of-momentum reference frame (CM) the equation $\overrightarrow{p_1} = -\overrightarrow{p_2}$ is valid by definition and it will turn into $\overrightarrow{u_1} = -\overrightarrow{u_2}$ if $p = p(u, m)$ is a bijective function. So, supposing $K = K(p) = K(u, m)$ to be a bijection too, conservation principles may be written as

$$\bar{p}_1 = -\bar{p}_2;\quad \begin{cases} \bar{p}_1 + \bar{p}_2 = \bar{p}'_1 + \bar{p}'_2 \\ K(p_1) + K(p_2) = K(p'_1) + K(p'_2) \end{cases} \Rightarrow \begin{cases} \bar{p}'_1 = -\bar{p}'_2 \\ 2K = 2K' \end{cases}$$

$$\begin{cases} p_1 = p_2 = p'_1 = p'_2 \\ K(p_1) = K(p_2) = K(p'_1) = K(p'_2) \end{cases}$$

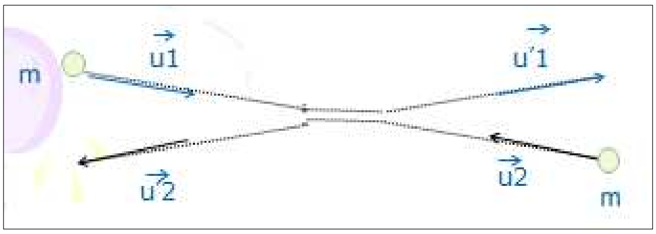

Figure VI.11. An elastic relativistic collision between identical particles in CM.

As the collision is elastic in CM, momenta's directions vary, but not their magnitude, so that $u_1 = u'_1 = u_2 = u'_2$, as depicted above. If the *deflection angle is small*, the y-component of momentum will be small too. In CM a total symmetry between the particles 1 and 2 is: the exchange 1 ↔ 2 does not make any mechanical

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[92] Kinetic energy is the only form of energy owned by the incoming and outgoing particles.

quantity vary. Thus each particle is covering the same transversal distance[93] in the *same coordinate time* Δt*, as well as in the same proper time* $\Delta\tau$*.* In fact, considering two light-clocks each in the proper frame of 'start' and 'end' events of the considered motion of one particle, each of them must beat the proper time. The two proper time intervals must be equal because they are related to the CM coordinate time by $\Delta t \ = \gamma(v)\ \Delta\tau,$ the speeds being the same.

The IF K, whose axes are named 'X' and 'Y', *moves with particle 1 along the x-axis of CM* instead. To better understand the characteristics of K, a sequence of IFs at increasing speed v_x with respect to CM may be considered. The components p_x of particle 2 – measured in every IF of the sequence – will be greater and greater, whilst those of particle 1 smaller and smaller … till the latter will become null in the K reference frame, in which p_1 is parallel to Y-axis and *it is small enough*[94] *to be thought as classic*. The outgoing momentum p'_1 is also parallel to Y-axis, owing to the symmetry of the collision in CM.

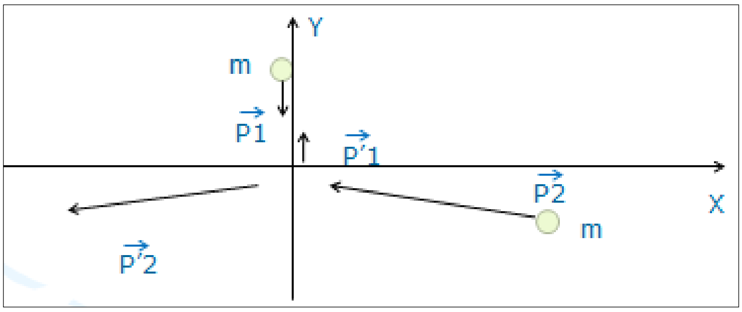

Figure VI.12. An elastic relativistic collision between identical particles in the K frame.

Since particle 1 is approximately non-relativistic, $p_1 \cong mu_1$ and $p'_1 \cong mu'_1$ in a very good approximation, m being **Newtonian inertial mass** (1st hypothesis: HP1). The final objective is working out the momentum expression for the relativistic particle. The 2nd hypothesis is formulated to that end: total transverse momentum is conserved and invariant (HP2). The sum of transverse momenta is null in CM after the collision, then it must be null in K: $\vec{p}'_1 = -\vec{p}'_{2Y}$. At this point the attention is put on *the invariants*: transverse displacement ΔY, total transverse momentum, proper time, and mass.

The invariance of mass is simply assumed for now, but it will be discussed at the end of this section.

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[93] The distance along the y-axis: it is transversal to the relative motion direction.

[94] This comes from the small deflection in CM.

It is useful to pass from CM to K. In CM the following identity is valid, where each subscript marks the respective particle:

$$m\frac{\Delta y}{\Delta\tau_1} = -m\frac{\Delta y}{\Delta\tau_2} \quad (7)$$

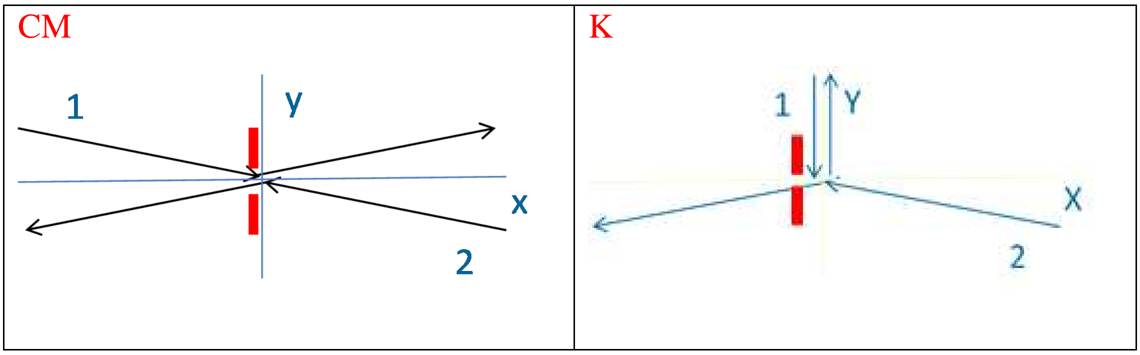
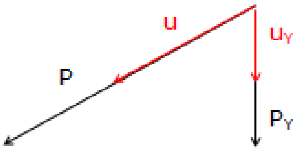


Figure VI.13. Comparison between the collision observed in CM and K frames. The red bars are aimed at focusing that the transverse displacement are invariant and conserved: $\Delta Y = \Delta y = \Delta Y' = \Delta y'$.

. Equality (7) must be valid in the frame K as well, in which proper time interval between the 'start' and 'end' events is the same for particle 1 and 2. However, *coordinate times are different in K because of the time dilation effect* $\Delta\tau_2 = \Delta t_2/\gamma$. The time dilation formula for the non-relativistic particle reduces to $\Delta t_1 \cong \Delta\tau_1$. So (7) changes into $m\,\Delta Y_1/\Delta t_1 = -m\,\gamma\,\Delta Y_2/\Delta t_2$ and, exploiting HP2, one is left with

$$p'_1 \equiv m\,\Delta Y_1/\Delta t_1 = -m\,\gamma\frac{\Delta Y_2}{\Delta t_2} = -p'_{2Y} \;\Rightarrow p'_{2Y} = m\,u'_{2Y}\,\gamma(u'_{2Y})$$

The last equation is valid solely for the Y-component of $\vec{p}'_2$. Nevertheless, $\vec{p}'_2$ and $\vec{u}'_2$ are parallel, so that the same relationship has to be valid for their magnitudes (similitude between triangles). In conclusion, the relativistic particle's momentum is expressed by



$$\boxed{\boldsymbol{p}'_2 = m\,\boldsymbol{u}'_2\,\gamma(u'_2)}$$

The Correspondence Principle has been utilized in the present argument. It is the principle according to which *a new theory has necessarily to come back to the previous one in the proper domain*: $v/c \ll 1$ in the case of relativistic mechanics and Newtonian mechanics. *This Principle allows extending the Newtonian (inertial) mass to relativistic mechanics*: Newtonian mass will enter into all relativistic dynamic expressions. However, it will assume an entirely new meaning.

Eventually, the result obtained in this particular TE is to be generalized[95] by induction: the relativistic linear momentum of a particle of mass *m* travelling at $\vec{u}$ is expressed by

$$\boxed{\vec{p} = \frac{m\vec{u}}{\sqrt{1 - \frac{u^2}{c^2}}} \Leftrightarrow \vec{p} = \gamma m\vec{u}}$$

It should be remarked that $\vec{u}$ denotes the *instantaneous velocity of the particle in an established inertial frame, anyhow it is moving*, while $\vec{v}$ denotes the *relative uniform velocity between different IFs*. The former is conceptually different from the latter. A shift from $\gamma(v)$ to $\gamma(u)$ occurred in the passage from time dilation formula to momentum expression. It is made feasible by **co-moving frames:** a sequence of reference frames in each of which a particle (object) is at rest at each instant of motion (D'Inverno, 1992; Bergia & Franco, 2001). The accelerated object may be associated to a different co-moving frame at each instant t_0, t_0+dt, $t_0+2dt, \ldots$, t_f of the motion, during an infinitesimal time interval *dt in which the object is assumed to move at uniform speed.* It is possible to deal with objects in non-uniform motion with respect to the LabIF in this way.

However, this does not entail that the objects' properties are conserved under accelerations. In particular, the object's mass *may vary* according to its structure/composition, for instance because the object emits electomagnetic energy. I shall assume in the following that **when a specific object is considered, its mass is the same before and after acceleration, since mass is an intrinsic property of the object itself**, like charge. Otherwise, the identity of the object would not be preserved.

VI.3.2.5. *Relativistic kinetic energy*

As momentum expression has changed, the one for kinetic energy also will, because of work-energy theorem $\Delta K = \langle u \rangle \, \Delta p$.

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[95] Few local elements have been used indeed.

The previously obtained result for momentum strongly suggests that *Lorentz's factor is a major function of instantaneous speed* in relativistic dynamics: it is frequently present inside relativistic equations and expressions for dynamical quantities. In the 'classical limit'[96] $\gamma \underset{c\to\infty}{\longrightarrow} 1$, and therefore a lot of relativistic formulas come back to Newtonian ones – in agreement with the Correspondence Principle – even though not all of them[97].

Kinetic energy is being *induced* using Bertozzi's data:

1. The last datum has to be rejected (just like the 5th one originally taken), since «some acceleration takes place within the Linac» in the first 1 m section of the covered distance[98] (Ireson, 1998).
2. We are adding the ordered pair $(0,0)$ corresponding to a trivial external constraint: when an electron is still in LabIF its kinetic energy must be null. This is being done in order to handle with four data, which are the minimum set for an effective linear interpolation.
3. If the classical trend is compared to the experimental data, a prominent gap may be effectively visualized, increasing with the ratio u/c (figure VI.15).
4. On the contrary, a *linear fit* of $K/(m_e c^2)$ against γ turns out to be *very good* (figure VI.16). The aim is indeed the search for *linearization*, since linear fit is the most used in any statistical analysis and the easiest to check visually. The best-fit straight line equation turns out to be $y = 1.1704\, x - 1.1628$. Therefore the data seem to suggest with a good approximation a relation like $K/mc^2 \cong A\gamma - A \rightarrow K \cong A\, m\, c^2\, (\gamma - 1)$, within experimental errors, where A is an empirical constant very close to 1.

The *Newtonian limit* of the interpolating function is being taken both to generalize the local obtained result both for determining the theoretical value of the best-fit slope A. The binomial theorem

$$(1 - X)^N \cong 1 - NX,\ X \ll 1$$

is worth using in order to approximate the function $\gamma(u) = 1/(1 - (u/c)^2)^{1/2}$ for $u \ll c$. The binomial formula is valid for fractional exponent too, even if this generalization is not trivial at all.

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[96] Actually, this limit does not exactly match the classical limit of SR, exactly like $h \rightarrow 0$ cannot be meant to be the one and the only classical limit of quantum mechanics.

[97] Some SR concepts are entirely novel, immeasurable with classical ones. For instance, it will be seen that total and rest energy go to infinity in the limit $c \rightarrow \infty$.

[98] The whole LINAC was used for the measure at 15 MV instead.

u/c	K/m_ec^2
0.867	0.978474
0.91	1.956947
0.96	2.935421
0.987	8.806262

It's worth plotting two dimensionless quantities, putting K/m_ec^2 in the y-axis. In fact

$[K] = [m_ec^2] = \mathrm{M}\,(\mathrm{L/T})^2$

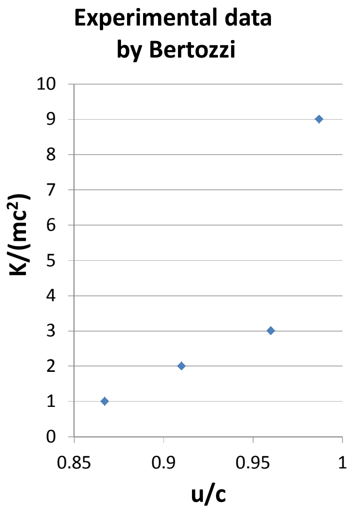


Figure VI.14. Observed values of normalized free-electron KE vs. normalized speed. The trend is clearly not linear, but it might be quadratic (according to classical dynamics): linearization is needed.

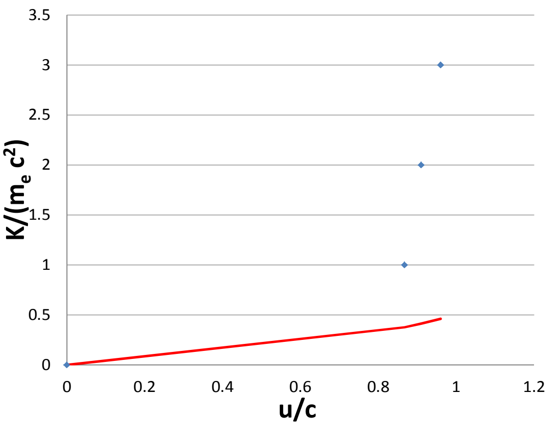


Figure VI.15: comparison between classical trend (red line) and Bertozzi's data joint to a physical constraint (blue points).

In this case $N = -\frac{1}{2}$ and $X = (\mathrm{u/c})^2$, thus

$$\gamma = \left(1-\left(\frac{u}{c}\right)^2\right)^{-1/2} \underset{\left(\frac{u}{c}\right)^2 \ll 1}{\cong} 1-\left\{-\frac{1}{2}\left(\frac{u}{c}\right)^2\right\} \qquad \Rightarrow \gamma \sim 1+\frac{1}{2}\left(\frac{u}{c}\right)^2$$

If the approximate equation for kinetic energy (KE) is hypothesized to be exact, it will become $K = A\,m\,c^2\,(\gamma-1) \sim A\,m\,c^2\left(1+\frac{1}{2}\left(\frac{u}{c}\right)^2-1\right) \Rightarrow K =$

$A\frac{1}{2}\, m\, u^2$, which *coincides with the classical expression for* A = 1. Thus the new general expression for KE is very unlike the classical one:

$$\boxed{K = m\, c^2(\gamma - 1)}$$

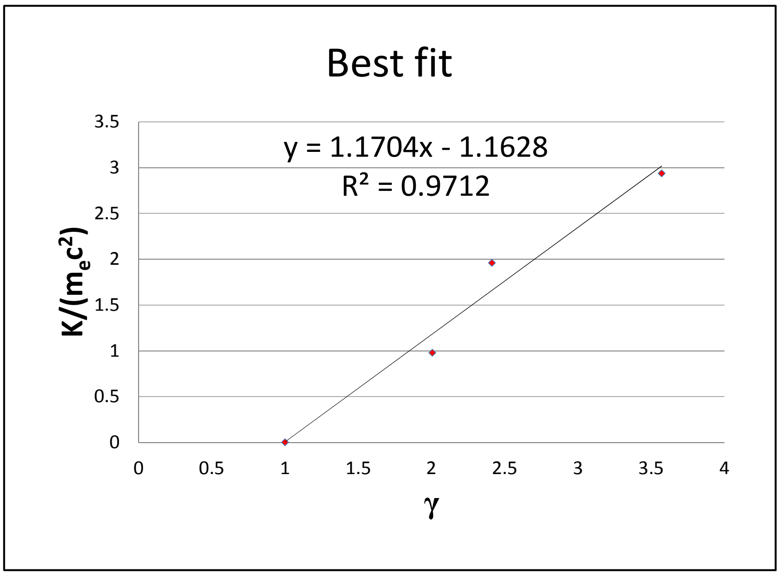


Figure VI.16. Best-fit straight line, its equation and the squared correlation coefficient.

The relativistic expression for the normalized squared speed as a function of KE may be easily obtained: $u^2/c^2 = 1 - [mc^2 / (m\, c^2 + K)]^2$. Two alternatives for a check are viable at this point:

a) This reverse function may be used for performing numerical calculations in which Newtonian and relativistic predictions are compared (Ireson, 1998);
b) The following limit of the function may be taken:

$$\lim_{K \to +\infty} \left\{ 1 - \left[\frac{mc^2}{mc^2 + K} \right]^2 \right\} = 1 \Rightarrow v = c$$

This result means that if work is done on a (massive) particle its KE will endlessly increase, but at the same time the speed will never reach *c*. For this reason "*high energy* accelerators" are mentioned in particle physics research instead of "high speed accelerators".

An alternate feasible way to the relativistic KE expression is its *deduction by means of the differential calculus*, inspired by Bergia and Franco (2001). It starts from the work-energy theorem $dK = u\, dp$, in which relativistic momentum expression is plugged, and then derivative and differential calculus are exploited.

$$dK = u\,dp = m\,u\,d(\gamma u) = mu\{(d\gamma)u + \gamma du\}\ (*)$$

Now du in the second term of the sum has to be written as *an expression with $d\gamma$ inside* for factorization.

$$\text{Since}\quad \frac{1}{\gamma^2} = 1 - \frac{u^2}{c^2} \Rightarrow -\frac{2}{\gamma^3}\frac{d\gamma}{du} = -2\frac{u}{c^2}$$

$$\Rightarrow \frac{d\gamma}{du} = \gamma^3\frac{u}{c^2} \Rightarrow d\gamma = \gamma^3\frac{u}{c^2}du \Rightarrow du = \frac{1}{\gamma^3}\frac{c^2}{u}d\gamma$$

$$(*)\ dK = mu\left\{u\,d\gamma + \gamma\frac{1}{\gamma^3}\frac{c^2}{u}d\gamma\right\} = m\,u\,d\gamma\left\{u + \frac{c^2}{u} - u\right\} = m\,c^2 d\gamma$$

If $dK = m\,c^2 d\gamma$ then the K.E. gained by an object initially still in LabIF accelerated up to the speed u must be $\Delta K = m\,c^2\Delta\gamma = m\,c^2(\gamma(u) - \gamma(0)) = m\,c^2(\gamma - 1)$

The K.E. of a still abject has to be null by definition: $K(0) \equiv 0$. Thus

$$\Delta K = K(u) - K(0) = K \ \Rightarrow K = m\,c^2(\gamma - 1)$$

This predicted theoretical expression has to be compared with all experimental data by Bertozzi – included $(0, 0)$ – by means of the inverse function. The function fits the data very well, as depicted in figure VI.17.

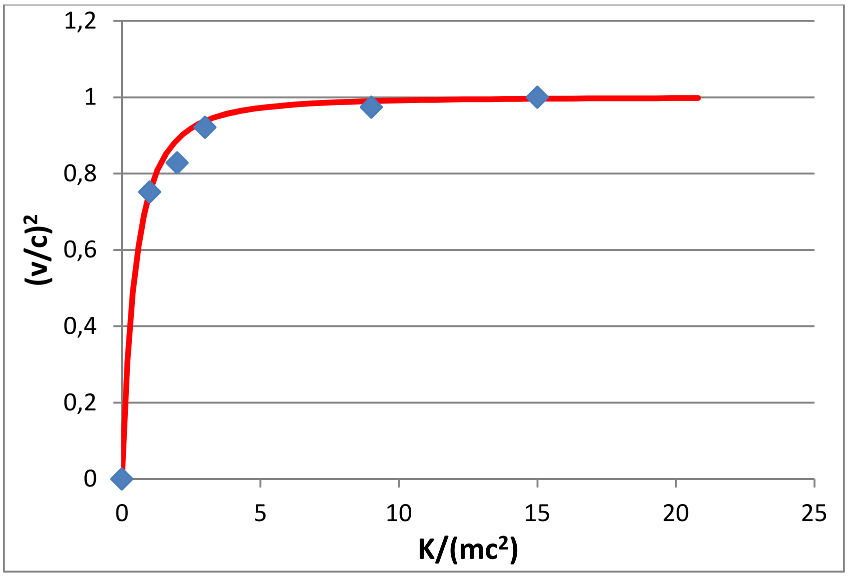


Figure VI.17.Best fit of the deduced expression for KE (red line) with the experimental data (blue points).

VI.3.2.6. *Radioactive β-decays*

Figure VI.18. Marie and Pierre Curie.

Historical foreword. In 1896 Becquerel discovered natural radioactivity by chance, while studying the 'rays' emitted by a uranium salt. He sought to identify if the emission were wavelike o particle–like. Marie Sklodowska (1867-1934) and Pierre Curie searched for effects by every known element of the periodic table until 1902, when they discovered two new chemical elements displaying natural radioactivity: *polonium* (Po) and *radium* (Ra). Thirty-two years later, Irène Curie and Frédéric Joliot obtained the first artificially induced radioactive decay (Krane, 1987; Motz & Weaver, 1991).

The particle character of atoms, and thus of matter, has been first given evidence by Pierre Curie using a leaf electroscope (figure VI.19). The radioactive sample expelled particles which ionized air, thus closing the circuit and making the positively charged leaves discharge and close. This was a visible effect; therefore it was supposed that the atoms of the sample were made of particles (discrete entities).

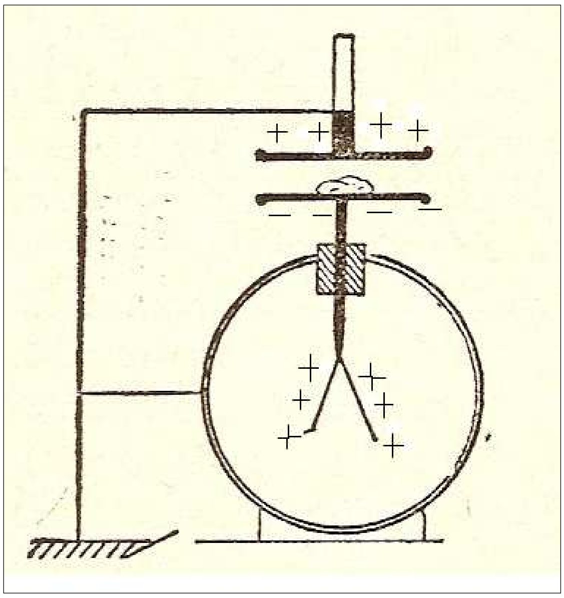
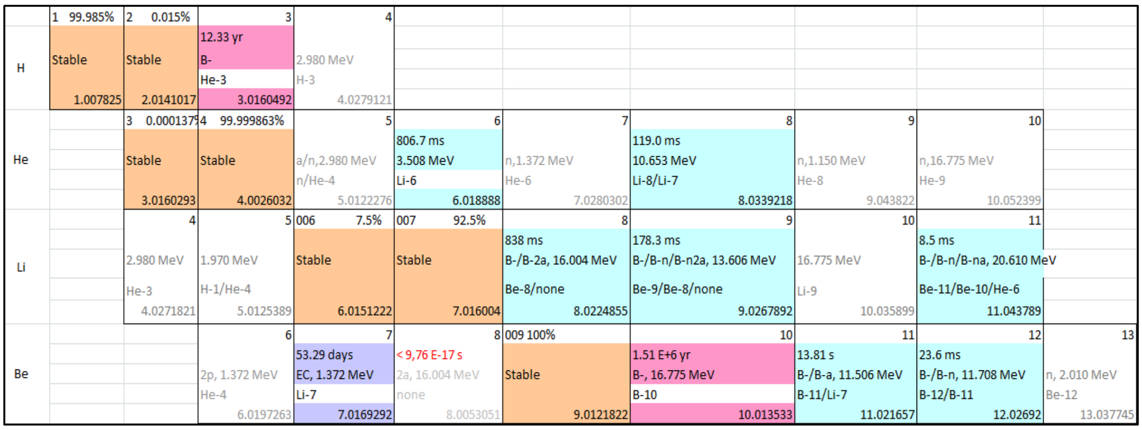


Figure VI.19. Sketch of the leaf electroscope used by Pierre Curie in its experiment for investigating the nature of radioactive emission. A radioactive sample is put on one plate of the capacitor.

Ninety-eight natural chemical elements have been found in the Universe[99] and twenty artificial ones have been artificially produced up to now. They are described by means of the number Z of protons and the number N of neutrons compounding the atomic nucleus; the sum A = N+Z is called mass number. The atoms of each single element are not identical: it turns out from experiments that atoms belonging to the same chemical species have different masses, due to the presence of isotopes. The single nuclear species is called "nuclide". It is specified that the conventional unit used in the following for atomic and nuclear mass is the atomic mass unit: $1\ \mathrm{a.m.u.} \equiv 1/12\, m\ ({}^{12}_{6}C)$. Not all matter is stable: several nuclides change into other ones by *nuclear transmutation or decay*. Some products will have different Z – new elements are thus created – and a mean *lifetime* ranging from μs to million years. Lifetime is the time a nuclide takes to decay. Because of the stochastic nature of decays, a better indicator is *half-life*: the time needed for half of the nuclei in a sample to decay (Krane, 1987).

The main features of half-life, mass and other properties of nuclides can be described synthetically by means of a table like the one depicted in figures VI.20 and VI.21. In the first line from the top of each cell there are mass number and, if the nuclide is stable, its relative abundance in nature. Half-lives are reported in the second line. The allowed type(s) of decay(s) of a nuclide together with the energy released in the favourite decay are in the third row; the daughter nucleus/i in the fourth. Atomic masses in a.m.u. are finally reported in the fifth (last) line. They are equal to nuclear ones at a high degree of approximation, because $m_e \cong 5 \cdot 10^{-4}\mathrm{a.m.u.} \ll m_p \cong m_n \cong 1\ \mathrm{a.m.u.}$, which is enough for the present purposes; no distinction will be made in this context.



H	1 99.985% Stable 1.007825	2 0.015% Stable 2.0141017	3 12.33 yr B- He-3 3.0160492	4 2.980 MeV H-3 4.0279121						
He		3 0.000137 Stable 3.0160293	4 99.999863% Stable 4.0026032	5 a/n,2.980 MeV n/He-4 5.0122276	6 806.7 ms 3.508 MeV Li-6 6.018888	7 n,1.372 MeV He-6 7.0280302	8 119.0 ms 10.653 MeV Li-8/Li-7 8.0339218	9 n,1.150 MeV He-8 9.043822	10 n,16.775 MeV He-9 10.052399	
Li		4 2.980 MeV He-3 4.0271821	5 1.970 MeV H-1/He-4 5.0125389	006 7.5% Stable 6.0151222	007 92.5% Stable 7.016004	8 838 ms B-/B-2a, 16.004 MeV Be-8/none 8.0224855	9 178.3 ms B-/B-n/B-n2a, 13.606 MeV Be-9/Be-8/none 9.0267892	10 16.775 MeV Li-9 10.035899	11 8.5 ms B-/B-n/B-na, 20.610 MeV Be-11/Be-10/He-6 11.043789	
Be			6 2p, 1.372 MeV He-4 6.0197263	7 53.29 days EC, 1.372 MeV Li-7 7.0169292	8 < 9,76 E-17 s 2a, 16.004 MeV none 8.0053051	009 100% Stable 9.0121822	10 1.51 E+6 yr B-, 16.775 MeV B-10 10.013533	11 13.81 s B-/B-a, 11.506 MeV B-11/Li-7 11.021657	12 23.6 ms B-/B-n, 11.708 MeV B-12/B-11 12.02692	13 n, 2.010 MeV Be-12 13.037745

Figure VI.20. The upper part of the nuclide map.

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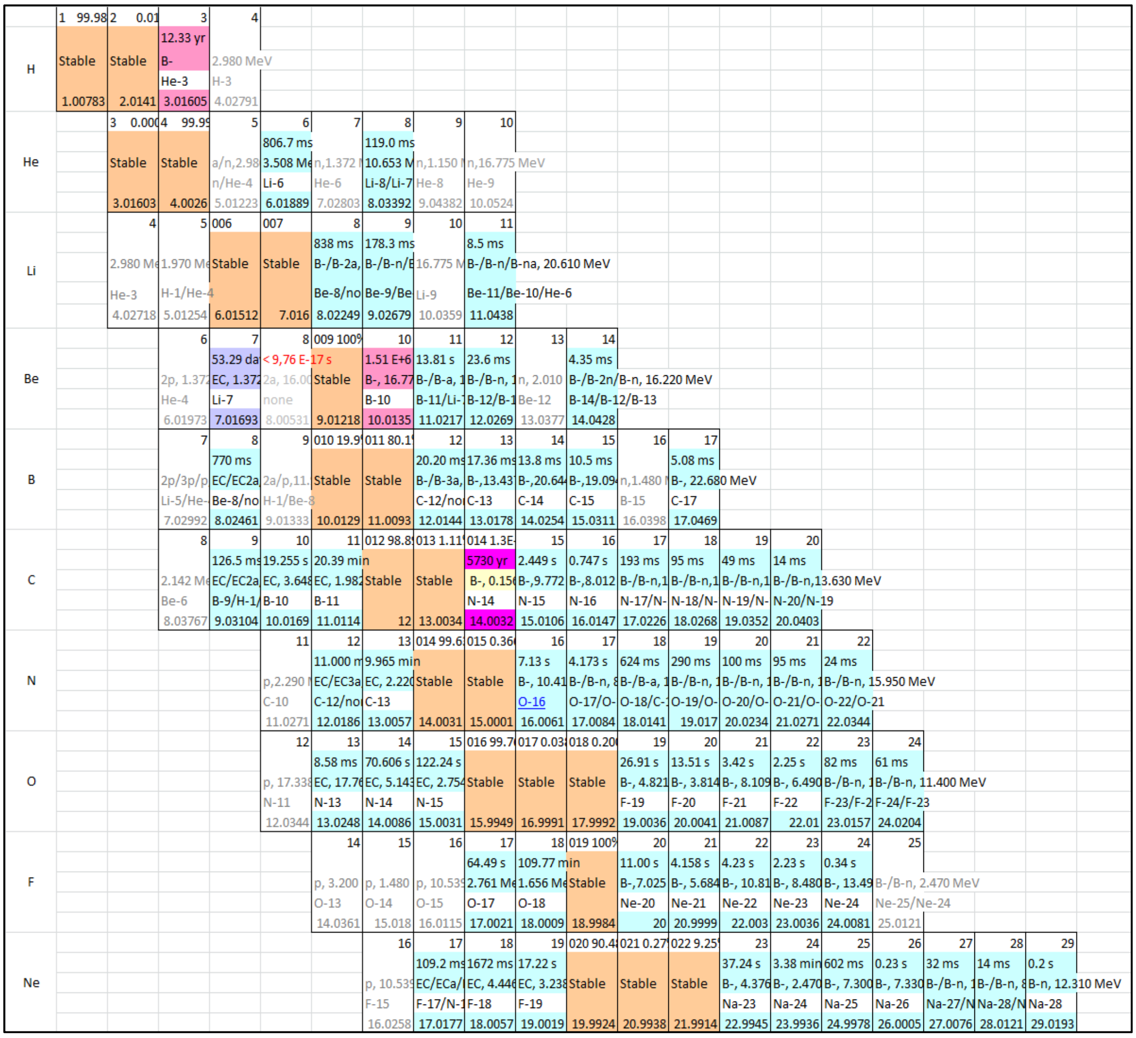
[99] They have been directly detected on Earth or by examining spectra of stars and supernova remnants.

The nuclide map may be read as *N vs Z plot* in which the number of neutrons grows from left to right as x-coordinate and the number of protons grows from up to down as y-coordinate. In the map those nuclides which decay within 1 s or less are coloured in light blue, the ones decaying within 1 day/ hour in violet (only Be-7 in the part of the map depicted in figure VI.21), the ones whose half-life is more than 1 year in pink (H-3, Be-10, C-14 in this part of the map). The second line of the orange cells is empty: these are *stable nuclides*, corresponding to *stable atoms*. Eventually, the grey nuclides have a *widely negligible half-life* because they are resonances. Their existence is allowed for a time interval greater then the minimum established by Heisenberg's uncertainty principle $\Delta E\ \Delta t \geq \hbar/2$. If the band half-width at half-height is Γ/2 and the measure of mean lifetime is τ, the Heisenberg inequality will change into $\Gamma\,\tau \geq \hbar \;\Rightarrow\; \tau \geq \hbar/\Gamma$ (Giacomelli, 2002). Resonances are obviously not considered.



	1 99.98	2 0.01	3	4																
			12.33 yr																	
H	Stable	Stable	B-	2.980 MeV																
			He-3	H-3																
	1.00783	2.0141	3.01605	4.02791																
		3 0.000	4 99.99	5	6	7	8	9	10											
					806.7 ms		119.0 ms													
He		Stable	Stable	a/n,2.98	3.508 Me	n,1.372	10.653 M	n,1.150	n,16.775 MeV											
				n/He-4	Li-6	He-6	Li-8/Li-7	He-8	He-9											
		3.01603	4.0026	5.01223	6.01889	7.02803	8.03392	9.04382	10.0524											
		4	5	006	007	8	9	10	11											
						838 ms	178.3 ms		8.5 ms											
Li		2.980 Me	1.970 Me	Stable	Stable	B-/B-2a,	B-/B-n/B	16.775 M	B-/B-n/B-na, 20.610 MeV											
		He-3	H-1/He-4			Be-8/no	Be-9/Be	Li-9	Be-11/Be-10/He-6											
		4.02718	5.01254	6.01512	7.016	8.02249	9.02679	10.0359	11.0438											
			6	7	8	009 100%	10	11	12	13	14									
				53.29 da	< 9,76 E-17 s		1.51 E+6	13.81 s	23.6 ms		4.35 ms									
Be			2p, 1.372	EC, 1.372	2a, 16.00	Stable	B-, 16.77	B-/B-a, 1	B-/B-n, 1	n, 2.010	B-/B-2n/B-n, 16.220 MeV									
			He-4	Li-7	none		B-10	B-11/Li-	B-12/B-1	Be-12	B-14/B-12/B-13									
			6.01973	7.01693	8.00531	9.01218	10.0135	11.0217	12.0269	13.0377	14.0428									
			7	8	9	010 19.9	011 80.1	12	13	14	15	16	17							
				770 ms				20.20 ms	17.36 ms	13.8 ms	10.5 ms		5.08 ms							
B			2p/3p/p	EC/EC2a	2a/p,11.	Stable	Stable	B-/B-3a,	B-,13.43	B-,20.64	B-,19.09	n,1.480	B-, 22.680 MeV							
			Li-5/He-	Be-8/no	H-1/Be-8			C-12/no	C-13	C-14	C-15	B-15	C-17							
			7.02992	8.02461	9.01333	10.0129	11.0093	12.0144	13.0178	14.0254	15.0311	16.0398	17.0469							
			8	9	10	11	012 98.8	013 1.11	014 1.3E-	15	16	17	18	19	20					
				126.5 ms	19.255 s	20.39 min			5730 yr	2.449 s	0.747 s	193 ms	95 ms	49 ms	14 ms					
C			2.142 Me	EC/EC2a	EC, 3.648	EC, 1.982	Stable	Stable	B-, 0.156	B-,9.772	B-,8.012	B-/B-n,1	B-/B-n,1	B-/B-n,1	B-/B-n,13.630 MeV					
			Be-6	B-9/H-1/	B-10	B-11			N-14	N-15	N-16	N-17/N-	N-18/N-	N-19/N-	N-20/N-19					
			8.03767	9.03104	10.0169	11.0114	12	13.0034	14.0032	15.0106	16.0147	17.0226	18.0268	19.0352	20.0403					
					11	12	13	014 99.6	015 0.36	16	17	18	19	20	21	22				
						11.000 m	9.965 min			7.13 s	4.173 s	624 ms	290 ms	100 ms	95 ms	24 ms				
N					p,2.290	EC/EC3a	EC, 2.220	Stable	Stable	B-, 10.41	B-/B-n, 8	B-/B-a, 1	B-/B-n, 1	B-/B-n, 1	B-/B-n, 1	B-/B-n, 15.950 MeV				
					C-10	C-12/no	C-13			O-16	O-17/O-	O-18/C-1	O-19/O-	O-20/O-	O-21/O-	O-22/O-21				
					11.0271	12.0186	13.0057	14.0031	15.0001	16.0061	17.0084	18.0141	19.017	20.0234	21.0271	22.0344				
					12	13	14	15	016 99.7	017 0.03	018 0.20	19	20	21	22	23	24			
						8.58 ms	70.606 s	122.24 s				26.91 s	13.51 s	3.42 s	2.25 s	82 ms	61 ms			
O					p, 17.338	EC, 17.76	EC, 5.143	EC, 2.754	Stable	Stable	Stable	B-, 4.821	B-, 3.814	B-, 8.109	B-, 6.490	B-/B-n, 1	B-/B-n, 11.400 MeV			
					N-11	N-13	N-14	N-15				F-19	F-20	F-21	F-22	F-23/F-2	F-24/F-23			
					12.0344	13.0248	14.0086	15.0031	15.9949	16.9991	17.9992	19.0036	20.0041	21.0087	22.01	23.0157	24.0204			
						14	15	16	17	18	019 100%	20	21	22	23	24	25			
									64.49 s	109.77 min		11.00 s	4.158 s	4.23 s	2.23 s	0.34 s				
F						p, 3.200	p, 1.480	p, 10.539	2.761 Me	1.656 Me	Stable	B-,7.025	B-, 5.684	B-, 10.81	B-, 8.480	B-, 13.49	B-/B-n, 2.470 MeV			
						O-13	O-14	O-15	O-17	O-18		Ne-20	Ne-21	Ne-22	Ne-23	Ne-24	Ne-25/Ne-24			
						14.0361	15.018	16.0115	17.0021	18.0009	18.9984	20	20.9999	22.003	23.0036	24.0081	25.0121			
							16	17	18	19	020 90.4	021 0.27	022 9.25	23	24	25	26	27	28	29
								109.2 ms	1672 ms	17.22 s				37.24 s	3.38 min	602 ms	0.23 s	32 ms	14 ms	0.2 s
Ne							p, 10.539	EC/ECa/	EC, 4.446	EC, 3.238	Stable	Stable	Stable	B-, 4.376	B-, 2.470	B-, 7.300	B-, 7.330	B-/B-n, 1	B-/B-n, 8	B-n, 12.310 MeV
							F-15	F-17/N-1	F-18	F-19				Na-23	Na-24	Na-25	Na-26	Na-27/N	Na-28/N	Na-28
							16.0258	17.0177	18.0057	19.0019	19.9924	20.9938	21.9914	22.9945	23.9936	24.9978	26.0005	27.0076	28.0121	29.0193

Figure VI.21. A more comprehensive view of the nuclide map.

It should be finally noticed that mass increases from left to right in the map. *Radioactive* decays are changes in the state of a nucleus occurring by emission of radiation, entailing a structural modification of the nucleus itself. Here *α, β* and *γ* 'radiations'/'rays' are considered: nuclei of ${}^{4}_{2}He$, electrons and high-energy electromagnetic radiation respectively. These 'rays' have to cross a magnetic or electric field for being identified and separated. *β-decay* process may be visualized moving diagonally downwards on the map. However, all diagonal displacements in which daughter's mass is larger than parent's are not allowed (see figure VI.21).

As mass number and charge are conserved, β-decay may be interpreted as a *neutron changing to proton with creation and emission of one electron.*

$$n \rightarrow p + e^{-} + X$$

Actually another particle *X* is emitted too, which allows energy and linear momentum to be conserved: a very weakly interacting particle *X* whose identity is not important for the present analysis.

To summarize, the necessary and sufficient criteria for *β-decays* are

- Z → Z+1 and N → N+1: a sort of "geometrical" criterionon the map;
- *m* (parent) > *m* (daughter): a "parametric" criterion.

On the other hand, *β-decay energy* is measured with proper instruments studying the decay products or the converse process. For instance, the value E = 0.1565 MeV was measured for the decay of ${}^{14}_{6}C$ in this way.

The previous qualitative exploration was thought in order to perform a quantitative analysis in which a *correlation between nuclide mass defect and released energy* were found out by the following steps:

a) Being Δm the atomic (nuclear) mass variation, $E/\Delta m$ ratio is calculated for several nuclides (*N* cases): very close numbers should be obtained;
b) This proportionality is visualized by plotting E against Δm;
c) The average ratio in MeV/a.m.u. is calculated;
d) If SI units are chosen, the ratio will turn out to be approximately c^2.

On the qualitative ground, the involved students should instead elaborate the following argument or something similar:

- Hp1: Mass appears and vanishes simply → mass is created from nothing and destroyed into nothing (*not likely*);

- Hp2: Something else is conserved, which *may* show itself as mass. Mass conservation law is actually a special case of a more general conservation law. *Hint* for this hypothesis: the kinetic energy of the electron! Where does it come from?
  - Either energy is not conserved… *but* all experimental results, included the ones in "everyday science" (for example friction), agree with this fundamental law…
  - …Or decay is a process that let energy transform from another form to kinetic one.
- Is it possible that the negative mass variation of nuclides is related to the positive energy variation of the electron?
- We can infer that mass represents (is a measure of) a particular form of energy, owned by the system, which decreases when mass does. Because of total energy conservation kinetic energy increases when this new kind of energy (actually rest energy) decreases.

VI.3.2.7. *The photon and its energy and linear momentum*

The referred photon phenomenology, the energy quantization hypothesis and the momentum induction are in section IV.7.

As for photon mass, the relativistic KE expression diverges in the limit for speeds approaching c: $K = m\,c^2(\gamma(u) - 1) \underset{u\to c}{\longrightarrow} +\infty$. Obviously no physical quantity may take an infinite value. Since c^2 is fixed, the only way out is assuming $\boxed{m = 0}$ for every entity moving exactly at c, so that an indeterminate form is obtained in the expression above $(0 \cdot \infty)$.

VI.3.2.8. *Absorption of two photons*

At this point a meaningful thought experiment can be run, in order to obtain a general expression for energy and go beyond the sole kinetic form. It was inspired by Fabri (2005).

Two photons absorbed by a body of mass *M* are considered. The process is described in two different inertial frames: K and K'. The latter is at rest with respect to the absorbing body, and the energies and the (opposite) momenta of the photon are labelled ε' and ε'/c respectively in it, so that the total linear momentum is *null* before and after the absorption. K is moving at speed u with respect to K', so photon energy and momentum are different: ε and ε/c respectively. In this case an angle θ is between the directions of light propagation (of photon momenta) and of relative motion, as shown in the picture below.

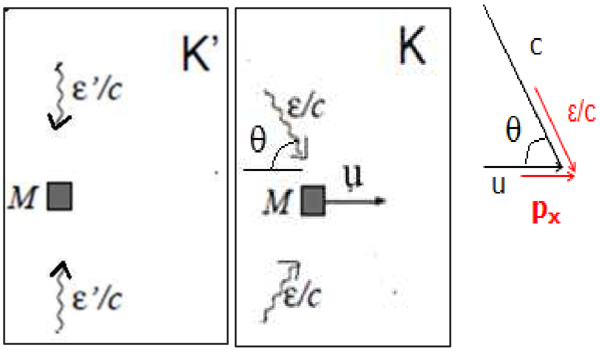


Figure VI.22. The process of photon absorption in the *initial state* observed in two different IFs. On the right, the angle θ between light and body speed's directions.

In order to apply momentum conservation for studying the process, relativistic total momentum measured in K *before the absorption* may be written

$$p = M\gamma u + \frac{2\varepsilon}{c}\cos\theta = M\gamma u + \frac{2\varepsilon}{c}\frac{u}{c}$$

The last passage may be justified by similitude between triangles as well, without using trigonometric functions. *After the absorption* the photons have disappeared, so we are left with

$$p' = M\gamma u'$$

Momentum conservation equation is thus $\boxed{M\gamma u + \frac{2\varepsilon}{c^2}u = M\gamma u'}$.

If initial and final states are compared in K', it will be seen that the absorbing body stands still; therefore its speed has to *remain the same in K according to any physical theory*: $u = u'$.

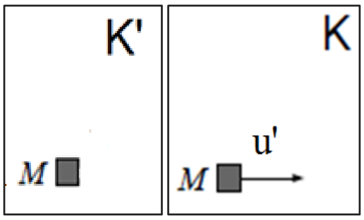


Figure VI. 23. Photon absorption in the *final state* observed in two different IFs.

The equation for momentum conservation becomes then

$$M\,\gamma\,u\;+\frac{2\varepsilon}{c^2}u=\;M\,\gamma\,u\;\Rightarrow\;\frac{2\varepsilon}{c^2}u=0\quad(8)$$

that is *physically impossible*: (i) if $u\;=\;0$ there is only one IF, (ii) if $\varepsilon\;=\;0$ there are no photons! The TE is meaningless in both cases, so there must be an error in the previous argument. The assumptions are correct, but one of them has not been explicitly discussed yet: *mass conservation,* a cornerstone of classical physics.

If this hypothesis is dropped and the other ones are kept, one is allowed to mark with an apex the body's mass after absorption, and equation (8) turns into

$$M\,\gamma\,u\;+\frac{2\varepsilon}{c^2}u=\;M'\gamma\,u\;\;\Rightarrow\;\;M+\frac{2\varepsilon}{\gamma c^2}=M'$$

Mass is not conserved according the last equation, which was logically deduced from relativistic momentum conservation and a simple passage between IFs in the description of a trivial physical phenomenon, assuming the Relativity Principle (RP). So this absorption of energy makes mass vary. The claim may be quantified in the following, exploiting the fact that $+2\varepsilon$ is the total energy variation of the body.

$$M+\frac{\Delta E}{\gamma c^2}=M'\Rightarrow\Delta M=\frac{\Delta E}{\gamma c^2}\qquad(9)$$

This is the **mass increase undergone by the body if the energy is varied while it is moving at constant speed**. In particular, if the body stands still in the laboratory $\gamma=1$, and then

$$\Delta M=\frac{\Delta E}{c^2}\qquad(10)$$

Equation (10) expresses the physical claim that **any increase of the body's energy ΔE keeping it at rest makes mass increase too**. The variation equation (10), which is equivalent to $\Delta E\;=\;\gamma\;\Delta M\;c^2$, comes out from the most general form of mass-energy equivalence, as the reader will see later on.

An object of negligible initial mass at rest in the LabIF is considered now ($m_i<\varepsilon$). The input of energy needed for making its mass increase until a value m is thus

$$\Delta E=\Delta M\,c^2=(m-m_i)c^2\xrightarrow[m_i\to 0]{}m\,c^2$$

The limit for null initial mass has been taken, assuming energy (variation) to be a continuous function of m. Therefore

$$\boxed{m\,c^2 \equiv E_0}$$

may be meant as the *energy needed for creating a particle of mass m at rest*. This is a good definition for *rest energy* E_0, which is nothing but the sum of every form of a particle energy at rest. This definition also matches Hecht's one (2011) for invariant mass: « The invariant mass of any object – elementary or composite – is a measure of the minimum amount of energy required to create that object, at rest, as it exists at that moment ». This is in agreement with the null mass of a single photon, because it does not exist at rest.

Particles are created in colliders in this way; one needs to

i. have enough available energy: at least $m\,c^2$ for creating a particle of mass m;
ii. have linear momentum conserved.

It may be stated that *mass represents rest energy* in Relativity (Einstein & Infeld, 1938), or more precisely that *mass measures the total amount of energy owned by a body/system/particle at rest* (Einstein, 1905a).

VI.3.2.9. *Relativistic energy*

Every argument has been carried out in a single IF – namely the LabIF – up to now, because all considered processes occurred in stationary conditions. Nevertheless, the examined object might move anyhow in the Lab IF, so *its KE in the LabIF is also to be considered*, by means of co-moving frames. The object's *total energy* may reasonably be defined by the sum of rest and kinetic energy:

$$E \equiv E_0 + K = mc^2 + m'c^2(\gamma - 1)$$

What about the masses m and m' measured in two different IFs? We can infer they have the same value in all IFs, since *all observers in them refer to the object's rest frame* and measure (inertial) mass at *low speed* by classical methods based on $F = ma$, possibly combined with Newton's third law or Hooke's law (Fabri 2005) So *mass* – Newtonian mass – *is a relativistic invariant*: $m = m' = m'' = \cdots$ As we have seen, this entails that mass is conserved under accelerations.

Thus $E = E_0 + K = mc^2(1 + \gamma - 1) = \gamma mc^2 \Longrightarrow \boxed{E = \gamma mc^2}$ (11)

in agreement with the previously deduced formula $\Delta E = \gamma \Delta M\, c^2$. For the sake of precision, this is not ΔE but a part of it, as it will be shown later. Equation (11) is the actual generalized expression for *mass-energy equivalence*.

Energy is dealt with by *two new basic ideas* in relativistic paradigm:

1. A particle system/single particle does not own all the diverse forms of energy of classical mechanics, but simply the *total energy* $E = \gamma m c^2$. If it is still in the LabIF – that is, if it has the same spatial position at every instant – then $\gamma = 1$ and its energy is $E_0 = m\, c^2$: the smallest energy associated to *the mere existence of a massive particle/system*, also called "zero-energy" because it is the energy at null speed.
2. Total energy is conserved in every physical process.

Therefore the words "total energy" and "energy" will be used interchangeably in the following.

Energy absorption or emission may alternatively bring to

- *Speed variation* when work is done on a body/system/particle (classical case);
- *Mass variation* when energy is varied at constant speed, including the zero speed case (purely relativistic case).

From the mathematical standpoint energy may be considered indeed as the product of three parameters: $m,\ \gamma(u)$ and c^2. Since the third one never varies in any physical transformation, energy variation is

$$\Delta E = c^2 (m\, \Delta\gamma + \gamma\, \Delta m).$$

The former term matches the first case above, the latter the second case.

VI.3.2.10. *Energy, momentum and invariant mass*

So the expression for relativistic momentum and energy are achieved:

$$\begin{cases} \vec{p} = \gamma(u)\, m\, \vec{u} \\ E = \gamma(u)\, m\, c^2 \end{cases}$$

The relativistic relationship among energy, momentum and velocity is worked out dividing the first equation by the second one. It is also useful for formally *deducing* the previously *induced* momentum-energy relation for a photon, so that generalization is scientifically guaranteed (Popper, 1934):

$$\frac{\vec{p}}{E} = \frac{\vec{u}}{c^2} \xrightarrow[u \to c]{} \frac{1}{c} \hat{u} \Rightarrow E = pc$$

Momentum and energy are *not* invariant, because of their dependency upon the measured speed. However, in general it is better to deal with *invariant quantities*, which are objective. Reasoning by analogy with time and space interval leads to calculate the difference of squares:

$$E^2 - p^2c^2 = (\gamma mc^2)^2 - (\gamma mu)^2c^2 =$$

$$= \gamma^2 m^2 c^4 (1/\gamma^2) = m^2c^4$$

$$\boxed{E^2 - p^2c^2 = m^2c^4} \quad (12)$$

This result is *not depending on the Lorentz factor.* Mass and light speed *in vacuo* are invariant, thus the quantity $E^2 - p^2c^2$ is too: mass is the analogue of proper time. This parallel will become apparent if time and space intervals are made explicit in the expressions for energy and momentum *in a generic IF*[100]:

$$\begin{cases} \vec{p} = \lim\limits_{\Delta t \to 0} \left[\dfrac{\Delta t}{\Delta \tau} \, m \, \dfrac{\Delta \vec{s}}{\Delta t} \right] = \lim\limits_{\Delta \tau \to 0} \left[\dfrac{\Delta \, \vec{s}}{\Delta \tau} \right] m \\ E = \lim\limits_{\Delta t \to 0} \left(\dfrac{\Delta t}{\Delta \tau} \right) c^2 m \end{cases}$$

$$\Longrightarrow \text{[finite variations]} \; E^2 - p^2c^2 = m^2c^2 \frac{(c\Delta t)^2 - (\Delta s)^2}{(\Delta \tau)^2} = m^2c^2 \frac{(c\Delta \tau)^2}{(\Delta \tau)^2}$$

$$= m^2c^4 = E'^2 - p'^2c^2$$

So the *invariance of* $E^2 - p^2c^2$ *is intimately related to the invariance of proper time*, namely to the fact that the difference of squared time and space intervals is *equal to* $\Delta\tau$ *in every IF*. This is due to the structure of relativistic space-time (Levrini, 2014), but this discussion might go too far.

Equation (12), the most important dynamic relation in Special Relativity, **may be used to define mass operationally**: mass is a relativistic invariant quantity[101] provided by combining energy and momentum measures in any inertial frame according to $m^2 = (E/c^2)^2 - (p/c)^2$. It is not necessary to refer to the rest IF anymore, so this is usually called *invariant mass*, especially in particle physics. It is conceptually separate from rest mass, even if they in fact coincide.

A visualization of the definitional equation is provided by the application of the Pythagorean Theorem to a right triangle whose catheti and hypotenuse

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[100] Primed quantities are meant as measured in a different IF.

[101] If the previous argument is reversed, and the invariance of $E^2 - p^2c^2$assumed, invariance of mass will assume a deeper meaning, related to invariance of proper time (namely space-time interval).

measure m, p/c and E/c^2 respectively (Okun, 2008b). This is the general case; special cases are being examined.

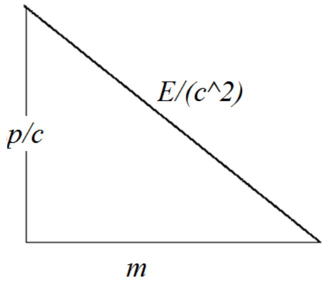

Figure VI.24a. The right triangle for visualizing the definition of mass in SR: general case (Okun, 2008b).

- *Object at rest*: $E \equiv E_0 = mc^2$. If the object stands still with respect to the observers in the LabIF, its momentum will be null, thus mass and (rest) energy will overlap: the triangle collapses to a horizontal segment.

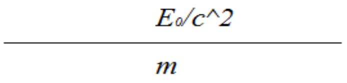

Figure VI.24b (Okun, 2008b).

- *Newtonian case*: $p/c << E/c^2 \sim m \Rightarrow E/(mc^2) = \gamma \sim 1$. Momentum is much smaller than energy, so that the horizontal cathetus is much longer than the other: hypotenuse and horizontal cathetus *almost* overlap.

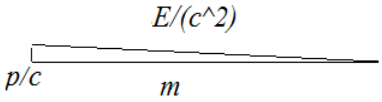

Figure VI.24c (Okun, 2008b).

- *Ultrarelativistic case*: $p/c \sim E/c^2 >> m \Rightarrow \gamma >> 1$. Speeds are so high that momentum is *almost* proportional to energy; mass is much smaller. For particles in LEP $\gamma = 10^5$, for protons in LHC $\gamma \sim 10^4$, for cosmic ray neutrinos $\gamma \sim 10^9 - 10^{10}$. Hypotenuse and vertical cathetus *almost* overlap.

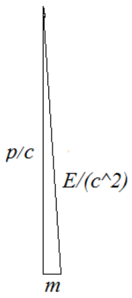

Figure VI.24d (Okun, 2008b).

- *The photon*: *p = E/c*. The triangle collapses to a vertical segment.

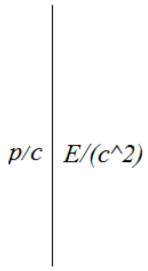

Figure VI.24e (Okun, 2008b).

If equation (12) is combined with the previously induced momentum-energy relation for the photon, it will be logical inferred that its *mass is null*. Therefore the former assumption proves correct.

$$\begin{cases} E^2 - p^2c^2 = m^2c^4 \\ E = pc \end{cases} \Rightarrow m = 0$$

This result was experimental confirmed at a higher and higher degree of accuracy (compare the end of IV.7).

VI.3.2.11. *Mass conservation and additivity (TE)*

Eventually, two colliding relativistic identical particles of mass *m* are considered, in order to gain an insight into the issue of mass conservation in SR (Feynman, 1964).

Before the *inelastic collision* the particle velocities are opposite in the LabIF, which will be named K; after collision a new particle is created at rest in K (figure VI.25). Only (i) Relativity Principle and (ii) linear momentum conservation are assumed.

The equation for the momentum conservation is $\gamma(w)mw - \gamma(w)mw = 0$, that is obvious and do not give us any information about the mass *M* of the product. For finding out the relationship among *M* and the other mechanic variables in a simple way it is necessary to consider an IF in which the product *is moving very slowly* or, equivalently, an IF K' *moving at Newtonian speed with respect to K*. K' is thought as a 'lift' (figure VI.25).

In K' the new object moves steadily with velocity **u** *orthogonal to* **w**, so that the velocity components of the initial particles in the direction of **u** are simply

u, which has to be a non-relativistic speed $u \ll w < c$. In this way, even if the initial velocity modulus in K' is not $u + w$ because classical velocity transformations are not valid anymore, that modulus is *very close* to $u + w$: momentum conservation equation in **u** direction in K' is

$$\gamma(u+w)\, m\, u + \gamma(u+w)\, m\, u = \gamma(u)\, M\, u \;\cong M\, u \quad (13)$$

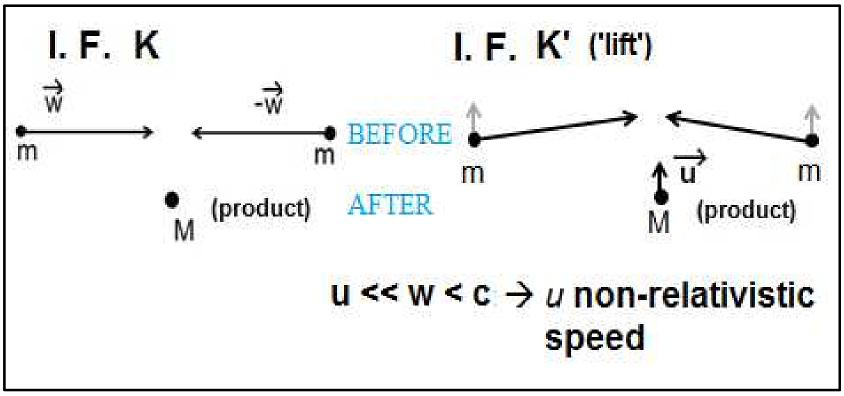

Figure VI.25. The relativistic perfectly inelastic collision seen in K and K'.

Then

$$M \cong 2\, m\, \gamma(u+w) > 2\, m$$

so that **mass is neither additive nor conserved, even though it is invariant:** these classical properties are disentangled in SR[102]. This approximate[103] result coincides with the right one, as shown by a trivial application of (total) energy conservation law in K:

$$\gamma(w)\, m\, c^2 + \gamma(w)\, m\, c^2 = M\, c^2$$

$$\boxed{M = 2\, \gamma(w)\, m > 2\, m}$$

The KE of the initial system, i.e. the whole of the two initial particles, *changes entirely into the product's rest energy*, which is also *mass* or, equivalently, the product's rest energy is 'represented' by its mass.

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[102] An invariant physical quantity is not necessarily conserved and/or additive, but in classical mechanics mass has all these properties, so that they tend to be considered as entangled.

[103] The correct relativistic calculation would have required to take into account that initial velocities in K' are the vector sum of the components, whose moduli are actually u and $w\sqrt{1-u^2/c^2}$, because of time Lorentz's transformation (u is the relative speed of K' too). Exploiting the Pythagorean Theorem the modulus of the initial velocities turns out to be $\sqrt{u^2 + w^2(1-u^2/c^2)} \approx \sqrt{u^2+w^2} = |\vec{u}+\vec{w}|$ if $u^2/c^2 \to 0$ and then it is possible to write $\gamma(u+w)$ in (13).

VI.4. Designed formative intervention experiments

Once the conceptual pathway had been set up, the next step was to build the educational pathway, which was then going to be tested by designing a series of formative intervention modules/experiments (t/l activities). According the standard EspB [22] a t/l activity includes

1. Prerequisites and scheduled duration of the activity;
2. Objectives and t/l strategies;
3. Conceptual pathway;
4. Instructional materials, namely slides (provided in appendix 4), visual tools (paragraph VI.4.2.4) and student worksheets (appendices 2 and 3).

VI.4.1. Prerequisites and scheduled duration

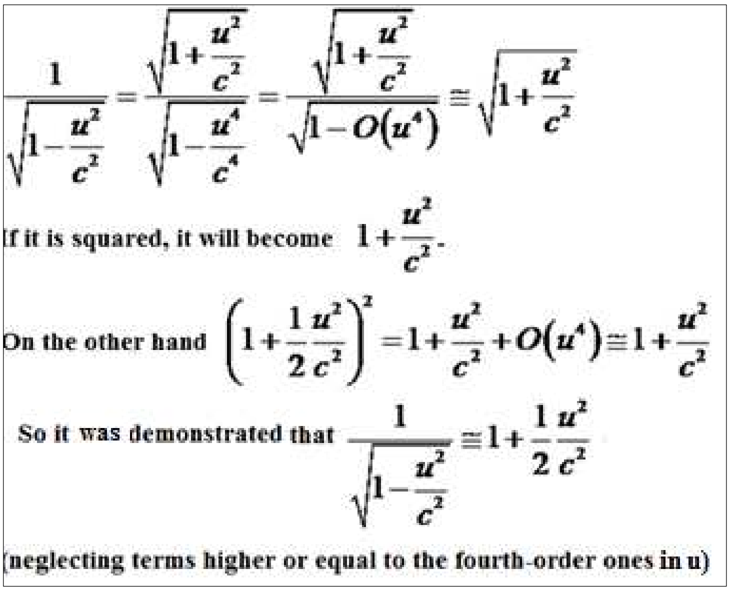
VI.4.1.1. *Classical module (common to the paths) (4 – 5 hours)*

1) Density-mass-volume relationship in fluid statics, regularly introduced through the definition of density as mass divided by volume;
2) Mass in Newtonian dynamics; Newton's laws of dynamics; circular uniform motion and therefore the notion of centripetal acceleration;
3) Kepler's laws, in particular the third one, and the weight definition $W = m\,g$;
4) Hooke's law;
5) The notion of collision entailing impulsive forces;
6) Basic chemistry, in particular state transitions, chemical solutions and redox;
7) General abilities:
    a. Table and plot processing;
    b. Search for regularities described by linear relations between the physical variables $\Delta v_1/\Delta v_2$ and N, i.e. velocity variation ratio against number of objects in a quasi-elastic collision experiment (see VI.2.1.2).

VI.4.1.2. *Relativistic four-vector module (7 – 8 hours)*

1) Definitions of kinematical and dynamical quantities in classical physics;
2) Second law of classical dynamics;
3) Notion of vector modulus;

4) Notion of energy as quantity defined unless an additive constant;
5) (Possibly) Compton effect;
6) Information technology (IT) and general abilities:
    a. Table and plot processing;
    b. Autonomous work with an applet;
    c. Search for regularities described by mathematical relations between the physical variables E and Δm (released energy and mass defect respectively) in the radionuclide β-decay.
7) Taylor series expansion, although the approximation $\gamma \sim 1 + \frac{1}{2}\left(\frac{u}{c}\right)^2$ may be obtained in an algebraic way too:



$$\frac{1}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{\sqrt{1+\frac{u^2}{c^2}}}{\sqrt{1-\frac{u^4}{c^4}}} = \frac{\sqrt{1+\frac{u^2}{c^2}}}{\sqrt{1-O(u^4)}} \cong \sqrt{1+\frac{u^2}{c^2}}$$

If it is squared, it will become $1+\frac{u^2}{c^2}$.

On the other hand $\left(1+\frac{1}{2}\frac{u^2}{c^2}\right)^2 = 1+\frac{u^2}{c^2}+O(u^4) \cong 1+\frac{u^2}{c^2}$

So it was demonstrated that $\frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \cong 1+\frac{1}{2}\frac{u^2}{c^2}$

(neglecting terms higher or equal to the fourth-order ones in u)

Figure VI.26. Calculations for finding the second-order approximation in u of γ.

VI.4.1.3. *Relativistic energetic phenomenological module (10 hours)*

I. Basics of Newtonian mechanics, in particular
    a. Kinetic energy expression;
    b. Energy conservation principles;
    c. $F = ma = dp/dt$;
    d. The classical velocity transformation law;
    e. Elastic collisions;
    f. Centre-of-mass reference frame (centre-of-momentum in SR);
    g. Definitions of kinematical and dynamical quantities
II. Thermodynamics and thermostatics

    a. Notion of internal energy;
    b. First law;
    c. Fundamental law of calorimetry

III. Mathematics
    a. Bijective function;
    b. Binomial theorem;
    c. Limits;
    d. (Possibly) derivative and integral calculus.

IV. Information technology (IT) and general abilities
    a. Autonomous work with an applet;
    b. Table and plot processing;
    c. Search for regularities described by mathematical relations between the physical variables E and Δm (released energy and mass defect respectively) in the radionuclide β-decay.
    d. (Possibly) Use of Excel or another calculus worksheet for data linear interpolation.

VI.4.2. Objectives and Teaching/Learning Strategies

Several indications and suggestions by scholars for valuing modern physics, fostering science literacy and innovation, and promoting innovation for modern physics literacy were summarized throughout the first and second chapter. It is being shown what was designed in order to follow these hints.

The most important educational contribution of modern physics is to *build formal thinking*. This occurs through the development of concepts and interpretive hypotheses *without a classical equivalent*. In the present path the concepts of this kind are invariance and isotropy of c; existence of an ultimate speed; non-invariance of simultaneity and of time intervals; proper time; space-time and momentum-energy unifications; four-dimensional non-Euclidean geometry of the Universe; Minkowski norm; rest energy and its relationship to mass; non-conservation and non-additivity of mass; β-decays as statistical processes; the concept of photon[104] and existence of massless entities; creation and annihilation of particles in collisions.

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[104] It is different from the simple Newtonian-like particle interpretation of light: such a concept has very likely never been dealt with in school practice before.

As concerns *innovation*, in order to follow the first suggestion by Sjøberg (2002) – dealing with non-specialist topics at high-school level, reported in II.1 – an academic treatment of the subjects covered in the worked-out teaching/learning pathway was avoided, for example limiting mathematical formalism to the minimum necessary and adapting it to the different target schooling levels.

In order to follow the second suggestion (highlighting the cultural aspects of science for broadening students' perspectives) a historical-epistemological reconstruction of the birth and conceptual evolution of classical mass by means of science text analysis was made: history of physics contributed considerably to the entire path design. Historical-epistemological reconstruction means, in this context, to choice some meaningful aspects of the historical scientific debate about a concept and to present how the concept itself evolved in different theories until nowadays, in order to show it from different perspectives and thus foster a critical reflection. This allows deeper learning than the one deriving from ordinary teaching, because *a discussion* is made instead of a mere assimilation of the concept by definitions or applications in exercises/problems. Galili (2012) claims indeed that a lot of students are motivated by the chance of participating to that live discussion of ideas; « By excluding controversy of knowledge and debate of conceptual refutation, we often lose such students, repelled by dogma and algorithmic applications ».

For instance, the historical controversy about the direction of vision in optics in ancient and medieval "science", as well as the Alhazen's theory of vision (11th century), have become materials for writing a short drama (de Hosson & Kaminski, 2007) in the form of a dialogue, entitled "Dialogue on the Ways that Vision Operates" with the same characters of the famous dialogue by Galileo. This inquiry-based teaching/learning sequence was experimented with 12 low-secondary students and gave positive learning outcomes in many cases. Differently from de Hosson, I did not write an original text on a historical controversy and my students did not identify with the characters of the drama, but I attempted to stimulate their reasoning upon the text of authoritative scientists, through step-by-step driving questions. Further, these pupils were from 16 to 19 years old in contrast with de Hosson's. However, my intention was basically the same: highlight scientific controversy and debate, for fostering interest and personal involvement. Moreover, I examined original historical texts for designing both classical and relativistic paths, being especially inspired by two Einstein's writings (1905a, 1935), and less by a third one (1905b), in order to adapt them to contemporary learners' needs, according to Mäntylä (2013) too. Finally, my research aim was *not* to compare historical development of mass and students' cognitive pathways, differently from de Hosson, but to exploit the former for finding the latter.

My approach when designing the *energetic relativistic path* has several similarities with the cognitive-historical one (Mäntylä, 2013), in which the historical development is not proposed again, but the experiments, modeling, and basic reasoning are taken from history of physics and adapted to current contents. In particular, experiments or though experiments tracing historical ones but educationally reconstructed were designed and used for *generative justification of knowledge* (Koponen & Mäntylä 2006), for ex. cart collisions, photon absorption.

The third Sjøberg's suggestion (to favour an active personal construction of meaning) was enhanced by the *cognitive qualitative and quantitative explorations* students had to undertake.

The fourth suggestion, essentially following a «historical line» (Arons, 1992), was a hint for

- going on from classical physics to SR and potentially General Relativity and Higgs mechanism, although not by narratives;
- dealing with crucial historical events for the conceptual development of mass, i.e. events which have progressively shaped and enlightened nature and facets of mass. Examples are (i) the birth of natural radioactivity, (ii) the hypothesis of photon and discovery of its physical properties; (iii) a re-mastered version of the first thought experiment on mass-energy equivalence (Einstein, 1905a; Fabri, 2005). The first (i) gave rise to the idea of atom as compounded by discrete entities and thus paved the way to atomic mass in its current meaning; the second (ii) brought to the concept of massless entity; the third (iii) had a very high heuristic value for the science community about the relativistic meaning of mass.

In a «historical line» all concepts are more or less *intimately interrelated*, which is important for an effective understanding. It has been recently discovered by educational psychology indeed that brain elaborates parts and wholes *simultaneously* (Caine *et al.*, 2009). Therefore it is important to train students to focus on partial and global vision of each topic *at once* by passing straightforwardly from details to framework and vice versa.

Eventually, the fifth suggestion (exploiting digital media) was followed by developing a *visual approach* in the path design: some rough multimedia resources were used.

Innovation for modern physics literacy was instead enhanced by:

- The ultimate speed experiment by Bertozzi (1964) and mass variation in β-decays, which served to warn students against the limits of validity of classical physics, as recommended by Gil and Solbes (1993);
- Questioning of absolute time and space (Gil & Solbes, 1993) by the comparison between the time intervals measured by two observers in different frames; in fact the travelled distance (space) cannot be separated from the elapsed time;
- Highlighting of the breaking aspects of relativistic with respect to classical mechanics, but with a stress on the correspondences at the same time: the Correspondence Principle was indeed used in several instances.

Since the internal coherence of the path is more important of the recourse to history, the focus on the latter was alternated with *non-historical phenomenological explorations*; the path was ideated indeed in order to gain conceptualization of experiment-theory interplay too.

VI.4.2.1. *Content-specific Objectives and Strategies*

The intention of the present work has been to take into consideration the fairly neglected *relativistic "dynamics"* under the educational viewpoint, in order to let high-school students understand its conceptual nuclei through a logically consistent path. More specifically, the pathway aims at building conceptual understanding of the physical meaning of relativistic linear momentum, kinetic energy, total energy, rest energy, invariant mass and, above all, of their relations. It draws on relativistic invariant quantities and exploits educational thought and real experiments; it shows both scientific modeling by induction both hypothetical-deductive methods for assuring logical consistency.

The ultimate aim is of course to let pupils construct *personal meaning-making* of this seemingly exotic matter. This was attempted by adapting the multiple concept of classical mass, the concept of energy, and the relativistic relationship between Newtonian inertial mass and energy for *being conceptually* *learned in the last three years of Liceo specializing in scientific studies*. The comparative examination of the concept of mass in three physical theories/models turned out to be necessary (or important at least) so as students gained sound knowledge and conceptual mastery of it. Actually, all of the following passages allow achieving a full learning with understanding of “mass” and the “problem of mass” in modern physics: from *quantitas materiae* to inertial and gravitational mass – the second one in Newton, Mach and Bridgman – then to the well-known equivalence expressed by $E_0 = mc^2$, to mass as related to momentum-energy density, determining space-time curvature (GR), and finally to effective mass.

VI.4.2.2. *Inquiry-based Learning by Representation Construction*

The active and personal intellectual involvement in the studied subject is necessary for an effective scientific learning, according to the constructivist tenets. In fact, the most recent definition of science literacy itself (compare I.3) entails a use of the achieved knowledge in an *inquiry process* that (i) brings students to increase knowledge autonomously by questioning and that (ii) allows to describe, explain and predict phenomena (Tytler *et al.*, 2013). Besides, science is not regarded as a set of formal theories in mathematical language[105], but as *a form of human knowledge next and related to the others*, joint to technology as concerns the environmental effects, and highly impacting on the cultural life of every active citizen. In fact, physics is culture (Levrini, 2014).

In order to enhance this active involvement and construction of personal meanings, *learning by inquiry* has been used for arranging both the tests administered during the 2013 and 2014 summer schools held in Udine, both the worksheets utilized in the 2012 winter school in Bard (AO), as well as the worksheets for several high-school classroom formative experiments in different Italian towns (see appendices 1, 2 and 3 for further details). The most important model of reference was *learning by construction*, but also *learning by discovery* was partially used, both in the form of sudden insight (compare I.2), triggered by a TE, both as long-lasting gradually developed invention, like the production of ideas during a group discussion.

Anyway, the basic idea was to guide students in a *step-by-step construction of new representations exploiting the owned conceptions*, in order to approach

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[105] It would have been a reductionist approach.

scientific concepts but respecting the pupils' cognitive styles at the same time. This may be done by explicit «challenges» to work out successive representations, following the perspective by Tytler, Prain, Hubber, & Waldrip (2013): « a particular approach to guided inquiry in science learning with a strong explicit emphasis on student-generated representational work through sequences of representational challenges accompanied by negotiation and refinement of the produced representation ». The attention of PER community to students' representational resources for meaning-making has indeed increased (Taber, 2011) and a growing literature stresses the power of refinement of explanatory models by class negotiation to achieve quality learning (Clement & Rea-Ramirez, 2008).

These negotiation and change necessarily presume a *revision of the held physical ideas*. Cognitive conflict was considered as the most significant strategy for triggering conceptual change in the classical approach (explained in II.6.2), but it proved to hinder an effective learning with understanding. It is necessary to foster *didactical* and, more generally, *cognitive continuity* instead for extending (assimilation) or reconstructing (accommodation) the whole interpretive framework of a student.

One of the strategies used in the planned formative intervention experiments was to let students explore phenomenology in which *mass is not conserved*, namely *nuclear decays* and *photon emission/absorption*, through guided inquiry or interactive lessons. Relatively high variations of atomic mass can be calculated indeed when dealing with decays ($\Delta m = 0.01 \div 0.001$ a.m.u.), thus stimulating a critical reflection in the learner. The ultimate speed experiment by Bertozzi (1964) also served to stimulate a revision at the subject matter level through observation. It is also worthwhile to introduce *analogies* and *metaphors* for making the new theory, Relativity in this case, more intelligible and plausible[106]. This was put into practice for example when four-vectors were defined by analogy with classical three-vectors. It has therefore been made «deliberate uses of bridging analogies» (Vosniadou *et al.*, 2008).

Understanding is improved if POE strategy (*Predict-Observe-Explain*) is used, a variant of which is PEC strategy (*Predict-Experiment-Compare*). The former was used in the 2014 summer school, by making students compare their prior motivated previsions for (i) the plot of Bertozzi's experiment and (ii) the

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[106] This approach is based on the metaphysical assumption that the symmetry of physical laws mirrors the ones of nature, which comes from scientific realism.

behavior of a moving light clock with the observed plot and light-clock simulation respectively. They were then asked to explain the (possible) *discrepancies* between their previsions and observations. The aim was to trigger a reflection on the conflicting classical and relativistic conceptions. PEC strategy was used in the experimentation with samples 3 and 4 in Udine instead (3^{rd} year of high school). The students were asked to foresee (i) a kinematical plot (velocity vs. time) and (ii) the dependence of the variation velocity ratio on the number of bars, thus *when a change was made*, in a collision experiment (see next section for details). So they had to explain again the difference between previsions and experimental results, but this time a real in-lab experiment had been run in front of them, and they were encouraged to identify relevant variables and, above all, to find regularities. A Real Time Lab (RTL) experiment was indeed performed, so that students could visualize any variable variation in real-time (Kearney, 2004; Sassi & Vicentini, 2008; Theodorakakos, Hatzikraniotis, & Psillos, 2010).

The devised RTL experiment is based on pre-existing educational materials for the study of motion through on-line sensors [19], in particular the worksheets C-10, C-11 and C-12. Those materials have been developed by the Naples PER group (Elena Sassi, Gabriella Monroy, Sara Lombardi and Italo Testa) and have been adjusted and validated in Udine in 2006, in order to foster innovation in physics education by means of pre- and in-service teacher training.

VI.4.2.3. *RTEI: quasi-elastic one-dimensional collision*

An educational real experiment follows, which was worked out to let students explore quasi-elastic one-dimensional collisions between macroscopic carts. A kinematic study is made with the help of real-time motion sensors; a Real-Time Experiment and Images (RTEI) is carried out, that is a type of RTL experiment exploiting the PEC strategy. A *qualitative exploration* phase is followed by *quantitative exploration*. The collision is provided by a magnetic interaction, rather than a mechanical one, in order to reduce damages.

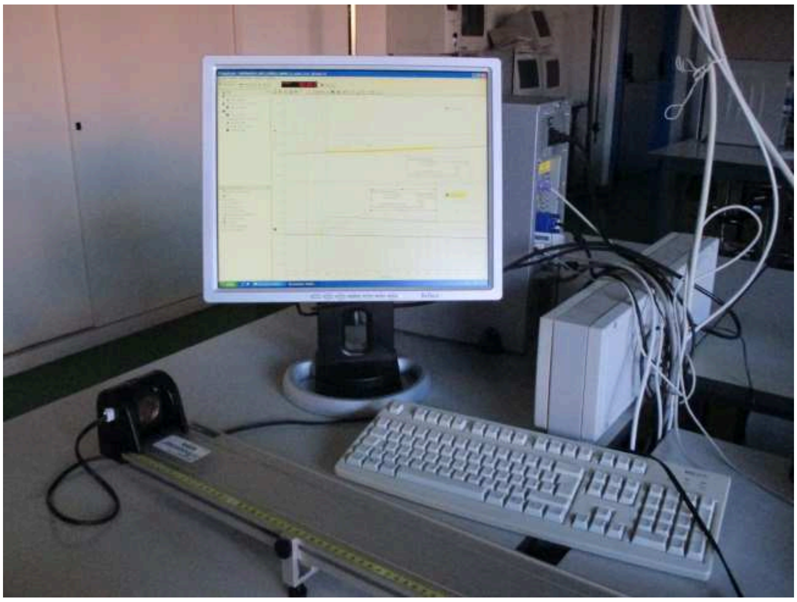
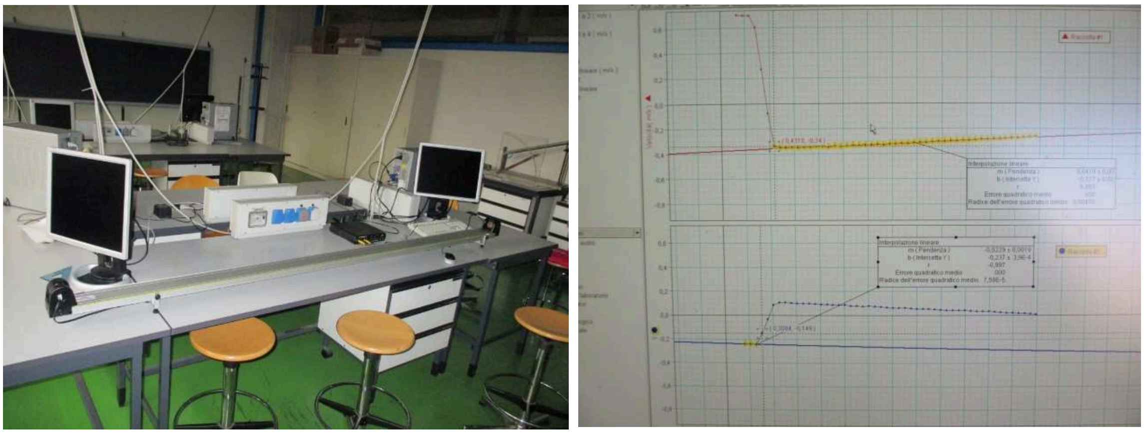


Figure VI.27. RTEI workbench.



Figures VI.28 and VI.29. Computer for online data acquisition and analysis (left); linear extrapolation of $v_{1,2}(t_{coll})$ on the pc screen (right).

In the first stage two empty (approximately) identical carts collide twice: at first one is standing at rest compared to the RTL sensors and the other is moving approximately steadily; later both of them are moving towards one another. A model for the collision is utilized, according to which the process has the *finite duration* Δt_{coll}, identified with the time interval in which the accelerations are much larger ($\Delta t_{coll} \cong 0.2$ s usually), and the instant of the collision (t_{coll}) is supposed to be in the middle of Δt_{coll}. The incoming and outgoing velocities $v_{1,2}(t_{coll})$ are extrapolated *as if each cart went on steadily*. A better accuracy in the collision description is not achievable, because data are being sampled with the frequency of 20 Hz; and even if the latter were much increased a discrete modelling of the process would be done anyway. The measure apparatus is shown in the images below. The sensors are being used for measuring *velocity* only, in order to obtain two overlapped real-time *velocity against time* plots.

Stages of data analysis:

- *Estimate* of the incoming and outgoing velocities of each empty cart – slowed down by friction – by reading them on the respective best-fit lines;
- *Calculation* of the variations Δv_1 and Δv_2 in the former velocity estimates and search for a relationship between them;
- *Calculation* of the ratio $\Delta v_1/\Delta v_2$ for the first case and for the collision of an empty cart against the other filled with one, two, three bars in succession.
- *Filling in* a summarizing table and *plotting* $\Delta v_1/\Delta v_2$ against N (number of bars, supposed identical, on the second cart), searching for a correlation and its justification in terms of the increasing bar number, having *implicitly* acknowledged that each cart is approximately equal to each bar for the purposes of this inquiry (for the sake of accuracy $m_{cart} \cong m_{bar} \sim 0.5$ kg).

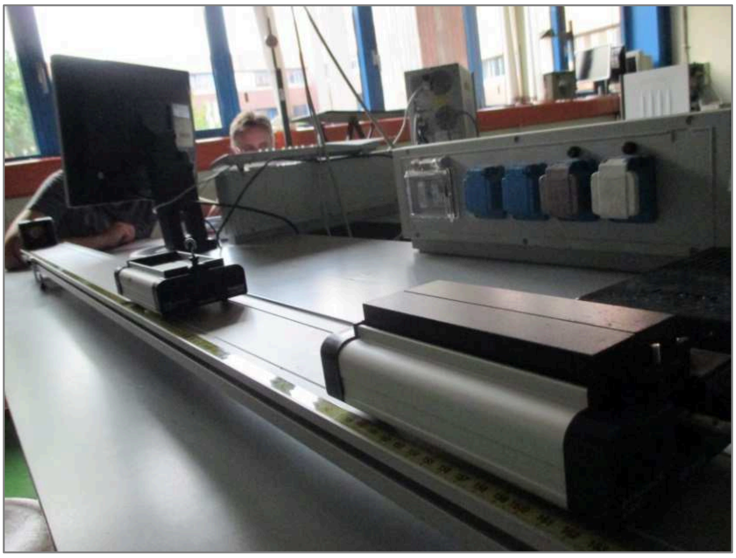


Figure VI.30. An instant before the collision of an empty cart against a cart charged with two bars.

VI.4.2.4. *Visualization*

Due to the importance of the development of formal thinking in science, *representation construction* has been extensively used in the present work, by means of *visualization* in particular.

A *visual approach* has been tried out indeed in the design of the t/l sequence by exploiting rough new technologies, owing (i) to the present emphasis on image study in much contemporary science and school science accordingly (Tytler *et al.*, 2013) on the one hand, and (ii) to the use of imagery in making sense of natural phenomena, i.e. *in physics reasoning* (Botzer & Reiner, 2005), by naïve students on the other hand. In this connection Gilbert (2005) reaffirmed the centrality of visualization to science learning owing to the importance of understanding how to move within and between different modalities of representation. Therefore images (very frequently), videos, online applets with simulations, a paper map of radionuclide, and some rough timed animations were used. The specific utilized visual tools are listed below.

Four-vector module: the students visualized a particle worldline and a photon worldline on a screen. Two videos[107] [38, 39] from the Cassiopea project were played in the formative intervention experiment 2. The on-line applet represented two mirror boxes with a light ray inside travelling back and forth, in order both to establish an operative definition of time through a periodic phenomenon (light clock) and to compare light paths in presence of relative box speed. Two applets were alternately used [16, 17]. The students visualized these simulations for constructing a mental qualitative representation of *time interval dilation* effect.

Energetic module: Bertozzi's experiment may also be found in its original version as PSSC video: "La velocità limite" [43]. Other PSSC videos[108], namely on the nature of photon as light *quanta* ("I fotoni" [40]) and particle-wave duality ("Interferenza dei fotoni" [41]) were shown during the formative intervention experiment 11, according to the numbering in table VII.2. The students also watched a brief historic-experimental video [37] explaining nature and a possible differentiation method for α, β, and γ radiations after the historical foreground to the part on β-decays in the experiment 6. Eventually, rough timed animations were used for the thought experiment on collisions introducing relativistic momentum, for stressing (i) the symmetry and simplicity of its description in CM and (ii) the paradoxical 'slowness' of the relativistic particle in K frame.

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[107] In the second video [39] it is asserted that mass grows with speed and that this fact is related to $E = mc^2$, which is also said to express (i) rest energy and (ii) indifferently mass or matter conversion into energy. Some clarifications are thus necessary.

[108] The PSSC video [42] was instead exploited for reconstructing the part on photon momentum.

VI.4.2.5. *WTL strategy and analysis of science texts*

The importance of *language* for learning is highlighted by a very wide educational literature. In particular, some socio-semiotic researchers like Halliday and Martin (1993), Unsworth (2001), and Bazerman (2009) claim that *analysing science texts and reproducing their structure in students' own words* is important for their scientific literacy (Tytler *et al.*, 2013), since they need to understand and use scientific genres in order to 'internalize' (Vygotskij, 1978) the structures the scientific community has developed for making a text scientifically relevant. This progressive construction of a specific technical language in historical and contemporary scientific arguments is considered by Halliday and Martin (1993) to be intimately related to the epistemic nature of science itself.

According to the Writing-to-Learn (WTL) strategy standpoint, any writing task entails a change in the writer's perspective, because a creative cognitive re-arrangement process of the previous elements of knowledge occurs, the final product of which is *more than the sum of the initial ideas*. When developing disciplinary (physics) cognition «previous learning that developed around the spontaneous concepts of everyday life (as Vygotsky described them) becomes reorganized and reintegrated within "scientific concepts," as Vygotsky called them, that are introduced and practiced through the genres of schooling, disciplines, and professions» (Bazerman, 2009). So *effective science reading* (Yore, 2000) *and writing foster domain-specific learning*, meant as cognitive reconstruction towards the accepted science (physics) concepts.

In 2008 Engström asserted that « Writing is a powerful dialogic means and a tool in the learning process, that makes the student sharpen the thinking and in an intelligent manner use appropriate concepts ». She recalled that there are two major lines of research about writing at the international level: "*Writing across the Curriculum*" (WAC) and "*Writing in the Disciplines*" (WID). The former concerns spontaneous writing and writing to inquiry, in order to favour the content appropriation. This kind of writing does not involve an addreesee of the text. It exploits WTL strategy intensively and covers all subjects in the *curriculum*. Data are collected by brief writing tasks like « What do you know about…? What is your opinion of…? What solution do you think is best? Why? These texts primarily serve as the student's thinking tool, discussion and dialogue in groups. » The latter line (WID) is the writing of a specific subject using its proper genre, for example a laboratory report. The student addresses his/her text to a practitioner.

For the reasons above, the classical conceptual path was built with excerpts from historical science texts inside. The fundamental aim was to let students learn the facets of classical mass by conceptual *comparison between different possibilities* (Galili, 2012), namely Newton's and Mach's approaches to inertial mass as well as 'measure of matter' (*quantitas materiae*) and gravitational 'charge'.

VI.4.2.6. *Patterns of discourse*

The pattern of classroom discourse develops along two dimensions: from authoritative to dialogic and from non-interactive to interactive (Mortimer & Scott, 2003), the interactive/dialogic approach being of course the most desirable one. Collective intelligence may be enhanced instead by a technique called «collective reasoning» (Cacciamani & Giannandrea, 2004). That technique was exploited in some formative experiment, by using the following tools:

- "mirroring" (which requires *empathy)*;
- Active listening;
- Writing of the agreements on the blackboard;
- Facilitating of the emergency of a higher viewpoint bringing to a synthesis.

I decided to move from the desk to a side chair during the activity, in order to highlight that my role was not dominant. I was intervening only for "mirroring" what the pupils were saying.

This social production of knowledge was also gained during other formative experiments by *Large Group Discussions* – by exploiting the interactive/dialogic approach – as well as involving the students in *small group discussions* and *group questions* on the most relevant concepts and phenomena descriptions shown in the activity. An *active* reasoning was stimulated to prevent pupils from becoming *que-seekers*, viz. from feeling the need of answering in the expected way, which occurs when the authoritative discourse is used.

VI.4.2.7. *Thought Experiments*

The TE has historically been a major tool for understanding Relativity, widely used by Einstein in the “consensus thought experiment” form (Gilbert & Reiner, 2000). I focused on “teaching thought experiments” (*ibid.*), useful to explore relativistic-speed world at school, whose laboratory facilities do not allow to reach that domain: a particle accelerator and detector would be necessary.

Data Analysis and Results

Fourteen formative intervention experiments were carried out in different educational settings. Their outcomes have been qualitatively analyzed in order to *triangulate,* looking for trustworthiness. These settings spanned from ordinary (randomized) classes of Liceo specializing in scientific studies to larger groups of talents from all over Italy attending summer schools in Udine. The whole duration of each activity was also diversified, thus supplying the involved pupils with different times for learning/understanding, which meant to *test learning/understanding at different depth levels.* Various instructional tools and data collection methods were tried out in the earlier five experiments, since a *tuning was needed* throughout the first stage; they then stabilized, as it frequently occurs in qualitative analysis. Moreover, the amount of calculated quantitative parameters grew when passing from *the tuning to the testing phase* (experiments 6 – 14), which is very frequent too.

VII.1. **Implementing the t/l proposals**

After having stated the general and specific research aims and designed a complete t/l path, the latter may be *iteratively tested and revised* by means of formative intervention experiments, according to the DBR tenets. This led to evaluation and improvement of the first path, then to the design of the second path, and eventually to the test and tuning of the latter.

First of all, the paths were split in 20 sub-modules each, according to table VII.1, in which the aim (activity) of each sub-module and the relative specific RQs (taken from section VI.1) are displayed, “C” standing for classical, “R” for relativistic. It is specified that *only the sub-modules which have been tested are reported here*. So the activities may have very different relevance into the path. For example, the activity R3, “exploration of a horizontal light-clock”, is a very smaller section of the path than R6, “running of the ‘photon in a box’ TE”; however, empirical results are available for both of them. Eventually, for the sake of clarity, the classical activities are grouped according to the conceptual nuclei individuated and labelled in section III.2 (recalled in the table below).

Classical module		
Sub-module activity	*Aims*	*RQs*
CN1) Newton's modern definition of mass: quantity of matter		
C1) Reading the excerpts from *Principia* on mass as quantity of matter and on inertia of mass	Conceptually understanding and distinguishing these facets of mass	RQ8C) How do students conceptualize inertial mass? Are they aware of the problems in its definition? RQ11C) How do students relate the facets of classical mass among them?
C2) Reading the excerpts by Mach: criticism to Newton's quantity of matter	Understanding the vicious circle pointed out by Mach	RQ4C) Are students aware of the vicious circle in the Newtonian definition of mass as *quantitas materiae*? If so, how do they express it?
CN2) Inertial mass in Newton and Mach		
C3) Examining the vicious circle in inertial mass Newtonian definition	*Idem*	RQ8C)
C4) Cart quasi-elastic collision experiment	Exploring a class of phenomena	RQ12C) Do students recognize the linear relationship between speed variation ratios and number of system components in cart collisions? If so, how do they justify it?
C5) Reading and reasoning on Mach's definition of inertial mass	Understanding Mach's solution to the problems in inertial mass definition	RQ2C)Which concept was actually activated through students' reflections and revisions of the classical aspects of mass, according to the research outcomes by Mullet and Gervais (1990)? Weight or mass? RQ8C)

C6) Relating it to the speed variation ratio of colliding carts	Recognizing in the performed exp. the possibility of a relative mass measure according to Mach's definition.	RQ13C) How, if so, do students relate the measured speed variation ratio to inertial mass ratio in Mach's definition?
CN3) Gravitational mass as evolution of inertial mass		
C7) Deduction of universal gravitational law starting from the second and third laws of dynamics combined with the third Kepler's law	Understand gravitational mass and its relation with inertial mass; make a rational meaning for the law expression	RQ9C) How do students conceptualize gravitational mass? Are they aware that it stems from inertial mass and in which way? RQ10C) What role(s) do students attribute to gravitational mass in universal gravitation law? How do they express it?
C8) Examination of the characters of gravitational force (also by an excerpt from *Principia*); account of Cavendish's experiment	Understand gravitational force in Newtonian theory	RQ10C)
C9) Gravitational force and weight, so theoretical and operational definitions or meanings of gravitational mass	Defining and estimating gravitational mass	RQ2C) RQ9C) RQ10C)
C10) Writing about mass facets and their relationship with the other quantities	Resuming and linking conceptions	RQ1C) How and in which contexts do students relate to and utilize the term "mass"? RQ2C) RQ3C) How do students link the terms "mass" and "weight"? Do they associate the words "quantity of matter" and "mass" (*ibid.*)? If so, how do they motivate these associations?

C11) Group discussion on problems in Newtonian definitions	Use of «collective intelligence»	RQ2C) RQ4C) RQ8C)
CN5) Matter and mass conservation and additivity		
C12) Description of mass conservation under chemical-physics transformations	Explaining mass conservation contextualized in each kind of transformation (IPS Group, 1967)	RQ6C) How do students conceptualize mass conservation law in the six most important typologies of chemical-physics transformations, namely spacetime translations, deformations, breakings, changes of state, solutions, and redox? RQ7C) Do students consider chemical transformations different from physical ones? If so, do they find mass conservation in chemical transformations more difficult to explain than in physical transformations? Are chemical transformations described using a linear reasoning?
C13) Reasoning about mass additivity	Understanding the meaning of this property and connecting it to conservativeness	RQ5C) Do students acknowledge conservation and additivity of classical mass after the proper formative intervention module?
C14) Group discussion and individual writing on the comparison among all mass facets	Use of the «collective intelligence»; resuming and linking the final conceptions	RQ2C) RQ11C)

Relativistic four-vector module		
Sub-module activity	*Aims*	*RQs*
R1) Comparison between *c* and the other speeds	Acknowledgment of role of *c*: it is invariant and cannot be summed	RQ1R) Do students recognize the role of *c* in Relativity, as for invariance and ultimate speed character? How do they express their final conceptions?
R2) Reasoning on simultaneity of light emission events in different IFs	Understanding simultaneity non-invariance in SR	RQ2R) Do students acknowledge that simultaneity is not invariant in SR? If so, how do they express the grasped concept?
R3) Light-clock exploration and comparison in two IFs	Qualitative understanding and quantitative deduction of time dilation, based on the two postulates. Acknowledgement of space-time structure of the Universe	RQ3R) Are time dilation and length contraction effects understood by students, either described as "distortion of perception", or not achieved at all? In the first and second cases, how do students characterize them? RQ4R) Do cognitive transfer about time dilation occur in students?
R4) Exploration of a horizontal light-clock	Qualitative and quantitative understanding of length contraction	
R5) Interpretation of the additive constant in the series expansion of total energy as rest energy; mass as modulus of energy-momentum	Stating mass-(rest) energy relationship: meaning of mass in SR	RQ6R) How do (skilled) students interpret the conceptual extension from mass in its classical meaning to mass as rest energy in the relativistic context?
R6) Running of the "photon in a box" TE	Supplying students with a qualitative and quantitative model for inertia of energy	RQ11R) Are thought experiments effective in relativistic context? RQ12R) Do they generate accommodation or assimilation?

Relativistic phenomenological module		
Sub-module activity	*Aims*	*RQs*
R'1) Introducing relativistic mechanics until relativistic kinetic energy expression	Make students accustomed to relativistic reasoning	RQ5R) Do students understand and/or learn the meaning of dynamical quantities in a new paradigm (SR)? In which ways do they reason about them?
R'2) Hands-on and minds-on exploration of β-decay phenomenology	Let students explore a class of phenomena in which mass is not conserved; let students inquire into the correlation between nuclear mass defect and product kinetic energy	RQ7R) How do students interpret and justify phenomena in which mass is not conserved? RQ8R) How, if so, do students correlate a phenomenology in which mass is not conserved to mass-energy equivalence?
R'3) (R'2) + searching for $E - \Delta m$ empirical correlation by plotting data from radionuclide table	*Idem*	*Idem*
R'4) Introducing the photon and stating its momentum-energy equation; running the photon absorption TE; defining rest energy; finding total energy expression.	Deducing mass-rest energy equivalence $E_0 = mc^2$; introducing the relativistic idea of energy	RQ6R) RQ7R) RQ11R) RQ12R) RQ13R)

R'5) Deducing mass-energy-momentum relation; invariance and non-conservation of mass	Explaining the intimate relation between energy and momentum in SR; formally deducing that mass cannot depend on the IF in which it is measured; formally deducing that *mass varies only at constant speed* with respect to LabIF	RQ10R) Do students discriminate between mass non-conservation and mass invariance in SR?
R'6) Running a relativistic inelastic collision TE between two identical particles	Deducing that mass is not additive	RQ9R) Do students acknowledge that mass is *neither* conserved *nor* additive in SR? In which ways do they express these physical claims? RQ13R)

Table VII.1. Activities (sub-modules) and corresponding aims and RQs for each module of the t/l paths.

A summary table is below, which supplies the framework of all the implemented formative experiments. They were held in places spread all over Italy. They were interactive tutorials in three summer schools in Udine (Northern Italy); a part of a course in the 2012 Bard winter campus (AO, Northern Italy); a t/l activity to prepare students for the 2014 International Masterclasses – a hands-on particle physics popularization initiative – at the University of Udine; nine formative experiments carried out in two ordinary classes in Treviso (Northern Italy), four ordinary classes in Udine, one in Cesena (Northern Italy), one in Cremona (Northern Italy), five in Ancona (Central Italy), two in Crotone (Southern Italy). The students of Bard came from each Italian region, and those participating in the 2014 Masterclasses from several cities/towns of Northern Italy: Bolzano, Udine, Tolmezzo (UD), Sacile (PN), Mirano (VE), Venice, San Donà di Piave (VE), Trento. The vast majority of the involved pupils were attending the fourth and fifth (last) class of Liceo specializing in scientific studies, while the formative activities 13 and 14 (table VII.2) took place with students of the third year. A few pupils taking part in the summer schools or 2014 Masterclasses were also attending Liceo specializing in classical studies or technical college.

The sample size was under the usual approximate threshold of 30 for the validity of central limit theorem (Cohen *et al.*, 2007; Ross, 2008) in 10/14 formative experiments; not much above (36) in 2/14 cases; remarkably above, i.e. 42 and 70, in the last 2/14 cases. However, *the background statistical noise is effectively too high in all of the experiments for allowing noteworthy inferences.*

The classical path was experimented in its earlier version in the 2011 summer school, and then in Crotone and in Cesena, until April 2013 (experiments 1, 2, 5); it was then reworked and refined, and its last complete version was experimented in two third classes of Liceo specializing in scientific studies in Udine in November 2014 (experiments 13, 14). The relativistic 4-vector module was tried out and revised in period from the 2011 summer school to the 2013 one (experiments 1 – 6); the sub-module on β-decays was also inserted in the activities in Crotone and Treviso. The energetic phenomenological module (including R'3, i.e. the refined sub-module on β-decays) was set out in the first two months of 2014 and it was intensively experimented at school from March to May: the period in which the offered topics are ordinarily dealt with. Finally, the module was experimented in the 2014 summer school.

The data of six formative experiments were chosen for the analysis, balancing classes selected according to cognitive skills with ordinary classes, the latter selected in schools in different Italian regions: Calabria (Crotone), Veneto (Treviso), and Fiuli (Udine). Namely, *experimentations 1 – 4, 10 and 11 were selected*: a deep and accurate analysis of some significant cases was preferred to a more extended but more simplified and less thoughtful analysis of the whole (large) amount of data, for time reasons too. They correspond to the blue-coloured rows in table VII.2

The chosen methods for data collection and analysis are various. The first span from historical-based worksheets (classical part) to IBL worksheets and pre/post tests for analysing long-term understanding too. The second (analysis methods) are centred on the identification of conceptual categories and their relative frequencies over the sample, along with identification of profiles. As regards experiments 1 – 4, the reason of this variety was to explore these methods, in order to find the one(s) that could fit the research better. As for experiments 6, 10, 11, the analysis methods were very similar[109], while the data collection ones –

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[109] The students' scores were replaced with the learning gain, which also takes into account the number of correct anwers, but furnishes hints on conceptual change in addition.

along with the contexts – were diversified for probing different aspects of learning process (*variation*) and finding common general aspects from the result comparison (*triangulation*). Variation of phenomenological setting and triangulation are indeed considered two procedures of generalization, even if it *is not strictly possible to generalize,* because it would require a not-feasible statistical inference of statements from which consequences for other specific situations might be deduced (Mayring, 2007).

#	Where and when	N (sample size)	Class	duration (h)	T/L proposals (formative intervention experiments)	Data collection methods	Total sheet number	Data analysis methods	
1	2011 Summer school, Udine (25-30 July)	42	IV e V	1.5	**C1, C2, C8, C9, C14, R3, R5**	Historical-theoretical worksheets (mass in classical physics), worksheets (mass in SR), post test	13	Operationally defined mutually exclusive conceptual categories; category frequencies; conceptual profiles (levels of physical representation). Phi test; correlation test	**TUNING**
2	Crotone (April 2012) °	36	V	10	**C1, C2, C5, C8, C9, C12, C14, R3, R5, R'2**	Historical-theoretical worksheets (mass in classical physics)	4	Conceptual categories; category frequencies	
3	Treviso (May 2012) °	27	V	2	**R'2, R6**	Pre-test, post-test (different, but with the same questions on (1) β - *decays* and (2) on the inertial behaviour of "photon + box" systems)	4	Conceptual categories; category frequencies	
4	2012 Winter school, Bard (AO, 14-16 December)	25	IV e V	4	**R1, R2, R3, R4**	IBL worksheets	9	Conceptual categories; category frequencies; students' scores*; phenomenographic profiles	
5	Cesena (April 2013) °	20	V	4	**C1, C2, C5, C8, C9, C12, C14, R3, R5**	Historical-theoretical worksheets (mass in classical physics) Pre-test, post-test (mass in SR)	8		
6	2013 Summer school, Udine (22-27 July)	36	IV e V	1 h 45'	**R1, R3**	IBL worksheets	8	Conceptual categories; category frequencies; students' scores*; phenomenographic profiles	**TESTING**
7	Ancona (19th of March, 2014) °	70	V	5	**R'1, R'3, R'4**	Answers to the questions on intermediate learning; IBL worksheet on radionuclides	4		
8	Cremona (5-6 th of March, 2014) °	22	V	10	**R'1, R'3, R'4**	Answers to the questions on intermediate learning; IBL worksheet on radionuclides (slightly larger version)	6		
9	Masterclass (13th of March, 2014)	24	V	3	**R'1, R'4, R'5**	Answers to the questions on intermediate learning	/		
10	Udine - sample 2 (May 2014) °	20	IV	6	**R'1**	Pre-test, post-test, post-test after six months; answers to the questions on intermediate learning; conceptual map	/	Conceptual categories; category frequencies; phenomenographic profiles; correlation tests; learning gain*	
11	Udine - sample 1 (27th of March - 22nd of May 2014) °	21	IV	≈ 20	**R'1, R'3, R'4, R'5, R'6**	Pre-test, post-test; post-test after seven months and half; answers to the questions on intermediate learning; conceptual map; IBL worksheet on radionuclides	7	Conceptual categories; category frequencies; phenomenographic profiles; correlation tests; learning gain*	
12	2014 Summer school, Udine (23-28th of June)	29	IV	5	**R'1, R'3, R'4, R'5**	IBL worksheets (POE strategy), Pre-test, post-test	14		
13	Udine - sample 3 (November - Dec 2014) °	23	III	5	**C1 - C14**	IBL worksheets (PEC strategy), post-test, test to control group	14		
14	Udine - sample 4 (November - Dec 2014) °	23	III	5	**C1 - C14**	IBL worksheets (PEC strategy), post-test, test to control group	14		

Table VII.2. Comprehensive framework of the features of each implemented formative experiment. The statistical parameters marked by * in the last column were also mediated over the questions and/or over the students' answer. The labels of the activities making up each experiment, taken from table VII.1, are reported in the sixth column. The classroom experiments, marked by °, were always carried out in Liceo specializing in scientific studies. The students of Masterclass and summer schools came from technical college and Liceo specializing in classical studies too.

VII.2. Study with talents: 2011 summer school

Forty-two high-school talents, aged 17 to 19 ($\langle age \rangle$ = 18 years), took part in my first formative intervention experiment. It was carried out in order both to *search for their ways of reasoning in progress* both to *evaluate their final understanding*. In fact, these kind of findings give a strong hint for the effectiveness of the t/l path.

The students were 23 boys and 18 girls coming from each Italian region, after a severe selection based on the arithmetic mean of their final marks in scientific subjects in the last two school years. Moreover, there were five additional participants at the summer school: University talents, one of whom took part in my tutorial. Almost all students were attending the fourth and fifth year[110] of Liceo specializing in scientific studies, four of whom in scientific-technological studies; only one student was specializing in classical studies (table VII.3).

The activity consisted indeed in an interactive tutorial with proposals for both *individual reflections* and *group discussions*. Each student was provided with and filled in some worksheets (a little booklet) outlining the whole path and including essay and multiple-choice questions (inner questions/problems, reported in table VII.4). The questions were group-like or individual ones, and the students answered individually to the “group questions” in the worksheets after each small-group discussion. Finally, they run a post-test.

VII.2.1. Administered Questions

The students answered the following inner and final questions:[111]

C1) « When does mass come into play in your everyday life? In which phenomena do you perceive its presence? »;

C2) « What physics sectors study these phenomena? »;

C3) « What do you mean when you talk about *quantity of matter*? »;

C4) « What other facets and definitions of mass do you know? »

R1) « Does the inertia of a body depend on its energy content, i.e. its own energy regardless its translational motion? »

R2) « *Relativistic mass* is mentioned in many textbooks. Explain what it is. »

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[110] Secondary school lasts five years in Italy, thus they were almost at the end of their studies.

[111] The administered worksheets and post-test are in appendices 2 and 1 respectively.

Name	Type of school	Region	Age	Class	Average of School Marks	Average of School Marks
					2009/10	2010/11
LUCA	LS	FVG	17	4	9,00	9,50
DARIO	LS	Piemonte	17	4	10,00	9,00
ANDREA	LS	Sicilia	17	4	10,00	8,70
VALENTINA	LS	FVG	18	4	8,67	8,83
MARTINA	LS	Veneto	17	4	9,33	9,33
LIVIA BEATRICE	LS	Lombardia	18	4	9,67	9,83
ROBERTO	LC	Puglia	18	4	10,00	9,67
CHIARA	LSB	Molise	17	4	9,20	9,67
BEATRICE	LS	FVG	18	4	8,33	9,33
ELISABETTA	LS	Lombardia	17	4	9,50	9,17
FRANCESCA	LS	Lombardia	19	5	9,67	9,33
ALBERTO	LS	Veneto	18	5	9,33	9,00
GIULIO	LS	Lazio	18	5	10,00	8,88
MANUELA	LS	Abruzzo	18	5	9,50	8,70
FRANCESCO	LS	Campania	18	5	9,67	8,80
VINCENZO	LS	Sicilia	19	5	9,67	7,50
ANDREA	LS	Liguria	17	4	8,80	9,25
ALBERTO	LS	FVG	19	5	9,67	8,67
JACOPO	LS	Piemonte	18	5	10,00	8,17
ROBERTO	LS	Calabria	19	5	10,00	8,00
DAVIDE	LS	Sicilia	19	5	9,67	7,67
CARLO	LS	Emilia Romagna	18	5	9,67	8,00
GABRIELE	LST	Lazio	17	4	10,00	7,67
ELENA CAMELIA	LS	Emilia Romagna	19	4	9,25	8,67
STEFAN	LS	Abruzzo	19	5	10,00	9,50
BARBARA	LS	Abruzzo	17	4	9,75	9,50
MAURIZIO	LST	Marche	18	4	9,00	9,67
CARMELO	LS	Sicilia	19	5	10,00	9,30
LIVIA VIORICA	LS	FVG	19	5	9,00	9,50
VALENTINA	LS	FVG	17	4	9,25	8,76
ALESSANDRO	LS	Puglia	18	4	9,25	9,25
MARCO	LST	Piemonte	18	4	9,75	8,38
MARGHERITA	LS	Marche	18	4	9,60	8,88
RITA	LS	Emilia Romagna	18	4	10,00	9,00
ROBERTO	LS	Basilicata	18	4	8,00	9,00
ISABELLA	LS	Sardegna	19	5	9,67	8,67
DAVIDE	LST	FVG	18	4	10,00	9,17
GIULIA	LS	Toscana	17	4	10,00	9,38
MIRIAM	LS	FVG	17	4	9,33	9,17
SABRINA	LSB	Veneto	19	5	10,00	8,75

Table VII.3. Initial data about the sample of 2011 summer school. “LS” and “LSB” stand for Liceo specializing in scientific studies; “LST” for Liceo specializing in scientific-technological studies; “LC” for Liceo specializing in classical studies. There are 2 additional students in the sample taken for analysis, one attending the III class of LS, the other being a University talent. (Marks run from 1 to 10 in Italian schools).

Classical inner questions	***Relativistic inner problems***
1) Until the XVIII century mass was essentially considered as "quantity of matter", also by Isaac Newton who in his *Principia Mathematica* (1687) wrote *(omissis)*. In the text above, which of the following concepts is prevalent in Newton? Mass, Body, Density or Volume? Why?	I. Soon after the relativistic path a *nuclear fission process* was examined, two examples of which having been provided. The students had to understand *where the huge quantity of released energy comes from*, since total energy is conserved.
2) Observe that masses m_1, m_2 in the Universal Gravitation Law play the same role than electrical charges. On the basis of this analogy, can you tell what the meaning of the word "gravitational mass" is?	II. A *two-particle collision* with particle creation was then analysed, the two particles being identical and the new one created at rest; in particular it was asked *which forms of energy were changing*.
3) Here the focus is that mass is no more the simple "quantity of matter" in Newton, although he enunciates it in that way: it is a concept in evolution in his mind. Is there a difference between the mass in gravitation and this one? Explain	
4) (group question) What are the conceptual differences ultimately among the notions of mass examined so far?	

Table VII.4. Questions and problems inside the worksheet administered to students (see table VII.2). For the complete worksheets please refer to appendix 2.

VII.2.2. Data: Analysis and Results

Both 'vertical' and 'horizontal' data analyses were carried out to *identify* individual and collective students' reasoning paths after instruction as well as to *probe the evolution* of individual student conceptions *in progress*.

The former type of analysis consisted in an examination of data from the whole of students, for finding the distribution of the replies to *each question* of the administered test in the whole sample. A number of «criterions of selection» (Krippendorf, 1989; Mayring, 2004; Zhang & Wildemuth, 2009), derived from the RQs, allowed to induce *categories*, whose relative frequencies were calculated.

The 'horizontal' analysis is instead a search for regular patterns throughout *all answers by each student*, in order both to find out their individual ways of looking at mass and mass-energy relation both to recognize the student profiles pointed out by Doménech, Casasús, Doménech, and Buñol (1993). These researchers classified the students' ways of looking at *classical mass* in five classes, having been in turn inspired by the semantic analysis of Gorodetsky, Hoz and Vinner (1986), and they called these classes *levels of 'physical representation'*. They are briefly explained below.

1. *Ontological*: mass is considered as a general property of matter or even identified with matter/bodies/particles; this is a pre-theoretical definition, for a theoretical framework is not yet developed. The definition is then *concrete*, in spite of appearance: it is implied by a concrete view of physical world. A typical example is *quantitas materiae.*
2. *Functional*: mass is identified with properties, tendencies or behaviours of the physical system. Examples are inertia for the inertial facet and heaviness for the gravitational facet. It may be also the measure of one of the properties owned by a body under well-defined conditions.
3. *Translational*: mass is completely identified with another related quantity, such as density/volume or weight (or energy in SR), at the pre-theoretical level.
4. *Relational:* mass is defined through precise conceptual relationships inside a formal theory made by mathematical laws (strict sense of the original paper). For instance, it may be supplied by F/a in the inertial sense and by either P/g or the inverted Universal Gravitation Law in the gravitational sense. This level was reduced to a sharp and clearly stated outline of *conceptual* relations, also when not mathematically formalized, in the data analysis reported here.
5. *Operational*: mass is considered as a quantity whose value is to be numerically found through experimental conceivable and explicit operations. Example: gravitational mass as the measure of a static (e.g. equal-arm) scales.

I extended these levels to SR. A conceptual profile may be found for each student by combining them.

Answers to the inner classical questions. The answers to question (1) showed an acknowledgment of the contents involved in the science text reading, even if with some variations. Density was considered by students as related to mass (18/42), and to – in their words – «*quantity of matter*» or «*substance*» (14/42). For instance, Luca replied: «*Because Newton uses it as reference point (valid for all bodies) to obtain the mass of each body*»; Carmelo replied instead: «*Because he speaks of quantity of matter in a volume, that is density or what he calls norma of each body*». The answers to question (2) allowed to found out that gravitational mass (m_g) was considered as a parameter describing an attractive interaction between bodies; the emphasis was on the body in 29/42 answers, while 11/42 students mentioned mass explicitly (Figure VII.2). In addition, other 11/42 referred to universal gravitation law, but never using formalism. As for question (3), in 26 cases the difference between inertial and gravitational mass was also expressed through a characterization of the latter with respect to the former. As regards inertial mass (m_i), the prevalent category (23/42) is well described by the following sentence: "Newton refers to inertial mass, which is the quantity governing the behaviour of bodies when accelerations/momentum variations (in collisions) are present". The concept was expressed in various modalities, most of the answers including «*The ability/property of a body in contrasting a variation in its state of motion / state of rest*». Other frequent answers were either «*the ability/property of a body in contrasting a variation of its state of uniform linear motion / rest*», or «*the ability/ property of a body in contrasting the change of state*».

The group question (4) lead eventually to several outcomes. The relative majority of the answering students (12/28) tried to give meaning to the concept of *quantitas materiae* in itself, whereas 8/28 fixed their attention to the circularity problem in Newton's definition and 8/28 (different) students just mentioned this facet of mass, without deepening its meaning. This data should be taken with a special care, as 14/42 students did not answer. A precise distinction between the definitions of gravitational and inertial mass was found only in 15/42, whilst confusion is present in 24/42, the former being considered as a "dynamical" quantity (a precise cause of motion is identified) and the latter as a "kinematical" quantity (all interactions are taken into account). The answers concerning m_i were grouped in four not-exclusive categories, from the most strictly scientific to the most functional (concrete) one, as explained in the caption of figure VII.1.

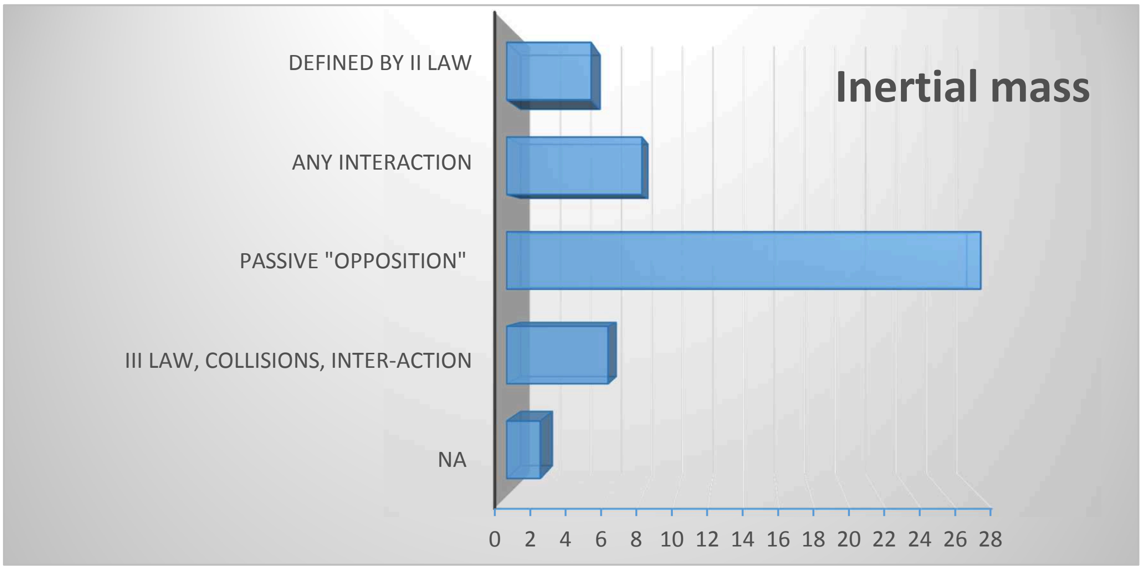

Figure VII.1. Non-mutually exclusive categories (7 cross-over) for the answers concerning inertial mass to the classical inner group question (4 in table VII.4). "Defined by II law" is a category operationally defined by the answers in which mass is seen as the constant / proportionality factor in the second law of dynamics; "Any interaction" by the answers in which the concept is extended from gravity to all interactions. The third category includes the answers in which inertial mass is considered as a property of the body, which 'resists'/ 'opposes' to something; the fourth category groups the replies in which mass is operationally defined *à la Mach* by the III law or reciprocal interaction/ collisions.

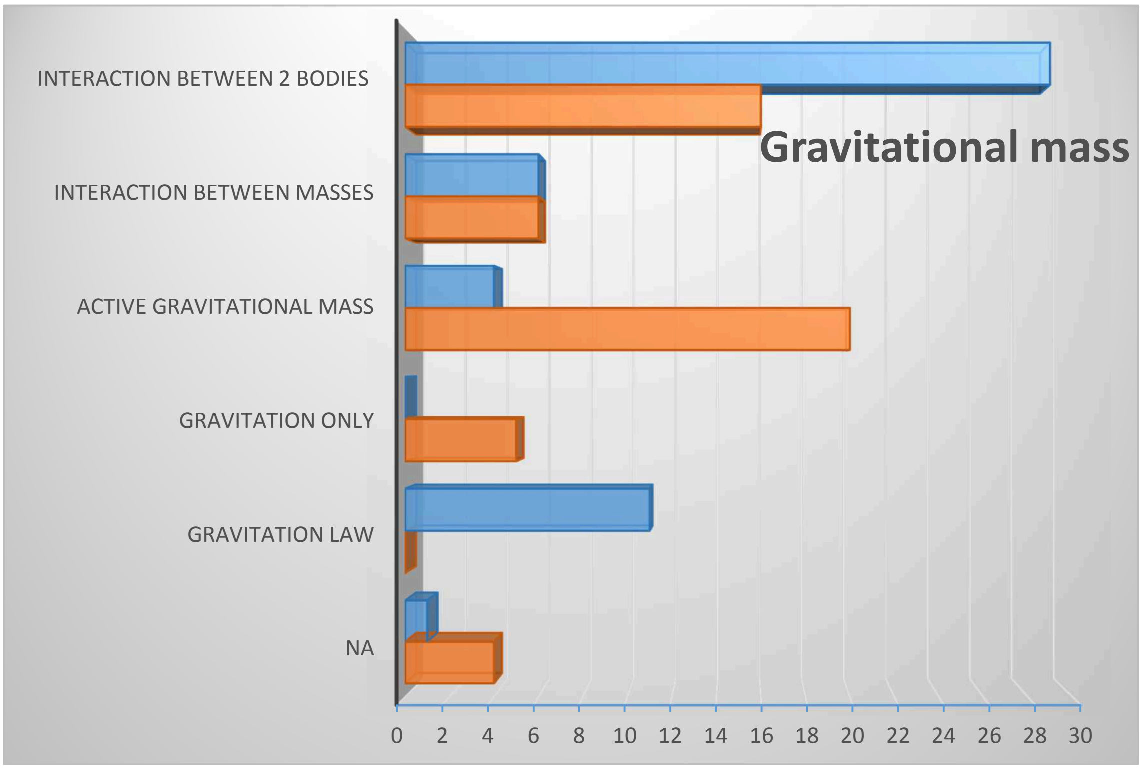

Figure VII.2. Change in the category discrete distribution from the second inner question (top blue histograms) to the inner group question (down red histograms). Not mutually exclusive categories (9 cross-over).

On the other hand, twenty-two students considered m_g as a property mediating/«*allowing*» an attractive/ gravitational interaction either (a) between two bodies (16/42) or (b) between masses (6/42). 20/42 highlighted m_g as (c) source of interaction, i.e. active gravitational mass, by typically writing «*property/capacity of generating a force*»; in addition 5/42 considered (d) m_g involved in gravitational interaction only, while m_i in all physical interactions. It has been found by comparison analysis (figure VII.2) that categories (a) and (b) became less and remain equally numerous respectively passing from the second question to the group question, while (c) turned out in a wider category and (d) stemmed. Eventually, the references to gravitation law vanished.

Answers to the inner relativistic questions. It came out from the analysis of the collision process that 7/42 students thought that kinetic energy and rest energy varied in the collision, whilst 4/42 mentioned kinetic energy only (Figure VII.3). Moreover, in the problem on nuclear fission 15/42 followed a type of reasoning like «*Total energy, but not mass, is conserved and kinetic energy varies, so rest energy also do; when E_0 varies, mass varies in the same sense*[112]*: mass-energy relationship is valid in variation form as well*». These results are however to be taken with a large grain of salt, because the most students did not answer at all (29/42 in the 1st case, 27/42 in the 2nd case).

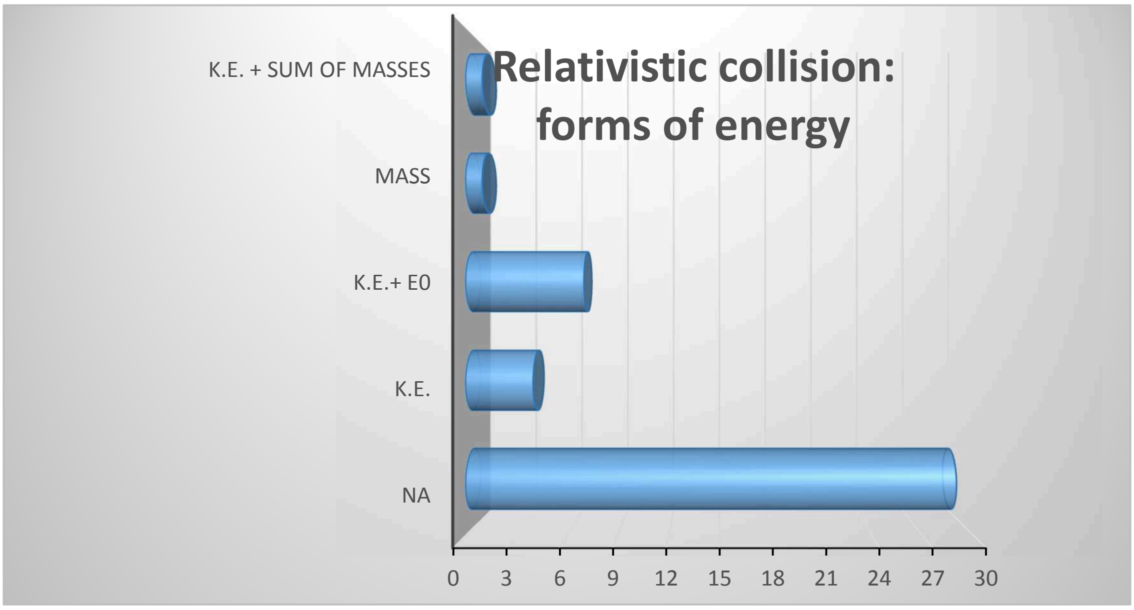

Figure VII.3. Forms of energy varying in a collision between two identical particles (inner relativistic question I).

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[112] Notice that the latter are the same words of Einstein's first paper (1905a).

Answers to the final test (classical part). The outcomes are summarized in figures 4 – 7. Neither these categories are mutually exclusive, nor are the "NA" categories considered. The phenomena referred to in familiar contexts (C1) were mainly *mechanical ones*, as depicted in the plots. Some students referred to mechanical *quantities* instead associated to mass, although "quantity of matter" was also mentioned. It is worth noting that 7/42 students indicated in the answers to C1 the *unique* dynamical phenomenon whose description in vacuum does not require mass: free fall. This means they have not understood the physical meaning of WEP essentially.

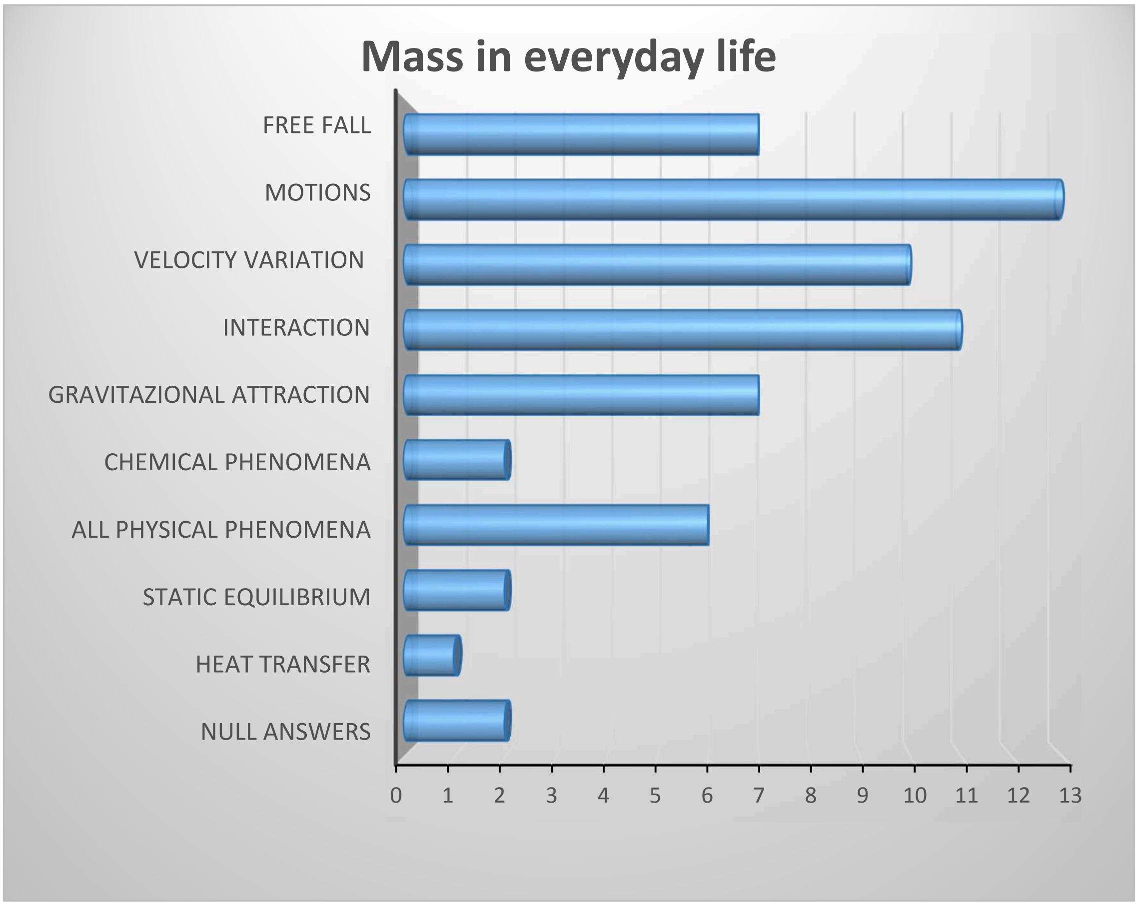


Figure VII.4. Typologies of evoked phenomena (answers to C1 in the final test). "Null answers" stands for answers not concerning the question. Non-mutually exclusive categories.

The most mentioned *theories* and physics sectors in the replies to C2 were dynamics (22/42), mechanics (17/42), and kinematics (10/42); SR played an important role (15/42) as well. On the other hand, there is awareness of the importance of mass in electromagnetism in few students (3/42) and no one is able to contextualize it in familiar phenomena. Finally, eight students did not distinguish physics sectors from theories.

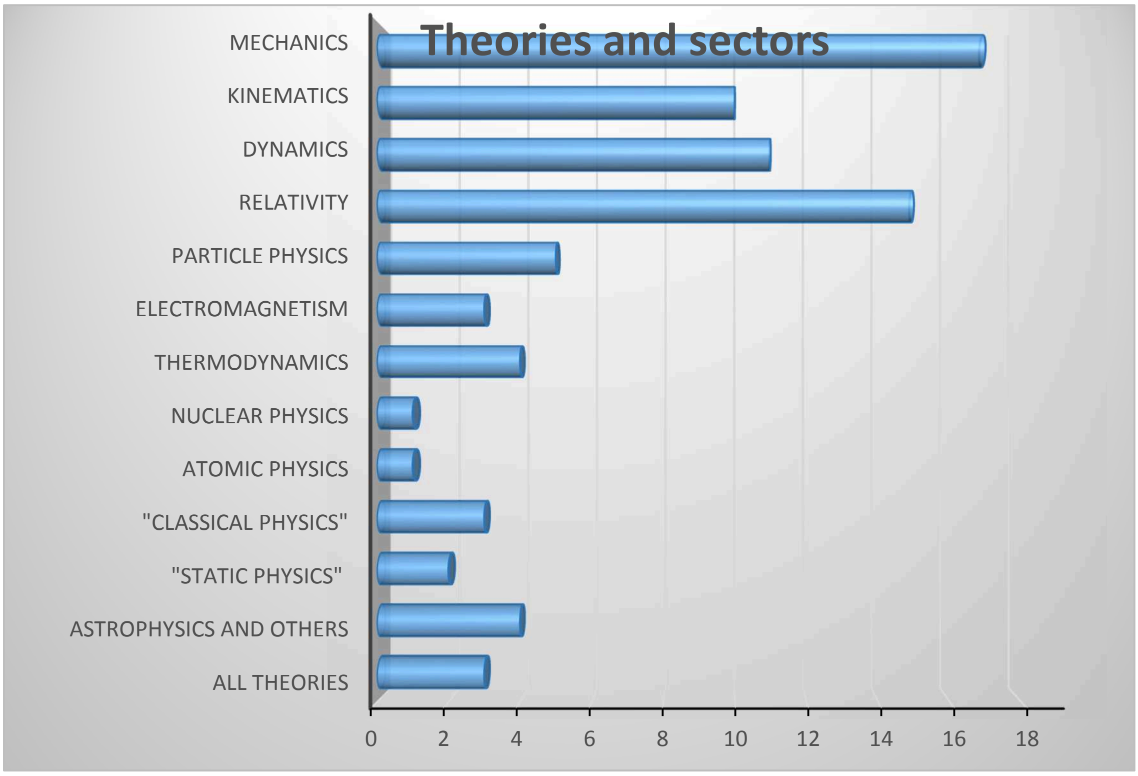


Figure VII.5. Physics theories and sectors concerning the phenomena previously recalled (answers to C2). Non mutually-exclusive categories.

From the answers to C3 emerges that 13/42 students identify quantity of matter correctly with mole, either in a general sense (ontological profile: 4/42), or as number of moles (relational profile: 9/42). Unfortunately, in 6/42 answers is present an identification with mass. As for C4, a meaningful quotation from an answer inserted in *quantitas materiae* category (5/42) is provided: «*Mass* [omissis] *can be derived from a formula* $m = \varrho/V$».

The results about *quantitas materiae* show that this classical pre-Machian conception of mass is rooted in some minds (6/42 answers for C3; 5/42 for C4). However, the analysis of video recordings allowed detecting that it is never mentioned in the oral answers to the first group question. Therefore it is not so much rooted.

Strikingly, 19/42 students only listed the meanings of mass in the replies above.

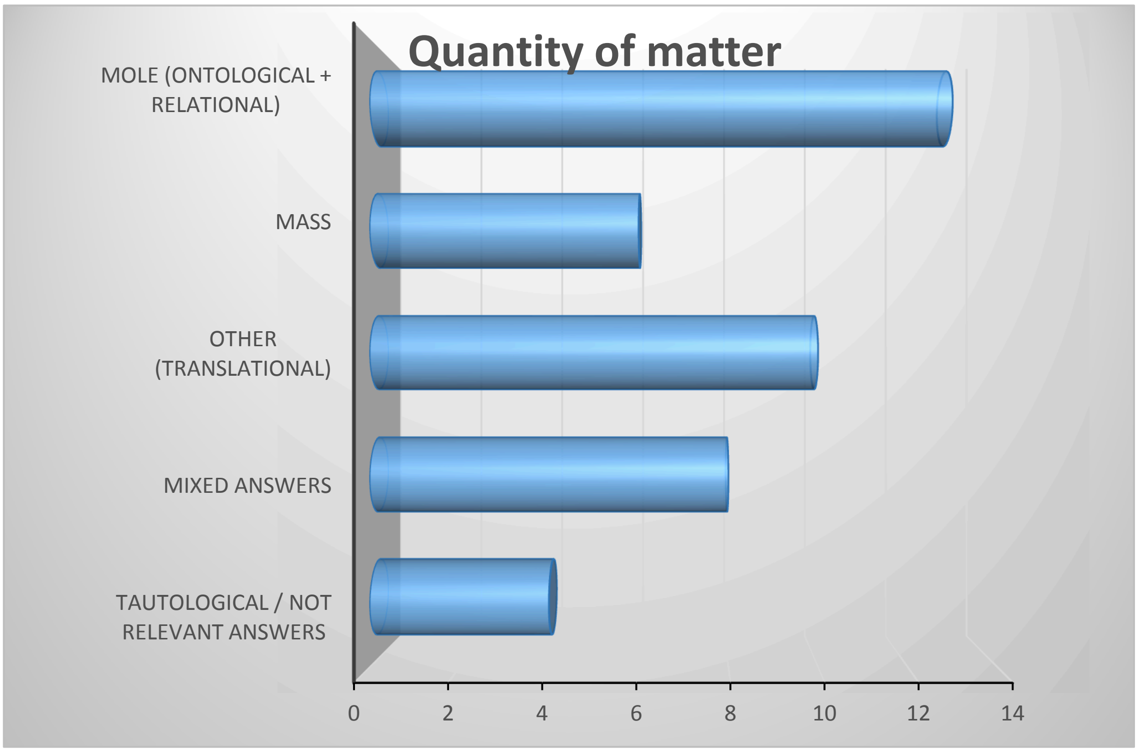


Figure VII.6. Students' conceptions of *quantitas materiae*. "Other" stands for N_0 (Avogadro's number), density, «*mass concentration in a given volume*», number of molecules or atoms or particles in a body (answers to C3).

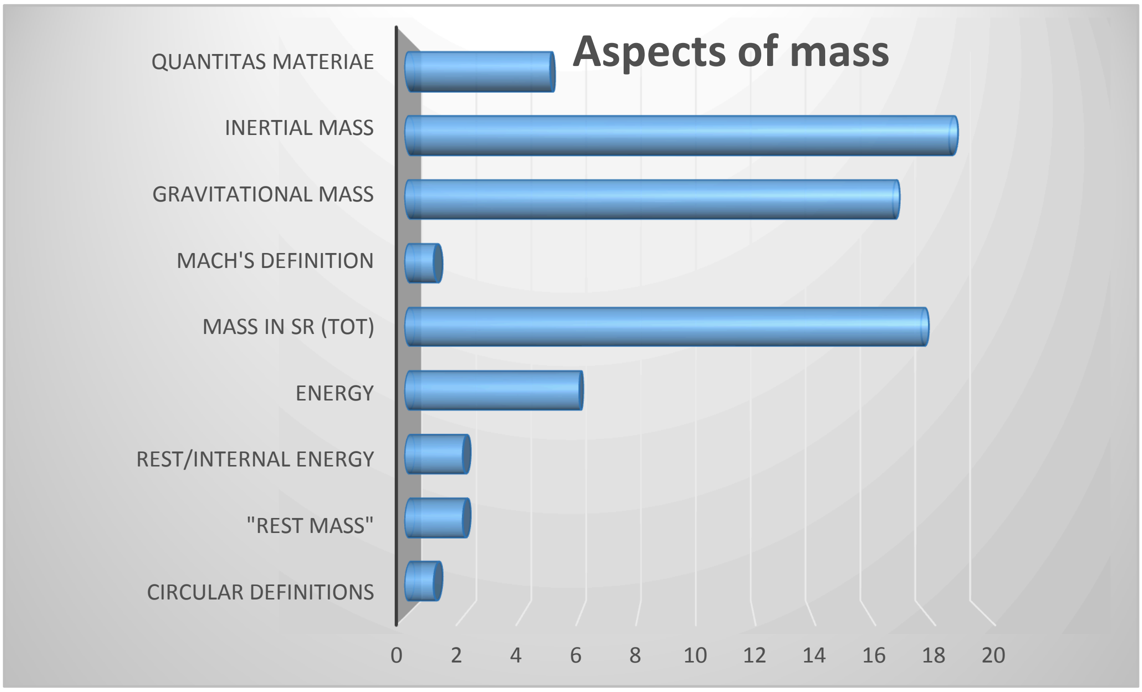


Figure VII.7. Facets of mass present in the answers to C4. Non mutually-exclusive categories.

Answers to the final test (relativistic part). R1: « Does the inertia of a body depend on its energy content, i.e. on its own energy, regardless its translational motion? » (Figure VII.8). Only 35/42 students answered to this question. No conceptual reference to the mass-rest energy equivalence was revealed in 40% of the answering sample (14/35). As expected, this lack was associated in several cases (20% of the total, 7/35) to the presence of the idea of "relativistic mass" in students, although our rationale had been brought on the ground of relativistic energy, just in order to let pupils distinguish between mass as rest energy and "relativistic mass". By contrast, conceptual reference to the equivalence mass-rest energy was present in 15/35 cases, mainly implicit or put into words: few explanations were made by means of mathematics. Five answers were labelled as "uncertain cases": they are either mere enunciations, or reference to a generic mass-energy relation, as well as not understandable sentences. Three of them answered: «*No, because kinetic energy does not affect rest energy*».

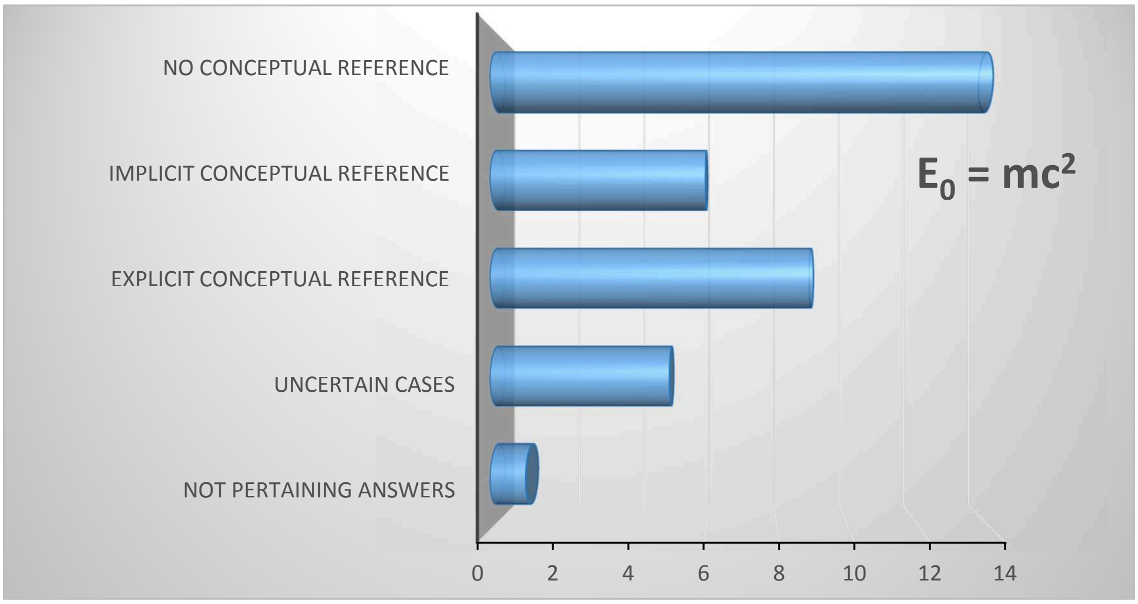


Figure VII.8. Conceptual reference to mass-rest energy equivalence (R1). 35/42 students answered to this question.

R2: «*Relativistic mass* is mentioned in many textbooks. Explain what it is» (Figure VII.9). Thirty-nine students answered. A meaningful example of answer in the category "mass at relativistic speed" is the following: «*That means that mass in motion at very high speed can become energy and vice versa*». According to this conception, one enters in the realm of relativity only at high speeds, and there mass in motion may change into energy. The correct definition – mass depending on speed – was given instead by 6/39 students, also using mathematical formalism; one among them even deduced the formula, with the aim of reducing relativistic expression for momentum to the classical one: « *Let us call the relativistic mass* m_r*. We want the classical expression of momentum to be valid with* m_r *instead of* m*. If we equal the expressions for* p_{rel} *we will obtain* $\gamma mv=m_r v$ *where* v *is the particle velocity, and then* $m_r = \gamma m$ ». It is a crucial issue in the debate on this topic. These six students learned the right concept of "relativistic mass" in its proper theoretical framework, although it is high inadvisable to use it both in physics both in physics education since it brings to misunderstandings (compare section V.4). Finally, 11/42 students used wrong terminology when replied to this question, which further witnesses the importance of language for teaching/learning.

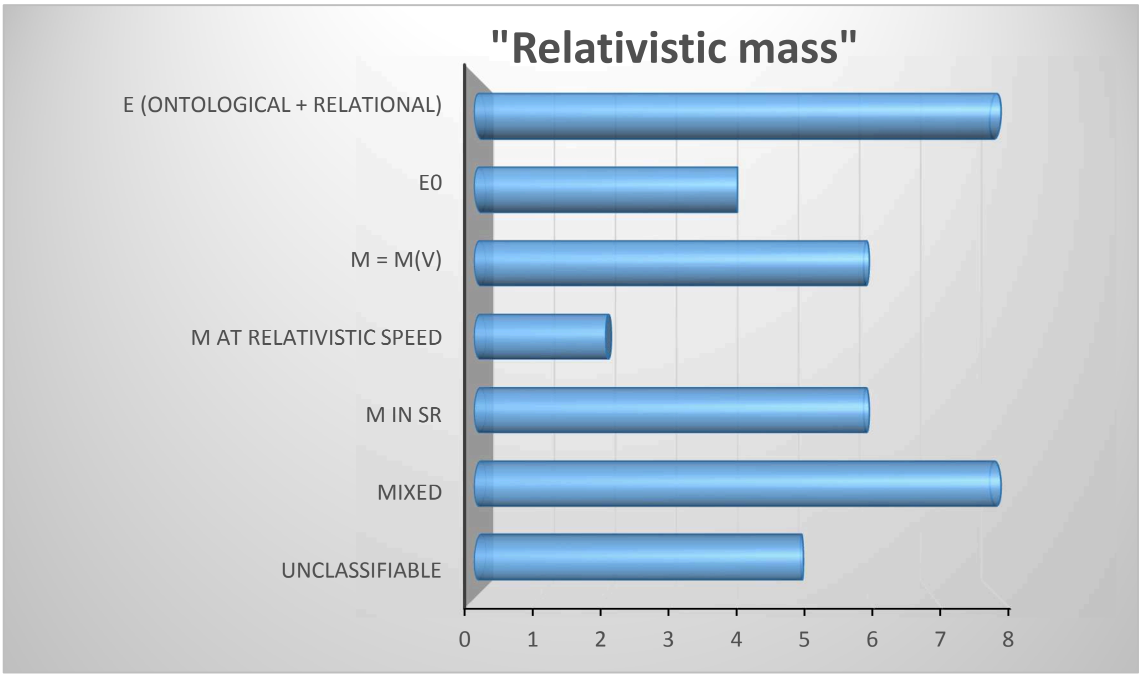


Figure VII.9. Conceptions of "relativistic mass" (answers to R2). The first category includes both a generic relation with energy (ontological level) and formulas connecting E, m, E_0 and m_0 in every possible way (relational level). Each mixed reply includes 2 categories at least.

VII.2.2.1. *Hypothesis testing and students' conceptual profiles*

The calculation of Pearson's rank correlation coefficient was performed, in order to evaluate if the following *null* associative hypothesis H_0 were supported:

"*There is no statistically significant correlation between the conceptual reference to mass-rest energy equivalence (mass is $m = E_0 / c^2$ in SR) and the presence of the "relativistic mass" conception (mass is $m_r = \gamma m_0$ in SR)*".

The procedures described by Cohen, Manion and Morrison (2007) were followed in measuring the association between two *ordinal* variables, named X and Y. Their values run from 1 to 5, according to the revealed level of rooting of conceptual reference to mass-rest energy equivalence (X) and the level of rooting and formalization of "relativistic mass" concept (Y). Further details are in table VII.8. A bubble graph of X against Y was plotted using the ordered pairs of that table, the radius of each bubble being proportional to the pair frequency. The level of statistical significance α was set at 0.05. *No significant correlation was found* between X and Y ($\rho_p = -0.2126$, critical value: $\rho_0 = 0.325$ for N=37 couples of data; $\rho^2 = 0.0452$). Accordingly, the probability that 37 measures of two uncorrelated variables yield $r \geq 0.2$ is 22 – 25% (Taylor, 2000).

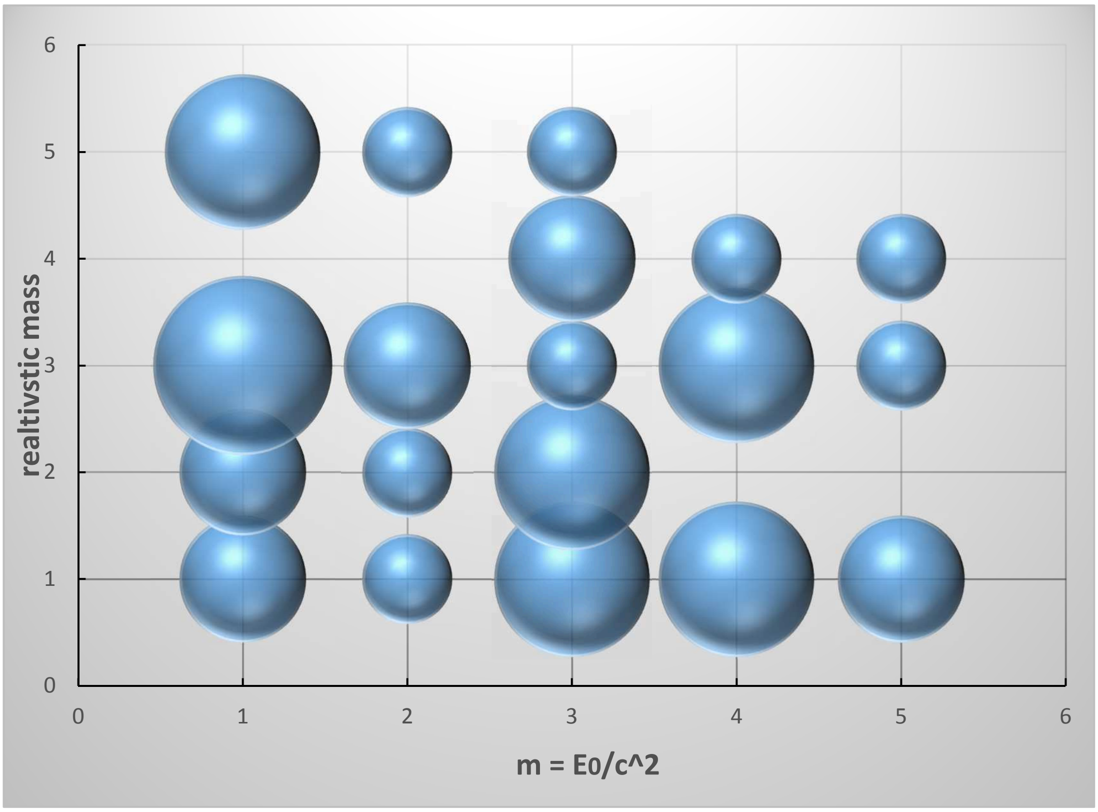


Figure VII.10. Ranks correlation depicted by a bubble graph.

Furthermore, a *phi-test* was used to search for a statistically significant correlation between the detection of a conceptual reference to mass – rest energy equivalence and the choice of "inelastic" (two-choice item) in the answers (25/42) to the relativistic inner question about collisions. The phi-coefficient was invented for binary variables. It always refers to a 2 × 2 table, said "contingency table", like the one below.

	Attribute 1	yes	no
Attribute 2			
yes		*a*	*b*
no		*c*	*d*

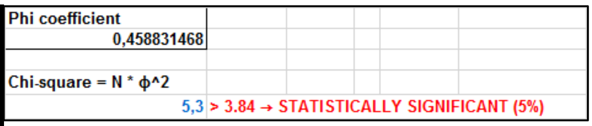
Table VII.5 (Cohen *et al.* 2007). An example of generic contingency table, where *a*, *b*, *c*, *d* indicate the frequencies of observation of the attributes.

The phi-coefficient was calculated by the following formula, a, b, c, d being the observation frequencies of each attribute ("elastic collision" and "reference to equivalence" in this case):

$$\phi = \frac{ad - bc}{\sqrt{(a + b)(c + d)(a + c)(b + d)}} = 0.46.$$

The findings are in the tables below. Phi-coefficient significance was evaluated by means of the correspondent significance of chi-square, because $\chi^2 = N\phi^2$, where $N = a + b + c + d$. «Muijs (2004) indicates that a measure of effect size for cross-tabulations, instead of chi-square, should be *phi*, which is the square root of the calculated value of chi-square divided by the overall valid sample size» (Cohen *et al.*, 2007).

COLLISION	inelastic	elastic	
Conceptual reference	10	0	10
No conceptual reference	9	6	15
	19	6	25



Phi coefficient	
0,458831468	
Chi-square = N * ϕ^2	
5,3	> 3.84 → STATISTICALLY SIGNIFICANT (5%)

Tables VII.6 and VII.7. 2 x 2 contingency table (left) and phi-coefficient for the correlation between the elasticity of the collision and conceptual reference to $E_0=mc^2$ (truly dichotomous variables).

	Level of presence of conceptual reference to mass- rest energy equivalence (ordinal scale)	Level of presence of “relativistic mass” concept (ordinal scale)
Student	1 absent; 2 very low (implicit); 3 low (implicit); 4 high (explicit); 5 very high (explicit)	1 absent (rest energy/ internal energy); 2 weak (mass in Relativity/mass at relativistic speed); 3 medium (energy), 4 strong (mass generically depending upon velocity), 5 very strong ($m_r = \gamma m_0$ /varying with reference frame)
StudentA	5	4
StudentB	2	1
StudentC	5	1
StudentD	3	1
StudentE	1	2
StudentF	3	4
StudentG	4	1
StudentH	2	3
StudentI	1	3
StudentL	1	5
StudentM	4	1
StudentN	1	2
StudentO	3	2
StudentP	1	3
StudentQ	4	4
StudentR	3	4
StudentS	3	2
StudentT	1	5
StudentU	4	1
StudentV	1	5
StudentZ	1	3
StudentAA	4	3
StudentAB	1	1
StudentAC	4	3
StudentAD	3	2
StudentAE	4	3
StudentAF	3	5
StudentAG	5	3
StudentAH	2	2
StudentAI	3	3
StudentAL	1	1
StudentAM	3	1
StudentAN	2	3
StudentAO	2	5
StudentAP	1	3
StudentAQ	5	1
StudentAR	3	1

Table VII.8. Ordinal variables for correlation analysis, extracted from the answers to R1 (left) and R2 (right) of the final test.

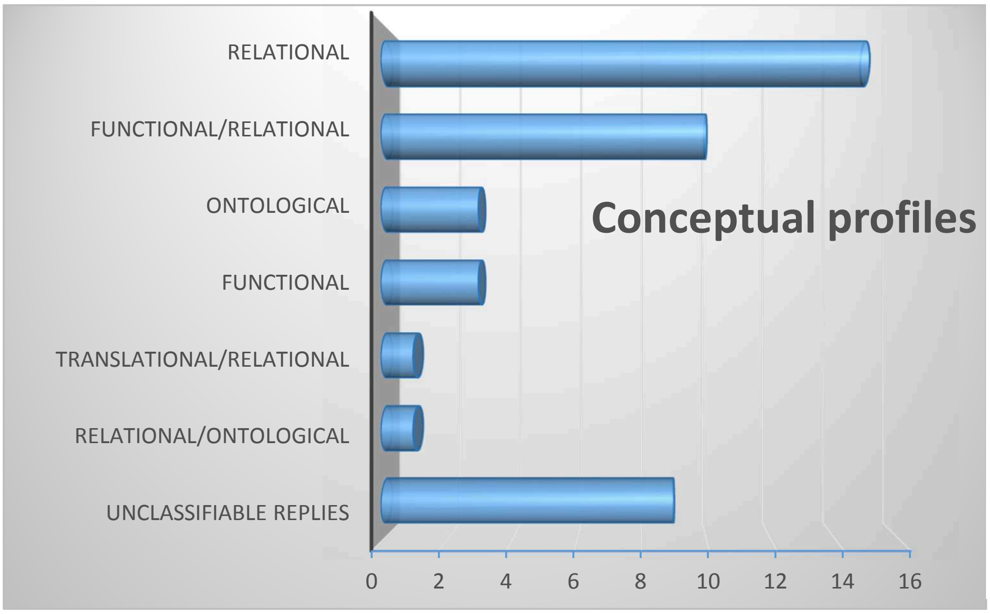


Figure VII.11. Students' profiles, found through a 'horizontal' analysis (i.e. carried out for each student separately). The categories are mutually exclusive. Some students have a sharp profile, others have a profile intermediate between two levels.

Eventually, results concerning the students' levels of physical representation (Doménech *et al.*, 1993) are shown in figure VII.11. The *relational* level was prevalent, as it was to be expected: it affects 64% of the sample (27/42). The functional level is halfway: a theoretical framework is, but in *implicit* form. It is globally rather common (13/42). To be noticed the small size of ontological (4/42) level; while only one (partially) translational and no operational profiles were found.

***Summary of the main findings*.**

a. Thirty-two students associated mass with mechanics in everyday phenomena according to the replies to C1 (final test).
b. Only 11/42 associated the label "gravitational mass" explicitly to universal gravitational law during the activity, although m_g were considered by 29/42 learners as a parameter describing a generic interaction between bodies (answers to the second classical inner question, see figure VII.2). Later on, in the answers to the inner group question, nobody mentioned gravitational law, while the greatest attention was paid to m_g as source of force/interaction (20/42).
c. It was found in the answers to the (classical) group question that pupils either tried to give an interpretation of *quantitas materiae* or focused their attention on the circularity problem concerning the latter, their attention being thus drawn to negative considerations rather than on the possibility of endowing this pre-theoretical facet of mass with meaning.

d. Inertial mass was understood consistently with Newton's II law by a few students (5/42) during the learning process (replies to classical group inner question, see table VII.4).
e. A difficulty in strictly distinguish gravitational from inertial facet is present in 24/42 learners *when they are asked to compare them*: in this situation the number considering inertial mass as 'resistance' is higher.
f. At the same time, 15/42 different pupils tended to fall in rigid patterns related to an action of the body in opposition to motion when they have to compare the facets of mass synthetically;
g. Finally, if the answer categories to questions C1 and C2 of the final test are compared it will be clearly seen that the number of students with a "holistic" vision of mass – i.e. mass perceived/considered as present everywhere – halves when changing from everyday phenomena to theories.

VII.2.3. **Discussion**

This formative intervention experiment was driven by two RQs, formulated during the path design (in VI.1.1 and VI.1.2 respectively):

RQ1C) *How and in which contexts do students relate to and use the word "mass"*?

RQ5R) *How do talents interpret the conceptual extension from mass in its classical meaning to mass as rest energy in the relativistic context (under the influence of the t/l path)*?

VII.2.3.1. *RQ1C*

Point (g) above indicates that ubiquity of mass was rationalised by one-half of the few people having perceived it (namely, 3/42 against 6/42). In other words, 6/42 replied that they perceived mass in every phenomena, and three of them brought this presence on a more rational ground, by mentioning all of physical theories explicitly. This outcome agrees with the totally or partially "relational" character of the vast majority of the sample.

The results from the classical group question, depicted in figure VII.1, show that a conception of *inertial mass as related to a passive opposition* either to an external stimulus or to a motion state variation prevailed. On the other hand, gravitational mass took on the *active role* of force/field source chiefly. So the comparison between the inner questions (2) and (4) concerning gravitational mass (figure VII.2) brings to the ensuing statement. The passage from a *strongly focused request for clarification* to *an open request for enucleating the main characters of gravitational mass* – by contrast with the other facets – causes a *sharp dichotomy* between the former and inertial mass, the first gaining an active role of interaction source and the second a passive role of inert opposition. At the same time, the emphasis on two-body interaction in the definition of gravitational mass decreases. Strangely, this effect does not occur when masses are explicitly referred to (6/42).

VII.2.3.2. *RQ5R*

First of all, seven students did not answer at all to the question R1 of the final test: « Does the inertia of a body depend on its energy content, i.e. its own energy regardless its translational motion? ». This is a common trend for the relativistic part: the highest recorded "NA" frequency is 29/42, that is more than 2/3 of the sample. The conceptual reference to mass as rest energy was identified in 15/35 cases (9/35 + 6/35), but it was absent in 14/35 (figure VII.8). The conception of "relativistic mass" is present in the half of these latter cases: it is either simply mentioned or explained as a quantity varying along with speed; for example: «*Yes. Mass summarizes, at rest, any energy type that that body may manifest and thus it is greater in a body at rest with respect to the same in motion, for instance*» (a sort of "converse relativistic mass"). A causal explanation cannot be offered, but it might be a hint to consider relativistic mass as provoking these disturbances. A final remark: although their sentence alone is correct, the three students who replied «*No, because kinetic energy does not affect rest energy*» showed to hold a conception in which rest energy is likely unrelated to mass.

The results on "relativistic mass" (R2 of the final test) indicate then that the notion of a speed-dependent mass, formally defined by $m_r = \gamma m_0$, was integrated in the relativistic paradigm by 14/39 answering students (figure VII.9, category "$m = m(v)$" joint to "m at relativistic speed" and to "m in SR"), only one of whom provided the correct definition, by searching for a momentum relativistic expression with the same form of the classical one[113]. This integration was lacking instead in 12/39 answers (category "E (ontological + relational)" joint to "E_0").

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[113] This is one of the major historical reasons for the introduction of the notion.

VII.2.3.3. *Correlation test*

The *null* hypothesis is supported by our ordinal data, so the *alternative* hypothesis of negative correlation is rejected for our sample: no statistical significant correlation was found between the level of presence of conceptual reference to $m = E_0/c^2$ and of "relativistic mass" notion in the answers, both quantified in a scale running from 1 to 5. A *modest size effect* is pointed out by $\rho_p = -0.2126$; ρ^2 calculation indicates besides that only 5% of variation in Y is affected by X variation; more precisely 5% of Y variation can be ascribed to a linear relation with X. Even more so, nothing is allowed asserting on *causality*, as with any correlation study. However, the top right part of the bubble plot (figure VII.10) is scarcely populated, so *a simultaneous effective understanding and mastery of the two conceptions seldom occurs at the qualitative level.*

Finally, the phi-test gave *positive outcome* with *moderate* effect size ($0.3 < \phi < 0.5$), but it has got a very local significance. In fact, it means that those students who are aware that rest energy is related to mass will tend to consider as inelastic a collision in which a new particle (and therefore mass) is created and vice versa. So the conception held by the 25/42 involved students proves *logically consistent, even though only under this very specific respect.*

VII.2.3.4. *Students' profiles*

First of all, it may be seen in figure VII.11 that *mixed categories* were formed, as students were not entirely associated to one level of physical representation (Doménech *et al.*, 1993) in some cases. Most students proved good at understanding and using formal language, often expressing concepts by means of formulas. However, five of them did not refer to any theoretical framework. The prominence of relational level confirms the overall very good scientific skills of the learners, indicating their ability in learning and organizing concepts belonging to a novel physics domain (SR concepts) in a consistent framework.

It is also important to stress that pupils were very good at *formalizing* on the whole. However, 50% of students (20/42) did not master the relationship $m = E_0/c^2$ well, on the basis of the relative previous results. The same and other previous results bring to suppose that the reason for the latter effect *might be* the conception of "relativistic mass", but this assertion needs to be tested.

VII.2.3.5. *Language*

Language also played an important role in the *proper understanding of mass in relativistic context*, which is intimately related to the theoretical framing of its conceptual relations with total energy, rest energy and "relativistic mass". The wrong answers of 7/39 students to the question R2 have to be ascribed to that factor indeed, namely to a mixing of

(a) proper use of terminology about "relativistic mass";
(b) "relativistic mass" (reasonably but incorrectly) considered as «*the mass in relativity*», which is referred to as «E_0/c^2» as well as «*mass in* $E = mc^2$;
(c) "relativistic mass" meant as mass at relativistic speed;
(d) emphasis on mass *variation* corresponding to energy variation.

Eventually, the use of *mass instead of mole* (6/42) for denoting quantity of matter (a pre-theoretical conception) may be due to the fact that they are used as synonyms in common language to denote "amount of stuff", in Italian at least.

VII.2.4. **Conclusions and considerations**

- The detected dichotomization of mass during the learning process, in which gravitational mass becomes a single source of force, is a *negative spinoff* of the t/l path, since the bridging analogy between Universal Gravitation Law and Coulomb's one, as well as Mach's definition, were aimed at strengthening the scientific idea of force as interaction. The effect had been perhaps due just to the analogy with electrical charges, which have neither active nor passive role. Without that support, the initial conception was very likely to re-emerge: active gravitational mass. This experimented approach *proved clearly to be ineffective*. More attention ought to be paid in stressing the symmetry between any pair of masses, just like symmetry between two charges.
- The results about the integration of the concept of "relativistic mass" in the relativistic paradigm, joint to the ones deriving from the "mixed" category (8/39 replies), indicate that 34/42 students *grasped that a change in mass meanings occurs in the passage to Relativity*. Nevertheless, this passage is not always toward the consensus view. In fact, nowadays most of the physics and physics education community consider "relativistic mass" as a redundant educationally misleading improper physical quantity. That conceptual revision

cannot be limited to *semantic aspects*, like students seem to believe. In fact, they limit themselves to mention terms without explaining them or integrating them in proper contexts.

- Roughly speaking, those students who tended to reason in terms of "relativistic mass" did not likely write of mass in SR as equivalent to rest energy and vice versa. If proofed, this would mean that "relativistic mass" hinders learning of the relativistic correct meaning of mass[114], but it is a simple hint once again.
- *The recorded reasoning paths on mass seem to be strongly affected by their learning topic areas.* In particular, both (i) a local view of mass in SR in a context defined by speed and (ii) a grasping of the concept of mass in SR limited to a "section" of physics were detected in the answers to R1 and R2 (final test). The second findings recall to the chapters of school textbooks, one of which frequently deals exclusively with Relativity. This proves the importance of designing an *integrated teaching across the discipline* (Fabri, 2005) once again, which is the core of vertical approach.
- Many previous results bring to suppose that the reason for bad mastery of mass-rest energy equivalence *might be* the conception of "relativistic mass", but this assertion needs to be tested. At the same time, the findings mirror a teaching practice in which much room is left for theoretical or mathematical approach, while the operational viewpoint is *completely ignored*: that level of physical representation was not present at all. So the Italian geographically representative sample appeared to be taught in a traditional non-active way. Nevertheless, the discrete overall bearing of the functional level (31%) indicates a more concrete and/or qualitative approach too.
- Eventually, the 42 talents showed on the whole good skills of understanding historical physics texts and writing about them. This is considered useful to improve domain-specific learning with understanding (Halliday & Martin, 1993; Yore, 2000; Unsworth, 2001; Bazerman, 2009) and to foster science literacy (Tytler *et al.*, 2013). A possible by-product of my research might be a small contribution to WID (*Writing in the Disciplines*) line of research, in which students have to write using a proper genre, and to WAC (*Writing across the Curriculum*) line, because most questions in the tutorials stimulate creative writing for inquiry or spontaneous writing on an intended topic[115].

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[114] Or at least the most widespread meaning in the scientific community.

[115] For example, questions of the type: "What do you know about…?"

VII.3. Classroom experiment (IV year) – Sample 1 and 2 (Udine)

A class of twenty-three students (sample 1) attending the fourth year of Liceo specializing in scientific studies (aged 17-18) carried out the *entire relativistic energetic t/l pathway*. Another class of twenty students (sample 2) attending the same year of carried out a formative experiment on *a section of it* instead, namely until the induction of relativistic kinetic energy (K.E.) formula. The t/l proposals were agreed with the teachers in order (i) to deal with relativistic dynamics thoroughly for getting to define and explain mass in SR (and contrast "relativistic mass") in the former case, (ii) to supply the learners with a first approach to relativistic dynamics in the latter case. The sample sizes[116], durations, performed activities, data collection methods and data analysis methods are summarized in table VII.2.

More specifically, the involved students filled in a pre- and almost identical post-test in both experiments (tables VII.9 and VII.10); they also answered to some intermediate interpretive written questions after each self-standing instructional unit in the second experiment (sample 2). Finally, a post-test for middle/long term learning was administered in both cases.

Learning and understanding were analysed both by *comparing* the administered pre- and post-tests both by analysing each post-test by itself. Pre/post comparisons were useful analyses of the occurred *conceptual change*, while post-tests were used for detecting the acquired knowledge and ways of arguing. Besides, the general intermediate questions were used to find pupils' intermediate explanatory models and problem solving skills in similar contexts. This allowed in turn probing how, if so, the expected reorganization of the framework theory occurred (Posner *et al.*, 1982; Vosniadou, Vamvakoussi, & Skopeliti, 2008). Besides, the students' real phenomenografic profiles before and after instruction were compared for each sample, and finally several statistical and educational parameters were calculated, as reported in table VII.2.

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[116] A different number of students is indicated in the table (sample1) because some of them were absent during the in/out test administration.

VII.3.1. Administered questions

	Question Text – Udine Sample 1	
Q1	"Does a particle with mass *m* at rest in the LabIF own energy, if it does not interact with any object (so that there are not force fields)? Justify your answer."	"Does a particle with mass *m* at rest in the LabIF own energy, if it interacts negligibly with any other object (for example a spaceship in the intergalactic space shielded from electrical and magnetic field)? Justify your answer."
Q2	"Consider observers endowed with every known measuring instrument. Imagine that each observer (each of whom in a different inertial frame) perform the measure of a given physical quantity and then write the result on a notebook. If all the observers find the *same value*, by comparing their notebooks, they will call *invariant* that quantity. *Thus a quantity is said invariant if its measured value is the same in every inertial frame, even if the measure is indirect*. Is mass invariant ?"	*idem*
Q3	"If a particle is accelerated, will its mass vary? Justify your answer."	*idem*
Q4	"Can I make the *energy* of a body increase *without a variation of its speed* with respect to the reference frame in which I stand? Yes, in this way… / No, because … Reply and explain"	"Can I make the *energy* of a body increase *without a variation of its speed* with respect to the reference frame in which I stand? Yes, in these ways (specify two at least)… B) In this case, does the body mass vary? / No, because … Reply and explain"
Q5	"Consider two objects moving against each other at *relativistic speed* with opposite directions. They collide and create a new object. Will the product mass be equal to the *mass sum* of the two initial objects? Explain your answer."	*idem*

Table VII.9. Questions of the initial (left) and final test (right).

	Question Text – Udine Sample 2	
Q1	“Consider an isolated physical system (body) moving unsteady” [*omissis*] In general, is there a physical quantity which is conserved, that is, which remains constant over time? Which one? Justify your answer”	“Why do we conclude that work-energy theorem is valid at high speeds too? Explain”
Q2	“Kinetic energy is: 1) The quantity expressed by the equation $K = ½ m v^2$; 2) The form of energy associated with a particle/mass-point’s motion state; 3) A contribution to total energy of a moving physical system to be considered for testing energy conservation; 4) A combination of the above (specify which one). Justify your choice”	“List main outcomes from Bertozzi’s experiment”
Q3	“Does work-energy theorem imply that *if a work is done on a body its velocity and/or kinetic energy can be limitless increased?* Explain”	“Why do we synchronize clocks with a light signal and not of another type, such as sound?”
Q4	“Does the same hold for *linear momentum*? Explain”	“An observer on a train and another on the station platform were given two identical clocks. Are first observer's heartbeats *really slowed down* for the other? Does the second observer feel his own heartbeats slowed down? Explain”
Q5a pre-Test	“Suppose that you observe on a train an object moving at 10 m/s with respect to you. Your mate on the station platform is estimating that the train moves at 25 m/s relative to him *in the same direction of object’s motion*. What is the object speed in the *inertial frame* of the platform? What formula did you apply to get it?”	“A bridge manipulator and a passenger on a train moving on a movable bridge were given two identical clocks. Is time interval between bridge opening and closing *different* for the train passenger with respect to the bridge manipulator? Explain”
Q5b post test	“Suppose that you observe on a train a propagating laser beam [*the rest of the text is completely analogous to that of 5a*]. Explain how you get your result.”	
Q6	“[*omissis*] Is duration of a phenomenon, such as the fall of something to the ground, the same for all observers? Reply and explains the assumptions made”	
Q7	“[*omissis*] Which of the following quantities are invariant and why? Time interval (duration); Kinetic energy; Momentum; Length; Light speed *in vacuo*; Speed of a sound produced by one observer; Mass”	

Table VII.10. Pre- and post-test questions (on the left) and intermediate questions (on the right).

VII.3.2. **Data: Analysis and Results**

Qualitative content analysis and correlation analysis were carried out. The significance level α for the latter was set at 0.05 for sample 1 and at 0.01 (high significance) for sample 2. Moreover, normalized learning gains were calculated. The use of this educational statistical parameter required three assumptions (Aslanides & Savage, 2013). I made these assumptions for simplifying this part of analysis: learning gain is useful to supply a quantitative hint for better observing and measuring learning progress. Of course, understanding is actually a multidimensional and complex process.

Assumptions:

1. A 'correct' answer means the knowledge of the corresponding concept;
2. A concept is either know or unknown;
3. All questions have the same relevance.

Pre/post-test comparison – Sample 1 (initial sample size: 21; final: 23)

Q1. Explanation of the energy owned by a non-interacting mass at rest in terms of mass-energy equivalence increased very much, as shown by figure VII.12. This category is defined as the set of answers like "*Energy given by* mc^2" (initial test); "*The particle with mass m has energy, in effect* $E_0=mc^2$"; "*When there is mass there is always an energy equivalent; mass is energy of a body at rest indeed*" (final test). Furthermore, the negative answers disappeared and the ones considering the owned energy as potential one decreased. The latter category is defined by sentences like "*The particle has potential energy although it does not interact with any object*"; "*Yes, it has energy of potential kind"* (initial test); "*It has potential energy given by the bond between the particles*" (final test). Accordingly, the learning gain is high ($g_1 = 0.63$).

Q2. The number of tautologies/inconsistencies for justifying a positive answer doubled, while the number of the ones for justifying a negative answer vanished. The correct category "A single frame/the rest frame" stemmed (5/23), operationally defined by: "*Yes, because the reference frame does not change, so even if speed is increased mass will remain invariant*"; "*Yes, in effect the mass measured in the inertial frame is the same measured in the laboratory, because for measuring m one always refers to the co-moving system*"; "*Yes, in effect one always refer to only one reference frame*". Finally, the reference to relativistic mass or “converse” relativistic mass (it decreases while speed increases) vanishes. The category “('converse') m_r” is defined by: “*No, because mass varies according to the reference frame; if I accelerate a particle up to light speed mass will vary indeed*”; “*No, because the mass of an accelerated body decreases, until one gets to the photon in which there is no mass*”; “*No, since relativistic mass is not invariant. Rest mass is invariant, but it decreases by accelerating until the photon (speed c), which has no mass”*; “*No, because mass can broaden or shrink according to the reference frame”*. The overall understanding is quite weak, as witnessed by $g_2 = 0.22$.

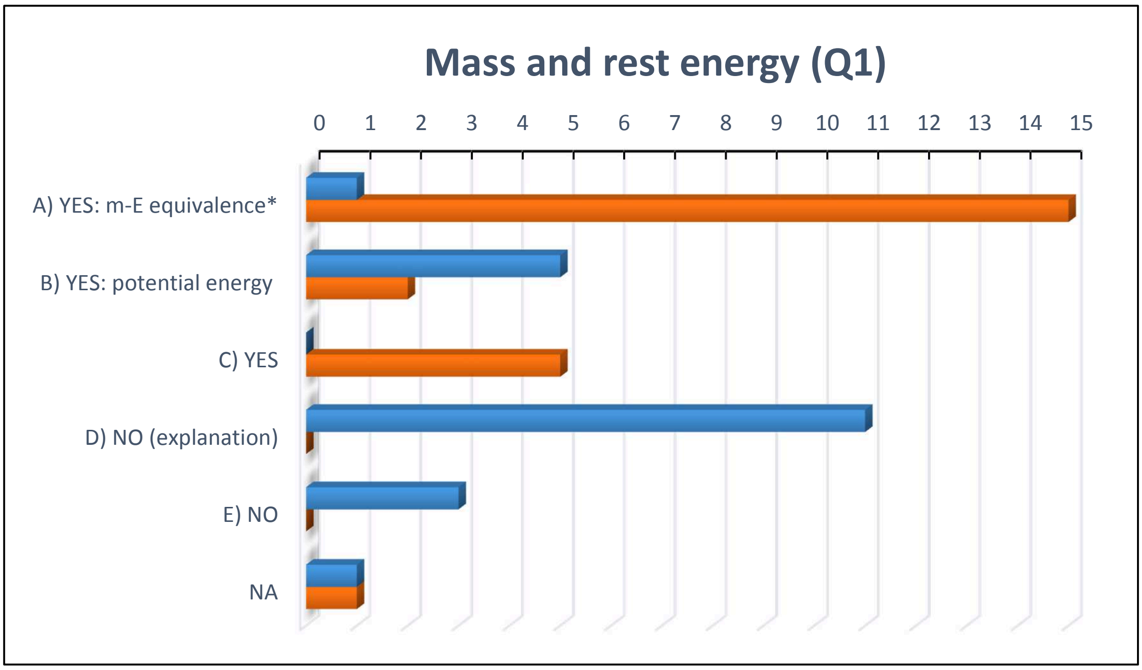

Figure VII.12. Answer category discrete distributions of pre- (up) and post-test (down) for Q1 (sample 1). The correct category is marked by an asterisk.

Q3. The reasons for which the mass of a particle is said to vary under acceleration change a lot between pre- and post-instruction. The broad categories “m is invariant” and “"Converse" m_r” disappear, while the (correct) category D stems and C increases very much. They are defined by the answers in the table below. The normalized learning gain is negative: $g_3 = -0.39$.

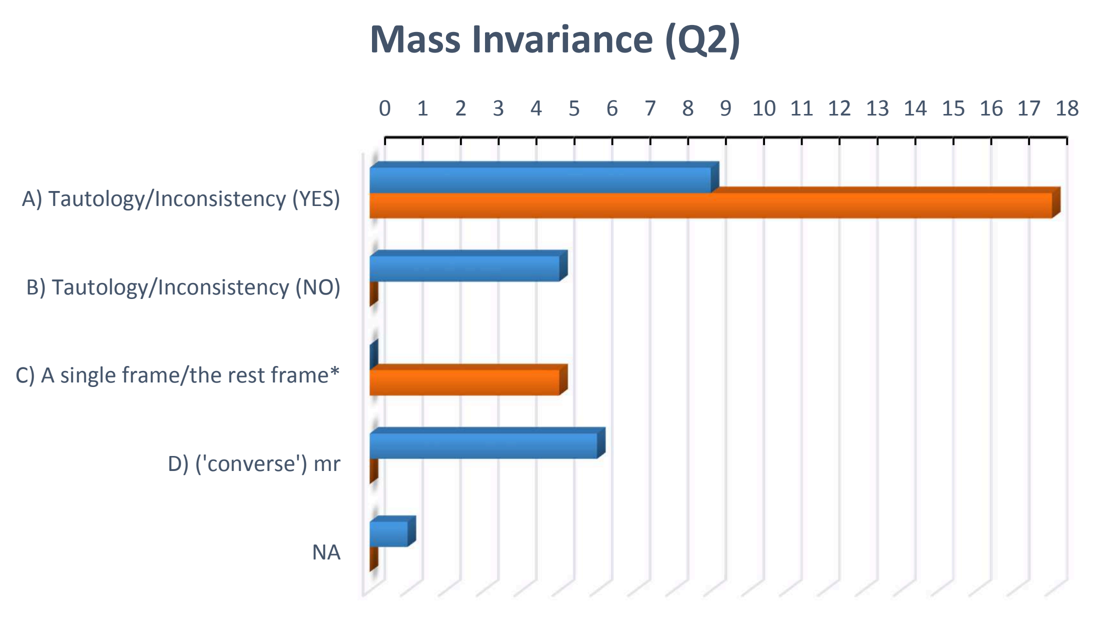

Figure VII.13. Category discrete distributions of pre- (up) and post-test (down) for Q2 (sample 1). The correct category is marked by an asterisk.

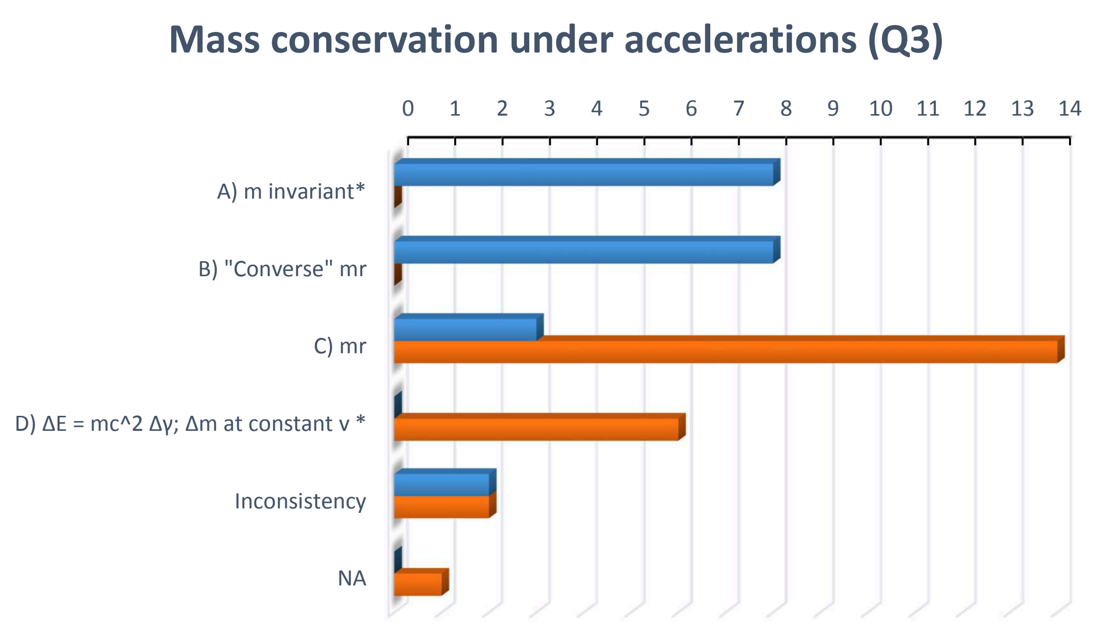

Figure VII.14. Category discrete distributions of pre- (up) and post-test (down) for Q3 (sample 1). The correct category is marked by an asterisk.

C) “m_r”	“*Yes, it might vary for speeds next to light’s*” (initial test)
	“*Yes because mass is energy of a still body, thus it varies by acceleration*” (final test)
	“*Yes, in effect if a particle is accelerated mass will vary according to the formula* $\Delta m = \Delta E$ (γc^2) (final test)
D) “$\Delta E = mc^2\ \Delta\gamma$; Δm at constant v”	“*No, since mass varies at constant speed only*”
	“*No, its mass does not vary, in effect* $\Delta E = mc^2\ \Delta\gamma$”
	“*No, since* $\Delta E = mc^2\ \Delta\gamma$”

Table VII.11. Operational definition of the categories C and D (Q3, sample 1).

Furthermore, the consistency of mass invariance conception before instruction was tested by calculating the correlation coefficient between the broad category “Yes” (any positive answer) to Q2 and “No” (any negative answer) to Q3 in the pre-test. A highly significant correlation was found ($r_{23} = 0.91$: very strong effect size, $r_0 = 0.56$; $r^2 = 0.83$). A more accurate test was made on the results after instruction, namely between the category C (“A single frame/the rest frame”) of answers to Q2 and “No” to Q3. A significant correlation was found ($r'_{23} = 0.51$: strong effect size, $r_0 = 0.42$; $r^2 = 0.26$).

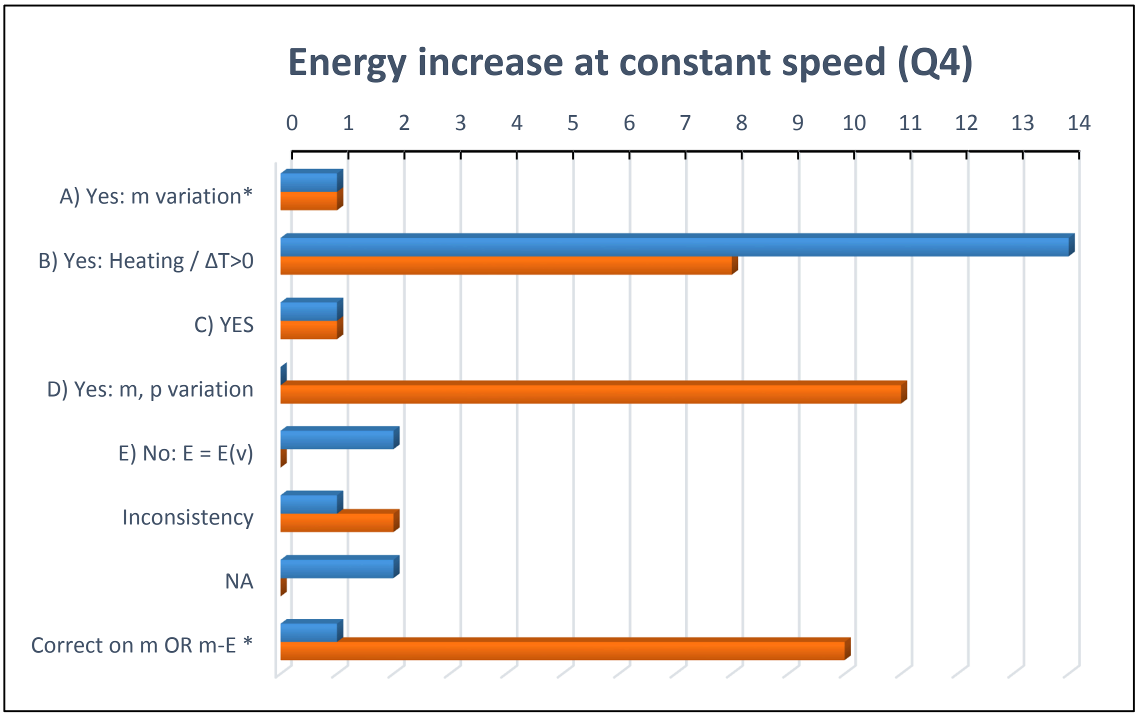

Figure VII.15. Category discrete distributions of pre- (up) and post-test (down) for Q4 (sample 1). The correct category is marked by an asterisk.

Q4. First of all, it is specified that the answers to this question are biased by my suggestion about heating: during the pre-test the students asked me repeatedly a hint for a way for making energy change, so I replied that heating could be a good method. This explains the prominence of category B, especially before instruction. Category B is defined by the answers of such as: "*By increasing internal energy by providing heating*", "*I can make its energy increase without varying the speed by increasing its temperature*" (pre-test); "*By providing heating*", "*heating, electromagnetic waves"* (post-test). Since only one students referred to mass variation before the experiment ("*No, because that can happen, but its mass varies in that case*") an additional point was inserted into the post-test corresponding question (see table VII.9). 10/23 correct answers were found by combining this result with the one on energy, giving rise to the category "Correct on m OR m-E". Examples: "*B) Yes, if a body is heated energy will increase and speed will remain the same, and its mass vary, even though a little bit*"; "*B) Yes, because speed is the same, energy increases and decreases, therefore the same must occur to mass*"; "*B) Yes, because speed is constant, but energy increases*". Therefore learning gain is fairly high for this question ($g_4 = 0.40$).

Q5. The categories A, B and E disappeared after instruction, while the broad correct category F (14/23) stemmed. So the learning gain in very high ($g_5 = 0.70$).

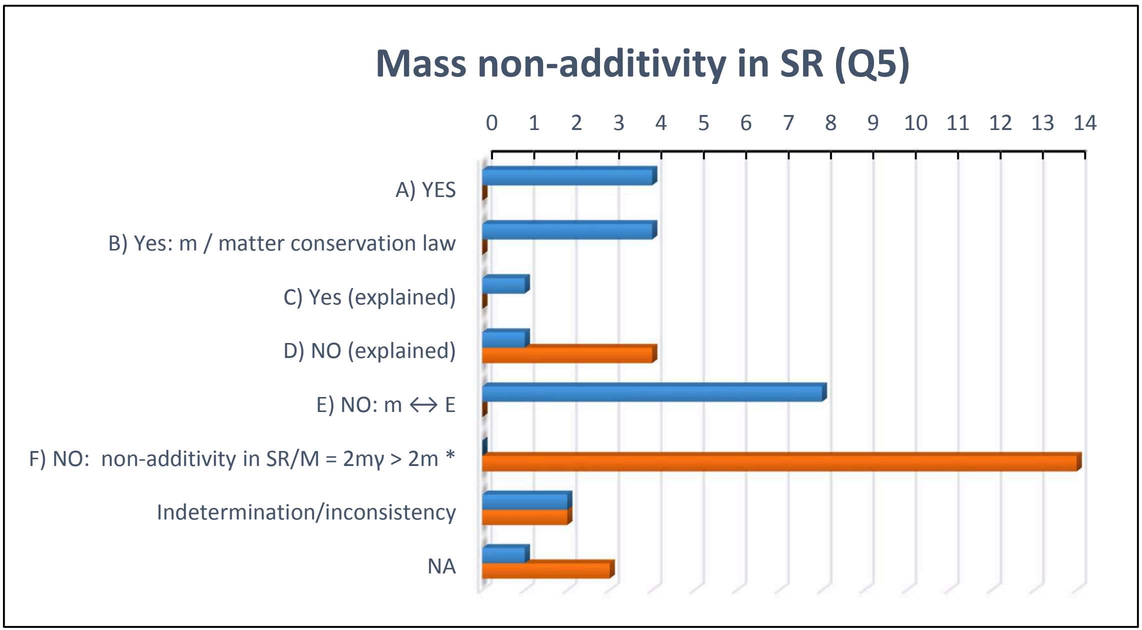

Figure VII.16. Category discrete distributions of pre- (up) and post-test (down) for Q5 (sample 1). The correct category is marked by an asterisk.

B)	*"Yes, since matter will remain the same even if the two objects become one"*
	"Yes, because according to the conservation mass law, the mass of products is equal to the mass of reagents"
E)	*"No, because a part of the masses may change into energy"*
	"No, because the mass of the created object will be greater, since the body acquires mass by decelerating"
	"No. Some mass can change into energy, and thus the final result may show a smaller mass, although by a small amount"
F)	*"The product mass will not be equal, mass is additive only in classical physics"*
	"No, it will be greater since $M = 2m\gamma_i > 2m$*"*
	"No, since mass is not additive"

Table VII.12. Operational definition of the categories B, E and F (Q5, sample 1).

The learning gain averaged over all questions is $\langle g \rangle = 0.31$: a fairly good value, indicating that 31% of top "understanding" was reached. In Relativistic Concept Inventory it came out $\langle g \rangle = 0.40$. Once again, these numbers are to be handled with care, because understanding is actually unique to the individual.

The correlations of Q5 with the other questions were searched for – namely a correlation test was made between two categories at a time – and a positive result was obtained in three cases. The correlation coefficient between the broad category "Yes with explanation" (any justified positive answer) to Q3 and "No with explanation" (any justified negative answer) to Q5 in the pre-test was calculated. A highly significant correlation was found ($r_{35} = 0.83$: very strong effect size, $r_0 = 0.56$; $r^2 = 0.69$). Thus a more accurate analysis was carried out in the pre-test answers, i.e. between (i) a category obtained by joining B (Q3) with C (Q3) and (ii) the category E relative to Q5. Another highly significant correlation was thus found ($r'_{35} = 0.75$: strong effect size, $r_0 = 0.56$; $r^2 = 0.56$). Finally, a significant negative correlation is between the categories "Yes" (Q1) and E (Q5) of the pre-test answers ($r_{15} = -0.50$: moderate/strong effect size, $|r_0| = 0.44$; $r^2 = 0.25$).

Pre/post-test comparison – Sample 2 (initial sample size: 20; final: 20).

Some knowledge acquisition after the proposed instruction was highlighted.

i. Kinetic energy acquired broader meanings: motion energy primarily, and/or essential term for energy conservation; furthermore the choices of its classical expression broke down in the answers to Q2 (figure VII. 17).

ii. Moreover, a category including some explained references to K.E. dropped too (e.g. « *K.E. is an energy expressed by the equation* $K= ½ m v^2$ *associated to particle motion* » or « *K.E. depends upon the body's mass and the speed which it's moving at* »): its frequency passed from 15/20 to 4/20.

iii. A category of answers to Q3 stemmed: 11/20 students reported that K.E. may be increased to infinity and speed be upper limited at the same time. Typical answers: « *No, energy can rise without limits but speed doesn't exceed light's*»; « *No, as speed never exceeds c though one keeps supplying energy to the particle and rise indefinitely its K.E.* »

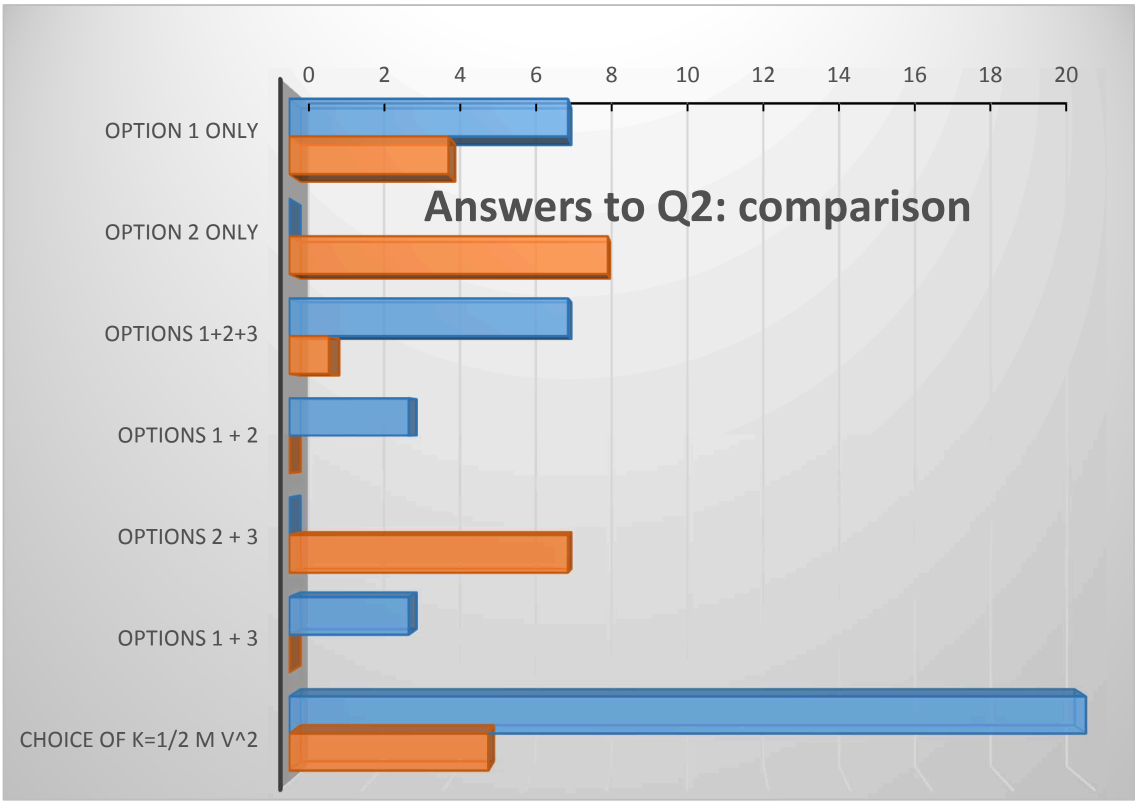

Figure VII.17.Category discrete distributions of pre- (up) and post-test (down) for Q2 (sample 2). Non mutually-exclusive categories.

iv. An analogous narrow category (4/20) originated for momentum (Q4). Example: «*Yes, since the value of p can approach to ∞ at relativistic speeds, but the speed v can't reach values over c*».

v. As for Q7, post-test, 19/20 pupils recognized *c* as relativistic invariant and 6/20 acknowledged the validity of the syllogism "if *c* is the speed limit, then *c* is invariant". Example: «*Light speed* in vacuo *yes, it's the limit speed and therefore it has precisely the same value in every inertial frame*». One-half sample answered 5b using the same syllogism, for instance « $3*10^5$ *km/s* → *Because a massive body can't exceed that speed!* » or « *The speed is close to c because c can't be exceeded*». A significant statistical correlation was found between the answers to 7 and 5b in which the syllogism was ($r = 0.65$: strong effect size, $r_0 = 0.57$; $r^2 = 0.43$).

vi. Before the path 2/20 and 3/20 pupils acknowledged the existence of a speed limit in items 3 and 4 respectively; 15/20 and 13/20 did the same after it.

vii. The answers to Q6 asserting time interval invariance went from 10/20 to 1/20, while the ones asserting non-invariance from 6/20 to 18/20.

viii. More specifically, prior to the path some conditions for measuring time intervals were added in eight answers of the first category, for instance «*the response time of each person is to be calculated*» or «*It will have the same value if the same measure units and the same reference frames are considered*». Furthermore, 8/20 students focused on the process of measure, 6/20 on the observers and 3/20 on certain phenomenon parameters: speed of fall, attraction forces, friction and height (categories in figure VII.18). After the path, time dilation effect and time dependency upon speed are referred to in overall 12/20 cases. Typical answers for the former: «*It depends on the observers' speed: if it's close to light's, time dilation will have to be considered* » or « *No, because time changes in different systems owing to duration dilation effect* »; for the latter: « *No, because the time measured by observers depends upon the travel speed and thus upon motion* ».

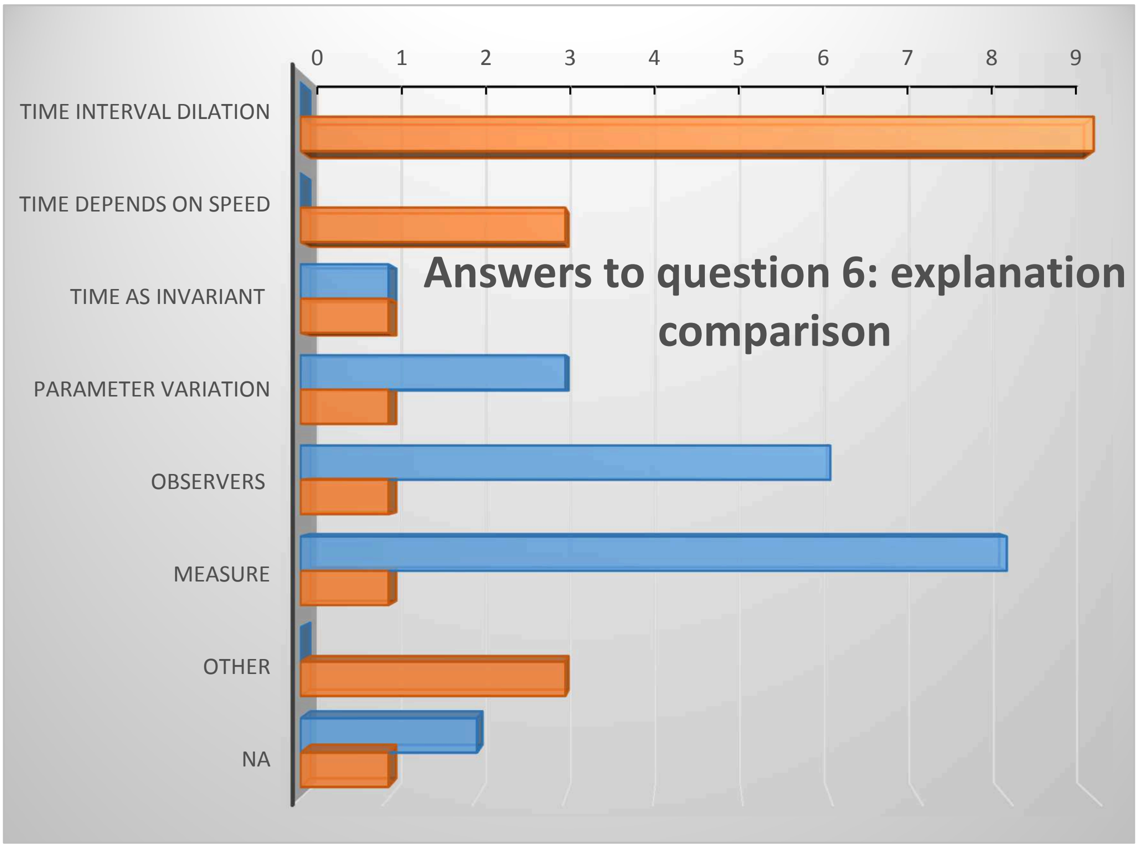

Figure VII.18.Category distribution for the answers to Q6 in pre- (up) and post-test (down) in sample 2.

g_1	-0.18
g_2	0.43
g_3	0.55
g_4	0.15
g_6	0.45
g_7	0.24
$\langle g \rangle$	$\mathbf{0.27}$

Table VII.13. Values of normalized learning gains for sample 2; g_5 is missing because Q5 is different before and after the formative experiment. The obtained mean value indicates a fairly good "understanding" was achieved.

Intermediate question analysis (sample 2 only). Twenty-three students were present at the question administration; the ones missing in these results answered off the point or did not at all. The fourth and fifth question, *Q4* and *Q5* (table VII.10) had been built up for probing cognitive transfer skills – which are necessarily entangled with sound mental model formation according to Tytler *et al.* (2013) – on time dilation. Cognitive transfer occurs when a student succeeds in solving problems in which the duration of any phenomenon is observed from different frames. In addition, *Q4* aimed at discriminating between descriptions of distortion of perception and real physical effect (Hewson, 1982; Posner *et al.,* 1982; Scheer *et al.*, 2001; Scheer *et al.*, 2002; Levrini, 2014). Results for *Q4*: transfer occurred in 15/23 cases, in 10 of which the effect is described as real; in 5/23 no transfer was found; in 2/23 it was not possible to determine if it occurred. Example for real effect: *"yes, since the observer is moving and thus time is slowed down. No for he's not in motion but still on the platform*»; for perceptive effect: "*they seem slowed down for the second but they aren't; no because he's still with respect to himself*». Results for *Q5*: transfer in 11/23 answers, no transfer in 1/23, uncertainty in 9/23.

At last, in the concept map explanation (before activity R3, table VII.1) 15/23 pupils wrote that Galilean coordinate/velocity transformations are not valid anymore and 6/23 claimed that K.E. classical expression is not valid.

VII.3.2.2. *Phenomenographic study – sample 1*

As an alternative to the method previously described, students' profiles may be determined by means of a phenomenographic analysis. Phenomenography is a kind of research aimed at eliciting the (necessarily) limited numbers of mental models held by a group of people (Marton & Booth, 1997). Its basic hypotheses are that (i) conceptions (meant as mental representations) stem from the interaction between the learner and everything he/she experiences and (ii) conceptions are detectable by means of language. These mental models generated by immediate experience allow and govern in turn the scientific observation and interpretation of phenomena (Greca & Moreira, 2002; Michelini, 2010a). The way in which each learner describes what happens to him/her may vary from one to another subject. The ways by which the involved students replied to the questions above («answering strategies») are classified according to three reasoning ways: declaratory[117] or «practical/everyday», «descriptive» and «explicative», according to Fazio, Battaglia, and Di Paola (2013); Pizzolato, Fazio and collegues (2014). They are ideal profiles of students; the profiles' percent relevance in each student of the sample is depicted in the two bubble graphs below, the first representing the initial state (pre-test, N=21), the second the final state (post-test, N=23). The three vertexes of the equilateral triangle represent the ideal profiles. «Practical/everyday» may indicate that either (i) a student describes the situation/phenomenon under examination by referring to his/her everyday experience using common language, without abstraction and/or modelling or (ii) that no reference is made to any relevant variable in the phenomenon description as well as (iii) that the answer is not scientifically correct at all and/or inconsistent and/or entirely uncorrelated to the question. This may be defined as the "zero level" of the answers. In the «descriptive» case, a student refers instead to the involved physical variables and their relations correctly, but he/she neither gives a scientific explanation for what happens nor works out a microscopic or macroscopic model. Eventually, in the «explicative» case a mechanism is elaborated to account for the considered phenomenon in physical terms; the invented model may include hypotheses or causal explanations.

In other words, each student gave 5 answers, each of which was judged as belonging to one of the three ideal profiles. So it was possible to find out the

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[117] I added this facet of the first reasoning way.

relative incidence of each category for each student in the initial and final state. For example, the first student (S1) wrote 3/5 declaratory or «practical/everyday» answers to the Q1, Q2, Q3 of the pre-test: "*The particle owns potential energy although it does not interact with any object*"; "*Yes, in fact it does not vary along with the reference frame"*; *"No, because mass does not change in any situation*". The 1st reply was considered as «practical/everyday» since a quantity is mentioned, but no description of the process is given; the second because the definition of invariant is merely repeated/expounded (declaratory answer); the third because it is a common-sense explanation. The 2/5 answers to Q4 and Q5 were considered «descriptive» instead: "*I can make its energy increase without a speed variation by increasing its temperature*" and "*Yes, the product mass will be equal to the mass sum of the two initial objects*" respectively. In fact, the first one offers an explanation, but it had been suggested by the tutor during the activity with just the same words. The last one identifies the relevant variables and describes what happens. So it was possible to assert that 60% of the detected answering strategies by S1 were «practical/everyday», 40% were «descriptive», and 0% «explicative». The percentages for the average student were also calculated for the pre-test: 50% «practical/everyday», 22% «descriptive», 23% «explicative»[118].

As for the post-test, S1 gave one everyday answer to Q2: "*Yes, in fact mass is independent of the reference frame*", while the other were descriptive: "*The particle of mass m own energy, in fact* $E_0 = mc^2$" (Q1); "*The particle mass varies according to* $\Delta m = \Delta E(\gamma c^2)$" (Q3); *"*$\Delta E = c^2 \Delta m$ *when mass varies;* $E = pc$ *when momentum varies. B) Yes because speed is the same"* (Q4); "*No, mass is additive in classical physics only*" (Q5). So 20% of the answers were everyday and 80% descriptive after instruction; ‹student›: 29% «everyday», 53% «descriptive», 13% «explicative». To complete the picture, two explicative answers by S2: "*Yes, since* $E_0 = mc^2$ *indeed (minimum energy associated to the existence of a particle with mass m, said 'zero-point energy')*" (Q1); "*Yes, in fact mass is invariant but it is not conserved. Because the mass found in the IF is the same which is found in the lab., since one always refers to the rest frame for measuring*" (Q2).

A sharp shift from the «everyday» to the «descriptive» vertex together with a rarefaction of the «explicative» zone is observable by comparing the bubble graphs. This trend is confirmed by comparing the average student percentages.

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[118] The total here and in the post-test is not 100% because some students did not answer.

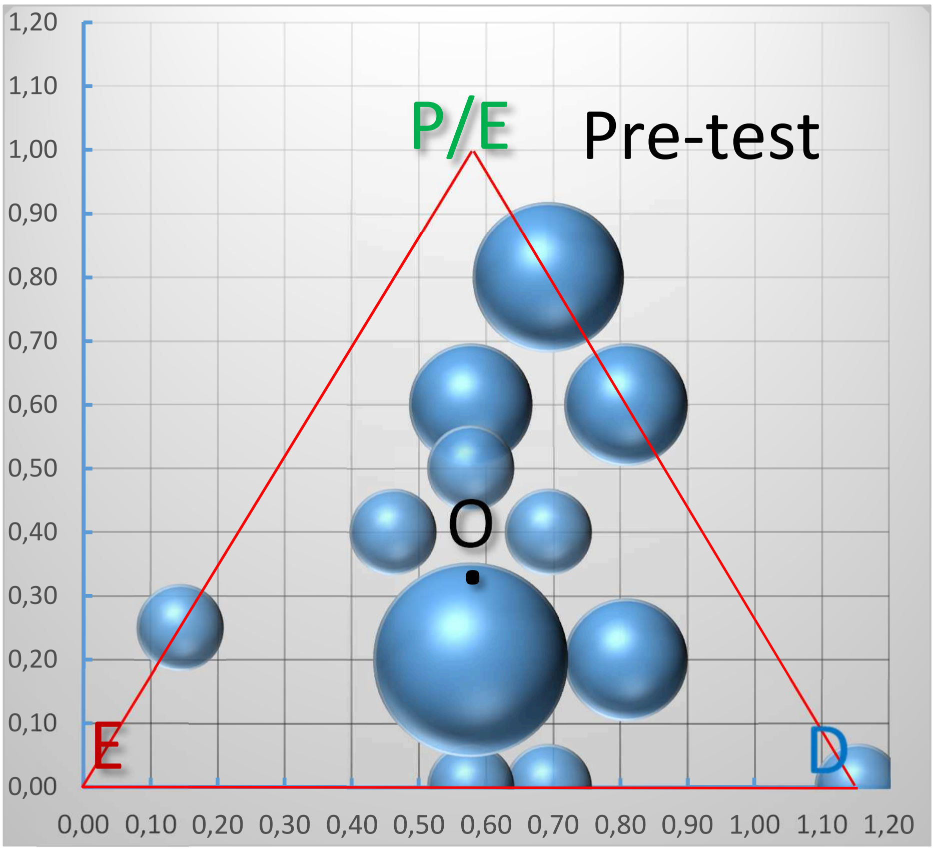

Figure VII.19. Bubble graph showing the "distance" of each group of *real student profiles* from the three ideal profiles in the vertexes *before* instruction. P/E stands obviously for practical/everyday, D for descriptive, and E for explicative. The radius of each sphere is proportional to the number of students in each group. This graph was built so that the distance of any bubble from the each side is equal to the fraction of the profile indicated in the opposite vertex. $O\ (0.58, 0.33)$ is the centre of the equilateral triangle.

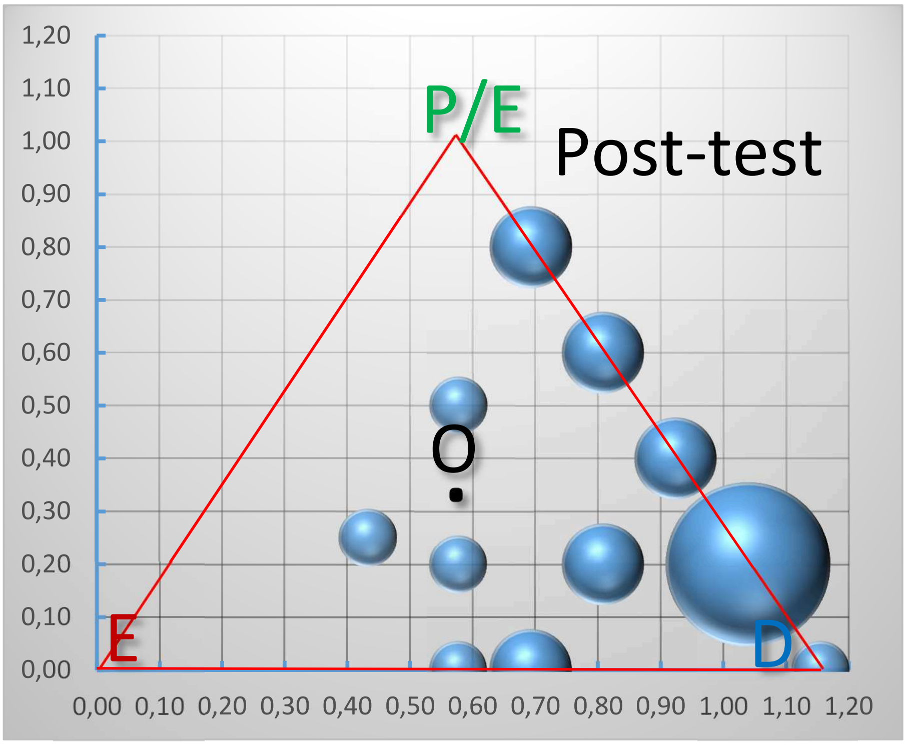

Figure VII.20. The same bubble graph of figure 19, but depicting the state *after* instruction.

VII.3.2.3. *Phenomenographic study – sample 2*

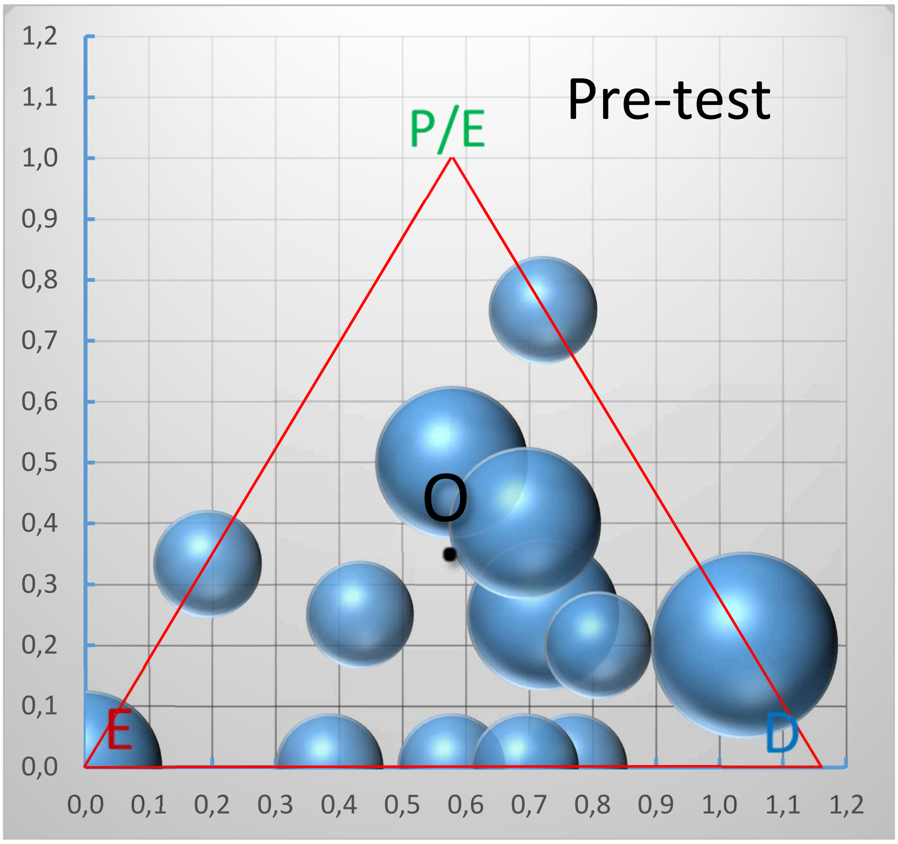

Figure VII.21. This and the next graph are obtained from the answers to questions Q1, Q3, Q4, Q6, and Q7, because the others were not apt to be classified according to these profiles. O (0.58, 0.33) is the centre of the equilateral triangle.

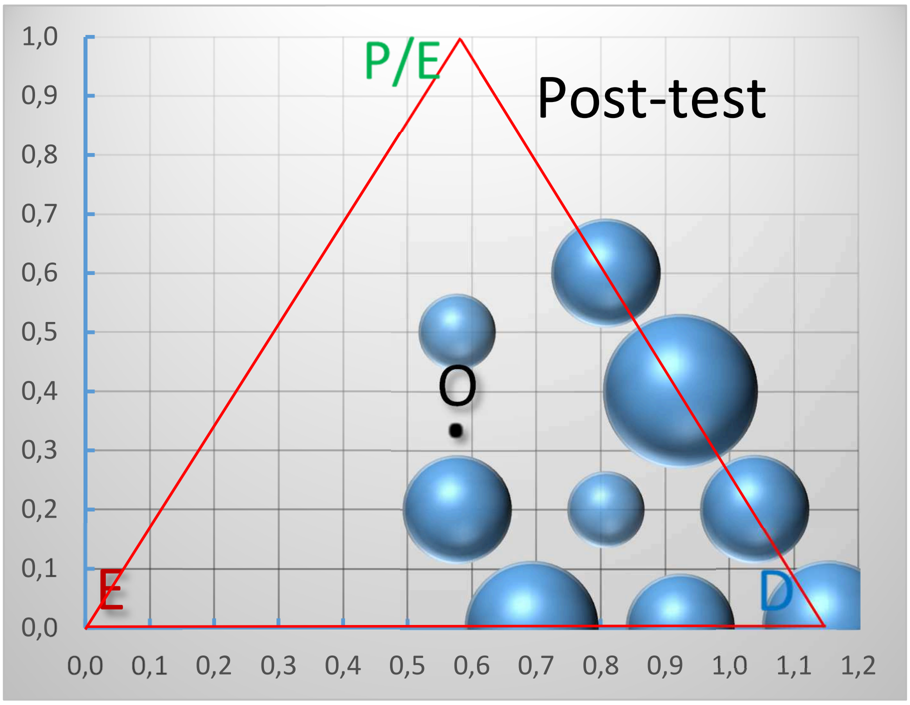

Figure VII.22. See previous figure.

	Pre-test	***Post-test***
P/E	*“A physical quantity which is conserved over time is weight”* (Q1) *“I can do work with value ‘$+\infty$’ in principle. But I will never use this value actually”* (Q3)	*“A physical quantity owned by the body which remains constant over time is mass, because it remains unvaried in every reference frame”* (Q1)
		“Yes” (Q4)
		“The duration of a phenomenon may assume different values for the hypothetical observers. What determines phenomenon durations is observer’s position in the reference frame indeed ” (Q6)
D	*“The physical quantity which is conserved in a body is mass”* (Q1)	*“The body cab increase its speed until it reaches a limit c, which is light speed”* (Q3)
	“*It depends upon the fact that all observers have the same measuring instrument, specifically calibrated* ”(Q6)	*“Not even momentum may increase without limit, since it depends upon speed which never exceeds c, that is light speed”* (Q4)
	“*It will have the same value if the same measure units and the same reference frame are used”* (Q6)	*“No, because there is time dilation with respect to moving objects*” (Q6)
E	*“No, since up to now light speed (c) has been demonstrated to be the maximum achievable speed, so* $K_{max}= 1/2\ mc^2$ (Q3) *“Yes. The falling time from a given height does not depend on body’s speed”* (Q6)	*“Only kinetic energy; speed has c as absolute limit, which cannot be overtaken, as Bertozzi’s experiment demonstrates”* (Q3)
		“The phenomenon duration has not got the same value for all observers, because it varies with speed. An observer with a very high speed will undergo ‘duration dilation’, thus time will slow down” (Q6)
		“Time interval is shorter because of time dilation, kinetic energy arising from speed, arising from time; momentum from the formula it is seen that it varies with speed, length varies along the x-axis only ” (Q7)
Average student	P/E: 20%; D: 38%; E: 24%	P/E: 21%; D: 64%; E: 14%

Table VII.14. Operative definition of the phenomenografic profiles for sample 2.

Before the formative experiment, the profile distribution is rather uniform, with the «everyday» zone emptier than the «explicative» one, while the most populated is «descriptive» zone, although by a small amount. After instruction an effect similar to that of sample 1 was found: a sharp shift towards the «descriptive» vertex, while the «everyday» zone remains almost equal; the distance of all bubbles from the «explicative» vertex in plot VII.16 is remarkable. This trend is confirmed by comparing the average student percentages.

VII.3.3. **Discussion**

VII.3.3.1. *Sample 1*

Almost all students (20/23) understood the idea that a non-interacting mass at rest does have energy (Q1); a large fraction of these (15/23) specified the reason was mass-energy equivalence. Since only one student supplied this explanation before instruction, the whole outcome on the point may be considered as very good.

Before instruction, furthermore, only 6/21 students stated that mass was invariant (Q2), but in terms of relativistic mass or "converse" relativistic mass (the conception of mass decreasing with speed, until becoming null at *c*). The formative module was useful to increase the positive answers (18/23) and let 5/23 students understand that the key of mass invariance stand in the reference to a unique IF: the rest frame.

On the contrary, reflecting upon mass conservation under acceleration (Q3) contributed to make most students (14/23) reason in terms of relativistic mass (only 3/21 answers belonged to this category in the pre-test), while no more direct mention was made of the cause of conservation: invariance (D'Inverno, 1992). However, a few students (5/23) acknowledged that mass varies exclusively at constant speed. Moreover, the positive outcome of the second correlation test points out that after the formative experiment a student who answered essentially to Q2 that mass invariance is due to its measure in the same IF for all was likely to answer that mass does *not* vary under accelerations and vice versa. Remarkably, the first correlation test tells us that this consistent reasoning was already present in more general terms at the beginning: who asserted that mass is invariant, independently of the reason, was very likely to assert that it remains the same under accelerations.

As for increasing energy at constant speed (Q4), a fairly spread understanding (10/23) of the fact that it is correlated to mass variation was achieved, since only one student associated that variation to mass variation before the path. The attention of a lot of students (14/21) was focused indeed on heating/temperature variation, because of the bias introduced by me. At the same time, half of the students (11/23) claimed after instruction that mass and momentum variation were the causes of energy increase, because of the mathematical relationships $E = \gamma mc^2$ and $E = pc$. Unfortunately, the second one is valid for the photon only!

The answers about mass additivity (Q5) gave the best learning results. An *actual conceptual change* occurred. In fact, before instruction some students wrote that mass was additive in relativistic collisions (9/21), while several other students claimed essentially that non-additivity was due to *change of mass into energy* (8/21). After instruction 18/23 students wrote that mass is not additive, 14 of whom explicitly justifying their assertion using non-additivity *in SR* or a formal explanation, by reporting the outcome of the relative TE in formulas.

It turned out from the other correlation tests:

- (before instruction) Who asserted with an explanation that mass varies under accelerations was very likely to assert that it is not additive ($r_{35} = 0.83$, $r_0 = 0.56$; $\alpha = 0.01$);
- (before instruction) Those who reasoned in terms of ("converse") relativistic mass tended strongly to claim that mass transforms into energy ($r'_{35} = 0.75$, $r_0 = 0.56$; $\alpha = 0.01$).

The *descriptive* profile prevails in most answers of the post-test (15/23); there are also 3/23 "intermediate" real profiles, 4/23 between descriptive and everyday, but no explicative at all, and 1/23 halfway between explicative and descriptive, but not everyday at all. The most significant datum, however, is that 15/23 real profiles (65%) *are not explicative at all.* The post-test has been considered since it should give a "measure" of the learning/understanding due to the pathway. The situation is even worse if initial and final tests are compared: a clear-cut shift occurred from the "neighborhood" of «explicative» vertex to the «descriptive» zone, along with a leaving from the «practical/everyday» vertex C and VII.20).

Therefore two effects have been revealed. Firstly, (i) a trend to pass from the description in familiar terms of what happens to a more precise scientific description. At the same time, (ii) the students tended to limit themselves to a description similar to the tutor's one after instruction, without attempting to formulate explicative hypotheses and models, unlike before the formative experiment. They appear more stimulated to invent or to gain an insight in the new concepts prior to instruction.

VII.3.3.2. *Sample 2*

K.E. took on its primary meaning and/or at the same time the role of part of a wider conserved quantity (total energy). The mathematical expression of K.E. also became conceptually separate from the classical one and related in a new (correct) way to speed. Thus students can be said to properly master the concepts embedded in a re-structured domain-specific knowledge framework. The same can be argued for the concepts of ultimate speed and c invariance. They were overall reported indeed by at least one-half answers *ex-post* to four different items. Furthermore, the syllogism "c is the ultimate speed $\Rightarrow$ c is invariant" come out to be strongly held by some students, since it is present in a statistically correlate way both in the answers to Q5b and Q7, as well as in the concept map explanation (frequency 9/23). On the other hand, it may be asserted that the duration dilatation effect has been *understood*, but not *learned*. In fact most students held a conception corresponding to the scientific concept after the path, but few ways of reasoning and no inclusion in any theory-like conceptual system were revealed. This was likely due to the *absence of any prior conceptual referent for the effect*, unlike the cases of K.E. and c. However, the last assertion requires a proof.

As regards phenomenological profiles, the post-test is considered at first for the same reasons of sample 1. The descriptive profile prevails in most answers of the post-test (15/20), just like in the former case. The "intermediate" profiles are 3/20; 2/20 between descriptive and everyday (nearer to everyday). Remarkably, 11/20 (about 50%) are not descriptive at all and, differently from the previous case, 3/20 are *pure descriptive profiles*. So the shift toward the descriptive vertex is sharper. An important difference is indeed that *the explicative zone was more* *"populated" and the everyday zone less "populated" before instruction* than in sample 1; in particular, 2/20 profiles in sample 1 were purely explicative.

So the first effect revealed in sample 1 *is not detected here*, while the second one is *more marked here*, as it is evident from the bubble graph comparison.

VII.3.4. **Conclusions**

A *randomized sample* was taken, which can be assumed as including «cases with all relevant attributes as in the population» (Mayring, 2007), although the sample size is below 30 (approximate threshold of the central limit theorem). No selection had been done indeed on the basis of student performance in physics. The effect size is strong enough for the size of sample 2 ($r = 0.65$) and sample 1 in three cases ($r_{23} = 0.91$, $r_{35} = 0.83$, $r'_{35} = 0.75$): Fischer (2013) recommends $r > 0.6$ for $N \approx 20$. So some significant context-dependent hints and conclusions may be drawn.

Sample 1. The students were (obviously) lacking of prior conceptual referents for rest energy, mass invariance (except for relativistic mass); energy increase at constant speed.

- A good understanding and mastery of the *relation between mass and rest energy* was achieved;
- All of students reply that mass is invariant, but 18/23 are not able to properly explain why. So a *weak understanding* was found, in agreement with the low learning gain value ($g_2 = 0.22$);
- The concept of mass conservation under acceleration was *learnt* in 6/23 cases only. Further, the understanding of the reason for mass invariance is positively correlated to the idea that mass does not vary under acceleration: the former paved the way to the grasping of the latter idea or vice versa (unlikely);
- As concerns energy increase, the answers are biased but a quite good *understanding* was detected after that a hint towards mass was furnished. Energy increase was also associated to momentum variation. It is perhaps necessary to stress more the difference between $E = \gamma mc^2$ and $E = pc$.
- A very good *learning* was gained on mass non-additivity. In particular, the inelastic collision TE proved effective in this context.
- Eventually, from correlation tests it turned out that relativistic mass was associated to a misconception and a correct answer before instruction.

As regards the profiles, my hypothetical explanation for students' behavior is that *once they have acquired the new knowledge, they think it is no more necessary to work on it*. Once again, this behavior mirrors the traditional teaching, made of frontal lesson and enrichment mechanisms to add up new information in a cumulative way. The students' interest and involvement in the afforded topics seems to have sharply decreased.

Sample 2. The worked-out teaching/learning path proved to be overall effective for the examined class: *conceptual change* for kinetic energy and vacuum light speed occurred; *understanding* of time dilation was detected in 12/20 answers *ex-post.* However, the students displayed globally to be good at reasoning and hypothesizing working mechanism about physical processes and concepts they had never been taught before, as well as at correctly describing processes never examined, but they tended to give up their explicative reasoning and divergent thinking (higher mental functions) after the topic explanation.

VII.4. Classroom experiment (last year) – Crotone

Another formative intervention module was carried out in the Liceo "Filolao" specializing in scientific studies in Crotone, as deepening offered to students for their A-levels final composition. The offered modules were t/l paths elaborated by the Physics Education Research Unit (PERU) of Udine. The whole thing was part of an Italian joint project for scientific literacy[119] in which the physics teachers of the classes had been involved.

My activities were sub-modules taken both from the classical and relativistic modules of the t/l path, according to table VII.2. Thirty-six 18 – 19 years old students took part to my module; they had not been previously selected and their marks were comparable with the regional average, so the sample was randomized.

The data analysis relative to the performed *classical module activities* (table VII.2) is reported here. As regard this path section, the working method consisted in the analysis of excerpts from historical texts and classroom discussions on them in the form of Large Group Discussion, using an interactive/dialogic approach (Mortimer & Scott, 2003; Cacciamani & Giannandrea, 2004). That method proved effective in involving pupils actively. The cognitive inquiry was carried out using a tutorial made of worksheets whose inner questions follows (the original sheets are in appendix 3).

VII.4.1. Administered Questions

Q1. « Which idea of mass stems from the contributions above? »

Q2. « Is that idea of mass complete? Justify your answer».

Q3. « Is there a difference between mass in gravitation and this mass [*by Mach, Ed.*]? Explain».

Q4. « Which dynamical laws does the relation $\frac{m_2}{m_1}=\frac{a_1}{a_2} \Rightarrow m_2=\frac{a_1}{a_2}m_1$ involved in Mach's discussion imply? »

Q5. (small groups work) « What are ultimately the conceptual differences among the notions of mass examined so far? »

Q6. « Mass is conserved in physics. Illustrate what this law entails for interactions and physical-chemical transformations, as for the measure of mass. »

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[119] It is called PLS (*Progetto Lauree Scientifiche*) and patronized by Ministry of Education.

VII.4.2. Data Analysis and Results

NAME	DESCRIPTION	*f*
QUANTITAS MATERIAE / W	Mass as a quantity given by $\rho \cdot V$, proportional to weight and measurable through it	19/36
GRAVITY	Focus on gravitational interaction only or, more generally, on gravitational facet of mass	11/36
PARAPHRASE	Summary of *Principia* / of the worksheet text	4/36
	NA	2/36

In addition, 4 students showed they have not understood the excerpts, in particular if the "primitive" quantity is ρ or V.

NAME	DESCRIPTION	*f*
MISSING m_i	Inertial facet of mass is missing	15/36
ONLY m_i	Only inertial mass is in quotations	4/36
OTHER	Not pertaining/ incomprehensible answers	2/36
	NA	15/36

Tables VII.15 and VII.16. Mutually exclusive categories for the answers to Q1 and Q2.

NAME	DESCRIPTION	*f*
PRIMITIVE QUANTITY	The difference stands in "primitive" quantity according to Mach or Newton (comparison)	15/36
VICIOUS CIRCLE	The vicious circle is highlighted	4/36
M ↔ GRAVITY	Mass is related more or less correctly to the gravitational force / "attractive force" / "gravitational field constant"	3/36
MACH: $\rho \neq \frac{m}{V}$	"In Mach density is not given by mass per unit volume"	1/36
	NA	13/36

NAME	*f*
Second law of dynamics	1/36
Third law of dynamics	5/36
Second and Third	28/36
Second or Third	1/36
NA	1/36

Tables VII.17 and VII.18. Mutually exclusive categories for the answers to the Q3 and Q4.

NAME	DESCRIPTION	*f*	DESCRIPTION	*f*
MACH (m_i) and NEWTON (*quantitas materiae*)	Quantity of matter and Mach's inertial mass explained by incomplete linguistic expressions, sometimes in disagreement with the consensus view.	15/36	Defining Mach's inertial mass through acceleration	12/36
			Ascribing to inertial mass the role of "cause of acceleration"	3/36
MACH (m_i) vs NEWTON (W)	Pragmatic-experimental character of Mach's approach – based on acceleration – *versus* the theoretical-definitional approach of Newton (drawing on weight in students' perspectives)	7/36		
Quantitas materiae vs DENSITY	*Quantitas materiae versus* density (defined by ρ = m/V)	4/36		
INTERACTION	Reference to "interaction between bodies"	1/36		
CRITICISM	Exposition of Mach's criticism to Newton	3/36		
	NA	6/36		

Table VII.19. Mutually exclusive categories for the answers to Q5. Two students have noticed in their answers (belonging to the first category) the *absence of mass in Relativity*.

The relevant concepts have been therefore grasped in 23/36 replies to the last question of the list above, even though in largely different ways.

The results by Mullet and Gervais (1990) have been taken into account at this point. In brief, a sample of 13-15 years-old students were found to own an intuitive concept of *weight* as $W = f(\rho, V, gravitation)$, activated by the terms "mass" and "weight" without distinction. The intuitive concept of *mass* as $m = f(\rho, V)$ is not lacking: it is triggered by the phrase "quantity of matter". Please refer to V.1.1 for details.

The following issue was considered: if the students having made reference to quantity of matter in Q1 – sometimes next to a concept related to gravity – mentioned either the gravitational facet of mass or gravity (directly or indirectly) in the answers to Q5. The outcome is depicted in the two-column histogram below.

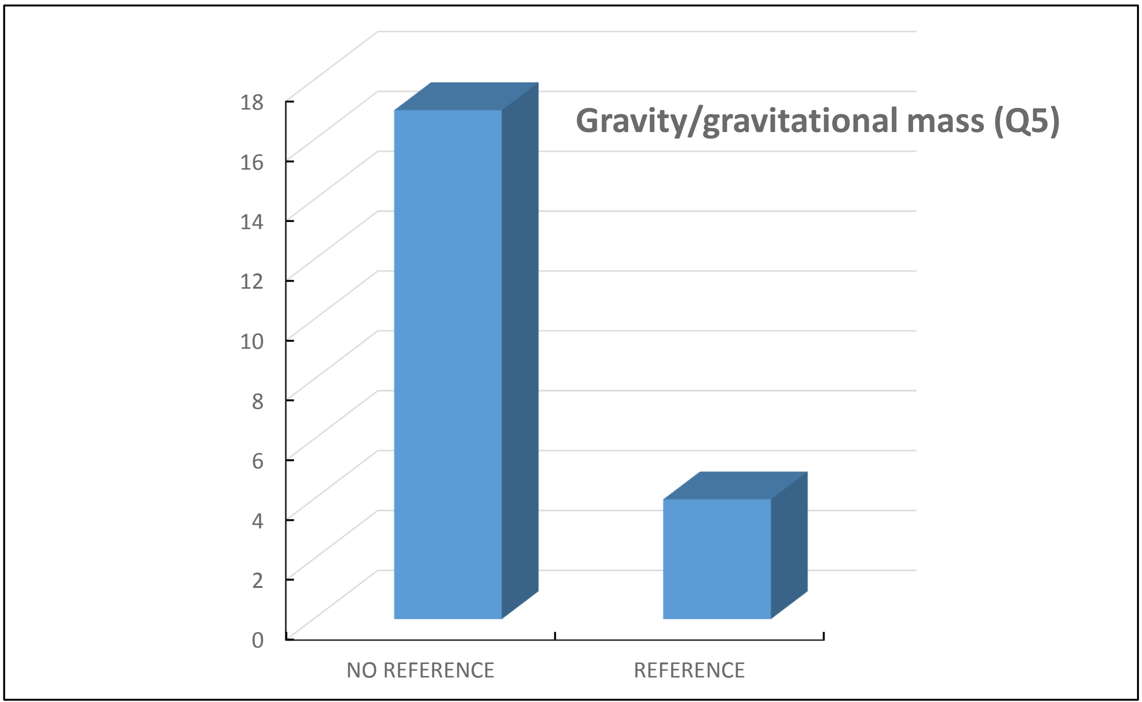


Figure VII.23. Number of students who referred or not referred to gravity/gravitational mass in the answers to the group question (Q5) *after having mentioned quantity of matter* in the answers to the first question (Q1); 21/36 made this mention when replying to Q1.

Results from question 6:

- (“additivity”) 3/36 students associate conservation to additivity of mass;
- (“transformations”) 5/36 students mention some of the chemical-physical transformations under which mass / mass measure / mass sum remains unchanged, namely spacetime translations, deformations, breakings, changes of state, solutions, and redox according to the IPS Group (1967);
- (“initial = final”) 3/36 highlight that the mass measure in the initial state is equal to the one in the final state;
- (“constancy”) 17/36 merely *assert* that mass / mass measure remains unvaried;
- (“copy/contradiction”) 5/36 merely rewrite or even contradict the question text.

Finally, it follows from data analysis that a lot of students *did not succeed in overtaking the level of simple description* in their answers, thus matching the descriptive phenomenographic profile: these replies are based on *pieces of superficial factual knowledge*.

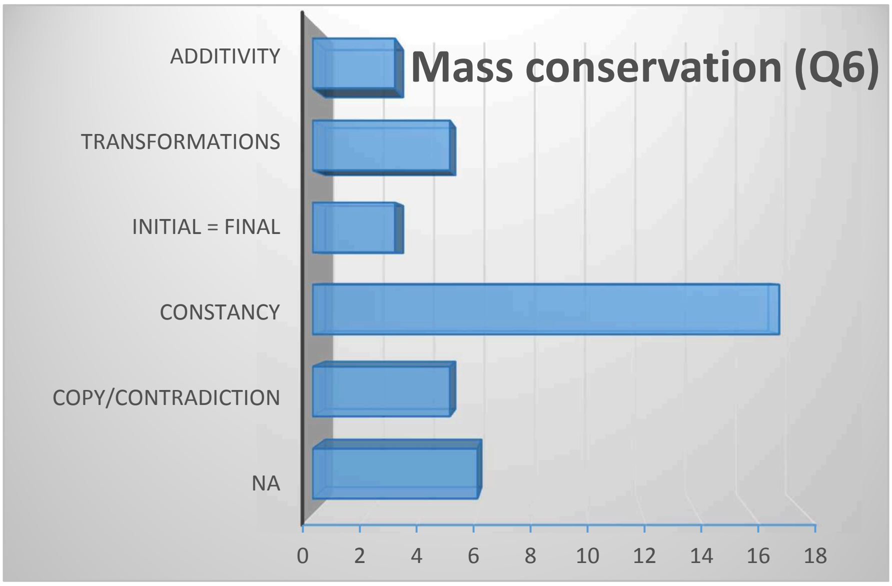


Figure VII.24. Answer categories for question 6. The categories were formed in such a way that they cannot be mutually exclusive: each student may hold two or more representations at once.

VII.4.3. **Discussion**

The collected data allow to discuss and draw some conclusions concerning two RQs formulated in VI.1.1 as a result of the prior educational literature examination:

RQ2C) « Which concept was actually activated through students' reflections and revisions of the classical aspects of mass, according to the research outcomes by Mullet and Gervais (1990)? Weight or mass? »

RQ6C) « How do students conceptualize mass conservation law in the six most important typologies of chemical-physics transformations? ».

Corresponding discussion:

RQ2C) The results of a comparison analysis between Q1 and Q5 ("group question") corroborate what Mullet and Gervais (1990) had found, as shown in figure VII.23: "quantity of matter" triggered the intuitive concept of mass, which *is not* a function of gravity. Ninety-one percent (33/36) of the sample did not mention gravity and/or the gravitational facets of mass when replying to Q5, characterizing it as $m = f(\rho, V)$; this facet also prevailed in the answers to Q1.

RQ6C) Mass conservation was stated by most students (17/36) in mere assertions, in which conservativeness and constancy overlap. The others – whose replies are of interest here – either expounded the phenomenology in which mass conservation is observable (5/36) or described it in terms of additivity (3/36), as well as considered mass as a state variable (3/36).

VII.4.4. **Conclusions**

- Conceptual reworking was detected only in a few replies. This was perhaps due to the difficulty of the rationale and to the innovative strategy.
- If the previous argument is reversed and Mullet and Gervais's conclusions assumed, it may be asserted that the *intuitive concept of mass was activated* by the implemented formative experiment.
- The objective of making students explain *how mass conservation occurs* was not reached: they did not go beyond the descriptive profile. However, a few interesting results came out, especially *the link between conservativeness and additivity* (3/36).

VII.5. Classroom experiment (last year) – Treviso

A further formative intervention experiment was carried out in Treviso, as part of the same Italian joint project for scientific literacy (PLS) including the experiment in Crotone. So the physics teachers of the two classes had been previously involved, and they had 'run' the light clock and two-photon emission (Einstein, 1905a) thought experiments with the aim of dealing extra-curricular special relativity topics at school. Information on the formative experiment may be found in table VII.2, as usual. I just recall here that the sample was made by 27 students; further, both *real* both *modelled phenomena* were presented and *discussion on them* stimulated – using data from β-decays and the "photon in a box TE" respectively. For dealing with the latter, a brief prior explanation of the Compton Effect in terms of momentum and energy conservation was necessary.

VII.5.1. Administered Questions

The initial test was composed by nine questions, while the final by four (appendix 1). Since the tests have two question in common, they were compared.

Pre-test:

Q1.« A spontaneous process called "β decay" occurs spontaneously in nature: $n \rightarrow p + e^{-} + \bar{\nu_e}$. The neutron can give rise to ("decay in") proton, electron and anti-neutrino. Leave out the last particle, whose mass is negligible for our purposes, and calculate the masses in initial and final states. m_n = 939.57 MeV/c^2 (initial state); m_p + m_e (+ m_ν) = 938.28 + 0.511 = 938.79 MeV/c^2 (final state). How do you interpret the mass variation undergone by the physical system?»

Q2.« As you learned from the 2-photons thought experiment by Einstein, the energy of a body at rest is related to its mass. In which way? »;

Q3.« What is the meaning of Einstein's equation $\Delta E = (\Delta m)\, c^2$? »;

Q4.« Consider an electrically charged body at rest in the laboratory, on which external electrical forces do work. This body will be accelerated and its energy will increase. Is it possible to assert that its mass will increase as well? Explain. »

Q5.« The following nuclear reaction occurs in nature (*fission*) [*omissis*]. The produced neutrons own a kinetic energy. Because of energy conservation principle, energy of nuclei will necessarily decrease. Is it possible to assert that system mass will also decrease? Justify. »

Q6.« How could the kinetic energy of a body be defined, both in classical and relativistic contexts? »

Q9. « On the basis of the developed path, are you able to suppose that light displays inertial properties in the interaction with a physical system? »

The questions Q7 and Q8 were not considered because the vast majority of students did not answer them.

Post-test:

Q'1. « A spontaneous process called "β decay" occurs spontaneously in nature: $n \rightarrow p + e^- + \overline{\nu_e}$. The neutron can give rise to ("decay in") proton, electron and anti-neutrino. Leave out the last particle, whose mass is negligible for our purposes, and calculate the masses in initial and final states. $m_n = 939.57$ MeV/c^2 (initial state); $m_p + m_e$ ($+ m_\nu$) $= 938.28 + 0.511 = 938.79$ MeV/c^2 (final state). How do you interpret the mass variation undergone by the physical system? »

Q'2. « On the basis of the developed path, are you able to suppose that light displays inertial properties in the interaction with a physical system? »

Q'3. « With regard to the two-photon experiment by Einstein, how is the momentum variation transmitted by the photon on the box correlated to the box constant acceleration? »

Q'4. « Why is it necessary to apply a greater force to the box with the photon inside in order to keep the same acceleration than the empty box? »

VII.5.2. **Data: Analysis and Results**

As for Q1, the expected link between explored phenomenology and mass-energy equivalence was found in 23/27 answers, although sometimes not directly (figure 25). The situation improved after the path, with the stemming of a new category. Nevertheless, mass was not related at all to energy in 4/27 answers.

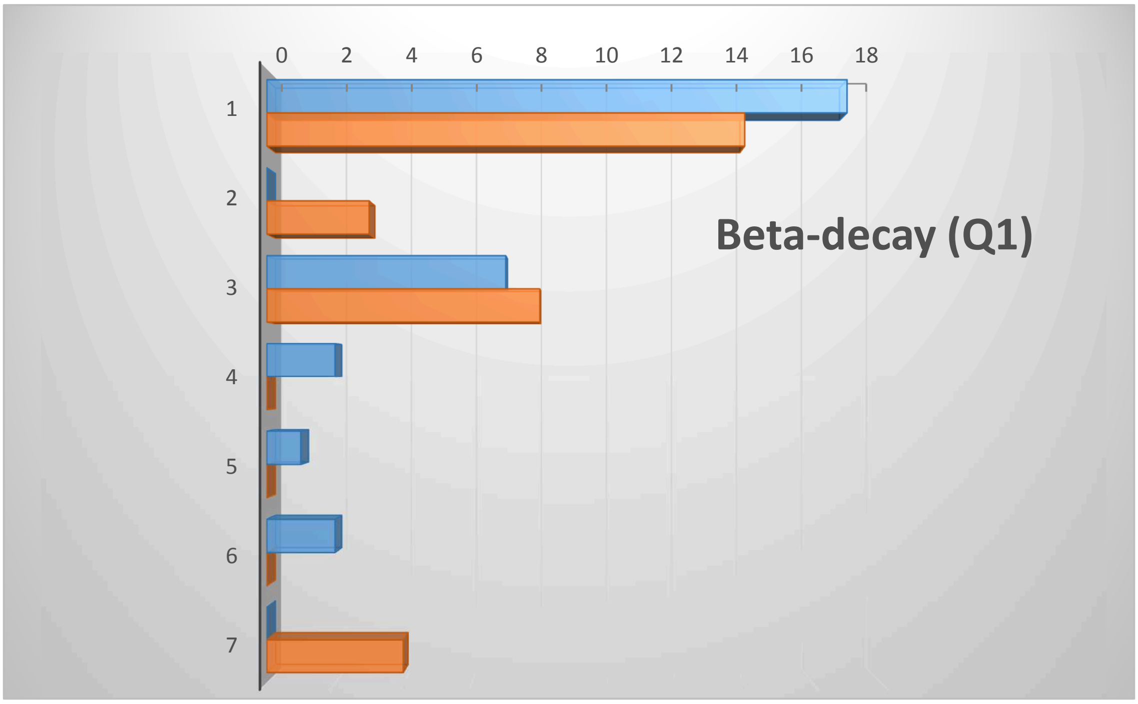


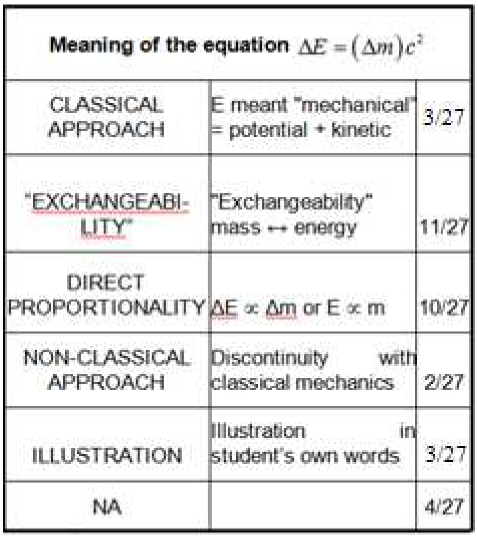
Figure VII.25. Outcomes from the compared analysis of Q1 (up) and post-test (down). The names of the not-mutually exclusive categories are in the legend below.

1. Mass variation due to its "conversion"/ "exchange" in energy;
2. Mass: measure of the energy "possessed" by a body;
3. Variation of mass because energy was released (before)/ products have K.E. (after);
4. Production of energy due to modification of the motion or of its direction;
5. Descriptions only (no explanations);
6. NA;
7. Mass variation due to $m_{measured}$ - $\Sigma m_{elements}$: "mass excess".

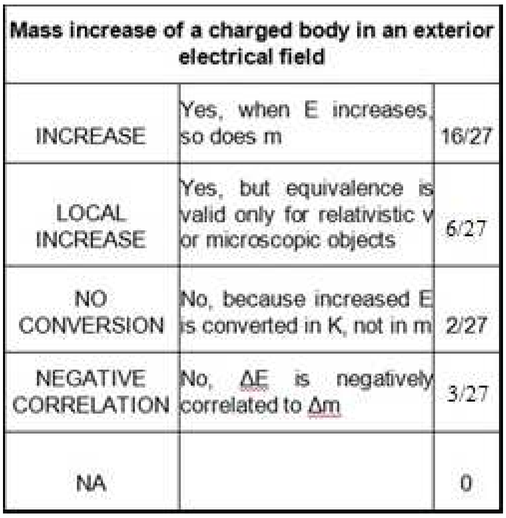
The expected link between explored phenomenology and mass-energy equivalence was found in 23/27 answers, although sometimes not directly. One negative point to underline: mass was not related to energy in 4/27 answers, perhaps owing to the phrase “mass excess”, similar to “mass defect”.

Link between rest energy and mass (Q2)	*f*
(Rest) mass is related to the energy of a body/ to emitted «*electromagnetic radiation*»	7/27
(Rest) mass is related to the internal energy/« *rest energy*» of a body	4/27
Energy variations are related to mass variations	2/27
NA	14/27

Table VII.20: 2nd question of pre-test (two-photon emission TE).

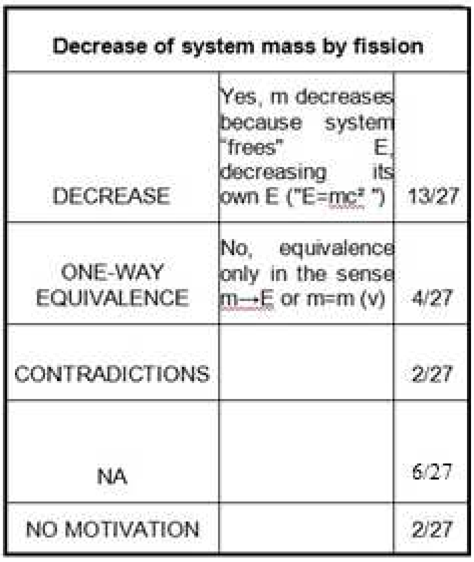


Meaning of the equation $\Delta E = (\Delta m)c^2$		
CLASSICAL APPROACH	E meant "mechanical" = potential + kinetic	3/27
"EXCHANGEABILITY"	"Exchangeability" mass ↔ energy	11/27
DIRECT PROPORTIONALITY	$\Delta E \propto \Delta m$ or $E \propto m$	10/27
NON-CLASSICAL APPROACH	Discontinuity with classical mechanics	2/27
ILLUSTRATION	Illustration in student's own words	3/27
NA		4/27

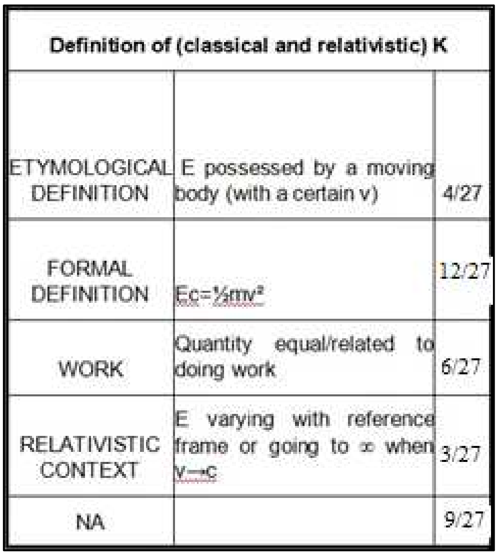


Mass increase of a charged body in an exterior electrical field		
INCREASE	Yes, when E increases, so does m	16/27
LOCAL INCREASE	Yes, but equivalence is valid only for relativistic v or microscopic objects	6/27
NO CONVERSION	No, because increased E is converted in K, not in m	2/27
NEGATIVE CORRELATION	No, ΔE is negatively correlated to Δm	3/27
NA		0

Tables VII.21 and VII.22: label, category definition and frequency (3rd and 4th questions of pre-test). In the replies to the Q3 owing to the third category ("direct proportionality energy") energy is also called «*radiation*», «*energy under the form of electromagnetic waves*», «*mechanical (potential and kinetic) energy*». Besides, 5/27 students explain Δ through a variation between states.



Decrease of system mass by fission		
DECREASE	Yes, m decreases because system "frees" E, decreasing its own E ("$E=mc^2$ ")	13/27
ONE-WAY EQUIVALENCE	No, equivalence only in the sense $m \to E$ or $m=m(v)$	4/27
CONTRADICTIONS		2/27
NA		6/27
NO MOTIVATION		2/27



Definition of (classical and relativistic) K		
ETYMOLOGICAL DEFINITION	E possessed by a moving body (with a certain v)	4/27
FORMAL DEFINITION	$E_c=\frac{1}{2}mv^2$	12/27
WORK	Quantity equal/related to doing work	6/27
RELATIVISTIC CONTEXT	E varying with reference frame or going to ∞ when $v \to c$	3/27
NA		9/27

Tables VII.23 and VII.24: label, category definition and frequency (5th and 6th questions of the pre-test).

It is easily seen that the students did not go beyond a simple description of Einstein's equation when answering to Q3. They did not give any example of physical phenomena being explained by it, so the *explicative* level was not reached.

Finally, it is pointed out that "no" answer frequency to Q4 (concerning mass invariance) is *significantly lower* than "yes" as regards Q5 (concerning mass non-conservation), both being scientifically correct in line of principle: 5/27 against 13/27.

Light clock Thought Experiment – outcomes

Light-clock TE	*f*
Light speed is an universal constant	10/27
Light speed does not vary when IF varies	0
Durations vary when IFs varies	11/27
Lengths do not vary when IFs varies	0
NA	5/27

Table VII.25: Non-mutually exclusive categories of answers to Q8: « You worked on an applet with a light clock. What may it be inferred from that thought experiment? »

"Photon in a box" Thought Experiment – outcomes

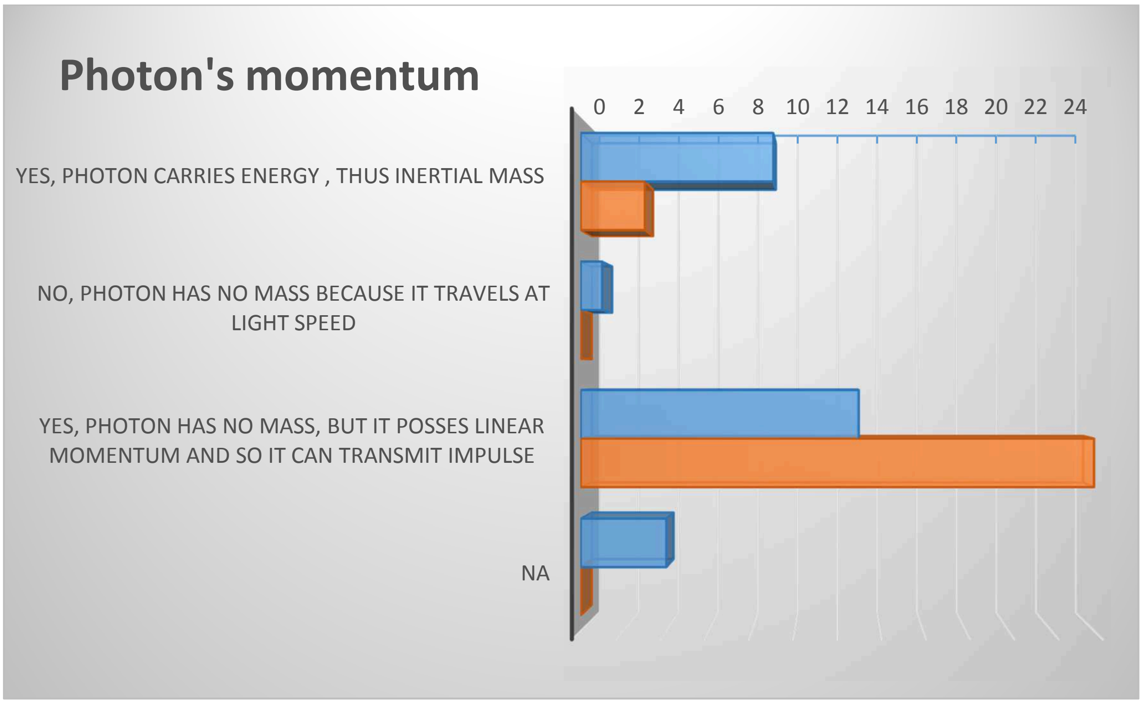


Figure VII.26. Comparison between the answers to Q9 (blue) and (identical) Q'2 (orange).

Why is it necessary to apply a greater *F* in order to keep the same *a*?	*f*
The student mentions properties/ inertial "behaviour" of the photon	7/27
Mentions (inertial) photon mass or a generic mass increase	7/27
Considers the effect due only to $h\nu/c \neq 0$	6/27
NA	7/27

How is transmitted impulse linked to box constant acceleration?	*f*
The student doesn't put any part of the logic deduction	8/27
The student inserts the II part only	3/27
The student inserts the entire deduction, but mixing up ideas in a wrong way	2/27
The student inserts the final formula only	3/27
NA	11/27

Tables VII.26 and VII.27. Answer categories to Q’3 and Q’4.

Category Name	**Category Description**	*f*
SUMMARY	The results and argumentations proposed in the t/l pathway are merely reported	7/27
ORIGINAL REASONINGS	The TE is used to introduce independent or different kinds of reasoning	12/27
"MANIFESTING EXISTING KNOWLEDGE”	The TE is used to strengthen and specify well-known notions	6/27
"INVENTING NEW KNOWLEDGE”	The TE is used for gaining new conceptions/concepts related to photon	12/27

Table VII.28. Use of “photon in a box” TE made by students. The categories are not mutually exclusive.

VII.5.3. **Discussion**

Beta-decays, light-clock and “photon in a box” TEs were the tested part of the t/l path. The data analysis was driven by the following content-oriented questions, written in section VI.1.2:

RQ6R) « How do students interpret and justify phenomena in which mass is not conserved? »;

RQ10R) « Are thought experiments effective in relativistic context? »

RQ11R) « Do they generate accommodation or assimilation? » More specifically:

a. « To what extent does the implementation of the light-clock TE allow students to recognize time dilation effect and its explanation? »
b. « What is the contribution of the “photon in a box” TE for learning qualitatively and quantitatively that a massless object shows inertial effect due to its energy inside a system? »

VII.5.3.1. *RQ6R*

« How do students interpret and justify phenomena in which mass is not conserved when put in the face of them? »

Sixty-three percent of the sample (17/27 students) replied that mass variation was due to its “*conversion*”/“*transformation*” in energy in the initial test. This is a widespread interpretation which does not grasp Einstein’s equivalence original meaning. Those students who replied that initial and final mass are different since “*energy was released in the process*” (7/27) are closer to Einstein actual meaning. Since the two categories had only one answer in common, that answer has to be subtracted from the sum. The intended reference to a correlation between mass variation and energy variation was therefore in 85% of the answers (23/27): a *good result.* The situation improved after the path for the reasons (i), (ii), and (iii) above. In the remaining 4/27 replies mass is not related to energy at all, very likely because of a distorted or not well contextualized understanding of the “*mass excess*” concept, to which these answers refer. These students remained anchored to the mass concept since it was the quantity under examination, without relating it to energy. So, even if they stated the existence of a relation between the variations of the two quantities, they did not show an actual mastery of them. As these pupils meant “*mass excess*” as a cause of the atomic mass variation, it is required to be more clear and precise in the explanation, through more emphasis on the role of released energy above all. Finally, tables VII.22 and VII.23 indicate that the students did not conceptually distinguish between mass invariance and conservation. The presence of the idea that mass increases whenever energy increases was also revealed, which is nothing but “relativistic mass” $m_r = E/c^2$. Once again, no causal or correlation explanation may be hypothesized, but it is a further hint of a concurrence between “relativistic mass” and learning disturbances. The gap between the correct answers to question 4 and 5 (2/27 against 13/27, see table VII. 22 and VII. 23) is likely due to the text of former joint to students’ lack in conceptualizing the absence of relationship between mass and kinetic energy of a macroscopic object. In fact, a generic increase of energy was referred to in the question text, as a consequence of the acceleration transmitted by electrical forces. It was then asked if it were possible to « assert that its mass will increase as well». Put it that way, a student will easily tend to think that mass *will* decrease, since they are equivalent and energy will. Actually, only kinetic energy will increase, while mass is equivalent to rest energy. So understanding of this crucial distinction appears *to not have been gained* and more stress on it is needed in instruction.

VII.5.3.2. *RQ10R and RQ11R*

« Are thought experiments effective in relativistic context? Do they produce accommodation or assimilation? »;

« To what extent does the implementation of the light-clock TE allow students to recognize time dilation effect and its explanation? »

« What is the contribution of the “photon in a box” T.E. for learning qualitatively and quantitatively that a massless object shows inertial effects due to its energy inside a physical system? »

The pre-test questions 7 and 8 concern another thought experiment: the educational adaptation of Einstein’s 1905 two-photon T.E. It had been previously dealt with by the physics teachers of the classes.

The learning outcomes concerning TEs are described in the following.

1. ***Two-photon emission***. 89% of the sample did not answer to Q7 at all. 78% of the answers to Q6 did not indicate an effective understanding of the experiment,

which involved the meaning of K.E. and the classical limit for the expression of relativistic K.E. *This TE turned out thus to be too difficult as for contents and exposition modality too,* in that class at least. The students did not well know what to do with the TE outcomes, and the 3/27 who answered limited themselves to *take note of them.* Those outcomes were considered by these pupils as "external" to themselves because somehow intrinsic to Nature: *an assimilation by enrichment mechanism occurred.* By the way, slightly better results were obtained through the third question, more general and formal. This indicates again the trend of reasoning in formal way in Italian schools.

2. ***Light clock***. 15/27 pupils gave correct and rightly motivated answers. The others chose incorrect alternatives (it was a multiple-choice question) very likely because conditioned by *meta-phenomenological considerations* on light speed, durations and lengths *induced by the question text* (see table VII. 25 and appendix 1 for further details). Furthermore, they gave incorrect explanations for their choices because they failed to acquire the novel reasoning modality proposed by the TE. I made this hypothesis since the experiment is conceptually simple.
3. ***Photon in a box***. The implementation of this TE allowed to deal with matter-radiation interaction and passed on two fundamental concepts: the massless light quantum gives rise to inertial effects (7/27 students, see table VII.26) *or* it has finite linear momentum (6/27 different students). A change in students' ways of relating to these phenomena thus occurred. In conclusion, *this TE proved a fairly effective educational tool* in the sense specified above and limited to those contents. In fact, the results depicted in table VII.16 show that it was mainly used either as *discovery tool* or as instrument for *a personal exposure of its outcomes by the students*.

VII.5.4. **Conclusions**

Generally speaking, the involved students showed a number of difficulties in consistently mastering the new concepts. However, positive learning outcomes were also found.

The presence of a «relativistic noise» (Pietrocola & Zylbersztajn, 1999) – disturbing an actual and complete understanding of mass-energy equivalence, as well as the part of relativistic mechanics in which it is inserted – comes out from data analysis. The widespread and repeated use of phrases or sentences like "*mass is the quantity of energy contained in a body*" / "*possessed by a body*", "*mass is related to the energy of a body*", "*exchangeability between mass and energy*" indicates that a scant knowledge of Relativity has been acquired (at school or from popularization books), which interferes with students' interpretative structures. The reason is that these pieces of information have been learned out of their context, thus pupils' «knowledge about Relativity does not offer an operative base for reasoning» (Pietrocola & Zylbersztajn, 1999). In other terms, the learners' relativistic shattered school knowledge creates «inert knowledge structures» (Whitehead, 1929; Michelini, 2005) and, moreover, misunderstandings comes from popularization books, sometimes containing even wrong information. The next step after this interference is usually the formation of a misconception, which nevertheless is not observable here.

What happens may be seen from a diverse perspective too: in the timescale of our formative experiment (about 2.5 hours) several students seemed to pass from an initial, lay model – although proper terms were being used – to a «synthetic model» (Vosniadou *et al.*, 2008), in which elements of the naïve conceptions and of the scientific concepts merged. In particular, one conception emerged after β-decays and "photon in a box" T.E. which might develop toward the consensus view. Mass variation is related to a generic "*conversion*"/ "*exchange*" or release of energy at the beginning, while after instruction (i) it is connected to a variation of kinetic energy of decay products by 8/27 students, (ii) the references to a generic ΔE decrease, and, above all, (iii) mass is said to be "*the measure of the quantity of energy owned by a body*" by 3/27 students: something recalling rest energy. It can be seen indeed in the distribution of figure VII.25 that the study of β-decay phenomenology makes plausible the existence of a new kind of energy intimately related to mass, even if for very few students. Moreover, the point (i) above entails the study of β-decays (and fission) phenomenology helps pupils in understanding that speaking of "kinetic energy" of products is more correct than using the generic term "nuclear energy" associated to mass variation, as it usually occurs.

VII.6. 2012 winter school

The present formative experiment consisted in a tutorial of about 4 hours as a part of the Relativity course scheduled in the winter campus [46] held in Bard (AO) from the 14th to 16th of December 2012. The talents had to choose among three different mandatory courses: Relativity, Astrophysics and Game Theory, held by University teachers and/or researchers on extra-curricular innovative topics. Expert secondary teachers and professionals in science popularization also contributed; several additional seminars for deepening were proposed. The target schooling level went from the fourth year of high school to the second University year. Differently from the previous case, however, *no prior selection had been done according to school merits*. So the students will be referred to as simply "students" or "pupils" herinafter.

My tutorial was made of 9 IBL worksheets on *relative motions and light speed* (3 sheets), *simultaneity in SR* (3 sheets), *kinematical effects in SR* (3 sheets). Almost all questions were open-ended, except for the second to last question (Q17) on length contraction. Twenty-two students filled in the worksheets[120] during the learning process, so that only the instant learning was probed. They also discussed all together if light speed *in vacuo* were relative to the reference frame and why.

This path section was meant for building up a t/l activity which allowed to answer the following research questions *a priori* (formulated in VI.1.2):

RQ1R) Do students recognize the role of c in Relativity, as for invariance and ultimate speed character? How do they express their conceptions?

RQ2R) Do students acknowledge that simultaneity is not invariant in SR? If so, how do they express the grasped concept?

RQ3R) Are time dilation and length contraction effects understood by students, or described as "distortion of perception" effect, or not achieved at all? In the first and second cases, how do students characterize them?

VII.6.1. Administered Questions

Q1. « In a swimming relay race a sound is produced underneath the water, when the first swimmers have already left. The sound wave speed with respect to the water at 20° C is 1482 m/s. a) What is the sound wave speed according to an athlete

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[120] The complete tutorial may be found in appendix 3, pagg. xlix – lviii.

swimming steadily at about 2 m/s with respect to the water? Explain. b) What is the sound wave speed for another still athlete waiting for leaving? Answer and justify. 2) Consider a hypothetical athletics competition (4 x 100 relay) *on the Moon*, where there is no atmosphere. One of the runners of the last distance brings a small torch turned on (A); a technician still in the runner's lane is turning on a second torch (B) while the last group is leaving. What speed will light from torch A propagate at, with respect to the athlete? What speed will light from source B propagate at, with respect to the technician? »

Q2. « Suppose A is in the Carthesian axis origin of a *reference frame* specified by *S*. Suppose too that B is in the axis origin of a *reference frame* specified by *S'*. If the source A is moving of rectilinear uniform motion towards B at velocity **v** with the same direction of the axis *x'*, what will the speed of a light ray emitted by A towards B be according to an observer in *S'*? Justify. »

Q3. « What are the possible answers to the previous question, emerged from the discussion? What hypotheses do they originate from? »

Q4. « There are two observers, A on the platform of a station, B halfway in a coach of a moving train. There are two light sources, corresponding to the coach front and back, turning on when they touch two *firing devices* on the rails. We stand at rest with respect to A, who receives two rays from the sources at once. Did the sources turn on at once? »

Q5. « In which way does A (defined as an "intelligent" observer in Relativity) reconstruct if the two lighting events were simultaneous? »

Q6. « Explain why it is necessary to make these "reconstructions" and "appraisals" of the signal emission instants, and why what it is directly perceived is not enough. »

Q7. « Can causality relationship ("cause comes before effect") be inverted in the passage from a reference frame to another? Answer and justify. »

Q8. « Since light speed is the same in any propagation direction (II postulate), the wavefront emitted by source 2 arrives to B **before** the one emitted by 1. Could this information be useful to B (according to A's reconstruction) in order to assess the time ordering of the two lighting events? Why? »

Q9. « What will infer B (according to A's reconstruction again) about the two events' simultaneity? »

Q10. « Will the observer C asses a *shorter, equal or longer* time interval than the one assessed by B between the two events? Explain. »

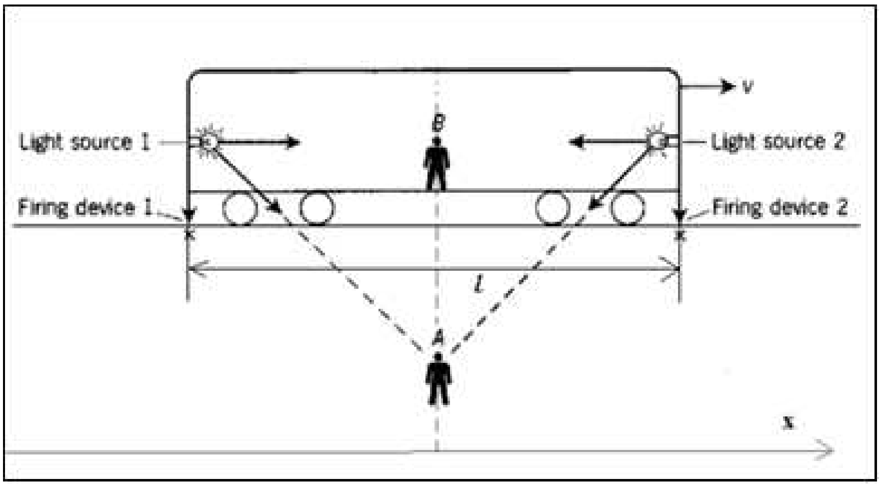


Figure VII.27. Picture of reference for the questions Q4 – Q9, concerning relativity of simultaneity, adapted from D'Inverno (1992).

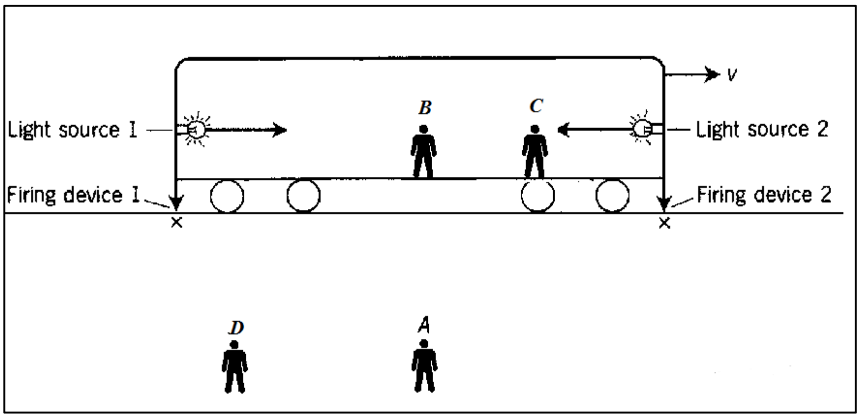
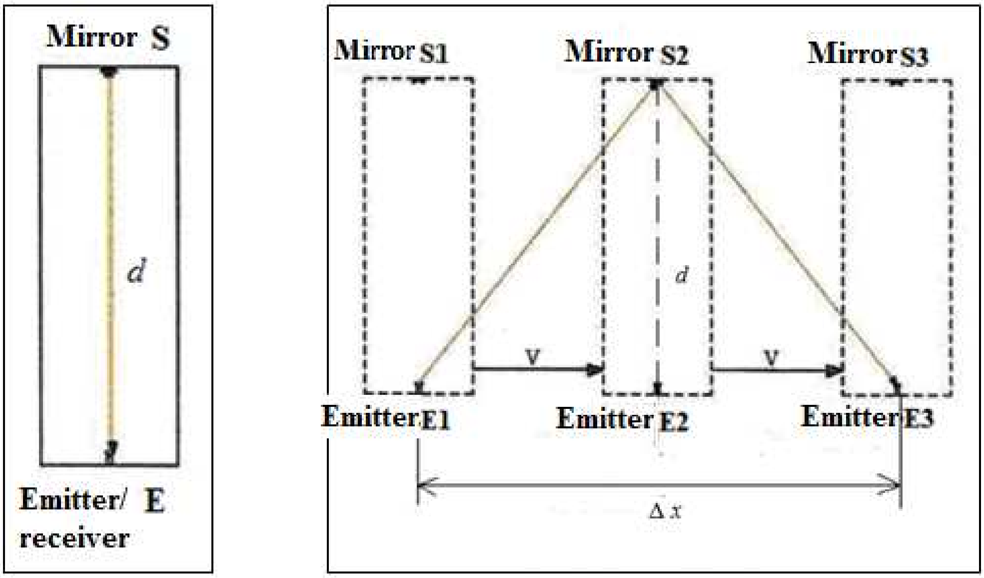


Figure VII.28. Picture of reference for Q10 (difference between observers and reference frames), adapted from D'Inverno (1992).

Q11. « A *light clock* is also in the coach, which is a device constituted by an emitter-receiver E with a mirror S at a distance *d* in orthogonal direction to the train motion. <u>*How long* does it take light for the back-and-forth path in the coach reference frame (in which B is)</u>? »

Q12. « According to the geometry in figure, is the time interval for the back-and-forth path measured by A's clock *equal* the one measured by B's clock? (Explain) »

Q13. « So we arrive to the time dilation formula (more exactly "duration dilation") rigorously and directly deducible from Lorentz transformations. »



Figures VII.29 and VII.30. Light clock at rest (as seen by the observer B, Q11) on the left and in motion (as seen by A, Q12) at three different instants on the right.

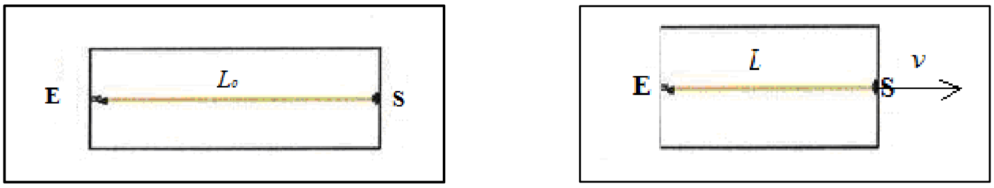
Q14. « Finally, let us discuss about the *length of a horizontal light clock* in the train. In the frame of A, the light ray completes the back-and-forth path in *more time/ less time* (multiple choice) than in B's, because of time dilation. Since light speed is invariant, the space between the walls must necessarily be _____________ than the one measured by B according to observer A, so the object ______________. »

Q15. « Quantitatively, we know that B measures as back-and-forth time ______________. »

Q16. « Let us examine what A calculates (with respect to whom the light clock moves with velocity v). »

Q17. « If the train should stop at whatever instant, the light clock length would … (multiple choice). »

Q18. « Explain (*ignore the circumstance that while arresting the train is not actually an inertial reference*). »



Figures VII.31 and VII.32. Still (left) and moving (right) horizontal light clock (Q14).

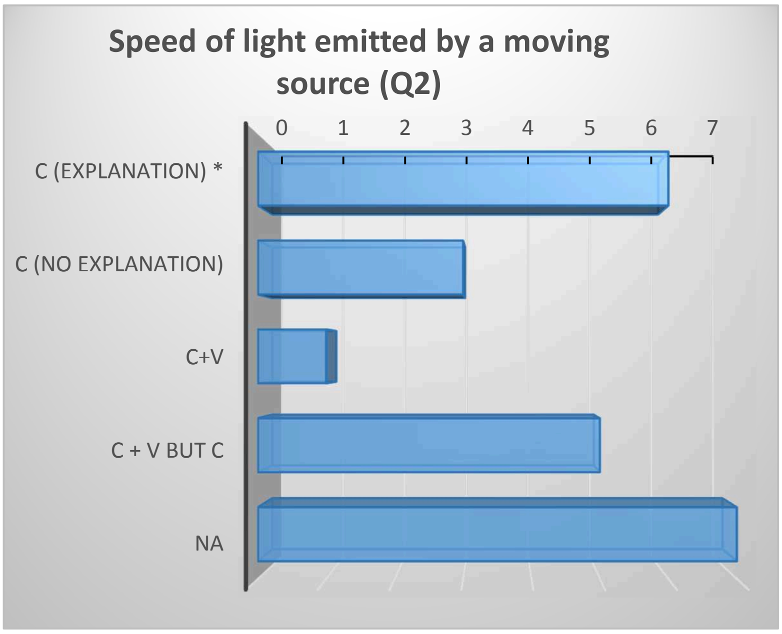
VII.6.2. **Data: Analysis and Results (categories)**

The answers to Q1 consisted in numerical results and, sometimes, a series of calculations, possibly justified. They were categorized as in the table below; to be noticed the subtraction of the sound speed in air from the sound speed in water in category C.

CATEGORY'S NAME (Q1)	**DESCRIPTION**	**OPERATIVE DEFINITION**	*f*
A) ONLY CORRECT RESULT	Only (correct) numerical result	*« 1) a) 1480 m/s, b) 1482 m/s; 2a) c, 2b) c »*	10/22
B) EXPLAINED CORRECT RESULT *	Correct result with explicit reference to the classical velocity composition law	*« 1) a) v =1480 m/s velocity difference (motion composition with the moving athlete as reference frame), b) v = 1482 m/s Speed of the wave moving in the reference frame of the still athlete; 2a) c, 2b) c »*	6/22
C) INCORRECT CALCULATIONS	Incorrect use of the classical velocity composition law	*« 1) a) 1482 m/s = v – 2 m/s, 1484 m/s = v. b) 1482 m/s - 344 m/s =…; 2a) c 2b) c»*	1/22
NA			5/22

Table VII.29. Conceptual categories for the answers to Q1 (classical velocity composition). The correct answer is marked by an asterisk.

Figure VII.33. Categories for the answers to Q2 (*c* invariance). The correct answer is marked by an asterisk.



Operative definitions of the main categories for Q2:

1) "c (explanation)": « *According to the classical mechanics laws it should be v+c, but since c is an ultimate it cannot be exceeded, so it is c* »; «*Always c anyway, which is invariant in any reference system. Since vacuum is not a medium* $(\varepsilon_0\ \mu_0)^{-1/2}$ *is an invariant* »; «*The speed of the light ray emitted by A is c because light does not travel in a medium*».
2) "c+v but c": «*Hypothetically: being an apparent velocity, B observs a value c+v. Correct answer: c*»; «$\vec{v} + c$ *since the light emitted at speed c has an initial velocity* $\vec{v}$*. It is actually c*»; «$v_S = c + v$ *in classical mechanics (c cannot actually be exceeded etc)* ».

CATEGORY'S NAME (Q3)	OPERATIVE DEFINITION	*f*
c constant *	« *c keeps constant in the different reference frames* »	1/22
c ultimate speed; c+v; (c-v) *	« *1) c → ultimate speed; 2) c + v → Newtonian mechanics (the most intuitive solution) ; 3) c - v* »; « *c → because it cannot be exceeded; c + v → because classical mechanics should work in this way; c-v* »	6/22
c (electromagnetism); c+v (classical mechanics) *	« *c+v according to the classical mechanics laws; c according to the electromagnetism laws*»; «*The answers may be c (taking account of – perhaps – the wave behaviour of light) or c+v, taking account of the source initial velocity (or inertial) which is summed to light's*»	2/22
c, c+v, c-v (no hypotheses)		3/22
NA		10/22

Table VII.30. Conceptual categories for the answers to Q3 (possible answers to Q2 and the hypotheses from which they stem). The correct answer(s) is/are marked by an asterisk.

CATEGORY'S NAME (Q4)	**DESCRIPTION**	**OPERATIVE DEFINITION**	f
A) YES: SAME DISTANCE *	Yes, because A stands in the midpoint (same distance) between the firing devices/sources	« *Yes because* $\Delta t = l/c \sim 10$ *ns, and A is at* $l/2$ $l/2$ »; « *Yes if he is halfway. If he were at different distances he could perceive different times* »; « *Yes, because he stands at the same distance from the sources* »	4/22
B) YES: SIMULTANEOUS TOUCH	Yes, because/if the firing devices are touched at once	« *Yes if the firing devices are touched at once* »; « *The sources light up at once due to the sensors which locate the train's position*»; « *Yes, because* F_2 *is at distance* $F_1 + l$ *and the equation of motion of* L_1 *is* $v*t$*, while* $L_2 = v*t + L_1$*, so it touches at the same time* »	6/22
C) NO: TIME DELAY	No, because the train goes on while light travels	« *No because the second light source lighted up before, since it is farther, because when A sees the EXACT train position, it has translated in the direction of the train* »	1/22
D) YES			2/22
E) YES BY DEF/HP		« *Yes, because the train is designed so that the distance of the light sources is equal to that of the firing devices »;* « *Yes, the sources light up at once by definition*»	5/22
A) + B)		« *Yes, because the two firing devices are touched at once, because A is exactly halfway between the two light sources and because light always travels at the same speed* »	2/22
NA			2/22

Table VII.31. Conceptual categories for the answers to Q4 (simultaneity of the lighting events for A). The correct answer is marked by an asterisk.

Question 5 is about the reconstruction of the lighting events by observer A. Operative definitions of the main categories:

A. "Geometrical) simmetry": « *Being placed just halfway, the observer A knows that light has to cover the same distance* »; « *He reconstructs by knowing that* $\Delta t = l/c$ *and by knowing his DISTANCE d* »; « $d_1 = d_2$ ».

B. "$\Delta t = 0$": « *He reconstructs if the two lighting events were simultaneous if he perceives them at once* »; « $\Delta t = 0$, *he perceives that difference between the different turn-on times is 0* »; « *Because he knows that* $\Delta\tau = 0$ ».

C. "Time calculation (v = s/t)": « *By performing calculations relative to* $v = s/t$ »; « *If they arrive to his eyes at the same instant or, if he is not halfway, by calculating the actual taken time* ».

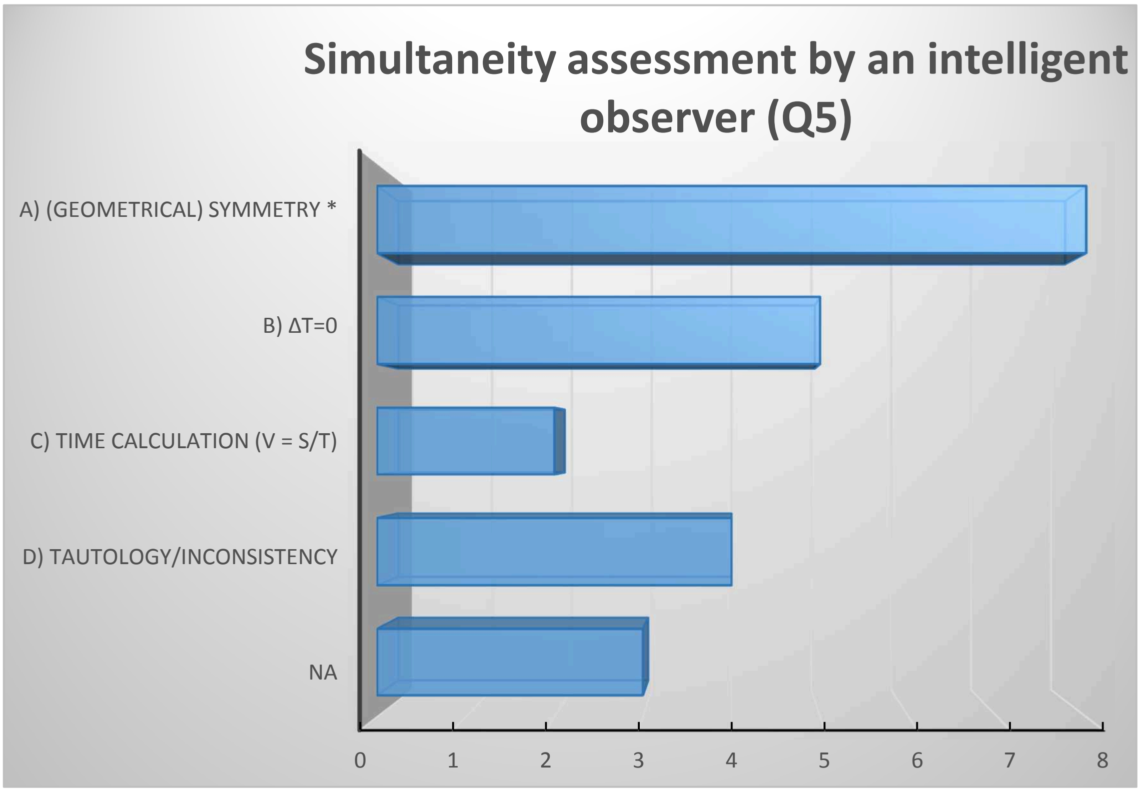


Figure VII.34. Categories for the answers to Q5. The correct answer is marked by an asterisk.

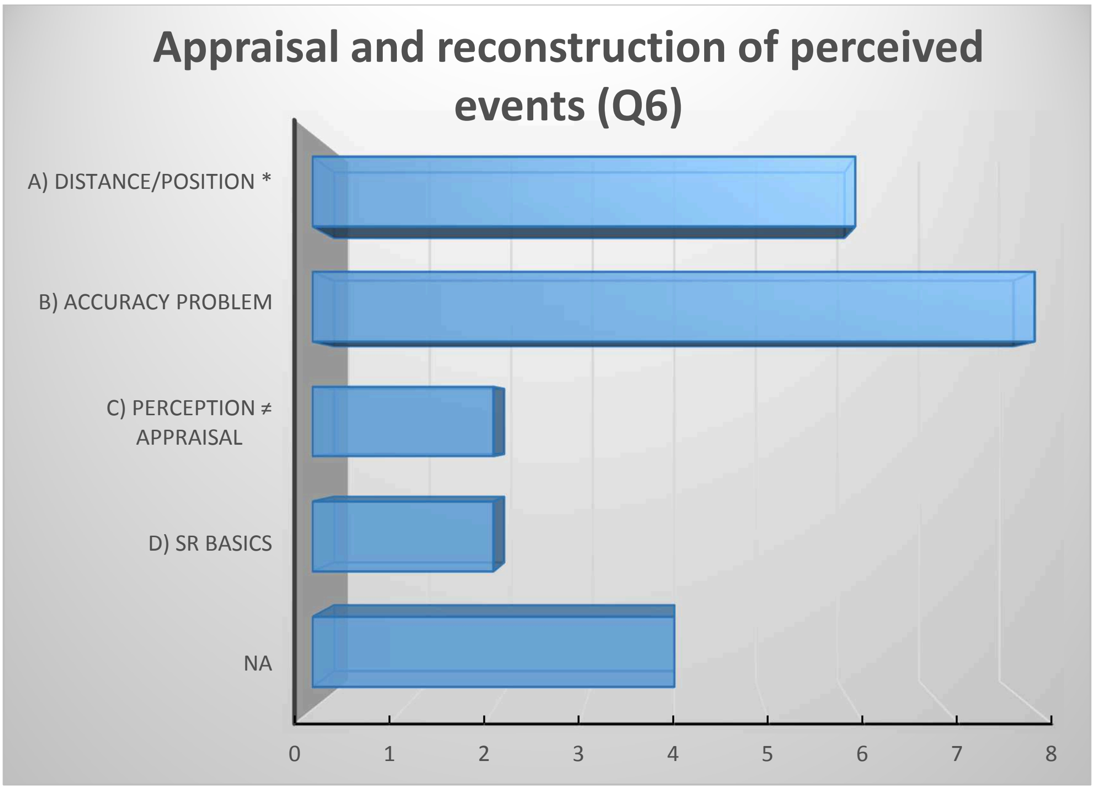


Figure VII.35. Categories for the answers to Q6. The correct answer is marked by an asterisk.

Question 6, operative definitions of the categories:

A. "Distance/position": « *Because if A is not at l/2 he will perceive different times, and so he will have to apply relativistic formulas in order to understand what the initial situation was* »; « *It is not enough since they depend on the relative position of A and B* ».

B. "Accuracy problem": « *If intervals smaller than one second could be revealed, different times could be perceived for light* »; « *It is necessary to make these reconstructions because the time intervals in which the phenomenon occurs are too short to be directly perceived* »; « *Only by knowing formulas reality may be actually understood; our perception does not allow us indeed to get too small differences* ».

C. "Perception ≠ appraisal": « *What is perceived is different from what is assessed. The reconstructions allow to identify oneself with the other person*»; « *Perception is just a phenomenon of the event and cannot describe the event using the correct appraisal* ».

D. "SR basics": « *Because otherwise one might think that light speed depends upon the coach speed* »; « *Because otherwise relativistic formulas and mathematical operations would not be taken into account* ».

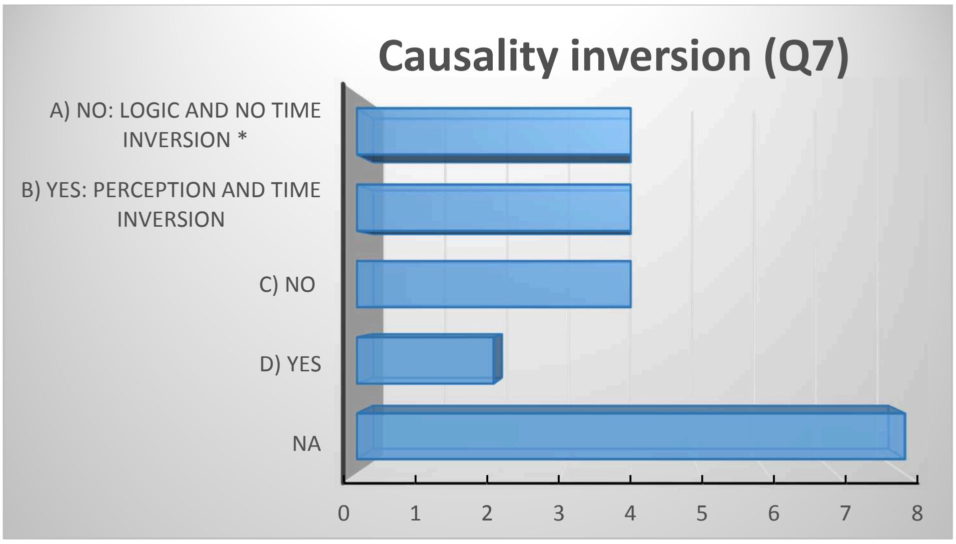


Figure VII.36. Categories for the answers to Q7. The correct answer is marked by an asterisk.

Question 7, operative definitions of categories A and B:

A. "NO: LOGIC and TIME INVERSION": «*No, since an image always arrives before the other, light speed being finite*»; «*No, for time cannot be inverted* »; « *Since it is a relation of logical character, it cannot depend upon the reference frame* ».

B. "Yes: PERCEPTION and TIME INVERSION": «*No, because what is cause in a frame might be effect in another*»; «*Yes, because effect may be perceived prior to cause* »; « *I will be able to perceive cause prior to effect if I am nearer to the effect, that is if information arrives to me prior to the effect* ».

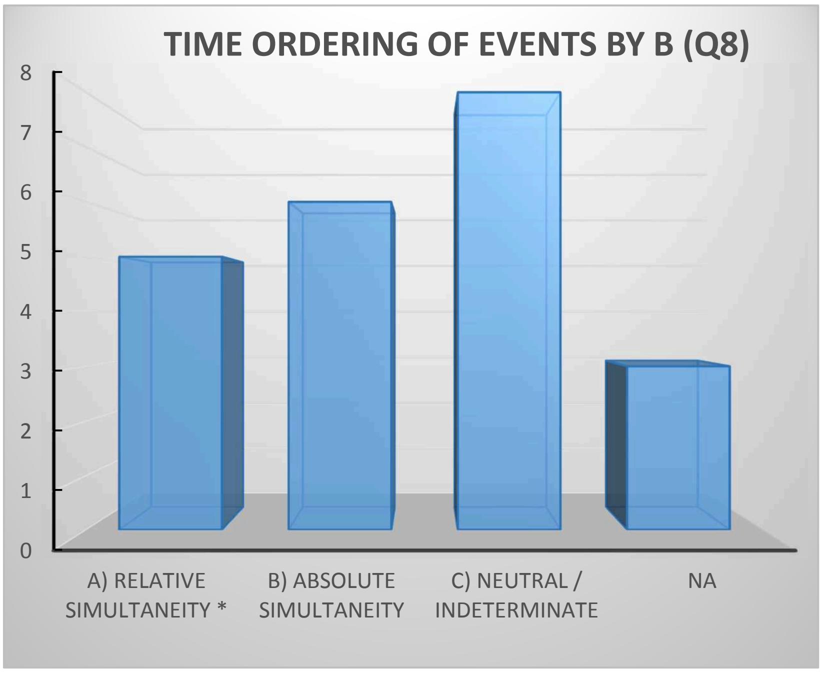


Figure VII.37. Categories for the answers to Q8 (assessment of the two lighting events' time ordering by B). The correct answer is marked by an asterisk.

Question 8, operative definitions of categories A, B, C:

A. "<u>RELATIVE SIMULTANEITY</u>": «*No because an absolute instant in which all events are or not are simultaneous does not exist* »; « *Yes, this information is useful to let A express a temporal judgement, since from knowing that the light of source 2 arrives to B earlier, he will deduce that on equal speed (c) there are two times from the event, according to B* ».
B. "<u>ABSOLUTE SIMULTANEITY</u>": « *Yes because from that fact he may decide the simultaneity of two events* »; « *Yes because it would allow him to understand what the actual situation is, knowing that the rays are actually simultaneous for B too*».
C. "<u>NEUTRAL/Indeterminate</u>": « *Yes because the displacement of B entails a time difference in light propagation*»; « *Yes, because the space between the source 2 and B is smaller than the one between source 1 and B* »; «*Yes, because [A] sees that the space is different, but the propagation speed is the same* ».

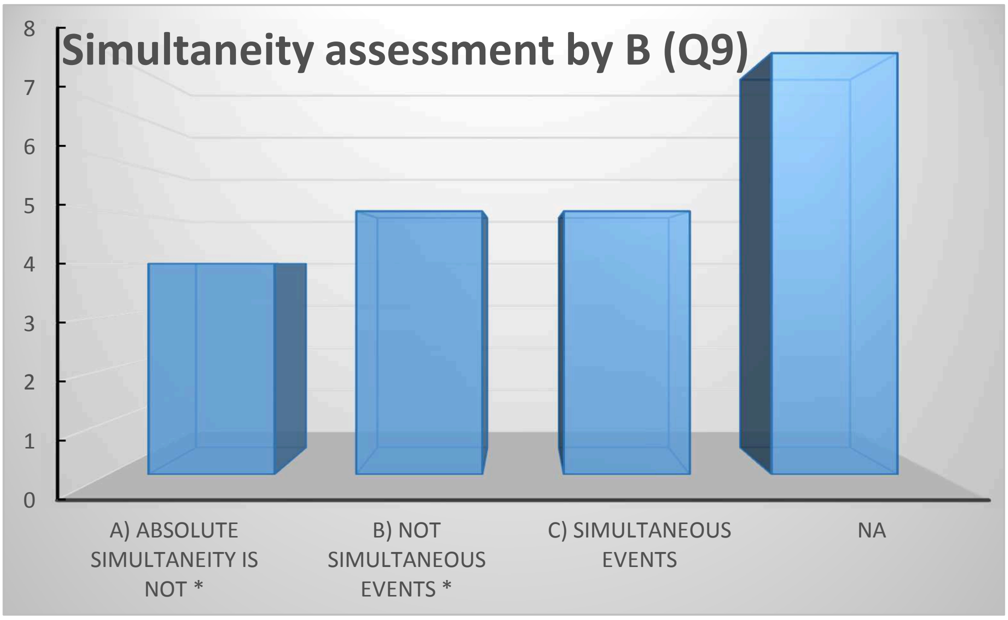


Figure VII.38. Categories for the answers to Q9 (assessment of the two events' simultaneity by B). The correct answers are marked by an asterisk.

Operative definitions of categories A, B, C (Q9):

A. «*It is relative to the chosen reference frame*»; « *Absolute simultaneity does not exist. If A sees the lighting as simultaneous he perceives they are not simultaneous for B. If B sees the lighting as simultaneous he perceives A≠*».
B. « *They are not simoultaneous, but the 2*nd *was turned up before the 1*st »; «*They are not simultaneous because B is moving for the A IF*».
C. « *Since he perceives that 2 lights up before but he knows that light travels less space, he will reconstruct that the event are simultaneous*».

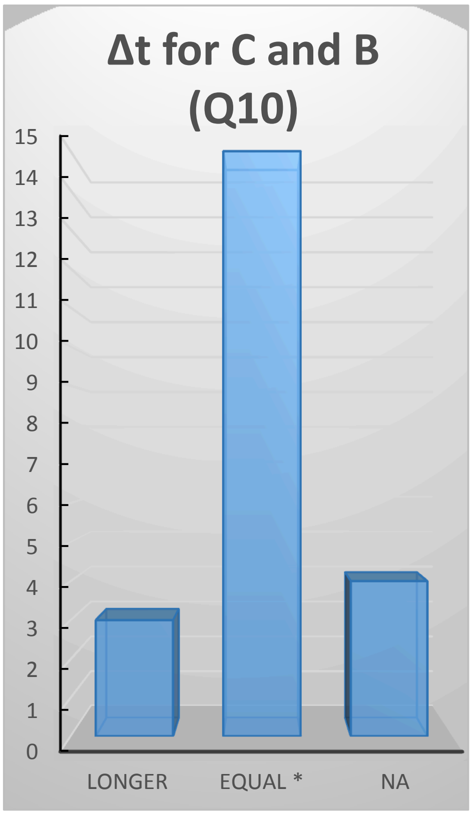


Figure VII.39. Frequencies of the multiple-choice answers to Q10 (comparison between the time interval appraisal by observers B and C, see figure VII.27). The correct answer is marked by an asterisk.

Operative definitions of categories A, B, C in figure VII.39 (Q10, about discrimination between observer and reference frame):

A. *«The interval is longer: the perceived space v*t given by the train motion must be added to the one due to the fact that light has to travel two different distances because of the non-central position»; «Longer: the difference* Δt_C *is conditioned by both v and the fact the distance C – lightsource2 is shorter. C will assess the interval of the source 2 as very smaller.* [the revision follows] *Since it is an appraisal, the two intervals are equal».*

B. *« An equal interval, since he is at rest with respect to the lamps he perceives them simultaneous like B does ».*

C. *« Equal because d/c »; « As regards perception* $\Delta t_C > \Delta t_B$*, but if an appraisal is made* Δt *is equal. A reconstructs that C, B perceive at the same instant ».*

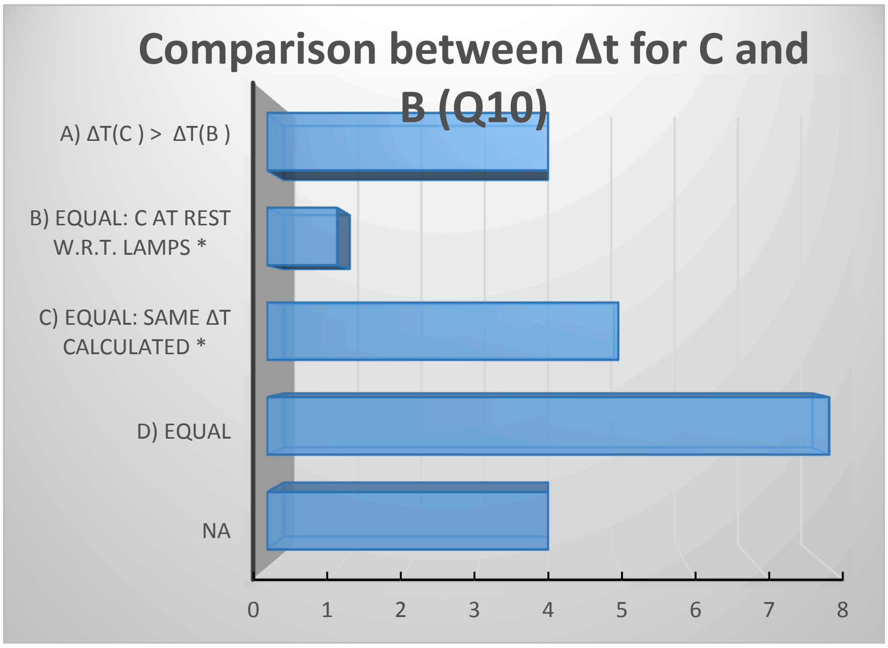


Figure VII.40. Categories for the cloze-test answers to Q10 (comparison between the time interval appraisal by observers B and C, see figure VII.27). The correct answers are marked by an asterisk.

Table VII.32. Conceptual categories for the answers to Q11 (proper time unit measured by a light clock). The correct answer is marked by an asterisk.

CATEGORY'S NAME AND DEFINITION (Q11)	*f*
2d/c *	18/22
d/c	1/22
NA	3/22

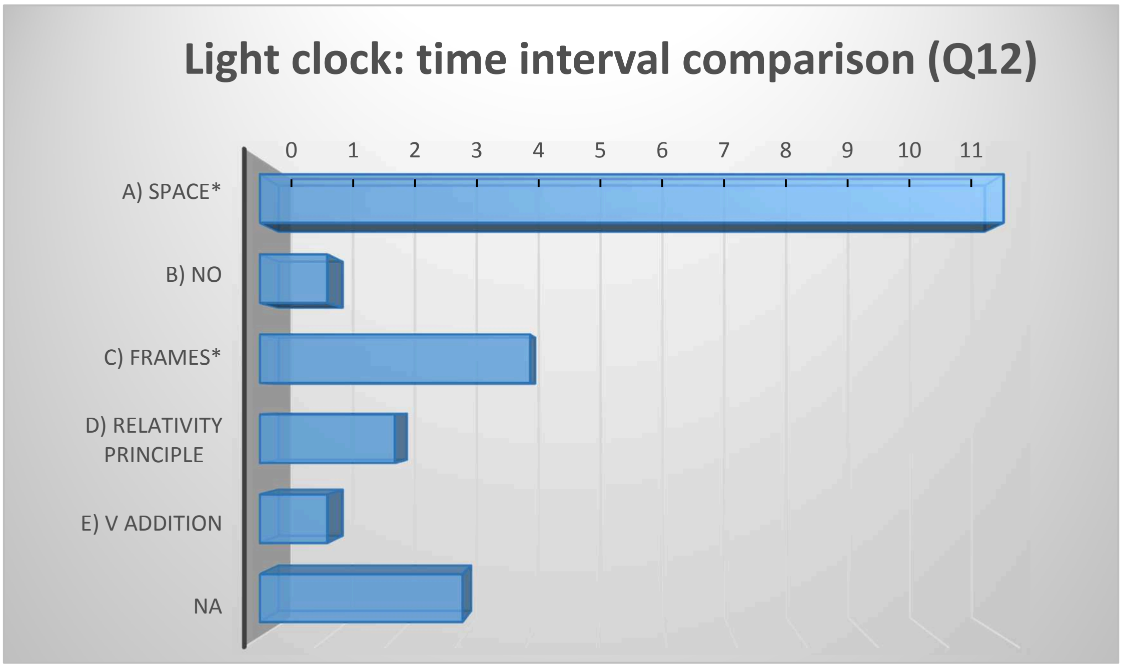


Figure VII.41. Categories for the answers to Q12 (comparison between the time intervals measured by two light clocks in relative motion, see figure VII. 29). The correct answers are marked by an asterisk.

Operative definitions of the main categories of answers to Q12 (explanations of the differences, when recognized, between the back-and-forth light paths in A's and B's clocks):

A. « *No, it is longer because there is more space to travel* »; «*No due to Pythagorean theorem*»; « *A is still in his reference frame, so the ray takes* $\Delta t = 2d/c$ *with respect to it;* $\Delta t' > \Delta t$ *is measured by B's clock since the ray has to travel a greater distance, being it in motion, and so (because the speed is =)* Δt *is longer*».

C. « *No since they are two different reference frames* »; «*No, it will not turn out to be equal, because A and B are in different reference frames and, considering high light speed, time depends on reference frame* »; «*No, because time varies according to speed. Also because A sees one of the two mirrors before* ».

D. « *If we consider that A assesses time with a mirror clock in his own IF (the platform) and B with a clock on the train, they must measure the same time one respect to the other: 2d/c*»; « *The actual measured time will be equal, entailing that space shrinks because each observer will respectively see the observed as moving more slowly* ».

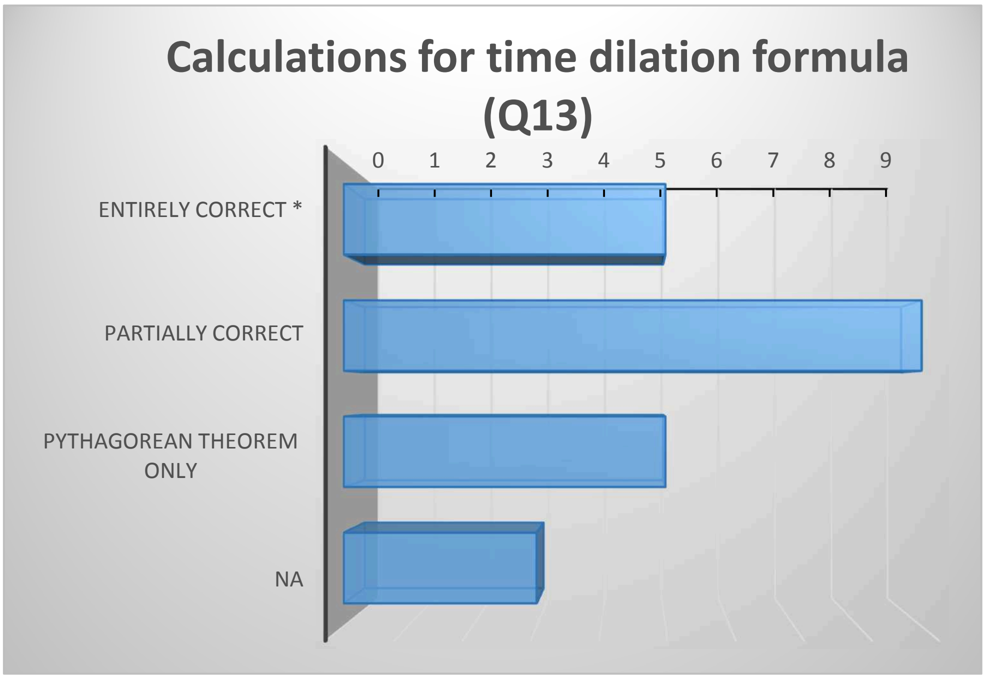


Figure VII.42. Assessment categories (reached levels) for the answers to Q13.

As for Q13, the correctness of the mathematical deduction of time dilation formula has been categorized according to three levels:

A. ENTIRELY CORRECT: the student performed the whole procedure correctly;
B. PARTIALLY CORRECT: the student used at least one kinematical relationship correctly, but he skipped steps / made corrections / made mistakes;
C. PYTHAGOREAN THEOREM ONLY: the student applied the Pythagorean theorem only.

ANSWERS TO Q14					
MULTIPLE CHOICE/1	*f*	**CLOZE-TEST/1**	*f*	**CLOZE-TEST/2**	*f*
MORE TIME, because of time dilation	10/22	LONGER	4/22	"shrinks" / "compresses" *	12/22
LESS TIME, because of time dilation *	6/22	SHORTER *	11/22	"expands "	1/22
NA	6/22	DIFFERENT *	1/22	NA	9/22
		NA	6/22		

Table VII.33. Relative frequencies for the multiple-choice and cloze-test answers to Q14 (length contraction as consequence of *c* invariance and time dilation in a horizontal light clock). The correct answers are marked by an asterisk.

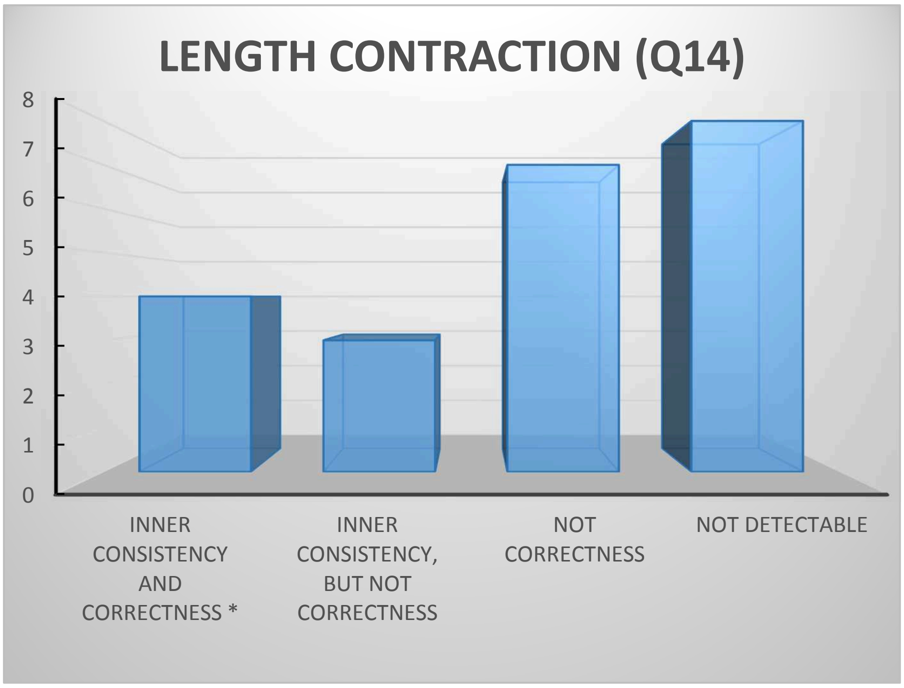


Figure VII.43. Check of internal coherence among the three answers to Q14, combined with their scientific correctness.

Table VII.34. Relative frequencies for the cloze-test answers to Q15 (back-and-forth time in the still horizontal light clock). The correct answer is marked by an asterisk.

CATEGORY (Q15)	OPERATIVE DEFINITION	*f*
A *	"2d/c"; " $2L_0/c$"; "2L/c"	13/22
B	"β"	1/22
C	"Δt"	1/22
NA		7/22

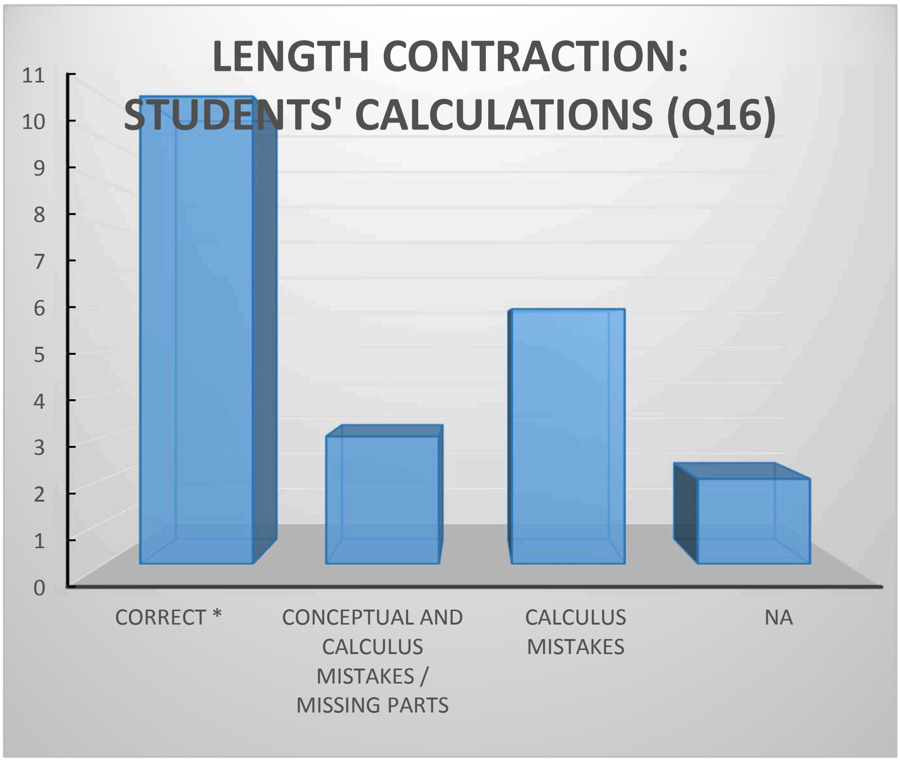


Figure VII.44. Assessment categories (reached levels) for the answers to Q16 (formal deduction of length contraction formula).

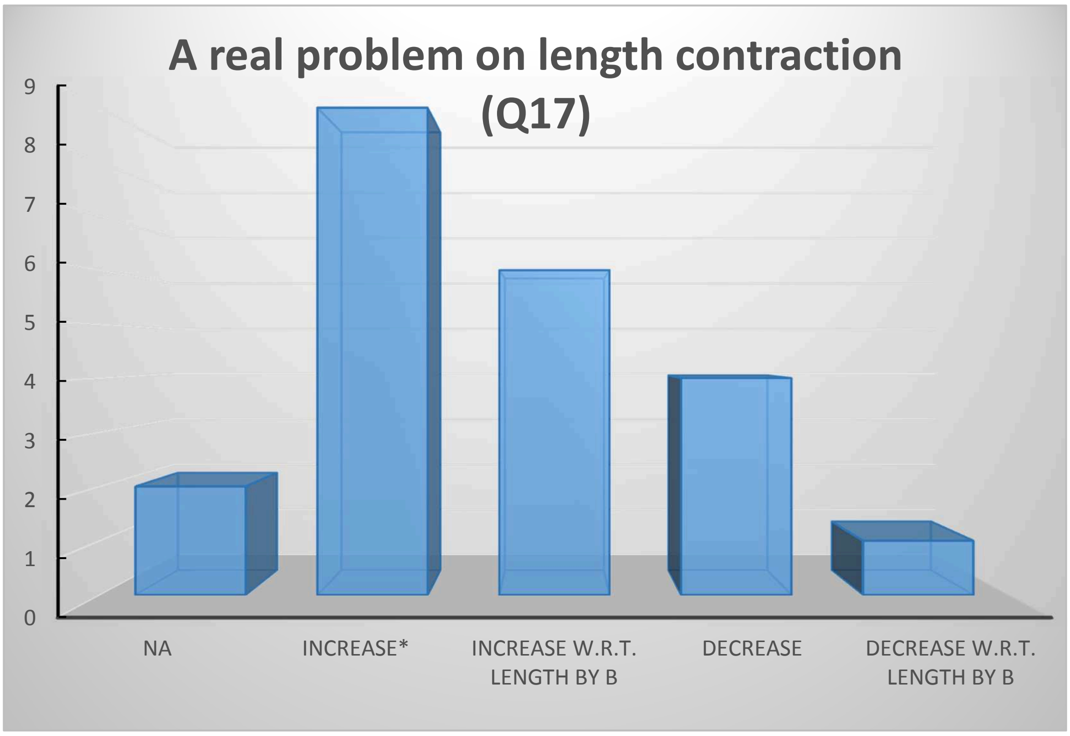


Figure VII.45. Frequencies of the multiple-choice answers to Q17 (conceptual application of length contraction effect: stopping of the train). The correct answer is marked by an asterisk.

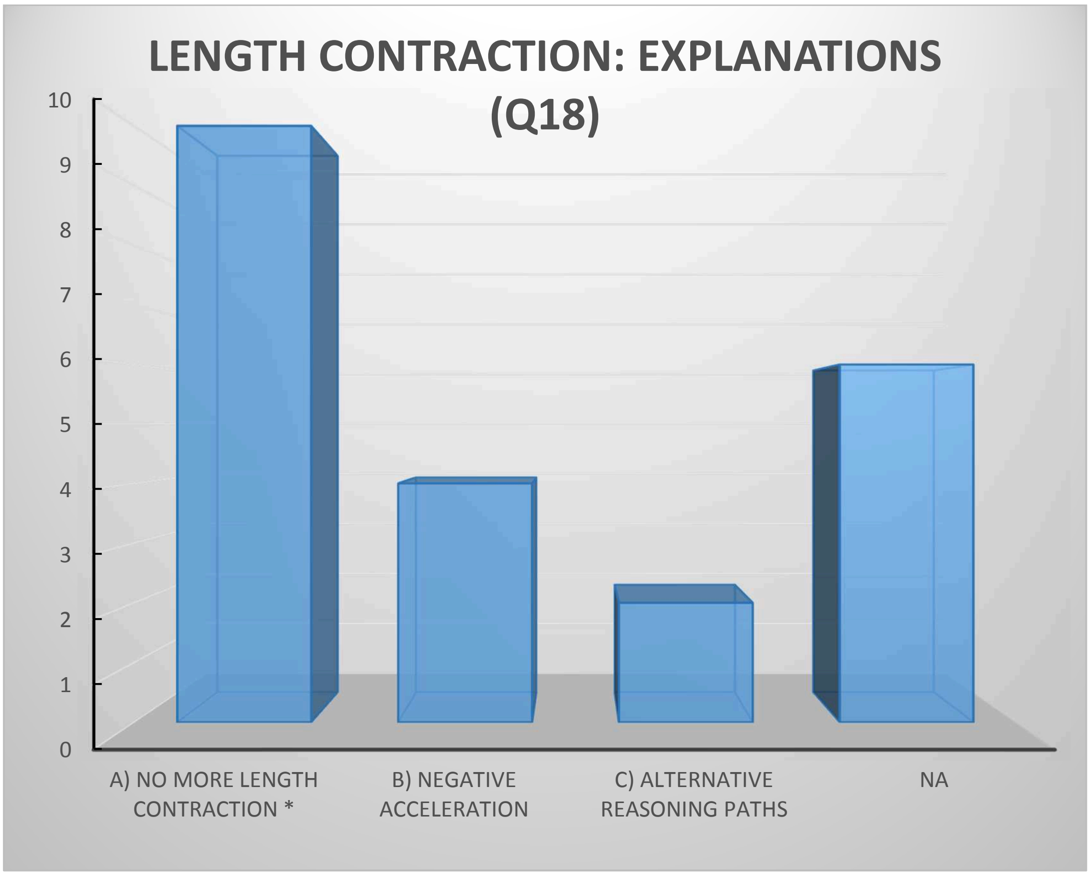


Figure VII.46. Categories of the answers to Q18 (justifications for the solution provided to the real problem of Q17). The correct answer is marked by an asterisk.

Operative definitions of the categories A, B, C (Q18):

A. *« It increases, because A sees it as contracted, while B sees its proper length. When the train has stopped,* $v = 0$ *so* $L = L_0$ *»; «When v tends to 0, γ tends to 1 and thus L tends to be equal to* L_0*; lengths match, from a contraction, and therefore it relatively increases »; « It cannot decrease neither according to A neither to B, since A is seeing it as contracted (it increases according to A indeed), while B is seeing it as still with respect to his reference frame. For this reason it cannot even increase according to B, but it increases according to A. B always sees it as long* L_0 *(MAXIMUM VALUE). A sees it as long* $L < L_0$ *when it is moving,* L_0 *(MAXIMUM VALUE) when it has stopped ».*

B. *« Because it decelerates »; « It increases because the speed decrease (and the stopping) entails a time dilation for an external observer ⇒ increase ».*

C. *« When the system travels at speed v, E behaves as if it were closer to S: at the time when* $v = 0$ *E does not "follow" S, and thus L increases »; « The lengths perceived outside the inertial frame will never be greater than the length* L_0 *».*

VII.6.3. Data: Analysis and Results (phenomenographic study and score distribution)

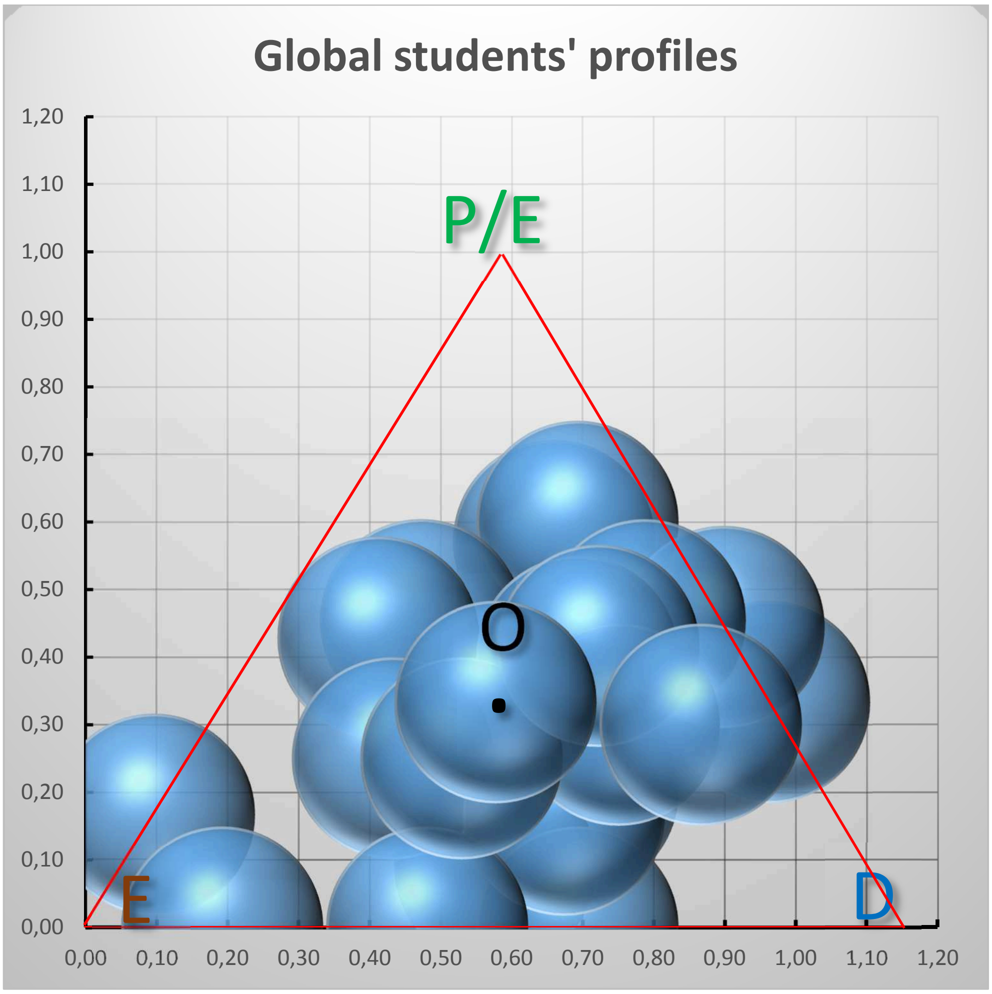


Figure VII.47. Bubble graph showing the "distance" of each group of *real student profiles* from the three ideal profiles in the vertexes, *during* instruction. These profiles take into account 12/18 questions.

Bubbles are all around the centre of the equilateral triangle $O\ (0.58, 0.33)$; they are slightly shifted toward the «descriptive» and «explicative» vertexes. Therefore most students' profile composition is either balanced or slightly unbalanced in favour of the latter profiles. In particular, 3/22 student profiles are not practical/everyday at all and 2/22 are very close to the explicative vertex: they are closer than any other profile is close to the descriptive and practical/everyday vertexes. Two profiles are not explicative at all, but more descriptive than practical/everyday. More generally, the descriptive zone is more densely populated than the explicative zone, while the practical/everyday one is more scarcely populated than the other two. The overall situation is in line with the calculated average percentages[121]: P/E: 30%; D: 38%; E: 32%.

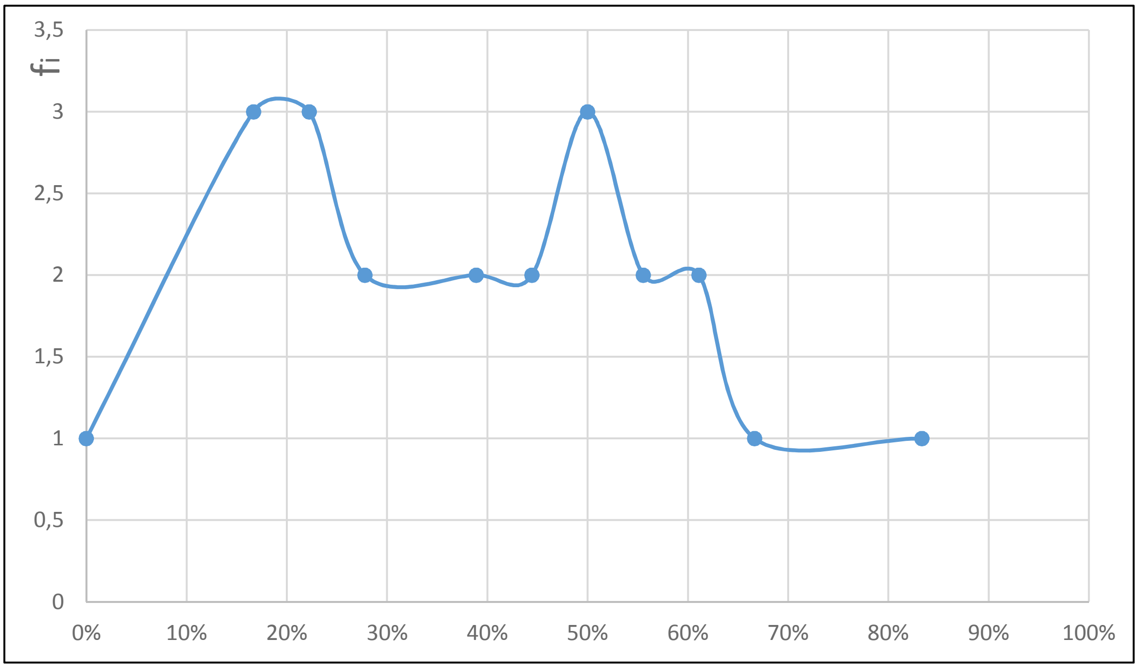


Figure VII.48. Students' score distribution.

The performance of the sample is characterized by scores under or equal to 50% (one-half of correct answers) in 16/22 cases and scores 56% and 61% in 4/22. The distribution is broadly divided in two more populated "zones": around 20% (this may be associated to low understanding) and around 50% (upper medium understanding). There are no scores higher than 83%. The mean score is 40%, against 71% and 56% in the RCI post-test and pre-test respectively (Aslanides & Savage, 2013).

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[121] The original average percentages were 22%, 29% and 25% respectively, with 28% of NA, which is comparable to the profiles' percentages and so too high. Therefore they were normalized.

VII.6.4. **Discussion**

VII.6.4.1. *Answer categories*

Q1. A correct use of classical composition law was found in all answering students but one; 6/22 also explained the result correctly.

Q2. Nine students (60% of the answering students) recognized beyond doubt that *c* is invariant – almost always implicitely – and 6 of them (40% of all the answering students) added a correct explanation. One of these six students explicitates *c* invariance referring quantitatively to the electromagnetic nature of light (« *Always c anyway, which is invariant in any reference system. Since vacuum is not a medium* $(\varepsilon_0\ \mu_0)^{-1/2}$ *is an invariant* »). These students assert either that *c* is the ultimate speed or that light travels in vacuum for justiying/deducing its invariance; an actual small argument is present in three cases. The replies of 5/22 students (33% of the answering sample) are <u>uncertain between *c* and *c*+v</u> instead, in the sense that they contain both. This outcome is due to a *difficulty in mastering the relationship between the classical velocity transformations*, learnt at school, and *the information that c does not vary / is the ultimate speed*, learnt either from other sources or at school outside the curriculum. Nevertheless, the latter information is ultimately prevailing on the former, in 3/5 answers. In the other two cases this isolated (correct) information risks to produce «<u>relativistic noise</u>» (Pietrocola & Zylbersztajn, 1999) disturbing the learning process.

Q3. Awareness of the three possible answers to Q2 was detected in 7/22 replies to Q3 (58% of the given answers). However, three of them did not contain any hypothesis for those possible anwers. The other four mentioned ultimate speed role for the answer "*c*" and classical laws for "c+v" and "c-v". On the other hand, the large-group discussion made 8/22 students (67% of the answering ones) definitely aware of the hypoteses. Two among them, different from the four above, contrasted an electromagnetic/wave-beaviour justification for "*c*" to a classical mechanics one for "c+v".

Q4. The answers grouped in the categories B, D, E and A + B (15/22 in total, 75% of the given answers) do not include <u>discrimination between the perception of an event and the event itself</u>. On the contrary, 4/22 from categories A and C argue using the finite speed of light and the role of distance. The last student limits himself/herself to distance: «*Yes, because the firing devices are at* *distance l*». Moreover, the answers included in B and A + B are explicitly based on the idea that simultaneity is something absolute, existing in itself. The correct answers are only 4/22 (20% of the given anwers), for various reasons. The others are either limited to Yes/No (2/22), or the problem solution is considered as obvious (5/22), or the correct conceptions of finite *c* and importance of distance for simultaneity bring to an incorrect conclusion (1/22), or a purely classical reasoning is done (6/22), or the classical and relativistic reasoning are mixed up (2/22), for instance « *Yes, because the two firing devices are touched at once, because A is exactly halfway between the two light sources and because light always travels at the same speed* ». "Relativistic noise" is present in the answer forming category C (« *No because the second light source lighted up before, since it is farther, because when A sees the EXACT train position, it has already translated in the direction of the train* »), while intermediate conceptions are present in the answers included in A+B.

Q5. The broadest category is the correct one at the same time, with relative frequency 8/22 (42% of the furnished replies): observer A, standing on the platform, reconstructs simultaneity by means of distance, together with a simmetry argument in 4/22 cases. So an assessment of time coincidence between two events is related to a spatial measure here, which is a first important step towards the conception of space-time reality. This attention to distance is also present in the 2/22 answers of category C, even though the empashis is on calculations here: « *By performing calculations relative to v = s/t* »; « *If they arrive to his eyes at the same instant or, if he is not halfway, by calculating the actual taken time* ». The learning outcome of this question is thus overall positive, although 5/22 students do not take distance into consideration at all: simultaneity of perception entails simultaneity of events directly for them.

Q6. Differently from the previous question, the importance of distance/position for the simultaneity assessment by A based on the reception of light signals *is recognized by 6/22 students only* (33% of the answering sample). Eight students focus instead on the *problem of accuracy in (time) measurements* owing to the finite value of *c*. Two students do not go beyond stating a difference between perception and appraisal, and the last 2/22 justify their replies invoking special-relativistic formulas/principles. So *most supplied* *justifications* (12/22) *either do not pinpoint the essence of the problem*, examining secondary aspects, or do not really explain it. The learning outcome is *not good* for this question.

Q7. This is a more general question, entirely overlooked by 8/22 students (NA). Eight of the remaining ones assert that causality cannot be inverted, while 6/22 just the opposite. Only 4/22 justified their answers, in both cases. So on the one hand I was comforted by the former, supplying meaningful reasoning like «*Since it is a relation of logical character, it cannot depend upon the reference frame*», but on the other hand I was worried by the latter, in which assertions are made like «*No, because what is cause in a frame might be effect in another*» or, worse, «*According to logics it cannot be inverted since the effect never preceds cause. However, this relation might be inverted by considering a reference frame in which time arrow is opposite to the ordinary time flowing*». This demonstrates that these four students have not understood the deep meaning of the finite value of *c* and its spinoff on physical phenomena. Anyway, most students did not justificate or answer at all (14/22), thus a diagnosis for the whole sample is hard.

Q8 and Q9 (relativity of simultaneity). It is well-known that simultaneity depends on the inertial frame in SR. I probed this point by Q8 and Q9. As regards Q8, relativity of simultaneity was grasped by 5/22 students (26% of the answering ones), while it was not grasped by 6/22 for sure (31% of the answering students). The other replies (8/22) did not allow to infer any conception of simultaneity held by the respective students. As for Q9, relativity of simultaneity was stated in 9/22 answers (64% of the given answers) by asserting either that absolute simultaneity does not exist or that the two lighting events do not remain simultaneous when passing from A's measurements to B's measurements. Conversely, 5/22 students (35% of the given answers) claimed just the opposite: the lighting events are simultaneous for B too. For instance: «*The two events occur at once, although he does not perceive them as simultaneous*» or «*Since he perceives that 2 lights up before but he knows that light travels less space, he will reconstruct that the events are simultaneous*»: the conception of "superiority" of the A's frame emerges in these examples, which derives from the idea of absolute space and time. Even the Galilean relativity of spatial coordinates is not considered as valid in the second sentence!

Q10 (observer and IF). This question was meant for probing if students overlapped observer and inertial frame, error revealed by De Hosson and colleagues (2010). First of all, the majority of the sample (15/22) answered correctly to the multiple-choice part. Moreover, 14/22 students (78% of the answering sample) wrote that the *time interval (null, in this case) between the two lighting events is equal for two observer at relative rest*. Only 4/22 claim that the observer who is not in the centre of the coach measures a longer time interval. *The sum of the two former results brings to consider the topic as understood*, even though 8/22 answers are without justifications.

Q11, Q12 and Q13 (time dilation). It may be seen in table VII.32 (previous paragraph) that all the answering students but one acknowledged the light-clock round-trip time at rest is equal to $2d/c$. This point was certainly easy, differently from Q12: the qualitative comparison between unit time intervals measured by light clocks in different frames. First of all, 2/22 pupils asserted that the two time intervals are equal, while 17/22 that they are different. The most common justifications have (correctly) been (i) a longer space/distance travelled by light in the moving light clock (11/22) and (ii) an argument founded on frames: the fact that they must be different (2/22) and/or time dilation effect itself (2/22), for example «*No, because time varies according to speed. Also because A sees one of the two mirrors before* ». The total relative frequency of correct answers is thus 15/22 (79% of the given answers): a good result. Conversely, two students exploited the RP to account for the equality between time intervals: a correct outcome gave rise to a wrong conclusion. This was due *to a lack of comparison in a single frame between two identical phenomena occurring in different frames*: both A and B are said to observe only their phenomenon each. As for Q13, the most commonly reached level (9/22, 47% of the supplied answers) has been "partially correct", i.e. the medium one, as it usually occurs, while 5/22 students performed a correct calculus.

Q14, Q15 and Q16 (length contraction: the effect). The findings from Q14 are *logically contrasting*, as shown by figure VII.42: inner consistency was found in 7/22 answers only. Ten pupils (63% of the answering sample) claimed that more time is needed for the back-and-forth path in a moving horizontal light clock than in a still one: *time dilation is misinterpreted*. On the other hand, 6/22 (37% of the answering sample) indicated that less time is needed: about *one third of the answering students chose the correct alternative, two thirds did not.* Furthermore, the moving clock is indicated as shorter than the one at rest by one half of all students in cloze-test/1, and 12/22 (all the answering students but one) wrote that the moving light clock shrinks in cloze-test/2. The very good outcome of cloze-tests is partly due to the supplied pictures of moving and still clock. Globally, only 4/22 answers were found consistent and correct, while 10/22 definitely not correct and 8/22 not detectable. *Therefore the overall outcome is not good*. As for Q15, rather easy, most students (13/22, 87% of the answering sample) supplied the correct answer. Finally, Q16 paralleled Q13. Although Q16 was more difficult than Q15, *one half of the students performed calculations correctly*. The increase of correct performances, compared to the case of time dilation (5/22 → 11/22), is remarkable; it is perhaps due to the cloze-test form of Q16: Q13 contained only some general hints.

Q17 and Q18 (length contraction: an application). An application of length contraction to a real situation was presented to the students in the form of a real open problem: when the train stops, what happens to its length (comparing initial and final instants)? It was thought in order to analyse the students' cognitive transfer skills. Nine students (45% of the answering sample) answered to Q17 (multiple-choice) that it increases. Remarkably, 6/22 students (30% of the answering sample) selected increase with respect to length by B. Therefore *the "converse effect" undergone by the train was understood, but sometimes not as an absolute one*. Actually, *the concept that there is only one reference frame after stopping is rather subtle*. An explanation was then required (Q18) for the answer given to Q17. Ten students (63% of the answering sample) argued essentially that *length contraction does not work anymore after stopping*, verbally or by mathematical formalism. This is a *rather good result*. Four students (25% of the answering sample) used speed increase as key justification instead.

VII.6.4.2. *Profiles and scores*

The analysis of profiles shows a very equilibrate distribution of the three types of reply over the sample. However, descriptive profile is prevailing on the others, as in many other experiments (in post-tests above all). At the same time, the students showed remarkable argumentative skills, since a number of them succeeded in providing a reasonable model-based explanation for the physical situation, as far as possible in short instant answers. This is important for science learning.

The score distribution shows a sample split in two parts, one of which reached an either medium (9/22 cases) or high (4/22 cases) level of understanding, to the extent at which understanding may be analysed by these scores (it is actually multidimensional). The other part (9/22) gave less than 30% of correct answers, which are around 20% in most cases (6/22). There were no excellent scores, likely because of the *difficulty of the afforded topics* or due to the *limits of our activity* and/or t/l *path.* Therefore the learning activity and, more generally, the learning path were useful only for about 50% of the students. They ran partially under this respect.

VII.6.5. **Conclusions**

I shall draw conclusions from the discussion above in the light of the RQs:

RQ1R) Do students recognize the role of *c* in Relativity, as for invariance and ultimate speed character? How do they express their conceptions?

RQ2R) Do students acknowledge that simultaneity is not invariant in SR? If so, how do they express the grasped concept?

RQ3R) Are time dilation and length contraction effects understood by students, or described as "distortion of perception" effects, or not achieved at all? In the first and second cases, how do students characterize them?

The conception of c as invariant was found to be strongly held by the sample, except for 5/22 students, who showed wavering; their uncertainty risks to produce "relativistic noise" (Pietrocola & Zylbersztajn, 1999). Moreover, the sample globally shows an adequate mastery of the basic issues concerning the contrast between *c* and any other speed.

The understanding and mastery of *relativity of simultaneity* was gradually increasing from the answers to Q4 to the answers to Q9. That conception was expressed by 9/22 students (64% of the answering sample) in the answers to Q9, i.e. at the end of the step-by-step progression towards relativity of simultaneity. However, the results relative to Q6 are *worse* than the ones relative to Q5: a focus on distance/space is present in 10/22 answers (53% of the answering sample) to Q5, but it is not present in most supplied justifications to the answers to Q6 (12/22, 67% of the answering sample). This is very likely due to the *question text formulation*. In this case, the attention is shifted from the ways of "reconstructing" simultaneity (Q5) to the difference between appraisal and direct perception by an observer (Q6). So 8/22 students (44% of the answering sample) focus on accuracy problems, since an appraisal requires accuracy.

The *distinction between observer and IF* proved clear to 14/22 students (78% of the answering sample), as it follows from the results relative to Q10. However, the mastery level is evaluable only in 6/22 cases: no justification is provided in the others.

Time dilation was *qualitatively* understood by most students in a proper way (15/22, 79% of the answering sample), but *quantitatively* understood by few of them: the formula was correctly deduced by 5/22. Length contraction was understood by 50% of students instead, as regards both *theoretical aspects* (11/22) both the proposed *application* (10/22). So the sample is splitted in two parts under this respect, exactly as it emerges from the score distribution. In addition, time dilation effect was *never* meant by students as a "distortion of perception" in the answers to the questions concerning time interval dilation (Q11 – Q13), unless the answers on simultaneity (Q4 – Q10). Considering that simultaneity and time interval measure are distinct concepts, this result brings to conclude that time interval dilation was *better grasped* than simultaneity non-invariance. It would be interesting to find out the reasons of this difference through further studies.

As regards the ways in which students characterized their conceptions, they may be elicited from the previous discussion joint to the operative definitions of answer categories.

Reflecting on results: overview

VIII.1. Conclusions

Several conclusions may be drawn from the research outcomes of the run formative experiments (i) about *student learning* and (ii) about the *design of a revised t/l pathway on these topics* as well. The experiments have been testing different combinations of sub-modules of the path (table VII.2) in different contexts and by means of different methods of data collection and analysis. So the difference among the obtained results and therefore among the specific drawn conclusions (chapter VII) is understandable, and the transferability level is rather low. Nevertheless, it is possible to individuate some *general aspects* deriving from common elements in the results.

First of all, I am highlighting the emerged conceptual knots and their origin. Most non-selected pupils showed *difficulties in mastering the key relativistic concepts effectively*, from relativity of simultaneity and length contraction (kinematics) to mass in classical physics and SR (dynamics). Time dilation was better understood than relativity of simultaneity during the 2012 winter school. It was also well understood in Udine (sample 2). Not only ordinary pupils, but also a number of talents had problems in conceptual learning with understanding of mass; by way of example, the conception of mass as quantity of matter was revealed in 6/42 participants to the summer school 2011. Remarkably, *"relativistic mass"* proved associated to either misconceptions or lack in understanding of mass in SR in several instances, according to the reviewed literature (chapter V). A performed correlation test on sample 1 (Udine) supported this hypothesis ($r'_{35} = 0.75$, $r_0 = 0.56$; $\alpha = 0.01$), but another on the same sample indicated a highly significant correlation between relativistic mass and mass non-additivity ($r_{35} = 0.83$, $r_0 = 0.56$; $\alpha = 0.01$). Another correlation test gave null result (2001 summer school), but the bubble graph supplied an interesting hint. Causality tests were never performed instead. Besides, the presence of a *«relativistic noise» hindering understanding* was repeatedly detected (Pietrocola & Zylbersztajn, 1999); it proved induced by relativistic «inert knowledge structures» (Whitehead, 1929;

Michelini, 2005) as well as by scant fragmented popular notions. An inquiry both into the origins of this knowledge fragmentation and into the nature of the detected «personal problematic contexts» (Pietrocola & Zylbersztajn, 1999) would be interesting for research.

The detected learning problems are due to four basic reasons. First of all, some aspects of the t/l path rationale and implementation need to be improved (*limits*). Secondly, the most common teaching practice in schools geographically spread over Italy proved *traditional*, using frontal lessons and generally *ex-cathedra* mathematical or theoretical approach; accordingly, operational approach tends to be neglected. Thirdly, the students of samples 1 and 2 in Udine *did not reason upon and creatively re-elaborate the concepts and/or worked out models* after instruction, probably because of affective and socio-cultural reasons, e.g. the lack of personal and social motivations, and the mentioned trend in teaching practice. This lack of motivation to reason and to actively use intellectual (good) skills was also detected in 16/42 answers to the intermediate group questions mentioning *quantitas materiae* supplied by the talents of 2011 summer school. Briefly put, they focused on the problem rather than on the solution. On the contrary, the students attending 2012 winter school (Relativity Course) were found to exploit their skills *during the activity*: the explicative profile affects about 1/3 of each student's answers on average. Fourthly, *language* turned out to condition learning, in particular the conceptual overlapping between "relativistic mass" and mass in Relativity, "relativistic mass" and mass at relativistic speed, "mole" and "mass"; "mass excess" and mass variation in β-decays (often called "mass defect"); (generic) "energy" and kinetic energy of decay products.

However, there have been *a number of positive learning results* under many respects. So the worked-out t/l path appears to have good potentialities (*values*) for overcoming some knots stated in the literature and the ones found by this research.

- Limits and aspects to improve. At first, the approach to gravitational mass in the formative experiments 1, 2 and 5 (tuning phase) has to be changed. In particular, the role of force as *reciprocal interaction* should be stressed, because of the dichotomization of classical mass detected in the answers to the 2011 summer school historical worksheets. The sub-module on mass conservation (C12 in table VII.1) was useful for obtaining *descriptions* rather than explanations/modelling by students in Crotone; it should be revised. As regards relativistic 4-vector module, it was considered too much difficult by 2011 talents as well as by students in Treviso, as shown by the high percentage of missing answers (69% and 89% respectively), possibly because of the high degree of formalization. Mass invariance was instead a critical point for the students belonging to sample 1 (Udine), to whom the relativistic phenomenological module was administered. Mass invariance was understood by only 5/23 pupils. Further, if it had been properly understood, the grasping of mass conservation under accelerations would have followed, according to the calculated correlation test ($r'_{23} = 0.51$, $r_0 = 0.42$; $\alpha = 0.05$). About energy increase at uniform speed (sample 1, Udine), since 11/23 students asserted that it is related to *both* mass *and* momentum variation, the very different physical meanings of total energy expression and photon energy-momentum equation should be highlighted. Broadly speaking, more stress is needed on the fact that *mass-energy relationship does not entail a "transformation"/ "conversion" of the former into the latter (and viceversa) or a mass variation associated to kinetic energy variation*[122]. The latter reasoning path was used by 16/27 students in Treviso when replying to a question on whether the mass of an accelerated body increases (« Consider an electrically charged body at rest in the laboratory, on which external electrical forces do work. This body will be accelerated and its energy will increase. Is it possible to assert that its mass will increase as well? Explain. »; the categories are in table VII.22). Besides, "mass excess" should be dropped or better clarified in the path, because it hindered the grasping of mass-energy relation in 4/27 students (Treviso). Finally, the approach to relativity of simultaneity adopted in experimentation #4 (Bard) did not allow a meaningful understanding, except for the fundamental fact that a generic assessment of time coincidence between events proves entangled with a measure of space/distance. This goes towards the idea of space-time as a unity and was present in 8/22 answers, therefore it is a value of the path.

- Values and proposals. The contribution of this t/l path to improve learning is highlighted by several strong points. The first is vertical perspective, whose need is witnessed by the results of 2011 summer school pointing out a diffuse learning of mass in SR affected by learning contexts, by asserting for example that "relativistic mass" «*means that mass in motion at very high speed can*

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[122] Mass is simply equivalent to rest energy, which transforms in other energy types.

become energy and vice versa». Then, thought experiments: both light clock and photon in a box (Treviso) as well as the perfectly inelastic relativistic collision for showing mass non-additivity (Udine – sample 1) proved effective in the contexts in which they were "run". Light-clock was also useful for *qualitative* understanding of time dilation in the 2012 Bard campus. Furthermore, the classical module activated the intuitive concept of mass as $m = f(\rho, V)$ (Mullet & Gervais, 1990) in 91% of the sample (answers to Q5, Crotone). Moreover, in the β-decay phenomenology exploration (Treviso) mass variation was related to kinetic energy of products by 8/27 students, and it was considered as «*the measure of the quantity of energy owned by a body*» by 3/27. For further research it would be interesting to examine the other form of "nuclear energy" – electromagnetic energy – using γ-decays. Eventually, a longer formative intervention module *in situ*, planned together with the involved teachers, would be necessary for reducing «relativistic noise».

VIII.2. **A final Consideration for further Developments**

The outcome of the "photon in a box" TE is essentially that *light displays inertial properties when it is part of a physical system*: when a photon interacts with the other components it behaves *as if* it has an inertial mass. If WEP is assumed, the photon will own an effective *gravitational* mass too and thus will interact with gravitational fields: light will undergo gravitational actions and its path will be curved by the presence of masses. Sometimes it is asserted that 'light weighs' for the sake of brevity. Einstein deduced this result by means of a «historical thought experiment» (Gilbert & Reiner, 2000) entailing a light ray propagating inside a free-falling lift in a gravitational field. It was observed by an inside and an outside observers.

Einstein's TE was entirely different from the "photon in a box" TE, but it was based upon WEP and mass-energy relation as well. A crucial point is the following (Einstein & Infield, 1938):

> But there is, fortunately, a grave fault in the reasoning of the inside observer, which saves our previous conclusions. He said: « A beam of light is weightless and, therefore, will not be affected by the gravitational field ». This cannot be right! A beam of light carries energy and energy has mass. But every inertial mass is attracted by the gravitational field, as inertial and gravitational masses are

> equivalent. A beam of light will bend in a gravitational field exactly as a body would if thrown horizontally with a velocity equal to that of light.

Another approach to the same idea is the following (PSSC, 1995). Two lifts with a light ray inside are considered, one accelerated upward with acceleration $-\vec{g}$ and one still with respect to a *uniform*[123] gravitational field of magnitude $\vec{g}$ for unit mass. The phenomena in the two associated IFs must forbid to distinguish between them, because of the Relativity Principle. Therefore the light ray is necessarily slightly curved downward in the second lift too.

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[123] This is an ideal condition never occurring in nature, but useful for this TE, which is 'run' in a special-relativistic framework, although gravity is taken into consideration. It is actually halfway between Special and General Relativity.

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APPENDICES

The appendices 1 to 3 contain the worked-out instructional materials administered in pilot formative intervention experiments planned for *evaluating* the implemented teaching/learning pathways. The tests for probing understanding and/or learning gain, compounded by essay questions and/or multiple-choice questions, are in the first appendix; they were administered prior and/or after each formative intervention experiment. Tutorials are in the second and third appendices; they served both for evaluating learning *in progress* both as didactical tools for students; some of them where structured according to the IBL strategy (appendix 3). The fourth appendix includes the text and picture contents of the slides showed during some formative intervention experiments for allowing students to better follow the line of discussion/reasoning[124]. These slides were arranged only for the relativistic four-vector and relativistic energetic (phenomenological) modules of the t/l path; two variants for each type are furnished. Eventually, appendix 5 includes the calculations for showing that the photon interaction by Compton scattering with the walls of a box influences the inertial behaviour of the entire system, along with a discussion on the educational relevance of this thought experiment. All the material is attached in the same chronological order of the formative intervention experiments as well as in its original format and language (Italian) for supplying a precise documentation.

1. Pre-tests and post-tests
2. Tutorials
3. IBL Tutorials
4. Slide contents
5. Calculi for the photon in a box (thought experiment)

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[124] The students of “sample 1” and “sample 2” in Udine were supplied with a paper print of the slides *after* each module and *before* each module respectively.

Appendix 1 – pre-tests and post-tests

➢ *2011 summer school (Udine): post-test*

TEST FINALE

Quando entra in gioco la massa nella tua vita quotidiana? In quali fenomeni ne percepisci la presenza?

____
____
____
____

Quali settori della fisica studiano questi fenomeni?

____
____
____
____

Che cosa intendi quando parli di *quantità di materia*?

____
____
____

Quali altre accezioni e definizioni di massa conosci?

____
____
____
____
____

In molti libri di testo si parla di *massa relativistica*. Spiega di cosa si tratta.

____
____
____
____
____

L’inerzia di un corpo dipende dal suo contenuto di energia, cioè dall’energia che possiede a prescindere dal suo moto di traslazione?

☐ Sì
☐ No

Spiega.

____
____
____
____
____

➢ *Treviso: pre and post-test*

PRE-TEST (time: 20 minutes)

1. In natura avviene spontaneamente un processo nucleare detto «decadimento β»:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Il neutrone può dare luogo a ("decadere in") protone, elettrone ed *anti-neutrino*. Tralasciamo quest'ultima particella, la cui massa è trascurabile ai nostri scopi, e calcoliamo le masse negli stati iniziale e finale.

$m_n = 939.57$ MeV/c^2 (stato iniziale)
$m_p + m_e (+ m_\nu) = 938.28 + 0.511 = 938.79$ MeV/c^2 (stato finale)

Come interpreti la variazione di massa che il sistema fisico ha subito?

2. Come hai appreso dall'esperimento mentale dei due fotoni di Einstein, l'energia di un corpo in quiete è legata alla sua massa. In che modo?

3. Qual è il significato dell'equazione di Einstein $\Delta E = (\Delta m)c^2$?

4. Considera un corpo elettricamente carico inizialmente in quiete rispetto al laboratorio su cui forze elettriche esterne compiano lavoro. Tale corpo subirà un'accelerazione e la sua energia aumenterà. Si può affermare che aumenterà anche la sua massa?
   ☐ Sì
   ☐ No
   Motiva

5. In natura avviene la seguente reazione nucleare (*fissione*):

$$n + {}^{235}U \rightarrow {}^{93}Rb + {}^{141}Cs + 2n,$$

indotta dall'urto di un neutrone termico (= avente energia cinetica dell'ordine di kT, dove k è la costante di Boltzmann) con un nucleo dell'isotopo 235 dell'Uranio in quiete nel sistema di riferimento del laboratorio. [A temperatura ambiente, l' energia cinetica dei neutroni è circa di 0.02 *eV*: sono chiamati "neutroni lenti"].
I neutroni prodotti possiedono un'energia cinetica. Per la conservazione dell'energia, quindi, l'energia dei nuclei deve diminuire. Si può affermare che diminuisce anche la massa del sistema?
   ☐ Sì
   ☐ No
   Motiva

6. Come si può definire l'energia cinetica di un corpo, sia in contesto classico che relativistico?
7. Quali elementi illustrati nell'esperimento dei due fotoni di Einstein potrebbero essere controintuitivi? *(retro del foglio)*

8. Hai eseguito un *applet* con l'orologio a luce. Che cosa si può dedurre da tale esperimento mentale?

   ☐ La velocità della luce è una costante universale;
   ☐ La velocità della luce non cambia al variare del sistema di riferimento (SdR);
   ☐ Le durate variano cambiando SdR;
   ☐ Le lunghezze non variano cambiando SdR.

   In definitiva, chiamiamo t_0 l'istante di comune *emissione*, per due SdR in moto relativo, del raggio luminoso. Coincide anche l'istante di *ricezione*?

   ☐ Si, nel SdR in moto perché la luce deve percorrere più spazio, ma "guadagna" tempo.
   ☐ No, perché nel SdR in moto passa meno tempo che in quello del laboratorio
   ☐ No, perché nel SdR in moto passa più tempo che in quello del laboratorio.

9. Sulla base del percorso svolto, puoi supporre che la luce manifesti proprietà inerziali nell'interazione con un sistema fisico?

   ☐ Si, il fotone trasporta energia e quindi massa inerziale;
   ☐ No, il fotone non ha massa perché va alla velocità della luce;
   ☐ Si, il fotone non ha massa, ma ha impulso e quindi può trasmetterlo.

POST-TEST (time: 10 minutes)

1. In natura avviene spontaneamente un processo nucleare detto «decadimento β»:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

   Il neutrone può dare luogo a ("decadere in") protone, elettrone ed *anti-neutrino*. Tralasciamo quest'ultima particella, la cui massa è trascurabile ai nostri scopi, e calcoliamo le masse negli stati iniziale e finale.

$m_n = 939.57$ MeV/c^2 (stato iniziale)
$m_p + m_e (+ m_\nu) = 938.28 + 0.511 = 938.79$ MeV/c^2 (stato finale)

   Come interpreti la variazione di massa che il sistema fisico ha subito?

2. Sulla base del percorso svolto, puoi supporre che la luce manifesti proprietà inerziali nell'interazione con un sistema fisico?
   ☐ Si, il fotone trasporta energia e quindi massa inerziale;
   ☐ No, il fotone non ha massa perché va alla velocità della luce;
   ☐ Si, il fotone non ha massa, ma ha impulso e quindi può trasmetterlo.

3. In riferimento all'esperimento dei due fotoni di Einstein, in che modo la variazione di impulso comunicata dal fotone alla scatola è correlata all'accelerazione costante della scatola?
4. Perché è necessario applicare una forza maggiore alla scatola con il fotone per mantenere la stessa accelerazione della scatola vuota?

➢ *Cesena: pre and post-test*

PRE-TEST (time: 15 minutes)

1. Conosci il funzionamento dell’orologio a luce. Che cosa si può dedurre dall’esperimento mentale del treno?
   [*può esserci più di una risposta corretta*]

☐ La velocità della luce è una costante universale;
☐ La velocità della luce non cambia al variare del SdR;
☐ Le durate variano cambiando SdR;
☐ Le lunghezze non variano cambiando SdR.
☐ Il quadrintervallo (intervallo spazio-temporale) non varia cambiando SdR.

Chiamiamo t_0 l’istante di comune *emissione,* per due SdR in moto relativo, del raggio luminoso. Coincide anche l’istante di *ricezione*?

☐ Sì, nel SdR in moto perché la luce deve percorrere più spazio che nell’altro, ma “guadagna” tempo.
☐ Sì, perché la velocità della luce non cambia al variare del SdR.
☐ No, perché due eventi simultanei in un SdR inerziale non lo sono in un altro in moto relativo.
☐ No, perché nel SdR in moto passa meno tempo che in quello del laboratorio.
☐ No, perché nel SdR in moto passa più tempo che in quello del laboratorio.

2. In fisica classica esistono vari tipi (o aspetti) della *massa*. Elencali.

Descrivi come si misura una di queste diverse «masse» a tua scelta (strumento e procedimento di misura).

3. a) Come definiresti l’energia di un corpo? b) Come si definisce *l’energia interna* di un corpo in termodinamica?

4. Qual è il significato fisico dell’equazione $E=mc^2$? Scrivi quello che conosci.

5. Fornisci un esempio di fenomeno fisico interpretabile tramite l’equivalenza massa-energia.

POST-TEST (time: 15 minutes)

1. Hai eseguito un *applet* con l’orologio a luce orizzontale e verticale. Quali delle seguenti affermazioni sono vere?
   [*può esserci più di una risposta corretta*]

☐ La velocità della luce è una costante universale;
☐ La velocità della luce non cambia al variare del SdR;
☐ Le durate variano cambiando SdR;
☐ Le lunghezze non variano cambiando SdR.
☐ Il quadrintervallo (intervallo spazio-temporale) non varia cambiando SdR.

In definitiva, chiamiamo t_0 l’istante di comune *emissione,* per due SdR in moto relativo, del raggio luminoso in un orologio a luce *orizzontale*. Coincide anche l’istante di *ricezione*?

☐ Sì, nel SdR in moto perché la luce deve percorrere più spazio che nel’altro, ma “guadagna” tempo.
☐ Sì, perché la velocità della luce non cambia al variare del SdR.

☐ No, perché due eventi simultanei in un SdR inerziale non lo sono in un altro in moto relativo.
☐ No, perché nel SdR in moto passa meno tempo che in quello del laboratorio.
☐ No, perché nel SdR in moto passa più tempo che in quello del laboratorio.

2. Come definiresti l'energia relativistica di un corpo? Come si raccorda questa definizione con quella di energia interna?

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3. Che cosa aggiunge la relatività al concetto classico di massa?

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4. Qual è il significato fisico dell'equazione $E_0 = mc^2$?

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5. Considera un corpo elettricamente carico, inizialmente in quiete rispetto al laboratorio, su cui forze elettriche esterne compiano lavoro. Tale corpo subirà un'accelerazione e la sua energia cinetica aumenterà. Si può affermare che aumenterà anche la sua massa?

☐ Sì
☐ No

Motiva

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➢ *2013 summer school (Udine): post-test*

Università degli Studi di Udine - Udine Unità di Ricerca in Didattica della Fisica

Scuola Estiva di Eccellenza di Fisica Moderna

Udine, 22-27 luglio 2013

"Applicazioni dell'Equivalenza Massa-Energia"

Cognome ______________________Nome____________________ Data____________________________

Problema 1 – Fissione

Considera la seguente reazione nucleare:

$$n + {}^{238}U \rightarrow {}^{143}Ba + {}^{93}Kr + 3n$$

indotta dall'urto di un neutrone *termico* (= avente energia cinetica dell'ordine di kT, con k costante di Boltzmann) con un nucleo dell'isotopo di Uranio. Un'altra possibile è

$$n + {}^{235}U \rightarrow {}^{93}Rb + {}^{141}Cs + 2n\,,$$

anch'essa indotta da un neutrone termico nell'urto con l'isotopo 235 dell'Uranio.

In entrambi i casi, la somma delle masse dei prodotti è un numero *minore* della massa del nucleo-padre: $\Delta M = -\,3.02 \cdot 10^{-28}$ kg . Sono due reazioni di *fissione* che avvenivano nelle prime bombe atomiche. Da dove deriva la quantità enorme di energia liberata (dell'ordine di $9 \cdot 10^{13}$ J per 1 *g* di massa mancante), se l'energia totale si conserva?

A. Considera la (3): la variazione di massa corrisponderà ad variazione di

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B. Come si ottiene la conservazione di *E* ?

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Esercizio: a temperatura ambiente, l'energia cinetica dei neutroni usati nella fissione è circa di 0.02 eV. Di conseguenza la loro velocità è relativamente *bassa*: sono detti "neutroni lenti".

Sulla base dell'espressione relativistica $E = \gamma m c^2$ per l'energia totale e della formula per γ trovata, calcola la frazione della velocità della luce $\frac{v}{c} \equiv \beta$ corrispondente all'energia cinetica K_{rel} = 0.02 eV (assumi $m_n c^2 = 1\,\mathrm{GeV}$)

Problema 2 – Urti particellari

Considera un urto fra due particelle cariche identiche di massa *m* che collidano con velocità uguali e opposte, creando una terza particella a riposo. L'urto è elastico o anelastico?

- ☐ Elastico;
- ☐ Anelastico.

Se vale la conservazione dell'energia totale $E = E_0 + K$ di un sistema dobbiamo scrivere:

$$2mc^2 + 2K = Mc^2 \quad \Rightarrow \quad M > 2m$$

dove *M* indica la massa della particella creatasi. Quello che si può misurare operativamente, se c'è, è la *variazione* di energia cinetica, tramite i calorimetri dei rivelatori di particelle. Qui quali forme di energia variano?

Che cosa ne deduci?

- ☐ L'energia totale finale è uguale a quella iniziale (è una legge fondamentale della fisica) e lo stesso vale per la massa. Nel bilancio sopra abbiamo trascurato l'energia elettromagnetica emessa.

☐ Si conserva l'energia totale, ma non la massa. Varia l'energia cinetica, quindi anche l'energia a riposo.

☐ La massa non si conserva, come si vede dal bilancio energetico. L'energia cinetica e l'energia totale invece si conservano.

Come definiresti quindi l'energia relativistica di un corpo? Come si raccorda questa definizione con quella di *energia a riposo?*

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Massa ed energia

La massa è una grandezza che si *conserva* (il suo valore non cambia con il passaggio del tempo) nelle trasformazioni chimico-fisiche?

☐ Sì

☐ No;

Motiva la tua risposta

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Considera un corpo elettricamente carico, inizialmente in quiete rispetto al laboratorio, su cui forze elettriche esterne compiano lavoro. Tale corpo subirà un'accelerazione e la sua energia cinetica aumenterà. Si può affermare che aumenterà anche la sua massa? Motiva

☐ Sì

☐ No

➢ *Udine – Sample 1: pre and post-test*

PRE-TEST su MASSA-ENERGIA (25 minuti)

1. Una particella di massa *m* a riposo nel SI del laboratorio, se non interagisce con alcun oggetto (quindi non ci sono campi di forze) possiede energia? Motiva la tua risposta.

2. Considera osservatori dotati di tutti gli strumenti di misura conosciuti. Immagina che ciascun osservatore (ognuno in un diverso sistema inerziale) compia la misura di una certa grandezza fisica e ne registri l'esito su un taccuino. Se tutti gli osservatori, confrontando i loro taccuini, vedono *che hanno ottenuto lo stesso valore*, chiamano tale grandezza *invariante*. *Quindi una grandezza è detta invariante se il valore misurato (anche indirettamente) è lo stesso in tutti i sistemi di riferimento*. La massa è invariante?

☐ Sì, infatti...

☐ No, perché...

3. Se accelero una particella la sua massa varia? Motiva la risposta.

4. Posso far aumentare l'*energia* di un corpo *senza farne variare la velocità* rispetto al sistema di riferimento in cui mi trovo ?

☐ Sì, in questo modo:

☐ No, perché...

5. Considera due oggetti che si muovono uno contro l'altro a *velocità relativistiche*, sulla stessa retta ma con versi opposti. Essi si urtano e generano un unico oggetto. La massa del prodotto sarà uguale alla *somma delle masse* dei due oggetti iniziali? Spiega la tua risposta.

POST-TEST su MASSA-ENERGIA (15 minuti)

1. Una particella di massa *m* a riposo nel SI del laboratorio, se interagisce in modo trascurabile con qualque altro oggetto (ad esempio in una navicella nello spazio

intergalattico schermata dai campi elettrici e magnetici) possiede energia? Motiva la tua risposta.

2. *Una grandezza è detta invariante se il valore misurato (anche indirettamente) è lo stesso in tutti i sistemi di riferimento*. La massa è invariante?

☐ Sì, infatti...

☐ No, perché...

3. Se accelero una particella la sua massa varia? Motiva la risposta.

4. Posso far aumentare l'*energia* di un corpo *senza farne variare la velocità* rispetto al sistema di riferimento in cui mi trovo?

☐ Sì, in questi modi (indicane almeno due):

In questo caso, la massa del corpo cambia? Rispondi e motiva.

☐ No, perché...

5. Considera due oggetti non interagenti che si muovono uno contro l’altro a *velocità relativistiche*, sulla stessa retta ma con versi opposti. Essi si urtano e generano un unico oggetto. La massa del prodotto sarà uguale alla *somma delle masse* dei due oggetti iniziali? Spiega la tua risposta.

➢ *Udine – Sample 2: pre and post-test*

PRE-TEST (30 minuti)

1. Considera un sistema fisico isolato che si muove di moto vario. Su di esso possono agire sia *forze conservative* (ad esempio la forza gravitazionale, elettrica, elastica,…) sia non *forze non conservative* (ad esempio gli attriti). In generale, esiste una grandezza fisica posseduta dal corpo che *si conserva*, cioè che rimane costante nel tempo? Quale? Motiva la risposta.

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2. L'energia cinetica è:

○ La quantità espressa dall'equazione K = ½ m v^2 ;
○ La forma di energia associata allo stato di moto di una particella/punto materiale;
○ Un contributo all'energia totale di un sistema fisico in moto che è necessario considerare per poter testare la conservazione dell'energia totale;
○ Una combinazione delle precedenti (precisa quale).
Motiva la tua scelta.

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1. Dal teorema dell'energia cinetica segue che *se si compie lavoro su un corpo, se ne può aumentare la velocità e/o l'energia cinetica senza limite*? Spiega.

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2. Lo stesso vale per la *quantità di moto*? Spiega

3. Supponi di osservare su un treno un oggetto che si muove alla velocità 10 m/s rispetto a te. Il tuo compagno sul marciapiede della stazione valuta che il treno si muove a 25 m/s rispetto a lui *nello stesso verso del moto* dell'oggetto. Qual è allora la velocità dell'oggetto nel *sistema di riferimento inerziale* del marciapiede? Quale formula hai applicato per ottenerla?

4. A ciascun sistema di riferimento inerziale può essere associato un *osservatore*, dotato di aste metriche e orologi, che misuri distanze (intervalli di spazio) e durate (intervalli di tempo). Quindi ciascun *osservatore* è in moto rettilineo uniforme rispetto a tutti gli altri. La durata di un fenomeno, ad esempio la caduta di un oggetto a terra, ha lo stesso valore per tutti gli *osservatori?* Rispondi e illustra quale/i assunzione/i hai fatto per rispondere.

5. Consideriamo ora osservatori dotati di tutti gli strumenti di misura conosciuti. Immagina che ciascun osservatore compia la misura di una certa grandezza fisica e ne

registri l'esito su un taccuino. Se tutti gli osservatori, confrontando i loro taccuini, vedono *che hanno ottenuto lo stesso valore*, chiamano tale grandezza *invariante*. *Quindi una grandezza è detta invariante se ne viene misurato lo stesso valore in tutti i sistemi di riferimento*. Quali delle seguenti grandezze sono invarianti e perché?

○ Intervallo di tempo (durata)
○ Energia cinetica
○ Quantità di moto
○ Lunghezza
○ Velocità della luce nel vuoto
○ Velocità di un suono prodotto da uno degli osservatori considerati
○ Massa

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POST-TEST (25 minuti)

1. Considera un sistema fisico isolato che si muove di moto vario. In esso possono agire sia *forze conservative* (ad esempio la forza gravitazionale, elettrica, elastica,…) sia non *forze non conservative* (ad esempio gli attriti). In generale, esiste una grandezza fisica posseduta dal corpo che *si conserva*, cioè che rimane costante nel tempo? Quale? Motiva

la risposta.

2. L’energia cinetica è:

○ La quantità espressa dall’equazione $K = \frac{1}{2} m v^2$;
○ La forma di energia associata allo stato di moto di una particella/punto materiale;
○ Un contributo all’energia totale di un sistema fisico in moto che è necessario considerare per poter testare la conservazione dell’energia totale;
○ Una combinazione delle precedenti (precisa quale).
Motiva la tua scelta.

3. Dal teorema dell’energia cinetica segue che *se si compie lavoro su un corpo, se ne può aumentare la velocità e/o l’energia cinetica senza limite*? Spiega.

4. Lo stesso vale per la *quantità di moto*? Spiega

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5. Supponi di osservare su un treno un raggio laser che si propaga. Il tuo compagno sul marciapiede della stazione valuta che il treno si muove a 25 m/s rispetto a lui *nello stesso verso della propagazione della luce laser*. Qual è allora la velocità di quest'ultima nel *sistema di riferimento inerziale* del marciapiede? Spiega in che modo hai ottenuto questo risultato.

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6. A ciascun sistema di riferimento inerziale può essere associato un *osservatore*, dotato di aste metriche e orologi, che misuri distanze (intervalli di spazio) e durate (intervalli di tempo). Quindi ciascun *osservatore* è in moto rettilineo uniforme rispetto a tutti gli altri. La durata di un fenomeno, ad esempio la caduta di un oggetto a terra, ha lo stesso valore per tutti gli *osservatori?* Rispondi e illustra quale/i assunzione/i hai fatto per rispondere.

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7. Consideriamo ora osservatori dotati di tutti gli strumenti di misura conosciuti. Immagina che ciascun osservatore compia la misura di una certa grandezza fisica e ne registri l'esito su un taccuino. Se tutti gli osservatori, confrontando i loro taccuini, vedono *che hanno ottenuto lo stesso valore*, chiamano tale grandezza *invariante*. *Quindi una grandezza è detta invariante se il valore misurato (anche indirettamente) è lo stesso in tutti i sistemi di riferimento*. Quali delle seguenti grandezze sono invarianti e perché?

- Intervallo di tempo (durata)
- Energia cinetica

○ Quantità di moto
○ Lunghezza
○ Velocità della luce nel vuoto
○ Velocità di un suono prodotto da uno degli osservatori considerati
○ Massa

➢ *2014 summer school (Udine): (identical) pre and post-test*

Università degli Studi di Udine - Udine Unità di Ricerca in Didattica della Fisica
Scuola Estiva per Studenti di eccellenza in Fisica Moderna
Udine, 23-28 giugno 2014

PRE e POST-TEST (15 minuti)

Cognome _______________ Nome _______________ Data _______________________

1. L'energia cinetica è

- ○ La quantità espressa dall'equazione $K = \frac{1}{2} m v^2$;
- ○ La forma di energia associata allo stato di moto di una particella/punto materiale;
- ○ Un contributo all'energia totale di un sistema fisico in moto che è necessario considerare nei bilanci energetici globali;
- ○ Una combinazione delle precedenti (precisa quale).

Illustra le ragioni della tua scelta in almeno 3 righe

2. Supponi di osservare su un treno un raggio laser che si propaga. Il tuo compagno sul marciapiede della stazione valuta che il treno si muove a 25 m/s rispetto a lui *nello stesso verso* della propagazione della luce laser. Qual è la velocità di quest'ultima nel *sistema di riferimento inerziale* del marciapiede? Illustra i ragionamenti compiuti. [*La luce laser viaggia in aria con ottima approssimazione a c = 300000 km/s rispetto alla sorgente*]

3. A ciascun *sistema di riferimento inerziale* è associato uno e un solo *osservatore*, dotato di aste metriche e orologi, che misura distanze (intervalli di spazio) e durate (intervalli di tempo). Quindi *ciascun osservatore* è in moto rettilineo uniforme rispetto a tutti gli altri. La durata di un fenomeno, ad esempio la caduta di un oggetto a terra da un'altezza fissata,

ha lo stesso valore per tutti gli *osservatori?* Illustra l'assunzione/i fatta/e per rispondere.

4. Quanta energia possiede una particella di massa m a riposo nel laboratorio, se interagisce in modo trascurabile con qualunque altro oggetto (ad esempio se si trova nello spazio intergalattico schermata da campi elettrici e magnetici)? Motiva in almeno 3 righe.

5. Se una particella viene accelerata la sua massa varia? Scrivi il tuo ragionamento.

6. È possibile misurare la massa di un oggetto in un sistema di riferimento inerziale nel quale esso è in moto a velocità relativistiche? Illustra una procedura valida allo scopo.

➢ *Udine – Samples 3 and 4: (identical) pre, post-test and test for control group*

Questionario sulla massa (40 minuti)

PREMESSA: dopo ogni domanda c’è una ‘scala di fiducia’. Indica il *livello di confidenza che hai nella risposta che hai dato* con una crocetta.

1. La differenza fondamentale fra i concetti di massa e di peso coinvolge la gravità. Spiega perché facendo uso sia di concetti sia di equazioni fisiche.

Stima quanto sei sicuro/a della tua risposta

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ho tentato poco fiducioso/a neutro fiducioso/a certo/a

2. “Quantità di materia” è sinonimo di massa ?

○ Sì, infatti... (spiega)

○ No, perché ... (illustra le differenze)

Stima quanto sei sicuro/a della tua risposta

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ho tentato poco fiducioso/a neutro fiducioso/a certo/a

3. La massa totale di un sistema si conserva nelle trasformazioni che esso può subire, cioè traslazioni spaziali, traslazioni temporali, deformazioni, rotture, passaggi di stato, soluzioni e ossido-riduzioni ? [Nota: *la massa di un sistema fisico si conserva per traslazioni temporali se rimane costante nel tempo*]

○ Sì, infatti... (spiega)

○ No, perché ... (illustra almeno un caso in cui la massa di un corpo non si conserva in una trasformazione)

Stima quanto sei sicuro/a della tua risposta

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4. In fisica una grandezza è detta "additiva" se *tale grandezza relativa a un oggetto composto è pari alla somma della stessa grandezza nei costituenti il composto.* Ad esempio il volume di un *corpo rigido* è additivo, perché il volume di un corpo composto

è pari alla somma dei volumi dei corpi costituenti il composto. La temperatura invece non è additiva, perché la temperatura di un oggetto non è uguale alla somma delle temperature dei suoi costituenti. La massa è additiva? Motiva in almeno 3 righe.

Stima quanto sei sicuro/a della tua risposta

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5. Prova a scrivere tre o quattro parole chiave (sostantivi) legate al concetto di massa nell'ambito dei fenomeni relativi al *peso* dei corpi o alla *gravità*. Prova poi a scrivere due o tre parole chiave relazionate al concetto di massa legato all'*inerzia*. Formula infine almeno tre frasi in modo che (1) l'insieme delle frasi contenga tutte le parole chiave e (2) ogni frase ne contenga almeno due. Le parole chiave dei due ambiti (peso/gravità e inerzia) devono essere mescolate il più possibile in ciascuna frase.

Stima quanto sei sicuro/a della tua risposta

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ho tentato poco fiducioso/a neutro fiducioso/a certo/a

6. In quale modo può essere definita in maniera rigorosa la massa inerziale? Specifica le grandezze fisiche in gioco.

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Stima quanto sei sicuro/a della tua risposta

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ho tentato	poco fiducioso/a	neutro	fiducioso/a	certo/a

7. In fisica vengono definite quindi tre diverse nozioni di massa: quantità di materia, massa gravitazionale e massa inerziale. Esprimi con parole tue le differenze concettuali fra di esse e come si relazionano fra loro in almeno 5 righe.

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Stima quanto sei sicuro/a della tua risposta

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ho tentato	poco fiducioso/a	neutro	fiducioso/a	certo/a

Appendix 2 – Tutorials

➢ *2011 summer school (Udine)*

PARTE I – LA MASSA IN FISICA CLASSICA

Fino al Settecento la massa fu considerata essenzialmente come *quantitas materiae*, anche da Isaac Newton, che nei suoi *Principia Mathematica* (1687) scriveva:

> La quantità di materia è la misura della medesima ricavata dal prodotto della sua densità per il volume. […] Aria di densità doppia, in uno spazio a sua volta doppio, diventa quadrupla; in uno triplice, sestupla. La medesima cosa si capisca per la neve e la polvere condensate per compressione e liquefazione. E la norma di tutti i corpi, che siano diversamente condensati per cause qualsiasi, è identica[…]. In seguito indicherò questa quantità indifferentemente con i nomi di corpo o di massa.

Nel testo sopra riportato, quale di questi concetti risulta essere fondante in Newton?

- ☐ Materia;
- ☐ Massa;
- ☐ Corpo;
- ☐ Densità.

Perché?

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Nel seguito compare un metodo per la misura della massa:

> Tale quantità diviene nota attraverso il peso di ciascun corpo. Per mezzo di esperimenti molto accurati sui pendoli, trovai che è proporzionale al peso, come in seguito mostrerò.

La massa dunque non è il peso, ma è **numericamente proporzionale ad esso**.
Nella VII proposizione del III libro dei *Principia* viene asserito che ogni corpo dell'universo ne attrae ogni altro con una forza "proporzionale alle varie quantità di materia che essi contengono".

Altrove viene enunciata la **proporzionalità quadratica inversa** fra tale forza e la distanza tra un "luogo" su un corpo celeste ed il centro di un altro. In tal modo si perviene alla legge che (scritta in termini moderni) ben conosci:

$$F_g = G\,\frac{m_1\, m_2}{r^2}\,.$$

Questa legge, in cui compare l'espressione per il modulo della forza gravitazionale, ha la stessa forma di quella di Coulomb

$$F_e = \frac{k\, q_1\, q_2}{r^2}.$$

Osserva che le masse m_1, m_2 nella legge di gravitazione universale hanno lo stesso ruolo delle cariche elettriche. Sulla base di questa analogia, sai dire qual è il significato del termine **"massa gravitazionale"** ?

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Quale sarà allora la mutua interazione di due masse gravitazionali?

☐ Una forza di tipo attrattivo;
☐ Una forza di tipo attrattivo in certi casi, di tipo repulsivo in altri.

A proposito invece della tendenza che Newton chiama "forza insita" nella materia, si legge:

> Questa forza è sempre proporzionale al corpo [*termine che l'autore usa come sinonimo di massa, n.d.r.*], né differisce in alcunché dall'inerzia della massa altrimenti che per il modo di concepirla. A causa dell'inerzia della materia, accade che ogni corpo è rimosso con difficoltà dal suo stato di quiete o di moto.

Il fisico e filosofo Ernst Mach (1838 – 1916) fa la seguente critica a Newton ne *La meccanica nel suo sviluppo storico-critico*:

> Per quanto riguarda il concetto di massa, osserviamo che la formulazione data da Newton è infelice. Egli dice che la massa è la *quantità di materia* di un corpo misurata dal prodotto del suo volume per la densità. Il circolo vizioso è evidente. La densità infatti non può essere definita se non come la massa nell'unità di volume. Newton si è reso conto che in ogni corpo è inerente una proprietà quantitativa che determina il movimento ed è diversa dal peso [...] ma non è riuscito a esporre questa conoscenza in modo corretto.

Qui viene messo in luce che **già in Newton** la massa non è più la semplice *quantità di materia*, nonostante egli la enunci in tal modo: è un concetto in divenire nella sua mente. C'è differenza con la massa legata alla gravitazione? Illustra.

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Mach ha un approccio completamente diverso da Newton: la sua definizione di massa è uno strumento **per organizzare i fatti** ottenuti dall'esperienza (*empiriocriticismo*).

> *Diciamo corpi di massa uguale quelli che, agendo uno sull'altro, si comunicano accelerazioni uguali ed opposte*. [...] *Se scegliamo il corpo A come unità di misura, attribuiremo la massa m a quel corpo che imprime ad A un'accelerazione pari a m volte l'accelerazione che esso riceve da A*. Il rapporto delle masse è il rapporto inverso delle accelerazioni preso con segno negativo. [...] Il nostro concetto di massa non deriva da alcuna teoria. Esso contiene soltanto la precisa determinazione, designazione e definizione di un fatto. La "quantità di materia" è del tutto inutile. [...] La mia definizione di massa è il

risultato di una ricerca volta a stabilire *l'interdipendenza dei fenomeni*.

Da un principio di simmetria, esteso a priori sulla base dell'esperienza dal caso di corpi identici a quello di corpi con caratteristiche differenti, si deduce la definizione sopra. Quale principio della dinamica classica è logicamente equivalente ad essa?

- ☐ I
- ☐ II
- ☐ III

Mach vede tale principio strettamente legato al concetto di massa.

Domanda di gruppo. In definitiva, quindi, quali sono le differenze concettuali fra le nozioni di massa esaminate finora?

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Elementi di moto armonico

Il moto di un punto materiale di massa inerziale m_i connesso all'estremità di una molla è regolato dalla legge di Hooke. Se la molla è ideale, il moto compiuto dal punto materiale è *armonico.*

La legge fenomenologica di Hooke può essere scritta nella forma

$$\ddot{x} = -\left(\frac{k}{m_i}\right)x,$$

equazione differenziale che ha soluzione del tipo

$$x = A\cos(\omega t + \phi),$$

con $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m_i}{k}}$ (1),

come risulta sostituendo la soluzione nell'equazione di partenza (metodo usato anche per verificare l'esattezza della soluzione ipotizzata). ω è chiamata *pulsazione* del moto armonico.

Pendolo semplice

Considera ora il moto di un pendolo semplice, rappresentato nella figura sotto. Se spostiamo il filo di un piccolo angolo ϑ rispetto alla verticale, il pendolo inizierà ad oscillare sotto l'azione della componente della forza peso ortogonale al filo, esprimibile come $m_g\, g\, sen\vartheta$, m_g essendo la massa gravitazionale.

- Perché nell'espressione della forza compare la massa gravitazionale e non quella inerziale?

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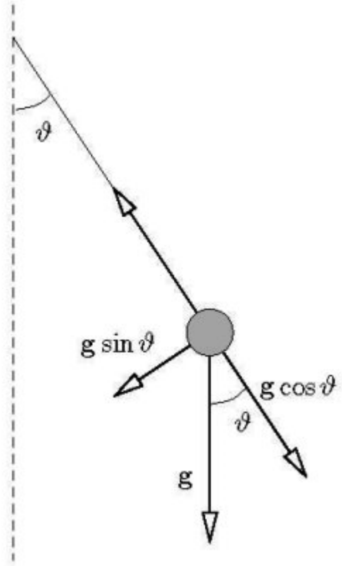
Ho trattato il moto di tipo *armonico* – fondamentale in fisica – perché ci fornisce un modello formale per il pendolo semplice (= pendolo in cui il filo, inestensibile, ha massa trascurabile rispetto al pesetto). Il pendolo semplice, per *piccole oscillazioni*, si muove infatti approssimativamente di moto armonico; l'entità del discostamento da quest'ultimo cresce con l'aumentare dell'ampiezza di oscillazione.

Per esplorare le relazioni concettuali fra inerzia e massa gravitazionale, consideriamo il rapporto $\frac{m_i}{m_g}$. Poiché abbiamo visto che i due concetti sono completamente separati, **supponiamo che tale rapporto sia differente per corpi di materiale differente e vediamo le conseguenze.**



Lo spostamento, invece che essere lineare come nel caso della molla, avviene lungo un arco di circonferenza; perciò la lunghezza *D(t)* dell'arco percorso può essere usata come variabile spostamento. Dato che $D=L\theta$

$$F = -m_g\, g\, sen\,\theta = m_i\, a = m_i\, L\frac{d^2}{dt^2}\theta\,.$$

Per *piccoli angoli*, si può fare l'approssimazione *sen* $\vartheta \approx \vartheta$; il limite superiore per l'angolo dipende dall'entità dell'errore relativo che vogliamo sul periodo. Per dare un'idea, se $\theta < 15°$ il periodo reale si discosterà da quello che ricaviamo con questa sostituzione per meno dello 0.5%.

L'equazione sopra diventa allora (usando la notazione con i "puntini" per la derivata)

$$\ddot{\theta} = -\left(\frac{m_g}{m_i}\frac{g}{L}\right)\theta,$$

in analogia con la trattazione della molla. Da quest'equazione, in cui chiameremo ω *frequenza angolare*, ricava un'espressione per T (*fai i calcoli sul tuo quaderno*).

$T =$ ____________________________

Confronta il tuo risultato con il *tutor*.

- Da quali grandezze modificabili sperimentalmente viene allora a dipendere il periodo?

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➢ Per piccoli angoli tutti i pendoli sono *isocroni*: prendendo oggetti con lo stesso volume, ma di materiale diverso, il periodo di oscillazione di un dato pendolo non cambia. Questo si può vedere con strumenti di misura del periodo ad alta precisione, entro i limiti degli errori sperimentali.

Domanda di gruppo. Data l'espressione che si ottiene per il periodo, quali conclusioni puoi trarre sul rapporto fra massa inerziale e gravitazionale?

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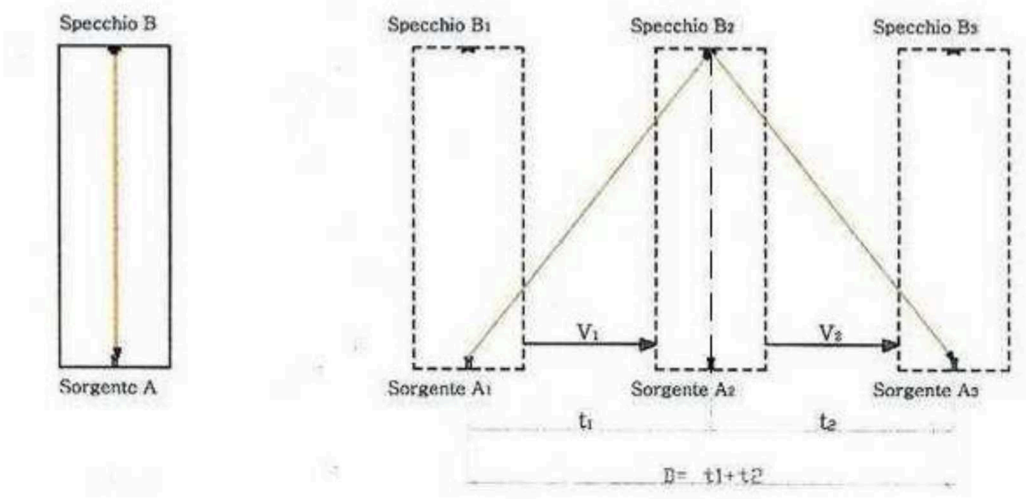
PARTE II – LA MASSA IN RELATIVITÀ

2. *Orologio a luce.*



Orologio a luce fermo

Orologio a luce in moto

Considera due sistemi di riferimento (SdR) in moto relativo rettilineo uniforme tra loro a velocità *v*, come visualizzato nell'Applet. Si possono associare infiniti osservatori, che misurino tempi e lunghezze, a ciascun sistema di riferimento. Nel SdR *K* è posta una sorgente luminosa A che emette luce visibile, la quale viene riflessa a distanza perpendicolare *h* da uno specchio piano B e ritorna al punto di partenza, percorrendo così il cammino a sinistra in figura in un tempo $\Delta t = \frac{2h}{c}$. Sia *K'* l'altro SdR; in esso il raggio di luce percorre un cammino obliquo; si viene così a formare il triangolo isoscele A1B2A3.

Nell'Applet compare il tempo $\Delta t'$ (*Jack's time*) che il raggio luminoso impiega per arrivare ad A3, come **misurato dall'osservatore rispetto cui l'orologio è in moto** (Jack)**, supponendo che la velocità della luce abbia lo stesso valore in *K* ed in *K'*. Tale intervallo temporale non è uguale a** Δt , misurato dall'orologio di Jill (*Jill's time*). La luce percorre infatti un tragitto maggiore (l'ipotenusa è sempre maggiore di un cateto) alla medesima velocità.

Svolgiamo i calcoli in *K'*: la sorgente si muove orizzontalmente di uno spazio che chiamiamo $\Delta x'$ (D in figura). Si ha allora

$$c\Delta t' = 2\sqrt{h^2 + \left(\frac{\Delta x'}{2}\right)^2} = 2\sqrt{\left(\frac{\Delta t}{2}c\right)^2 + \left(\frac{\Delta x'}{2}\right)^2} = \sqrt{(c\Delta t)^2 + (\Delta x')^2}.$$

Tenendo conto del fatto che $\frac{\Delta x'}{\Delta t'} = -v$, dove *v* è la velocità di *K'* rispetto a *K*, si trova la relazione fra $\Delta t'$ e Δt. Svolgi i calcoli sul tuo quaderno.

Risulta $\boxed{\Delta t' = \gamma \Delta t}$

scrivi l'espressione per il fattore di Lorentz: γ = _________________________________.

Di conseguenza, dalla semplice postulazione dell'**invarianza** di *c* (vale a dire che *poniamo* che tale valore sia indipendente dal SdR in cui è misurato) segue qualcosa di nuovo riguardo al passaggio del tempo in due SdR in moto relativo uniforme fra loro: la *dilatazione dei tempi*.

3. In Relatività si parla di **"evento"**, riferendosi a qualcosa che è avvenuto in un preciso punto dello spazio tridimensionale e ad un determinato istante. L'istante e la posizione (tempo e spazio rispettivamente) in cui avviene qualcosa, pur rimanendo distinti, sono uniti fra loro dalla formulazione di Einstein, a tal punto che viene stabilita l'esistenza dello *spazio-tempo*. Quest'ultimo è stato interpretato geometricamente dal matematico Hermann Minkowsi.

4. L'intervallo temporale fra due eventi legati ad uno stesso oggetto, misurato in un SdR solidale ad esso, è detto intervallo di *tempo proprio.* Nel caso dell'orologio a luce tale intervallo $\Delta\tau$ è quello misurato nel SdR *K*: si ha dai calcoli precedenti

$$(\Delta s)^2 \equiv (c\Delta\tau)^2 = (c\Delta t')^2 - (\Delta x')^2 \quad (2)$$

Se considerassimo un altro SdR si avrebbe ancora la stessa espressione, pur con distanze spaziali ed intervalli temporali diversi. Perciò il tempo proprio ha un valore indipendente dal SdR in cui viene misurato: è **un invariante relativistico.**

Dall'espressione della dilatazione dei tempi segue che l'intervallo di tempo proprio è legato all'intervallo Δt fra gli stessi eventi, ma misurato in un SdR in moto con velocità *v* rispetto al SdR solidale, dalla relazione

$\Delta\tau =$ ______________.

5. Nello spazio euclideo lo **spostamento** è rappresentato, in coordinate cartesiane, dal trivettore $(\Delta x, \Delta y, \Delta z)$. Il suo modulo, vale a dire la distanza fra due punti legati dal moto di un oggetto fisico, non dipende dal particolare SdR in cui è calcolato: è un *assoluto*. Analogamente si può introdurre un **quadrivettore** nello *spazio-tempo*: $(c\Delta t, \Delta x, \Delta y, \Delta z)$. La sua norma (l'equivalente del modulo di un vettore cartesiano) si indica con Δs e risulta **invariante** dalla definizione sopra: essa coincide con l'intervallo di tempo proprio, a meno di un fattore *c.* Estendendo la (2) alle dimensioni spaziali *y*, *z* abbiamo

$$(\Delta s)^2 \equiv (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2,$$

per coordinate *t, x, y, z* generiche. La coordinata temporale viene moltiplicata per *c* per ragioni dimensionali.

Osserva il piano in figura: in ordinata è rappresentato il tempo *ct* ed in ascissa una sola dimensionale spaziale, *x*. Esso è detto diagramma di Minkowski. Perché le rette a 45° vengono chiamate “rette luce”?

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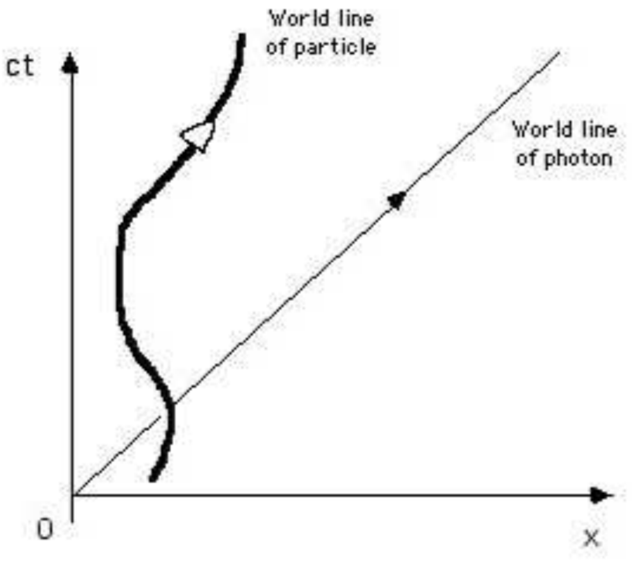
- Nota la differenza importante fra la “distanza” nello spazio-tempo geometrizzato di Minkowski e la distanza nello spazio euclideo, cui sei abituato. Nella *metrica* minkowskiana si **sottrae** al termine temporale la somma dei quadrati delle componenti spaziali al termine temporale. Non è una semplice somma di quadrati.

6. Vogliamo ora costruire un quadrivettore che descriva il moto di una particella di massa *m* nello spazio-tempo; in quest’ultimo non si parla di traiettorie ma di **linee d’universo** (vedi figura). Tale quadrivettore avrà una direzione nello spazio-tempo; sarà quella del quadrivettore spostamento?

   ☐ Sì
   ☐ No

Spiega il tuo ragionamento al *tutor*.



7. Come possiamo costruire il nuovo quadrivettore, che sarà analogo all’impulso newtoniano $\vec{p} = m\frac{d\vec{x}}{dt}$? La derivata è il limite di un rapporto fra differenze finite. In ambito classico facciamo una divisione fra un **vettore di modulo invariante**, lo spostamento, ed uno **scalare invariante**, il tempo (classicamente l’intervallo temporale fra due eventi è sempre lo stesso, *in qualunque stato di moto lo si misuri*), per poi fare il limite $\Delta t \to 0$.

8. Analogamente, in Relatività dovremo fare il rapporto fra un **quadrivettore di norma invariante** e uno **scalare invariante** nello spazio-tempo; quest'ultimo ovviamente non potrà più essere un tempo generico *t.* Il tutto va moltiplicato per la massa.

   Prova a scrivere il rapporto opportuno per ciascuna componente, compresa quella temporale, che nella fisica classica manca (quest'ultima va moltiplicata per *c* per ragioni dimensionali)

$$quadrimomento = m\left(\rule{1.5cm}{0.4pt}, \rule{1.5cm}{0.4pt}, \rule{1.5cm}{0.4pt}, \rule{1.5cm}{0.4pt} \right) = (\ldots\ldots\ldots, \ldots\ldots\ldots, \ldots\ldots\ldots, \ldots\ldots\ldots)$$

Facciamo ora il *limite newtoniano* delle espressioni ottenute: calcoliamo i valori che assumono per velocità dei punti materiali di modulo *u<<c.* In tal modo ci riconduciamo ad un ambito a noi noto, così da poter "riconoscere" queste quantità.

Attenzione: qui ho cambiato il simbolo per la velocità da *v* ad *u* perché stiamo analizzando la cinematica di corpi animati da moto vario, non rettilineo uniforme. Perciò il simbolo γ viene ad assumere un significato diverso da prima: ora immaginiamo un passaggio fra un SdR istantaneamente in quiete con il corpo ("riferimento comovente") ed il SdR in cui ci si pone.

Segui i calcoli del *tutor*.

La 2°, 3° e 4° componente nello sviluppo per basse velocità hanno la forma della quantità di moto classica. Estendendo euristicamente al caso relativistico, possiamo porre come forma per l'impulso relativistico l'espressione $\vec{p}_{rel} =$ ________________.

La 1° componente è la più interessante per noi: il secondo termine è l'espressione dell'energia cinetica classica, perciò possiamo interpretare tutta l'espressione come **energia totale** *E*, per ragioni di omogeneità. Quindi $E \underset{u<<c}{\equiv} mc^2 + \frac{1}{2}mu^2 \Rightarrow E - E_0 \underset{u<<c}{=} \frac{1}{2}mu^2$, dove E_0 è la costante additiva arbitraria che compete ad ogni definizione di energia. Facendo la generalizzazione possiamo assumere come espressione per l'energia cinetica relativistica

$$K_{rel} \equiv E - E_0 = \gamma mc^2 - mc^2 = mc^2(\gamma - 1).$$

In tal modo abbiamo attuato l'identificazione (3)

$$\boxed{E_0 = mc^2}$$

e l'abbiamo generalizzata al caso relativistico. **Di conseguenza la somma di tutti i contributi all'energia di un corpo, escluso quello cinetico, è data dalla *massa* totale del corpo stesso, a meno di un fattore c^2 (equivalenza massa-energia). Alternativamente, la massa è uguale a tutta l'energia di un corpo quando è in quiete ("energia interna").** Numericamente, c'è il fattore c^2 fra le due quantità, ma in opportune unità di misura può essere reso pari a 1.

Esercizio

Considera la seguente reazione nucleare:

$$n + {}^{238}U \rightarrow {}^{143}Ba + {}^{93}Kr + 3n\,,$$

indotta dall'urto di un neutrone termico (= avente energia cinetica dell'ordine di *kT*) con un nucleo dell'isotopo di Uranio.

Un'altra è

$$n + {}^{235}U \rightarrow {}^{93}Rb + {}^{141}Cs + 2n\,,$$

anch'essa indotta da un neutrone termico (a temperatura ambiente, la loro energia cinetica è circa di 0.02 *eV*: sono detti "neutroni lenti") nell'urto con l'isotopo-235 dell'Uranio.
In entrambi i casi, se si fa la somma delle masse dei due nuclei prodotti essa risulta *minore* di quella del nucleo di Uranio: $\Delta M = 3.02 * 10^{-28}$ kg < 0.

Sono due reazioni di *fissione*, che avvenivano nelle prime bombe atomiche. Da dove deriva la quantità enorme di energia liberata (dell'ordine di $9 * 10^{13}$ J per ogni *g* di massa mancante), considerato che l'energia totale si conserva?

C. Considera la (3): la variazione di massa corrisponderà ad variazione di ____;

D. Quindi come si ottiene la conservazione di *E* ?

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Urti

Considera un urto fra due particelle di massa identica *m* che collidano con velocità uguali ed opposte, creando una terza particella a riposo.

L'urto è elastico o anelastico?

- ☐ Elastico;
- ☐ Anelastico.

Se vale la conservazione dell'energia totale $E = E_0 + K$ di un sistema dobbiamo scrivere, per quello che abbiamo definito,

$$2mc^2 + 2K = Mc^2 \quad \Rightarrow \quad M > 2m\,,$$

dove con *M* indichiamo la massa della particella creata.

Quello che si può misurare, se c'è, è la *variazione* di energia cinetica. Qui quali forme di energia variano?

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Che cosa ne deduci?

- ☐ L'energia totale finale è uguale a quella iniziale (è una legge fondamentale della fisica) e lo stesso vale per la massa. Nel bilancio sopra abbiamo trascurato l'energia elettromagnetica emessa.
- ☐ Si conserva l'energia totale, ma non la massa. Varia l'energia cinetica, quindi anche l'energia a riposo E_0. Quando varia E_0 la massa varia nello stesso senso: la (3) vale anche nella forma $\Delta E_0 = (\Delta m)c^2$.
- ☐ La massa non si conserva, come si vede dal bilancio energetico. L'energia cinetica si conserva, quindi $E_0 = E - K$ si conserva.

Additività

Quali grandezze fisiche "additive" conosci?

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La massa inerziale è fra queste?

- ☐ Sì
- ☐ No;

Perché ?

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Un nucleo atomico ha massa uguale, inferiore o superiore alla somma delle masse dei neutroni e dei protoni che lo compongono? _______________________.

Giustifica la tua scelta

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9. *Domanda di gruppo*. Quali sono gli aspetti del concetto di massa che cambiano rispetto a quello classico?

➢ *Crotone*

PARTE I – LA MASSA IN FISICA CLASSICA

Fino al Settecento la massa fu considerata essenzialmente come “quantità di materia”, a partire da Isaac Newton, che nei suoi *Principia Mathematica* (1687) scriveva:

> La quantità di materia è la misura della medesima ricavata dal prodotto della sua densità per il volume. […] Aria di densità doppia, in uno spazio a sua volta doppio, diventa quadrupla; in uno triplice, sestupla. La medesima cosa si capisca per la neve e la polvere condensate per compressione e liquefazione. E la norma di tutti i corpi, che siano diversamente condensati per cause qualsiasi, è identica […]. In seguito indicherò questa quantità indifferentemente con i nomi di corpo o di massa.

Nel seguito compare un metodo per la misura della massa:

> Tale quantità diviene nota attraverso il peso di ciascun corpo. Per mezzo di esperimenti molto accurati sui pendoli, trovai che è proporzionale al peso, come in seguito mostrerò.

La massa dunque non è il peso, ma è **numericamente proporzionale ad esso**; tramite il peso Newton determinava la massa.

Nella VII proposizione del III libro dei *Principia* viene asserito che ogni corpo dell’universo ne attrae ogni altro con una forza “proporzionale alle varie quantità di materia che essi contengono”.

Altrove viene enunciata la **proporzionalità quadratica inversa** fra tale forza e la distanza tra un “luogo” su un corpo celeste ed il centro di un altro. In tal modo si perviene alla legge che (scritta in termini moderni) ben conosci:

$$F_g = G\,\frac{m_1\,m_2}{r^2}\,.$$

Il peso è una forza di tipo gravitazionale. Per corpi in prossimità della superficie terrestre si può introdurre l’intensità (costante[125]) di campo gravitazionale $g \equiv \frac{GM_T}{R^2} \quad \Rightarrow \quad F_g = P = m_g g$

Questa legge, in cui compare l’espressione per il modulo della forza gravitazionale, ha la stessa forma di quella di Coulomb

$$F_e = \frac{k\,q_1\,q_2}{r^2}\,.$$

Osserva che le masse m_1, m_2 nella legge di gravitazione universale hanno lo stesso ruolo delle cariche elettriche.

Quale idea di massa emerge dai 3 contributi riportati?

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[125] A meno di variazioni (piccole in percentuale) della distanza R dal centro della Terra.

Tale idea di massa è completa? Motiva la risposta

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A proposito di quella tendenza che Newton chiama "forza insita" nella materia si legge :

> Questa forza è sempre proporzionale al corpo [*termine che l'autore usa come sinonimo di massa, n.d.r.*], né differisce in alcunché dall'inerzia della massa altrimenti che per il modo di concepirla. A causa dell'inerzia della materia, accade che ogni corpo è rimosso con difficoltà dal suo stato di quiete o di moto.

Il fisico e filosofo Ernst Mach (1838 – 1916) fa la seguente critica a Newton ne *La meccanica nel suo sviluppo storico-critico*:

> Per quanto riguarda il concetto di massa, osserviamo che la formulazione data da Newton è infelice. Egli dice che la massa è la *quantità di materia* di un corpo misurata dal prodotto del suo volume per la densità. Il circolo vizioso è evidente. La densità infatti non può essere definita se non come la massa nell'unità di volume. Newton si è reso conto che in ogni corpo è inerente una proprietà quantitativa che determina il movimento ed è diversa dal peso […] ma non è riuscito a esporre questa conoscenza in modo corretto.

Nei 3 contributi viene messo in luce che **già in Newton** la massa non è più la semplice «quantità di materia», nonostante egli la descriva in tal modo: è un concetto in divenire nella sua mente. Quali differenze trovi fra l'aspetto riportato da Mach e la massa che compare nella legge di gravitazione? Illustra.

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Mach ha un approccio completamente diverso da Newton: la sua definizione di massa è uno strumento **per organizzare i fatti** ottenuti dall'esperienza (*empiriocriticismo*).

> *Diciamo corpi di massa uguale quelli che, agendo uno sull'altro, si comunicano accelerazioni uguali ed opposte*. […] *Se scegliamo il corpo A come unità di misura, attribuiremo la massa m a quel corpo che imprime ad A un'accelerazione pari a m volte l'accelerazione che esso riceve da A*. Il rapporto delle masse è il rapporto inverso delle accelerazioni preso con segno negativo. […] Il nostro concetto di massa non deriva da alcuna teoria. Esso contiene soltanto la precisa determinazione, designazione e definizione di un fatto. La "quantità di materia" è del tutto inutile.

Alla base di questa definizione di massa vi è un principio di simmetria.
In termini matematici

$$\frac{m_2}{m_1} = \frac{a_1}{a_2} \quad \Rightarrow \quad m_2 = \frac{a_1}{a_2} m_1$$

Quale/i principio/principi della dinamica classica implica quest'ultima relazione ?

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Mach considerava tale/i principio/i strettamente legato al concetto di massa.

Domanda di gruppo. In definitiva, quindi, quali sono le differenze concettuali fra le nozioni di massa esaminate finora?

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In fisica classica la massa si conserva. Illustra che cosa comporta questa legge in caso di interazioni e trasformazioni chimiche e fisiche,

1. riguardo alla *misura* della massa

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2. riguardo alle proprietà della massa come grandezza scalare

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➢ *Cesena*

PARTE I – LA MASSA IN FISICA CLASSICA

Fino al Settecento la massa fu considerata essenzialmente come “quantità di materia”, a partire da Isaac Newton, che nei suoi *Principia Mathematica* (1687) scriveva:

> La quantità di materia è la misura della medesima ricavata dal prodotto della sua densità per il volume. […] Aria di densità doppia, in uno spazio a sua volta doppio, diventa quadrupla; in uno triplice, sestupla. La medesima cosa si capisca per la neve e la polvere condensate per compressione e liquefazione. E la norma di tutti i corpi, che siano diversamente condensati per cause qualsiasi, è identica […]. In seguito indicherò questa quantità indifferentemente con i nomi di corpo o di massa.

Nel seguito compare un metodo per la misura della massa:

> Tale quantità diviene nota attraverso il peso di ciascun corpo. Per mezzo di esperimenti molto accurati sui pendoli, trovai che è proporzionale al peso, come in seguito mostrerò.

La massa dunque non è il peso, ma è **numericamente proporzionale ad esso**; tramite il peso Newton determinava la massa.

Nella VII proposizione del III libro dei *Principia* viene asserito che ogni corpo dell’universo ne attrae ogni altro con una forza “proporzionale alle varie quantità di materia che essi contengono”.

Altrove viene enunciata la **proporzionalità quadratica inversa** fra tale forza e la distanza tra un “luogo” su un corpo celeste ed il centro di un altro. In tal modo si perviene alla legge che (scritta in termini moderni) ben conosci:

$$F_g = G\,\frac{m_1 m_2}{r^2}\,.$$

Il peso è una forza di tipo gravitazionale. Per corpi in prossimità della superficie terrestre si può introdurre l’intensità (costante[126]) di campo gravitazionale $g \equiv \frac{GM_T}{R^2} \quad \Rightarrow \quad F_g = P = m_g g$

Questa legge, in cui compare l’espressione per il modulo della forza gravitazionale, ha la stessa forma di quella di Coulomb

$$F_e = \frac{k\,q_1\,q_2}{r^2}\,.$$

Osserva che le masse $m_1\,, m_2$ nella legge di gravitazione universale hanno lo stesso ruolo delle cariche elettriche.

Quale idea di massa emerge dai 3 contributi riportati?

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[126] A meno di variazioni (piccole in percentuale) della distanza R dal centro della Terra.

Tale idea di massa è completa? Motiva la risposta

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A proposito di quella tendenza che Newton chiama "forza insita" nella materia (oggi denominata "inerzia") si legge :

> Questa forza è sempre proporzionale al corpo [*termine che l'autore usa come sinonimo di massa, n.d.r.*], né differisce in alcunché dall'inerzia della massa altrimenti che per il modo di concepirla. A causa dell'inerzia della materia, accade che ogni corpo è rimosso con difficoltà dal suo stato di quiete o di moto.

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Domanda di gruppo. In definitiva, quindi, quali sono le differenze concettuali fra le nozioni di massa esaminate finora?

In fisica classica la massa si conserva. Illustra che cosa comporta questa legge in caso di interazioni e di trasformazioni chimiche e fisiche,

2. riguardo alla *misura* della massa

3. riguardo alle proprietà della massa come grandezza scalare

➢ *Udine – Sample 2*

1. Abbiamo svolto un percorso concettuale che ci ha portato a comprendere la *non validità* delle trasformazioni di Galileo. Illustralo con una mappa concettuale, scrivendo i sostantivi più importanti collegati da verbi; spiega la mappa chiarendo che cosa hai imparato.

2. Perché sincronizziamo gli orologi con un *segnale luminoso* e non di altro tipo, ad esempio sonoro?

3. Abbiamo dotato l'osservatore sul treno e quello sul marciapiede di due orologi identici.

a. I battiti del cuore del primo osservatore risultano *realmente rallentati* per il secondo?
b. Il secondo osservatore sente i propri battiti rallentati?
Spiega.

4. Abbiamo dotato di due orologi identici un manovratore di ponti e un passeggero su un treno in corsa sul ponte.
a. L'intervallo di tempo fra l'apertura e la chiusura di un ponte mobile *è differente* per il passeggero del treno rispetto al manovratore di ponti?
Spiega.

NOME	COGNOME

1. Scrivi una ragione per la quale la quantità di moto non può essere scritta a velocità relativistiche nella forma classica $p = m\,v$.

2. Spiega la differenza fra il $\gamma(v)$ che compare nella formula di dilatazione delle durate e il $\gamma(u)$ che compare nell'espressione della quantità di moto.

Appendix 3 – IBL Tutorials

➢ *2012 winter school (Bard, AO)*

Moti relativi e velocità della luce

1) In una gara di nuoto a staffetta viene emesso un suono sotto il pelo dell’acqua, quando i primi nuotatori sono già partiti. La velocità dell’onda sonora a 20°C *rispetto all’*acqua è 1482 m/s.

   a) Qual è la velocità dell’onda sonora per un atleta che nuota ad una velocità costante di circa 2 m/s rispetto all’acqua (a sinistra in figura)?

   (spiega)

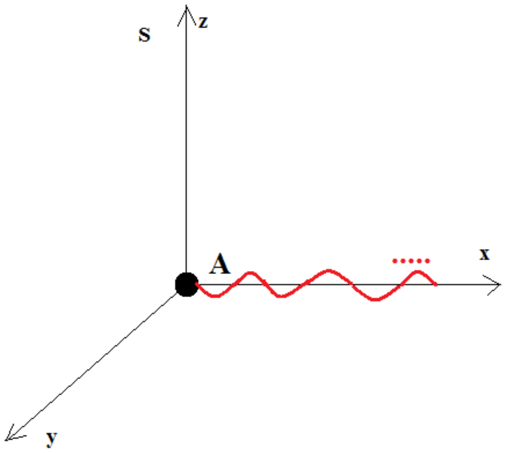
   b) Qual è la velocità dell’onda sonora per un altro atleta fermo che aspetta di partire (a destra in figura)? Rispondi e motiva:

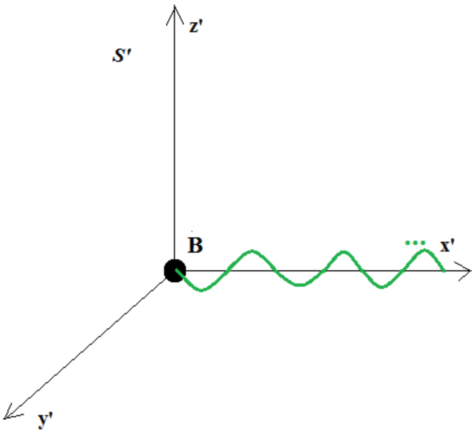
2) Considera un’ipotetica gara di atletica (staffetta 4 x 100) *sulla Luna,* dove non c’è aria. Uno dei velocisti dell’ultimo tratto porta con sé una piccola torcia elettrica accesa (A); un tecnico fermo nella corsia dell’atleta accende una seconda torcia (B) quando parte l’ultima batteria.

a. Con quale velocità si propaga la luce dalla torcia A rispetto all’atleta? ____________________

Con quale velocità si propaga la luce della sorgente B rispetto al tecnico? ____________________

b. Poniamo che A sia nell’origine degli assi cartesiani in un *sistema di riferimento* che indichiamo con *S*. Poniamo inoltre che B sia nell’origine degli assi cartesiani in un *sistema di riferimento* che indichiamo con *S’*, come raffigurato sotto.





3) Se la sorgente A si muove di moto rettilineo uniforme verso B, con velocità **v** parallela e concorde all’asse *x’*, qual è la velocità di un raggio luminoso emesso da A in direzione di B per un osservatore in *S’* ?

i. Rappresenta la situazione con un disegno schematico:

ii. Rispondi e motiva

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4) Quali sono le possibili risposte alla domanda 3, emerse dalla discussione? Da quali ipotesi derivano?

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Simultaneità in Relatività Ristretta

1) Vi sono due osservatori, A sul marciapiede di una ferrovia, B a metà vagone di un treno in corsa (vedi figura sotto). In corrispondenza della testa e della coda del vagone vi sono due sorgenti luminose, che si accendono quando toccano due dispositivi (*firing devices*) sui binari.

2) Ci poniamo in quiete con A, che riceve simultaneamente due raggi dalle sorgenti.

a) Le sorgenti si sono accese simultaneamente? (spiega)

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b) In che modo A (definito osservatore "intelligente" in Relatività) *ricostruisce* se i due eventi di accensione sono simultanei ?

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c) Spiega perché è necessario fare queste "ricostruzioni" e "valutazioni" degli istanti di emissione dei segnali e non è sufficiente quello che si percepisce o si stima direttamente.

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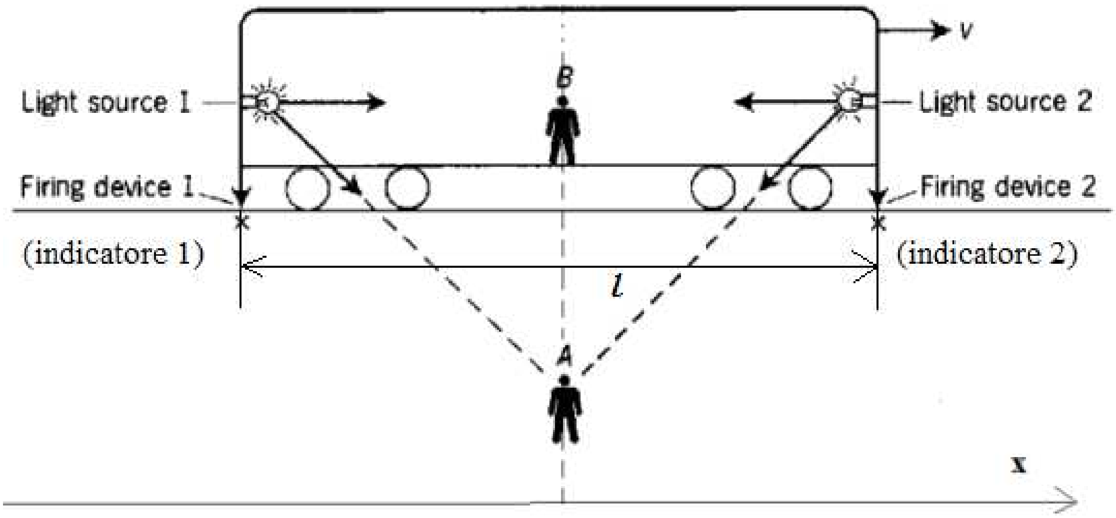

5) In Relatività un *sistema di riferimento,* o semplicemente *riferimento*, è costituito (1) da una classe di osservatori – dotati di regoli e orologi – in tutti i punti dello spazio, e (2) da una terna di assi cartesiani ortogonali.

Si può anche immaginare come un reticolo infinitamente esteso di orologi collegati da regoli ortogonali fra loro. In Relatività Ristretta si prendono in considerazione soltanto i *riferimenti inerziali*, cioè quelli nei quali è valida la I legge della dinamica classica.

6) La relazione di causalità ("la causa viene prima dell'effetto") può essere invertita nel passaggio da un sistema di riferimento inerziale ad un altro? Rispondi e motiva

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7) Poiché la velocità della luce è la stessa per i due versi di propagazione (II postulato), il fronte d'onda emesso dalla sorgente 2 arriva a B *prima* di quello emesso da 1.

- Può essere utile a B questa informazione (sempre secondo la ricostruzione di A) per dare un giudizio sull'ordinamento temporale dei due eventi di accensione? Perché?

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- Quali ragionamenti / operazioni eseguirà allora B per *giudicare se* i due eventi di accensione sono simultanei?

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- Che cosa concluderà B (sempre secondo quanto ricostruito dal nostro osservatore A) in merito alla simultaneità dei due eventi?

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8) Consideriamo altri due osservatori C (sul treno) e D (a terra). C non è a metà del vagone e non passa davanti a D all'istante t_0, come si vede in figura. Il tutto è valutato da A.

- Entrambi gli osservatori A e D *percepiscono* i segnali simultaneamente? (Spiega)

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- Entrambi gli osservatori A e D *valutano* che le accensioni avvengono simultaneamente?

Spiega____

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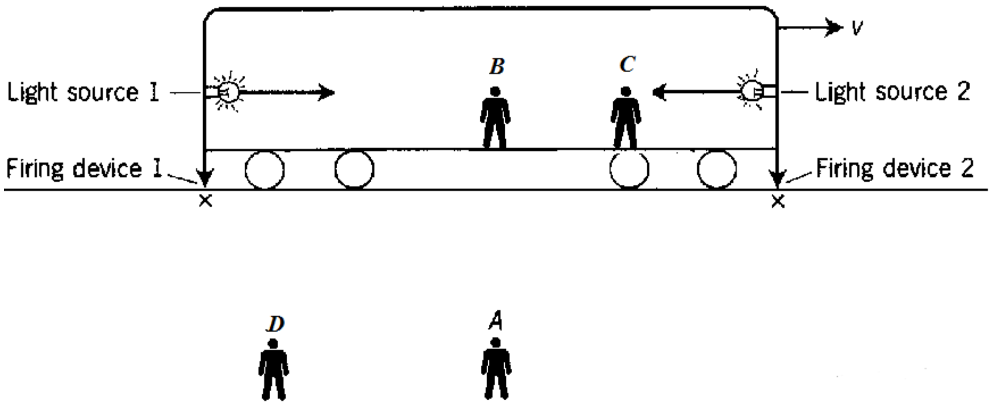

- Entrambi gli osservatori B e C *valutano* che le accensioni *non* avvengono simultaneamente?

Spiega____

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- L'osservatore C valuta come intervallo temporale fra i due eventi un intervallo *minore, uguale o maggiore* di quello valutato da B? Spiega.

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Effetti cinematici in Relatività Ristretta

Q19. Nel vagone dell'esperimento precedente vi è anche un *orologio a luce*, dispositivo costituito da un emettitore-ricevitore E insieme a uno specchio S ad una distanza *d* in direzione ortogonale al moto del treno.

Orologio a luce fermo

Quanto tempo impiega il raggio luminoso per il percorso di andata e ritorno nel sistema di riferimento del vagone (cui appartiene B)? $\Delta t =$ ______________________________

10) Per l'osservatore a terra (A) considerato in precedenza le posizioni di interesse dell'orologio a luce sono quelle corrispondenti agli eventi di *emissione* (1), *riflessione* (2) e *ricezione* (3) del raggio di luce.

Sulla base della geometria in figura, l'intervallo temporale per il percorso di andata e ritorno misurato dall'orologio di A *sarà uguale* a quello misurato dall'orologio di B? (Spiega)

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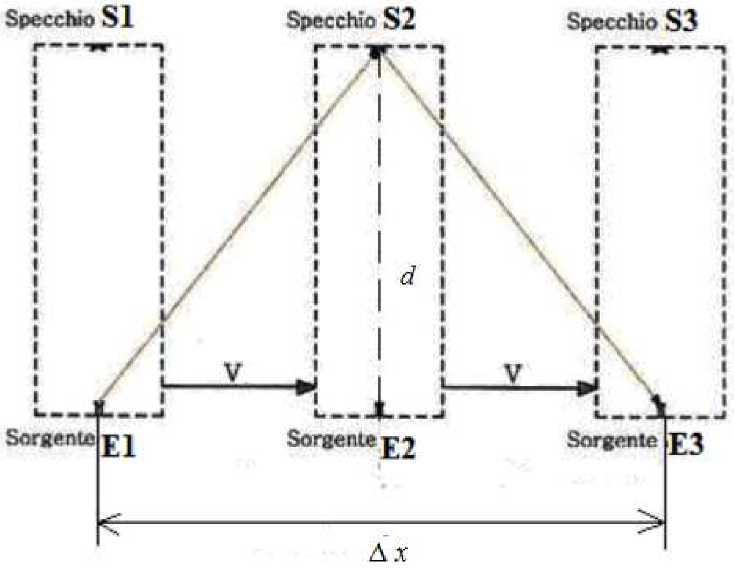

Orologio a luce in moto

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L'osservatore A a terra vede passare davanti a sé l'orologio a luce con velocità *v* (la stessa del treno). Indichiamo con Δx la distanza di cui si muove orizzontalmente l'orologio (e quindi la sorgente) nell'intervallo $\Delta t'$ di andata e ritorno del raggio *misurato dall'osservatore a terra*. Trova la relazione fra $\Delta t'$ e Δt in funzione di *v*, grandezza che caratterizza il moto relativo fra i due riferimenti. (*Suggerimento: confronta il tempo di andata e ritorno della luce misurato nel sistema mobile con quello misurato da A, che deriva dall'osservazione dello stesso fenomeno da terra).*

$$c\Delta t' = \underline{\hspace{8cm}}$$

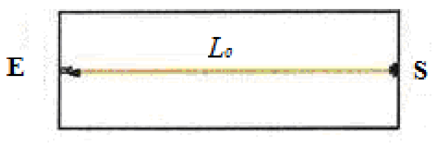
$$\frac{\Delta x}{\Delta t'} = v \quad \Rightarrow \quad \left(c\Delta t'\right)^2 = \underline{\hspace{5cm}}$$

$$\left(\Delta t'\right)^2 = \underline{\hspace{8cm}}$$

$$\Delta t' = \underline{\hspace{4cm}}\ \Delta t$$

Si perviene così alla formula di dilatazione dei tempi (più propriamente “dilatazione delle durate”) deducibile rigorosamente e direttamente dalle trasformazioni di Lorentz.

11) Ragioniamo infine sulla *lunghezza di un orologio a luce orizzontale* a bordo del treno.



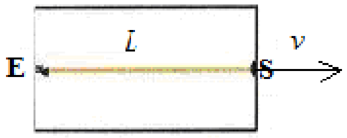
Orologio a luce fermo

Nel riferimento di A, il raggio di luce impiega per compiere il percorso di andata e ritorno:

☐ *più tempo* che in quello di B, a causa della dilatazione dei tempi;

☐ *meno tempo* che in quello di B, a causa della dilatazione dei tempi.

Poiché la velocità della luce è invariante, per l’osservatore A lo spazio fra le pareti deve necessariamente essere _____________ di quello misurato da B, quindi l’oggetto si ________________



Orologio a luce in moto

> *“Eliminato l'impossibile, ciò che resta, per improbabile che sia, deve essere la verità”.*
>
> (*Sherlock Holmes ne* Il segno dei quattro, *Arthur Conan Doyle*)

Quantitativamente, sappiamo che B misura come tempo di andata e ritorno_________________. Vediamo che cosa calcola A (rispetto cui l’orologio a luce si muove con velocità *v*):

- Andata: $c\Delta t_1' = L + v\Delta t_1' \Rightarrow \Delta t_1' = \underline{\hspace{3cm}}$;
- Ritorno: $c\Delta t_2' = \underline{\hspace{3cm}} \Rightarrow \Delta t_2' = \underline{\hspace{4cm}}$.
- Andata e ritorno:

$$\Delta t' = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{6cm}} = \underline{\hspace{6cm}} = \frac{2L}{c}\gamma^2$$

.

D’altra parte la dilatazione dei tempi si scrive $\Delta t' = \underline{\hspace{3.5cm}}$. Eguagliando i membri a destra delle due equazioni si ottiene $L = L_0/\gamma = L_0\sqrt{1-\frac{v^2}{c^2}} < L_0$.

12) Se il treno si fermasse in un istante qualunque, la lunghezza dell’orologio a luce:

☐ Aumenterebbe;

☐ Aumenterebbe rispetto a quella misurata da B;

☐ Diminuirebbe;

☐ Diminuirebbe rispetto quella misurata da B.

Giustifica (*trascura che durante la frenata il treno non è in realtà un riferimento inerziale)*

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➢ *2013 summer school (Udine)*

Università degli Studi di Udine - Udine Unità di Ricerca in Didattica della Fisica

Scuola Estiva di Eccellenza di Fisica Moderna

Udine, 22-27 luglio 2013

“Origine e significato dell'Equivalenza Massa-Energia in fisica moderna”

Cognome ______________________Nome____________________ Data________________

Premessa – Moti relativi e velocità della luce

1. Un passeggero è seduto su un treno che viaggia a una velocità costante di 80 km/h e lancia una pallina da tennis a 3 km/h nella direzione del moto e con verso concorde a esso. Qual è la velocità della pallina misurata da un passeggero fermo sulla pensilina di una stazione mentre il treno passa? E quella misurata da un corridore che si muove a 10 km/h in verso opposto al moto del treno? Spiega

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2. Considera un’ipotetica gara di velocità *sulla Luna,* dove non c’è atmosfera. Uno dei velocisti porta con sé una piccola torcia elettrica accesa (A); un tecnico è fermo nella corsia dell’atleta e ne possiede una seconda (B).

a. Con quale velocità si propaga la luce dalle torce A, B rispetto a ciascun possessore? ____________________.

b. Quale velocità misura il tecnico per la luce di A quando l’atleta gli passa davanti a 8 m/s nella direzione e verso di propagazione dell’onda luminosa? Motiva

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3. Quali considerazioni puoi fare sulla base delle risposte alle domande precedenti? Illustra

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1. **I postulati**

Per uscire dalla situazione critica in cui ci troviamo, possiamo tentare una scelta *estetica*, non dettata da necessità logiche (*euristica positiva*).

Poniamo che valgano i seguenti postulati, su cui Albert Einstein fondò la sua teoria nel 1905:

I. **Principio di relatività** (P.R.) in forma teorica e sperimentale:
«Tutte le leggi fisiche (non solo quelle meccaniche) hanno la stessa forma in tutti i sistemi di riferimento inerziali»;
«Esperimenti (di qualsiasi natura) condotti nelle stesse condizioni in diversi riferimenti inerziali danno gli stessi risultati».

II. **Principio di invarianza di *c***. «La velocità della luce nel vuoto *c* è l'unica il cui valore non dipende dal sistema di riferimento in cui si misura, né dalla direzione di propagazione» ⇔ «La luce si propaga isotropicamente a velocità *c* in ogni sistema di riferimento inerziale.»

In Relatività un *sistema di riferimento,* o semplicemente *riferimento*, è costituito da una *classe di osservatori* – dotati di *regoli metrici* e *orologi* – in tutti i punti dello spazio e da una *terna di assi cartesiani* ortogonali. Esso si può anche immaginare come un reticolo infinitamente esteso di orologi collegati da aste metriche ortogonali fra loro. In Relatività Ristretta si prendono in considerazione soltanto i *riferimenti inerziali*.

2. **Sincronizzazione**

Come possiamo allora sincronizzare due orologi nello *stesso* riferimento inerziale? E un reticolo?

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3. **Orologio a luce**

Considera due riferimenti in moto relativo rettilineo uniforme tra loro a velocità v, come visualizzato nell'*applet*. Si possono associare infiniti potenziali "osservatori" a ciascun riferimento, che misurino *durate* e *lunghezze spaziali* con gli orologi e i regoli di cui sono dotati. Analizziamo dapprima i fenomeni dal riferimento *K′* in cui il nostro apparato è fermo (detto "riferimento proprio").

In basso è posta una sorgente luminosa A che emette luce visibile, la quale viene riflessa a distanza perpendicolare *h* da uno specchio piano B e ritorna al punto di partenza, percorrendo

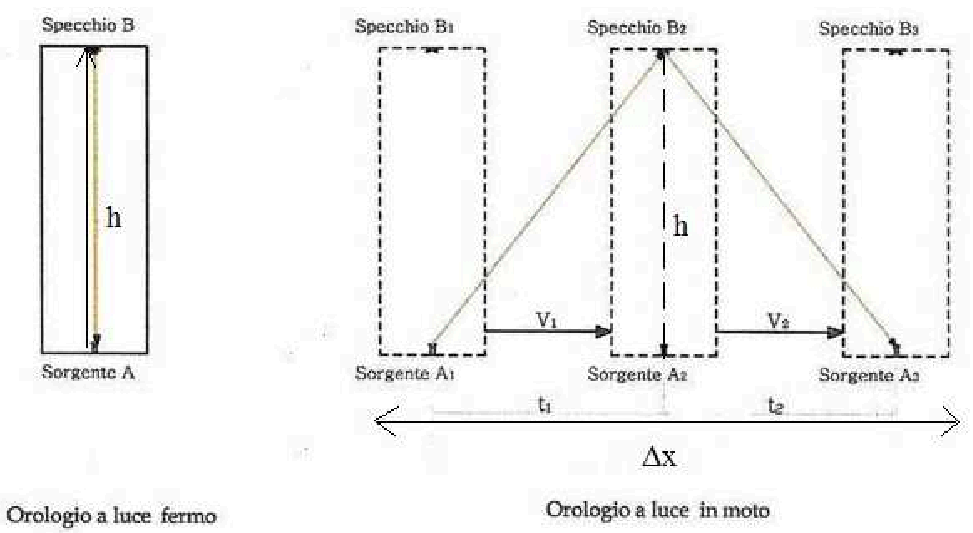

Figura 8.1 rappresentazione dell'esperimento per la spiegazione della dilatazione del tempo con un orologio a luce secondo l'ipotesi relativistica

così il cammino di andata e ritorno in un tempo $\Delta\tau = 2h/c$. In Relatività si ragiona in termini di "eventi": qualcosa che è avvenuto in un preciso punto dello spazio a un determinato istante, come lo scoccare di una scintilla, l'inizio dell'esplosione di una supernova, l'invio di un segnale da una pulsar, il passaggio di una particella da una data posizione, eccetera. L'intervallo temporale fra due "eventi" che avvengono nella stessa posizione è chiamato intervallo di *tempo proprio* $\Delta\tau$.

Sia *K* un riferimento generico rispetto al quale *K′* è in moto uniforme. Per fissare le idee pensa a due *osservatori* in riferimenti diversi: il primo su un treno in viaggio a velocità costante v rispetto a una stazione, il secondo sulla pensilina che osserva il treno passargli davanti. Dal primo l'orologio a luce sarà sempre visto fermo, dal secondo in moto con velocità v: esso percorrerà uno spazio orizzontale Δx in un tempo Δt .

Nell'*applet* compare Δt (*Jack's time*), coincidente con il tempo impiegato dal raggio per arrivare ad A3 in figura, come **misurato dall'osservatore sulla pensilina** (Jack)**, supponendo che la velocità della luce abbia lo stesso valore in *K* e in *K'*.** Tale intervallo temporale *non è uguale* a quello $\Delta\tau$ che il raggio impiegherebbe per compiere lo stesso percorso in un orologio fermo a terra. Se quest'ultimo è in moto, il cammino geometrico della luce è maggiore: l'ipotenusa è sempre più lunga di un cateto.

- <u>Domanda di gruppo</u>: le *misurazioni* che confrontiamo sono compiute tutte dall'osservatore a terra. Perché non consideriamo le misure di due osservatori in riferimenti differenti? *(Discussione a gruppi di 6-7 unità e sintesi collettiva)*

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La sorgente viaggia a velocità $\mathrm{v} = -\Delta x/\Delta t$ rispetto alla pensilina. Quanto impiegherà il raggio di luce, visto da terra, a compiere andata e ritorno? Ci interessa ricavare la relazione matematica fra Δt e $\Delta\tau$ tramite il teorema di Pitagora.

$c\Delta t = 2\sqrt{h^2 + \left(\frac{\Delta x}{2}\right)^2} =$ ____

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Risulta (1) $\boxed{\Delta t = \gamma\, \Delta \tau}$

Scrivi l'espressione del fattore di Lorentz γ =
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Dalla semplice postulazione dell'*invarianza* di *c* (si *pone* che essa sia indipendente dal riferimento in cui è misurata) segue qualcosa di nuovo riguardo alla relazione fra intervalli temporali in riferimenti diversi: la **dilatazione delle durate**.

4. Quadrintervallo

L'istante e la posizione (il tempo e lo spazio rispettivamente) in cui avvengono i fenomeni, pur rimanendo distinti, sono uniti fra loro dalla formulazione di Einstein: viene teorizzata l'esistenza di uno *spazio-tempo*. Quest'ultimo è stato interpretato geometricamente nel 1908 dal matematico Hermann Minkowski. Dai calcoli svolti risulta

$$(\Delta s)^2 \equiv (c\Delta \tau)^2 = (c\Delta t')^2 - (\Delta x')^2 \text{ (2)},$$

dove Δs rappresenta il "quadrintervallo", cioè *l'intervallo spazio-temporale fra due eventi*. Se considerassimo un altro riferimento in moto (un altro treno a velocità differente) si avrebbe ancora la stessa espressione, pur con distanze spaziali e intervalli temporali diversi? Perché? (*Suggerimento: qual è l'intervallo più breve fra due eventi?)*

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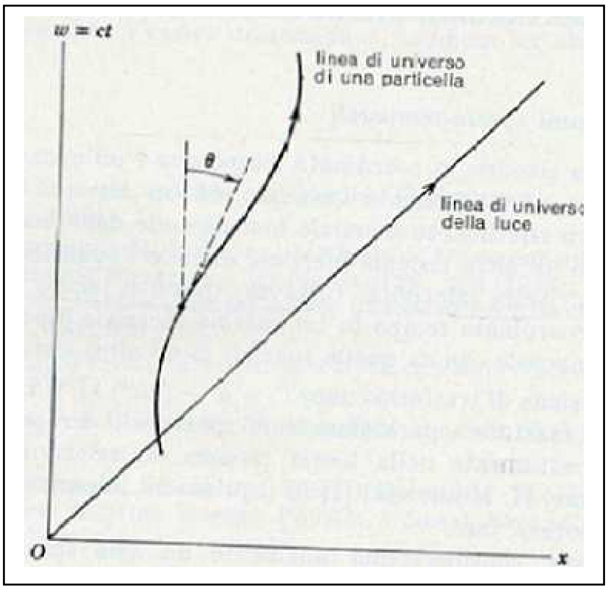
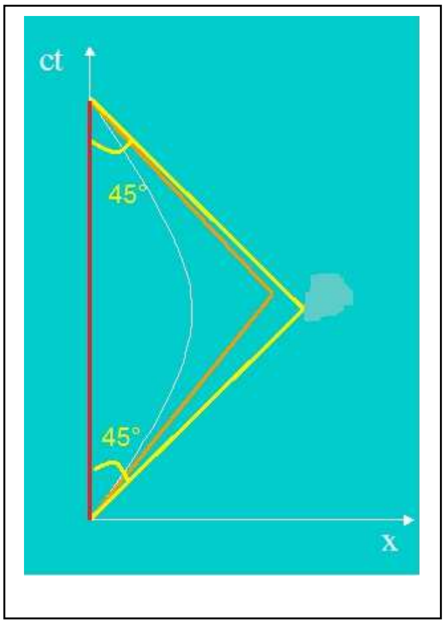
5. **Quadrispostamento**

Nello spazio euclideo lo “spostamento” è rappresentato, in coordinate cartesiane, dal vettore $(\Delta x, \Delta y, \Delta z)$. Il suo modulo, distanza fra due punti legati dal moto di un oggetto fisico, non dipende in meccanica classica dal particolare riferimento in cui è calcolato: è un *assoluto*. Analogamente si può introdurre un *quadrivettore nello spazio-tempo di Minkowski*: $(c\Delta t,\ \Delta x,\ \Delta y,\ \Delta z)$. La sua norma[127] è data da

$$(\Delta s)^2 \equiv (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 ,$$

per coordinate *t, x, y, z* generiche, ottenuta estendendo la (2) a tre dimensioni. La coordinata temporale viene moltiplicata per *c* per rendere la prima componente omogenea alle altre.
Osserva il piano di Minkowski in figura: in ordinata è rappresentato il tempo *ct* e in ascissa una sola dimensionale spaziale, *x*. Nota l’importante differenza fra la “distanza” nello spazio-tempo geometrizzato di Minkowski e nello spazio euclideo. Nel primo caso *si sottrae* al termine temporale la somma dei quadrati delle componenti spaziali al termine temporale, invece di sommarla. Questo è un tratto distintivo della geometria *metrica* dello spazio: essa **non può essere euclidea**. È chiamata a volte “pseudo-euclidea”.





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[127] L’equivalente del modulo di un vettore cartesiano.

Esercizio. “Quale linea d’universo descrive la traiettoria corrispondente al minor tempo proprio (grafico a destra)? Perché?”

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6. **Quadrimpulso**

Vogliamo ora costruire un quadrivettore che descriva la dinamica di una particella di massa *m* nello spazio-tempo. In fisica classica, per ottenere la *velocità* si dividono le componenti dello spostamento – vettore di modulo invariante – per l’intervallo di tempo coordinato Δt, scalare invariante per trasformazioni di Galileo: classicamente l’intervallo temporale fra due eventi è sempre lo stesso, *da qualunque stato di moto lo si misuri*. Poi si prende il limite $\Delta t \to 0$.

Analogamente, in Relatività dovremo fare il rapporto fra un **quadrivettore** e uno **scalare invariante**, che non può essere Δt. Perché? Quali sono allora il quadrivettore e l’unico invariante utilizzabili ?

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Spiega il significato di considerare il limite $\Delta\tau \to 0$ d’ora in poi

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Possiamo allora definire la **quadrivelocità** $\lim\limits_{\Delta\tau\to 0}\left(\frac{c\Delta t}{\Delta\tau},\frac{\Delta x}{\Delta\tau},\frac{\Delta y}{\Delta\tau},\frac{\Delta z}{\Delta\tau}\right)$

Se la moltiplichiamo per la *massa inerziale newtoniana*, otterremo l’equivalente relativistico della quantità di moto: il **quadrimpulso**. Si noti che l’utilizzo in relatività della massa classica – invariante nel passaggio tra riferimenti – è un’*ipotesi* della dinamica relativistica. Il quadrimpulso è perciò dato da

$$m\lim_{\Delta\tau\to 0}\left(\frac{c\Delta t}{\Delta\tau},\frac{\Delta x}{\Delta\tau},\frac{\Delta y}{\Delta\tau},\frac{\Delta z}{\Delta\tau}\right)=\lim_{\Delta\tau\to 0}\left[m\left(\frac{c\Delta t}{\Delta\tau},\frac{\Delta x}{\Delta\tau},\frac{\Delta y}{\Delta\tau},\frac{\Delta z}{\Delta\tau}\right)\right]=$$

$$=\lim_{\Delta\tau\to 0}\left[m\left(c\frac{\Delta t}{\Delta\tau},\frac{\Delta x}{\Delta t}\frac{\Delta t}{\Delta\tau},\frac{\Delta y}{\Delta t}\frac{\Delta t}{\Delta\tau},\frac{\Delta z}{\Delta t}\frac{\Delta t}{\Delta\tau}\right)\right]=\lim_{\Delta\tau\to 0}\left(\gamma mc, m\frac{\Delta x}{\Delta t}\gamma, m\frac{\Delta y}{\Delta t}\gamma, m\frac{\Delta z}{\Delta t}\gamma\right)=$$

$$=\left(\gamma mc, \gamma mu_x, \gamma mu_y, \gamma mu_z\right)$$

Chiarisci come siamo arrivati alla forma finale. In particolare, perché la massa può entrare nell'operazione di limite (primo passaggio)?

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Prendiamo ora il *limite newtoniano* della prima componente: calcoliamo i valori che assume per velocità dei punti materiali di modulo[128] $u \ll c$. In tal modo ci riconduciamo ad un ambito a noi noto, così da poter "riconoscere" queste quantità tramite il *principio di corrispondenza* con la meccanica classica.

$$\text{Si può dimostrare che } \gamma = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = 1 + \frac{1}{2}\frac{u^2}{c^2} + O\left(u^4\right)$$

Perciò $\gamma mc^2 \underset{\frac{u}{c}\to 0}{=} mc^2 + \frac{1}{2}mu^2$, a meno di termini del quarto ordine in u.

Possiamo ora dare alla componente temporale del quadrimomento il significato di energia totale relativistica, che sarà data dalla somma di un'energia cinetica relativistica ed un'energia non classica: $E = mc^2 + K_{rel} = E_0 + K_{rel} = \gamma mc^2$.

Perché tutti i termini devono avere la dimensione di un'energia?

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In tal modo abbiamo attuato l'identificazione (3):

$$\boxed{E_0 = mc^2}$$

La somma di tutti i contributi all'energia di un corpo, escluso quello cinetico, è data dalla sua **massa**. Alternativamente, la massa è uguale a tutta l'energia di un corpo quando è in quiete, detta **"energia interna"** o **"energia a riposo" (equivalenza massa-energia).** Numericamente c'è il fattore c^2 fra le due quantità, ma in opportune unità di misura può essere reso pari a 1 e adimensionale.

L'ipotesi di Einstein che tutta l'energia a riposo contribuisca alla massa ha *carattere*

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[128] Attenzione: qui ho cambiato il simbolo per la velocità da *v* ad *u* perché stiamo analizzando la cinematica di corpi animati da moto vario, non rettilineo uniforme. Perciò il simbolo γ assume un significato diverso da prima: ora immaginiamo un passaggio fra un riferimento istantaneamente in quiete con il corpo (riferimento «comovente»») e quello del «laboratorio» in cui si conducono le misure.

estetico: non segue logicamente da quanto visto, perché mancano i termini di ordine superiore nello sviluppo dell'energia. A rigore, l'energia non-cinetica potrebbe contribuire soltanto in parte alla massa; tuttavia l'ipotesi formulata non è stata mai invalidata dagli esperimenti.

Una delle conseguenze più affascinanti dell'equivalenza consiste nel fatto che l'energia delle onde elettromagnetiche deve avere una massa, anche gravitazionale. La luce dev'essere perciò soggetta alla gravità e deviare dalla sua traiettoria rettilinea in presenza di corpi massivi!

➢ *Ancona e Cremona*

DECADIMENTO DI RADIONUCLIDI

1. Sulla Terra sono stati trovati 96 **elementi chimici** naturali e ne sono stati prodotti artificialmente 22. Essi vengono descritti mediante due quantità: il numero Z di protoni e il numero N di neutroni presenti nel nucleo atomico. Gli atomi di uno stesso elemento *non* sono tutti uguali fra loro, perché si vede dai risultati sperimentali – riassunti nella mappa – che atomi della stessa specie chimica hanno **valori diversi di massa,** indicati in basso a destra in ogni casella in unità di massa atomica (*u.m.a.*)[129]. Quest'ultima è misurata sperimentalmente con metodi spettrometrici. Queste differenze indicano l'esistenza di atomi, detti **"isotopi"***,* che differiscono tra loro per **il numero di neutroni**, pur avendo lo stesso Z: la differenza non è più *chimica* ma *nucleare.* Nella mappa sono rappresentate tutte le *singole specie nucleari* osservate, dette "nuclidi".
2. I protoni e neutroni sono chiamati collettivamente "nucleoni", poiché fanno parte del nucleo. Il loro numero totale (necessariamente intero) è detto *numero di massa* A = N + Z ed è indicato in alto a destra nelle caselle.
3. Non tutta la materia è però stabile: dopo un intervallo temporale più o meno lungo – dai *μs* ai milioni di anni – una parte dei nuclidi si trasforma in altri (*trasmutazione nucleare*) che possono avere diverso Z, creando così nuovi elementi. Il "tempo di vita medio" di un nuclide è una media statistica dei tempi nei quali ne è stato visto decadere un numero statisticamente significativo[130]. Esso è indicato nella seconda riga dall'alto di ogni casella e la sua presenza indica innanzitutto che un nuclide **"decade"**. Sono colorati in azzurro i nuclidi che decadono in tempi inferiori o dell'ordine di 1 s, in violetto quelli che decadono in ore/giorni, in fucsia quelli che hanno tempi superiori a 1 anno solare. In quanto tempo decadono quelli colorati in arancio?

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   Quanti nucleoni hanno il Li-7, il C-14, l'U-235 ?

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   Gli elementi in grigio hanno una vita media talmente piccola che vengono considerati

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[129] 1 u.m.a. ≡ 1/12 m(C-12) per convenzione internazionale. La massa atomica è approssimabile a quella del nucleo, poiché $m_e \cong 10^{-4}\,\text{u.m.a.} << m_p \cong m_n \cong 1\,\text{u.m.a.}$

[130] Questo perché ovviamente ogni nuclide esistente in natura decade in un intervallo temporale diverso da tutti gli altri: ci può essere un nucleo di Li-9 che decade in 10 s ed un altro in 1 ms: in media il nuclide vive 178.3 ms.

totalmente instabili. Non li consideriamo nella nostra trattazione.

È importante notare che la massa atomica *cresce da sinistra a destra nella mappa*. Perché?

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4. I **decadimenti radioattivi** consistono nel mutamento della configurazione di un nuclide con emissione di particelle (dette 'radiazioni' o 'raggi') *α, β, γ*: rispettivamente nuclei dell'atomo He-4, elettroni, radiazione elettromagnetica ad alta energia. Per riconoscere il tipo di decadimento occorre far passare i 'raggi' in un campo magnetico o elettrico. http://www.youtube.com/watch?v=4OkR-B4BpvA
5. Le mappe di radionuclidi possono essere lette come **diagrammi N – Z**: in *ordinata vi è il numero atomico Z* crescente dall'alto verso il basso, in *ascissa il numero di neutroni N* crescente da sinistra a destra. Nella seconda riga all'alto della casella del nuclide si trovano il/i *decadimento/i permesso/i* e *l'energia rilasciata* dal decadimento preferenziale; in quella subito sotto il *nucleo/i figlio/i* (= il prodotto del decadimento del nuclide stesso). Per esempio: in quali elementi può decadere il Be-11, con che tipo di decadimento e con quanta energia liberata?

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6. Il Carbonio-14 non è un nucleo stabile: presa una certa quantità, ne decade in media *la metà* in 5730 anni circa ("tempo di dimezzamento"). Questo fenomeno è utilizzato per datazioni archeologiche. Il C-14 decade *β* in N-14. Com'è variato Z?

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   Come sono variati A e N?

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Esplicita le variazioni di Z, N, A per altri 2 nuclidi a tua scelta che decadono *β*.

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Di conseguenza, come ci si deve spostare sulla mappa per riconoscere il processo?

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Perciò una possibile interpretazione a livello sub-nucleare di quello che avviene è sintetizzabile con lo schema

$$n \rightarrow p + e^{-} + X \ .$$

Il nuclide figlio deve avere un *protone in più* del nuclide genitore per compensare la carica negativa dell'elettrone, in modo che la carica complessiva del sistema si conservi. Poiché il numero di massa A si conserva nel processo, *un neutrone del nucleo deve mutare in protone*, con creazione ed emissione di un elettrone e di una particella *X*, che interagisce pochissimo con la materia. La sua natura non è importante per la nostra analisi[131].

Quale altra proprietà dei nuclidi varia?

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Consideriamo ora il F-17 e il F-18: sulla base del meccanismo individuato, dovrebbero decadere in Ne-17 e Ne-18 rispettivamente. Perché ciò non avviene?

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Che cosa accade all'O-16 e O-17? Che cosa ha questa situazione in comune con quella precedente?

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[131] In fisica subnucleare la particella elementare *X* è chiamato *antineutrino*.

7. Riconsidera il decadimento $^{16}\mathrm{C} \rightarrow {}^{16}\mathrm{N}$ e gli altri scelti in precedenza. Quale può essere un nuovo criterio *necessario e sufficiente* perché un decadimento *β* tra due nuclidi sia permesso?

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8. L'**energia** complessivamente emessa nel decadimento è misurabile con opportuni strumenti che esaminano i prodotti della reazione, o è comunque valutabile studiando la reazione inversa. In questo modo è stata misurata per il decadimento del Carbonio-14 un'energia rilasciata pari a $0.1565 \text{ MeV} = 0.1565 * 10^6 \text{ eV}$ [1 eV è l'energia che acquisisce un elettrone accelerato dalla tensione di 1 V]. In quale forma è liberata l'energia?

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4. Considera ora la seguente lista di radionuclidi e calcola <u>per quelli che decadono *β*</u> il rapporto $E/\Delta m$, dove *E* è l'energia emessa nel processo e *Δm* la variazione di massa corrispondente.

Nuclide	***E*** (MeV)	m_{padre} (u.m.a.)	m_{figlio} (u.m.a.)	$\Delta m = m_{figlio} - m_{padre}$ (u.m.a.)	$E/\Delta m$ (______)
He-6	3,508	6,018888	6,0151222	-0,0037658	931,542
Be-11	11,509	11,0216611	11,0093054	-0,0123557	__________
B-13	13,437	13,01778022	13,00335484	__________	__________

Nuclide	***E*** (MeV)	$\Delta m = m_{figlio} - m_{padre}$	$E/\Delta m$
O-19			
B-17			
F-23			
N-21			
O-24			

- Disegna un **grafico *Δm – E*** su carta millimetrata

Che cosa noti?

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Calcola $\left\langle E/\Delta m \right\rangle = \sum_{i=1}^{\cdots} \left\{ E_i / \left(\Delta m \right)_i \right\} =$

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Discuti con i tuoi compagni sul significato fisico del risultato ottenuto, anche alla luce di quello che conosci su massa ed energia. Il professore farà da moderatore raccogliendo le principali idee emerse. Scrivi le tue conclusioni alla fine della discussione.

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Il fattore calcolato ha unità di misura $\mathrm{MeV/u.m.a.}$ Di quale grandezza fisica ti aspetti sia il valore? Argomenta secondo quanto discusso in gruppo, utilizzando l'analisi dimensionale.

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Converti il fattore in unità SI ____.

- ***2014 summer school (Udine)***

Università degli Studi di Udine - Udine Unità di Ricerca in Didattica della Fisica
Scuola Estiva per Studenti di eccellenza in Fisica Moderna
Udine, 23-28 giugno 2014

S1 – Esperimento di Bertozzi:accelerazione di elettroni

Cognome ____________________Nome__________________ Data______________

1. Come ti aspetti che vari la *velocità al quadrato* degli elettroni se viene gradualmente aumentata la differenza di potenziale del Van De Graaff da 0 fino a 15 MV? Secondo un andamento

☐ Lineare; ☐ Quadratico; ☐ Esponenziale crescente; ☐ Asintotico (asintoto orizzontale); ☐ Asintotico (asintoto verticale); ☐ Radice quadrata; ☐ Altro (specifica) ____________________

Su che cosa si basa la tua previsione ?

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2. Quale tipo di andamento mostrano i dati di Bertozzi ?

☐ Lineare; ☐ Quadratico; ☐ Esponenziale crescente; ☐ Asintotico (asintoto orizzontale); ☐ Asintotico (asintoto verticale); ☐ Radice quadrata; ☐ Altro (specifica) ____________________

3. La tua previsione è stata confermata? In caso negativo, come spieghi la discrepanza rispetto ai risultati dell’esperimento? Discuti per 3 minuti con due compagni e scrivi almeno 3 righe

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4. Scegli l'implicazione corretta fra le seguenti e argomenta la tua scelta, esplicitando il tuo ragionamento in almeno due righe.

☐ c è la velocità limite $\Rightarrow$ c è invariante;
☐ c è invariante $\Rightarrow$ c è la velocità limite;
☐ c è la velocità limite $\Leftrightarrow$ c è invariante .

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5. Il percorso concettuale svolto ci ha portato a comprendere la *non validità* delle trasformazioni di Galileo delle coordinate e delle velocità. Illustralo nel riquadro con una **mappa concettuale**, scrivendo i *sostantivi* principali collegati da *verbi*. Spiega la mappa nelle righe sottostanti chiarendo che cosa hai imparato.

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Università degli Studi di Udine - Udine Unità di Ricerca in Didattica della Fisica

Scuola Estiva per Studenti di eccellenza in Fisica Moderna

Udine, 23-28 giugno 2014

S2 – Orologio a luce

Cognome ____________________Nome__________________ Data______________

1. Perché gli orologi vengono sincronizzati con un *segnale luminoso* e non di altro tipo, ad esempio un'onda sonora?

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2. Considerato che *c* è invariante, come sarà *l'intervallo di tempo di un percorso A/R* (la nostra unità di tempo) *in un orologio a luce in moto,* misurato dal marciapiede ?

☐ Più lungo di quello misurato da un orologio a luce identico posto sul marciapiede;

☐ Uguale a quello misurato da un orologio a luce identico posto sul marciapiede;

☐ Più breve di quello misurato da un orologio a luce identico posto sul marciapiede.

Motiva la tua previsione in almeno tre righe

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3. La previsione è in accordo con la simulazione osservata? In caso negativo, come spieghi la discrepanza? Discutine con due tuoi vicini per 3 minuti e scrivi almeno 4 righe

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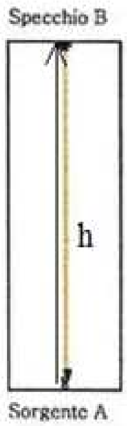
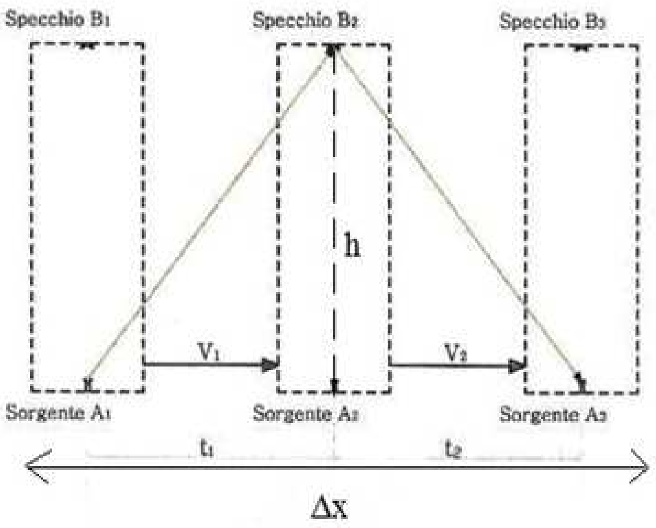
4. Considera due sistemi di riferimento inerziali (SI) in moto relativo a velocità v, come visualizzato nella simulazione. Si possono associare a ciascun SI infiniti potenziali “osservatori” che misurino *durate* e *lunghezze spaziali* con orologi e regoli. Analizziamo dapprima i fenomeni dal riferimento *K′* in cui il nostro apparato è fermo .



Orologio a luce fermo



Orologio a luce in moto

In basso è posta una sorgente luminosa A che emette luce visibile, la quale viene riflessa a distanza perpendicolare *h* da uno specchio piano B e ritorna al punto di partenza, percorrendo il cammino A/R in un intervallo temporale $\Delta\tau = 2h/c$. In Relatività si ragiona in termini di “eventi”: qualcosa avvenuto in un *preciso punto dello spazio* a un *determinato istante*, come lo scoccare di una scintilla, l’inizio dell’esplosione di una supernova, l’invio di un segnale da una pulsar, il passaggio di una particella in una data posizione, eccetera. L’intervallo temporale fra due eventi che avvengono nella stessa posizione è chiamato intervallo di *tempo proprio* $\Delta\tau$.

Sia *K* l’altro SI; un osservatore in *K* vedrà il proprio orologio a luce sempre fermo, mentre vedrà l’altro (identico) in moto con velocità v; esso percorrerà uno spazio orizzontale Δx in un tempo Δt .

5. L’orologio a luce viaggia a velocità $v = -\Delta x/\Delta t$ rispetto al marciapiede. Quanto impiegherà il raggio di luce, visto da un osservatore a terra, a compiere andata e ritorno? Prova a **ricavare la relazione matematica fra** Δt e $\Delta\tau$ **in funzione della velocità relativa con il teorema di Pitagora**.

$c\Delta t = 2\sqrt{h^2 + \left(\frac{\Delta x}{2}\right)^2} =$ ____

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Risulta $\boxed{\Delta t = \gamma \Delta \tau}$

Scrivi l’espressione del fattore di Lorentz γ = ________________________________ .

6. Supponi di aver dotato l’osservatore sul treno e quello sul marciapiede di due orologi da polso identici per misurare le pulsazioni cardiache.

☐ Il secondo osservatore misura i propri battiti rallentati?

☐ I battiti cardiaci del primo osservatore risultano *realmente rallentati* per il secondo ?

Spiega in almeno 3 righe:

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7. Nei romanzi di fantascienza il problema di far arrivare una nave spaziale con equipaggio in un altro sistema solare è stato narrativamente risolto in molti modi. Uno di questi è accelerare la nave per farla viaggiare a velocità relativistiche. Illustra perché

questo dovrebbe permette all’equipaggio di completare il viaggio in un tempo accettabile.

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Università degli Studi di Udine - Udine Unità di Ricerca in Didattica della Fisica

Scuola Estiva per Studenti di eccellenza in Fisica Moderna

Udine, 23-28 giugno 2014

S3 – MASSA-ENERGIA

Cognome ____________________Nome__________________ Data_______________

1) *Da che cosa deriva l'assurdo* nell'esperimento mentale dell'assorbimento di 2 fotoni?

a) Abbiamo supposto (anche implicitamente) la conservazione di 3 grandezze fisiche nel processo: _____________; ____________________; ____________________.

b) Quale delle tre ipotesi deve allora cadere? Per scoprirlo, prova a immaginare un urto anelastico analogo in cui il blocco viaggia a velocità newtoniane e nel quale si conficcano due proiettili di massa m. Il ragionamento funziona in questo caso? Discuti per 3 minuti con due compagni ed esplicita i tuoi *ragionamenti* e *conclusioni* in almeno 4 righe

2) Supponendo che la __________________ non si conservi e *imponendo la conservazione delle altre due quantità*, prova a continuare l'esperimento mentale dei due fotoni. Considera che 2ε è l'energia complessivamente assorbita dal blocco, quindi è anche la sua *variazione di energia* ΔE.

3) L’energia a riposo si definisce come ________________________________ e si esprime come ________________.

Finora abbiamo ragionato in un solo SI; prendiamo in considerazione l’energia dovuta **al moto del SI** solidale all’oggetto rispetto al SI del laboratorio: essa è **l’energia cinetica del corpo** nel SI del laboratorio.

Scrivine nel riquadro l’espressione

Per ottenere **l’energia totale** di un sistema dovrò quindi sommare il contributo di energia a riposo a quello cinetico: K + E_0.

In entrambi i termini della somma compare la massa. Possiamo supporre che sia dello stesso oggetto, ma *assumerà lo stesso valore* se misurata quando l’oggetto si muove a velocità costante o se misurata mentre l’oggetto è in quiete nel laboratorio ? Argomenta la tua risposta. Esegui poi il calcolo dell’energia totale.

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- Poiché la massa in relatività sembra possedere properietà differenti da quella classica, controlliamo se essa è *additiva* analizzando un urto anelastico.

4) Due oggetti identici di massa ***m*** inizialmente non interagenti si muovono uno contro l’altro a velocità relativistiche di modulo ***w*** uguali e opposte nel SI K. Essi urtano e

generano un nuovo oggetto di massa ***M.*** Che velocità avrà quest'ultimo misurata in K e perché ?

Se consideriamo solo la descrizione del processo dal SI K, non troviamo però una relazione fra le masse ***m*** dei due oggetti iniziali e quella ***M*** del prodotto.
Consideriamo allo scopo un SI K' (ipotetico ascensore) che si muova con velocità -***u*** molto piccola, diretta verso il "basso", ortogonale alle velocità $\boldsymbol{w}$.

5) Raffigura nel riquadro sottostante le due situazioni – prima e dopo l'urto – con l'urto visto da K a sinistra e visto da K' a destra

6) Non è difficile scrivere la conservazione della quantità di moto (q.d.m.) in K

In K' considera invece la conservazione della *componente della q.d.m. lungo la direzione di* $\boldsymbol{u}$. Poiché $u \ll c$, la componente "verticale" della velocità delle particelle *prima dell'urto* è $\mathbf{u}$. Percio $P_i^{tot} = \gamma_i mu + \gamma_i mu$, dove *i* sta per "iniziale".
D'altra parte *dopo l'urto* avremo $P_f^{tot} = \gamma_f M\, u \;\cong$ ________________ (considera che la velocità "verticale" apparente dell'oggetto finale non è relativistica).
Mettendo insieme le due cose risulta

Come puoi descrivere il processo in termini di bilancio di massa e di energia totale dei due oggetti (almeno 3 righe) ?

- ***Udine – Samples 3 and 4***

LA MASSA IN NEWTON

Nella scienza moderna il concetto di massa venne inizialmente considerato dal suo inventore Isaac Newton come *misura della quantità di materia*; egli scriveva infatti nei *Philosophiae Naturalis Principia Mathematica* (1687):

> La quantità di materia è la misura della medesima ricavata dal prodotto della sua densità per il volume. […] Aria di densità doppia, in uno spazio a sua volta doppio, diventa quadrupla; in uno triplice, sestupla. La medesima cosa si capisca per la neve e la polvere condensate per compressione e liquefazione. E la norma di tutti i corpi, che siano diversamente condensati per cause qualsiasi, è identica […]. In seguito indicherò questa quantità indifferentemente con i nomi di corpo o di massa.

A proposito invece di quella tendenza che Newton chiamava “forza insita” nella materia, oggi denominata “inerzia”, si legge:

> Questa forza è sempre proporzionale al corpo [*termine che l’autore usa come sinonimo di massa, N.d.R.*], né differisce in alcunché dall’inerzia della massa altrimenti che per il modo di concepirla. A causa dell’inerzia della materia, accade che ogni corpo è rimosso con difficoltà dal suo stato di quiete o di moto.

Scrivi una definizione di questa grandezza fisica con parole tue.

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Quale equazione fra quelle sotto è legata alla quantità di materia? Quale invece si rifà alla particolare accezione di inerzia riportata nel brano sopra? *Motiva le **due** risposte nel riquadro.*

☐ $P = m\,g$

☐ $d = m / V$

☐ $S = m_{fluido}\,g$

(Spinta di Archimede)

☐ $F = m\,a$

☐ $p = m\,v$

☐ $K = ½\,m\,v^2$

Molti corpi celesti si muovono attorno ad altri su *orbite circolari* – in generale ellittiche, cioè a forma di ellisse, ma noi consideriamo solo le orbite circolari, un sottoinsieme di quelle ellittiche.

Ad esempio, un pianeta di *massa inerziale* m_i ruota attorno a una stella di massa inerziale $M_i >> m_i$ a causa dell'*equilibrio* fra la *sua inerzia*, che tende a farlo muovere di moto rettilineo uniforme, e *una forza centripeta* che tende a farlo cadere sulla stella. È grazie al bilanciamento di queste due "tendenze" se la Terra non cade sul Sole o la Luna sulla Terra. La "massa inerziale è la grandezza fisica cui si riferisce Newton nel secondo brano.

Per studiare questa forza centripeta assumeremo la validità di

1. La II legge della dinamica $F = ma$;
2. La III legge della dinamica o "principio di azione e reazione": $\mathbf{F_{12}} = -\mathbf{F_{21}}$ (equazione vettoriale);
3. La III legge di Keplero, estesa ad un sistema stellare qualunque: «il cubo della distanza di un pianeta dalla stella attorno cui ruota diviso per il quadrato del periodo di rotazione non dipende dal particolare pianeta: tale rapporto è una costante», in termini matematici $r^3 / T^2 = k$.

Scriviamo l'accelerazione centripeta del pianeta in moto circolare:

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 \left(\frac{r^3}{T^2} \right) \frac{1}{r^2}$$

Il termine fra parentesi richiama la terza legge di Keplero. Ricaviamo l'espressione per la forza centripeta $F_c = m_i a_c$ sostituendo l'equazione di tale legge. Il risultato è la forza che *la stella esercita sul pianeta*:

$$F_c = m_i \frac{v^2}{r} = 4\pi^2 \frac{k\ m_i}{r^2}$$

Quale legge di proporzionalità vi è fra modulo della forza e distanza pianeta-stella?

- ☐ Proporzionalità diretta;
- ☐ Proporzionalità quadratica;
- ☐ Proporzionalità quadratica inversa;
- ☐ Proporzionalità inversa.

Perciò se due corpi celesti si avvicinano fra loro la forza

- ☐ Aumenta;
- ☐ Diminuisce;
- ☐ Aumenta o diminuisce a seconda del segno delle grandezze fisiche presenti.

Ogni corpo che viene attratto ne *attrae altri* a sua volta secondo la ____, infatti l'interazione è reciproca. Perciò anche il *pianeta esercita una forza sulla stella*, seppur molto inferiore a quella della stella su di esso.

Quale sarà l'espressione della forza e perché?

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Sai spiegare perché una palla lasciata libera cade sulla Terra e non viceversa?

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Perciò l’espressione generale della forza di *reciproca interazione* conterrà il prodotto delle masse:

$$F_g = \frac{4\pi^2 k'' M_i m_i}{r^2} = G\frac{M_i m_i}{r^2},$$

avendo introdotto la costante $G = 4\pi^2 k''$

Tale forza di attrazione reciproca è detta “gravitazionale” e in tal modo **la massa acquisisce un significato gravitazionale.** G = 6,67*10^{-11} N m^2 / kg^2 è chiamata *costante di gravitazione universale.* La legge sopra viene estesa a tutti i corpi: diventa la *legge di Gravitazione Universale.*

G è stata trovata attraverso la misura accurata della (debole) forza gravitazionale tra due coppie di sfere di piombo di volume differente ai capi di una bilancia di torsione (esperimento di Cavendish, 1794). La misura diretta dell’angolo di rotazione permise di risalire all’intensità della forza; essendo le masse delle sfere e la loro distanza (lunghezza del braccio della bilancia) note, si poté risalire con notevole precisione alla costante di gravitazione, invertendo la legge sopra: $G = F_g\, r^2/(M\, m)$.

La forza gravitazionale è sempre attrattiva ?

☐ Sì, infatti

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☐ No, perché

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Nella VII proposizione e Corollario del III libro dei *Principia* viene esplicitamente asserito che

> esiste una forza di gravità che interessa tutti i corpi, proporzionale alle varie quantità di materia che essi contengono [...] La forza di gravità, di cui risentono le varie parti uguali di un qualunque corpo, va come l’inverso del quadrato della distanza tra le varie particelle.

In un altro brano Newton fa ancora riferimento alla proporzionalità quadratica inversa fra tale forza e la distanza tra un «luogo» su un corpo celeste ed il centro di un altro.

- Quali idee di massa sono emerse finora? *Illustrale evidenziando le relazioni con le altre grandezze.*

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- La legge di Gravitazione Universale può essere utilizzata per *definire* la massa?

  ☐ Sì, in questo modo:

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  ☐ No, perché...

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Il *peso P* è una forza di tipo gravitazionale. Per corpi in prossimità della superficie terrestre si può introdurre **l'intensità *g* di campo gravitazionale**:

$$g \equiv \frac{GM_T}{R^2} \quad \Rightarrow \quad F_g = P = m_g g$$

. M_T è la massa della Terra ed R il suo raggio: g risulta costante per *piccole variazioni di altitudine*, cioè variazioni trascurabili rispetto al raggio terrestre.

Come si può quindi *stimare* la massa gravitazionale di un corpo in un laboratorio terrestre?

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Il fisico e filosofo **Ernst Mach** (1838 – 1916) fa la seguente critica a Newton ne *La meccanica nel suo sviluppo storico-critico*:

> Per quanto riguarda il concetto di massa, osserviamo che la formulazione data da Newton è infelice. Egli dice che la massa è la *quantità di materia* di un corpo misurata dal prodotto del suo volume per la densità. Il circolo vizioso è evidente. La densità infatti non può essere definita se non come la massa nell'unità di volume. Newton si è reso conto che in ogni corpo è inerente una proprietà quantitativa che determina il movimento ed è diversa dal peso […] ma non è riuscito a esporre questa conoscenza in modo corretto.

- Da che cosa deriva il circolo vizioso? In particolare, quale grandezza è primitiva per Newton, cioè è quella a partire da cui le altre grandezze sono definite, e perché? (1)Massa; (2)Densità; (3)Volume.

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Nei brani emerge che, già per Newton, la massa non è solo la "quantità di materia", nonostante egli la utilizzi per definirla; la massa è un concetto in divenire nella sua mente.

URTI DI CARRELLI: SCHEDA ESPERIMENTO

I due sensori sono posizionati all'estremità della guida in modo che ognuno di essi misuri la velocità di un carrello (a campionamento, con frequenza 20 Hz). Entrambi i carrelli vengono posti inizialmente a circa 40 cm di distanza dai rispettivi sensori. *Una breve spinta è data ai carrelli in modo da farli urtare reciprocamente.*

1. A sensori spenti, *prevedi il grafico velocità-tempo* v(t) dei carrelli scarichi nel sistema di riferimento del laboratorio dall'istante in cui vengono spinti fino a quando vengono fermati dopo l'urto. L'asse x è orientato positivamente dalla parte sinistra della rotaia a quella destra. Riporta le unità di misura negli assi di tempo e velocità. Spiega brevemente il *motivo della tua previsione*.

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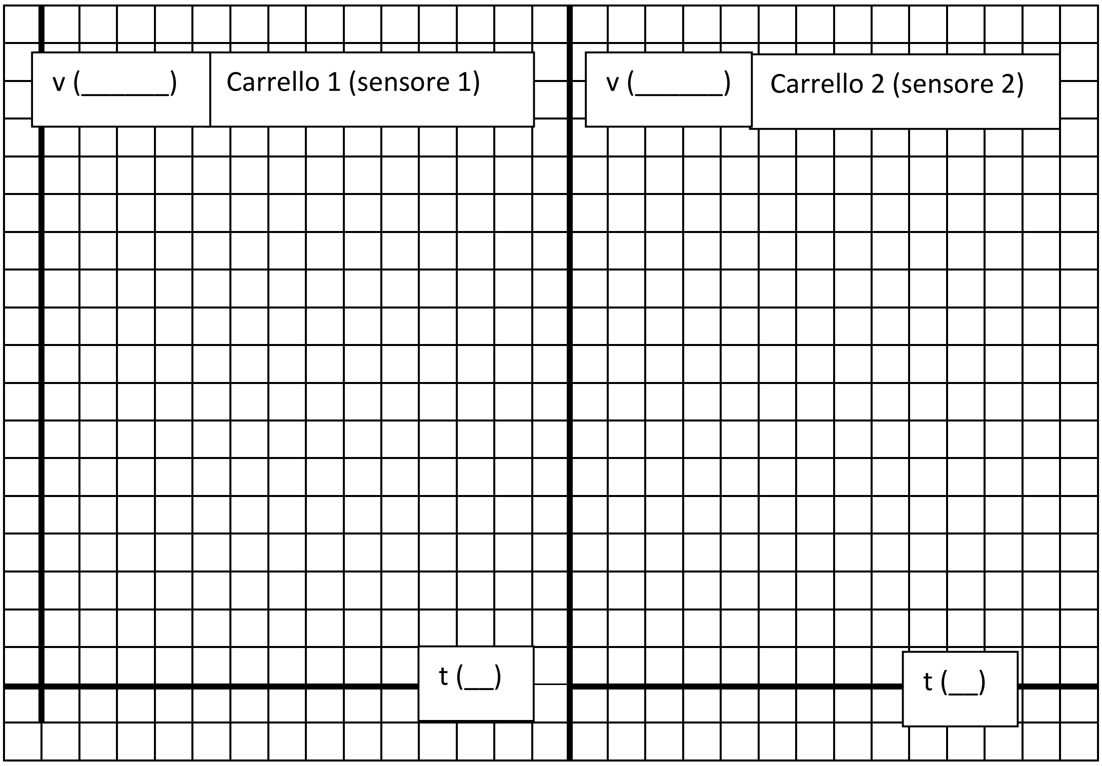

2. Il docente fa urtare i *carrelli scarichi* fra loro due volte. Osserva sul monitor il grafico velocità-tempo che si compone. Cerca di spiegare le ragioni delle differenze fra il grafico sperimentale e quello che avevi previsto; evidenzia poi in che cosa sono simili (*analogie* e *differenze*).

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3. Consideriamo come *istante dell'urto* t_{urto} il punto medio dell'intervallo temporale Δt_{urto} nel quale la pendenza dei grafici diventa molto ripida. Qual è la causa di questo cambiamento di pendenza?

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4. Il docente stima la velocità iniziale e finale di ciascun carrello tramite interpolazione lineare al calcolatore, individuando $v(t_{urto})$ con un cursore. Riporta $v_1(t_{urto})$ e $v_2(t_{urto})$ in unità del SI in tabella.

	Carrello 1	Carrello 2
PRIMA dell'urto		
DOPO l'urto		
Δv		

5. Che relazione si osserva fra Δv_1 e Δv_2 ? ____.

   Calcola il **rapporto $\Delta v_1/\Delta v_2$** ______.

6. Vengono misurate con il metodo esposto le variazioni di velocità Δv_1 e Δv_2 con il carrello 1 scarico e *il carrello 2 con 1 barra, 2 barre e poi 3 barre* successivamente; le barre sono supposte identiche. Il rapporto **$\Delta v_1/\Delta v_2$** in questi tre casi in che relazione sarà con quello ottenuto per i carrelli scarichi? (1) Minore di prima; (2) Uguale a prima; (3) Maggiore di prima; (4) Minore di prima (dipende dalla configurazione); (5) Maggiore di prima (dipende dalla configurazione) . Se il rapporto dipende dalla configurazione delle barre, prova a prevedere la dipendenza motivando.

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URTO	**$\Delta v_1/\Delta v_2$**
0 barre – 0 barre	
0 barre – 1 barra	
0 barre – 2 barre	
0 barre – 3 barre	

7. Utilizzando i grafici forniti calcola il rapporto $\Delta v_1/\Delta v_2$ nei vari casi (inserisci unità SI).

   a) carrello scarico contro 1 barra: Δv_1 =________, Δv_2 =__________, $\Delta v_1/\Delta v_2$ =_________.

   b) carrello scarico contro 2 barre: Δv_1 =________, Δv_2 =_________ , $\Delta v_1/\Delta v_2$ =_________.

   c) carrello scarico contro 3 barre: Δv_1 =________, Δv_2 =__________, $\Delta v_1/\Delta v_2$ =_________.

   Inserisci i rapporti nella seguente tabella riassuntiva.

8. Avevi previsto l'andamento dei dati in tabella? Scrivi sotto quali aspetti l'andamento sperimentale osservato differisce da quello previsto e in che cosa invece è simile (*analogie* e *differenze*).

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9. Nella pag. seguente disegna un grafico del rapporto $\Delta v_1/\Delta v_2$ in funzione del numero N di barre nel carrello carico. I dati presentano una correlazione? Se sì, *quale* e come si potrebbe giustificare in termini del *numero di componenti* dei sistemi che urtano?

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PROBLEMI DELLA MASSA NEWTONIANA

A ben vedere, un circolo vizioso molto simile a quello trattato tra massa, volume e densità esiste anche nella definizione usuale di *massa inerziale*. Essa si può considerare come la misura della capacità di un corpo di opporsi ad una variazione della sua velocità quando viene compiuto lavoro su di esso.

1) Con quale equazione ti è stata definita la massa inerziale?

2) Se in tale equazione compare la massa, dev'essere possibile determinare con precisione, quindi *misurare*, le altre grandezze fisiche. Quali sono?

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3) Illustra una procedura per misurarle *che non coinvolga la massa*

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4) Trai le conclusioni dai punti precedenti

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Mach ha un approccio diverso da Newton: la sua definizione di massa è uno strumento **per organizzare i fatti dell'esperienza,** in accordo con la sua visione della scienza: l'*empiriocriticismo*.

> *Diciamo corpi di massa uguale quelli che, agendo uno sull'altro, si comunicano accelerazioni uguali ed opposte*. […] *Se scegliamo il corpo A come unità di misura, attribuiremo la massa m a quel corpo che imprime ad A un'accelerazione pari a m volte l'accelerazione che esso riceve da A*. Il rapporto delle masse è il rapporto inverso delle accelerazioni preso con segno negativo. […] Il nostro concetto di massa non deriva da alcuna teoria. Esso contiene soltanto la precisa determinazione, designazione e definizione di un fatto. La "quantità di materia" è del tutto inutile.

Alla base di questa definizione vi è un principio di simmetria. In termini matematici essa si scrive

$$\frac{m_2}{m_1}=\frac{a_1}{a_2} \quad \Rightarrow \quad m_2=\frac{a_1}{a_2}m_1$$

Con questa definizione viene evitato il circolo vizioso riguardante la massa inerziale?

☐ Sì infatti____

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☐ No perché____

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Se consideriamo intervalli di tempo Δt uguali per le accelerazioni impresse e ricevute nell'urto la relazione sopra si può scrivere nella forma

$$m_2/m_1 = \Delta v_1/\Delta v_2$$

Questa relazione può essere utilizzata per *estendere la definizione di Mach all'esperimento svolto*:

$$\mathbf{m_x = m_1 \Delta v_1/\Delta v_2}$$

dove m_x può essere la *massa da misurare* e m_1 la massa unitaria di un oggetto campione.

Riformula la definizione di Mach nel contesto dell'esperimento, illustrando il ruolo delle grandezze considerate e le loro relazioni [*Suggerimento: quanto vale la massa del carrello1 rispetto a quella delle barre ? E quella del carrello2 ?*]

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Una definizione *operativa* di una grandezza fisica è data dal gruppo di operazioni (misure) da eseguire con determinati strumenti per determinarne il valore numerico. Scrivi allora una definizione operativa di massa inerziale che utilizzi urti elastici centrali.

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Nota che è possibile utilizzare la definizione di Mach in un qualunque sistema di riferimento in moto rettilineo uniforme rispetto a quello in cui si è effettuata la misura. Perché? (Ricorda la trasformazione di Galileo delle accelerazioni)

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Questo ci permette di definire univocamente il valore della massa inerziale di un corpo? *Motiva.*

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- Domanda di gruppo*. Discuti per 5 minuti in gruppo con due tuoi compagni vicini a te, poi rispondi individualmente alla domanda.* **Evidenzia gli aspetti problematici dei concetti newtoniani di massa come misura della quantità di materia e come misura dell'inerzia.**

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CONSERVAZIONE E ADDITIVITÀ DELLA MASSA

In fisica classica *la massa si conserva*. Descrivi come ciò avviene in ciascuna delle sei più importanti tipologie di trasformazioni chimico-fisiche: *traslazioni spazio-temporali, deformazioni*, *rotture, passaggi di stato, soluzioni e ossido-riduzioni.*

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In fisica una grandezza è detta "additiva" se *tale grandezza relativa a un oggetto composto è pari alla somma della stessa grandezza nei costituenti il composto.* Ad es. il volume di un *corpo rigido* è additivo, perché il volume di un corpo composto è pari alla somma dei volumi dei corpi costituenti il composto. La temperatura invece non è additiva, perché la temperatura di un oggetto non è uguale alla somma delle temperature dei suoi costituenti. La lunghezza di un'asta rigida è additiva, mentre la velocità di un corpo rigido e la densità non sono additive.

a) La massa è additiva? Ricorda le misure nell'esperimento con i carrelli (punto 9) e *motiva.*

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b) Quale implicazione logica è vera fra le seguenti ? *Motiva.*

☐ Grandezza additiva ⇒ grandezza conservativa (rispetto alle 5 trasformazioni);
☐ Grandezza conservativa ⇒ grandezza additiva;
☐ Grandezza conservativa ⇔ grandezza additiva;
☐ Nessuna delle precedenti.

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Nel nostro percorso abbiamo complessivamente incontrato tre accezioni di massa: "quantità di materia", massa gravitazionale e massa inerziale, quest'ultima definita in termini delle sue *proprietà* da Newton e in termini *operativi* da Mach.

- Domanda di gruppo. *Discuti per 5 minuti in gruppo con due tuoi compagni vicini a te, poi rispondi individualmente alla domanda.* **Illustra le differenze concettuali fra tutte le nozioni di massa esaminate**.

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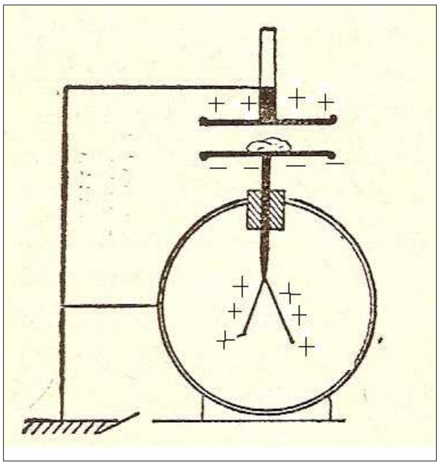
Appendix 4 – Slide contents

➢ *Treviso (4-vector rationale)*

1. **<u>L'equivalenza massa-energia</u>** (Emanuele Pugliese, Lorenzo Santi)
2. **<u>Atomo non indivisibile</u>**

   Esperimento con elettroscopio a foglie di P. Curie → evidenza di particelle che vengono espulse dall'atomo e ionizzano l'aria, chiudendo così il circuito, facendo scaricare le "foglie" precedentemente caricate positivamente → effetto visibile. Quindi <u>l'atomo dovrebbe essere composto dalle particelle (se sono tali) che vengono rivelate</u>. ⇒ **conferma di modello atomico discreto.**
3. **<u>Elettroscopio a foglie carico</u>**



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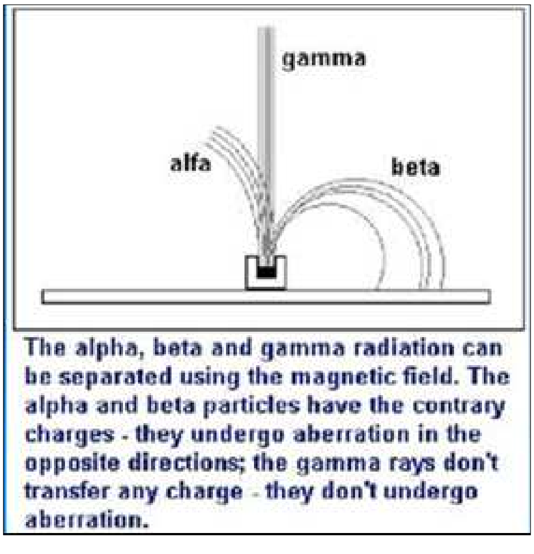

The alpha, beta and gamma radiation can be separated using the magnetic field. The alpha and beta particles have the contrary charges - they undergo aberration in the opposite directions; the gamma rays don't transfer any charge - they don't undergo aberration.

5. Principali decadimenti

La radioattività presuppone la trasmutazione di un elemento in un altro. Nel 1911 Rutherford introduce l'idea di *nucleo* atomico, composto da unità elementari (p, e^-, γ), attorno al quale orbitano gli elettroni. Attenzione: questo modello "planetario" dell'atomo non è quello attuale[132]. L'emissione radioattiva è conseguenza del decadimento di un nucleo "padre" in un nucleo "figlio". Ciò può avvenire in vari modi; i 3 più frequenti sono

a. Espulsione di un nucleo di elio (particella α);
b. Emissione di un elettrone (particella β);
c. Emissione di un fotone (raggio γ).

6. Massa atomica

Per la misura della massa atomica si possono utilizzare vari metodi, fra cui la spettrometria di massa, che sfrutta il differente raggio di curvatura di particelle con masse differenti e stessa carica, combinando le equazioni

$$m\frac{v^2}{R} = qvB$$

[per campi magnetici ortogonali al moto della particella]

$$qE = qvB \Rightarrow v = \frac{E}{B}$$ **(condensatore in campo magnetico ortogonale)**

Altri metodi: rapporto fra masse di 1 mole di elementi diversi e divisione per N_A. Come unità di misura viene scelta convenzionalmente 1 u = 1/12 m(^{12}C).

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[132] Una carica in moto circolare o ellittico è accelerata, quindi irraggia e perde energia: gli elettroni dovrebbero cadere sul nucleo in una piccola frazione di s.

7. **Mappa nuclidi (N-Z) colorati secondo la vita media**

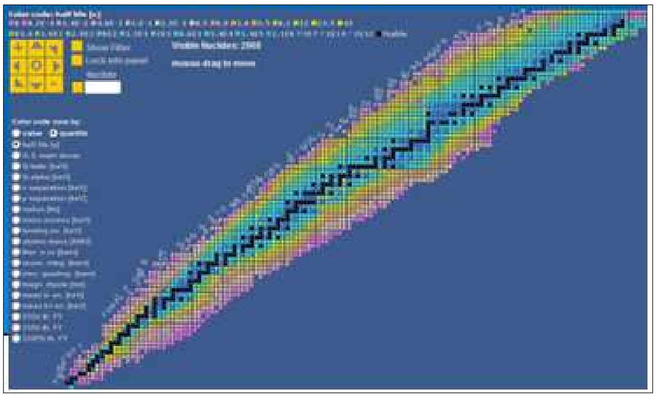


Image from Live Chart of Nuclides [28].

8. **Mappa nuclidi colorati secondo l'eccesso di massa**

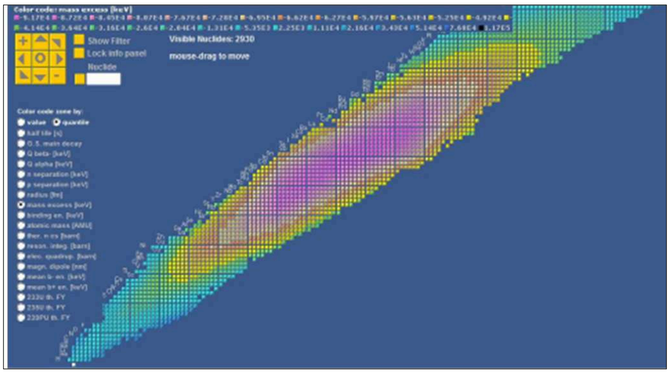


Image from Live Chart of Nuclides [28].

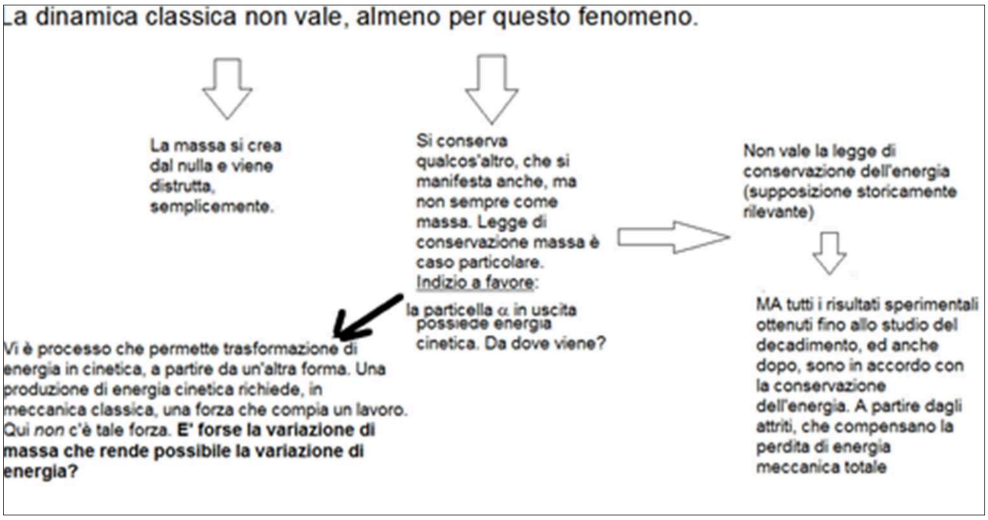
9. **Mappa nuclidi**

Un nuclide è la singola specie nucleare, mentre un nucleo generico non tiene conto della differenza di n dello stesso elemento. Dal confronto fra la mappa con le vite medie dei nuclidi e quella con l'eccesso di massa atomica si vede che quest'ultimo aumenta man mano che ci allontaniamo dalla zona dei nuclei stabili. Prova a calcolare sulla mappa cartacea l'aumento o la diminuzione di massa atomica in conseguenza di un decadimento β (spostamento in diagonale, in alto a sinistra nella mappa digitale, in basso a sinistra in quella cartacea).

10. **Problema: nel decadimento *β* la massa non si conserva**

Dalla dinamica classica sappiamo che la massa si conserva: come si risolve la contraddizione? Forse la *dinamica* classica non vale, almeno per questo fenomeno (vedi slide seg.). Esiste nuova fisica che interpreti anche questa fenomenologia?

11.



La dinamica classica non vale, almeno per questo fenomeno.

La massa si crea dal nulla e viene distrutta, semplicemente.

Si conserva qualcos'altro, che si manifesta anche, ma non sempre come massa. Legge di conservazione massa è caso particolare.
Indizio a favore:
la particella α in uscita possiede energia cinetica. Da dove viene?

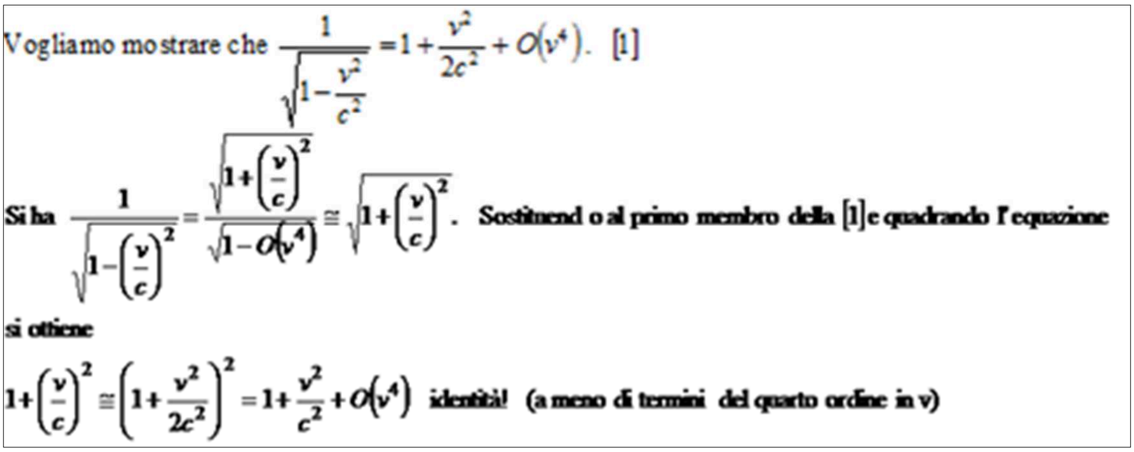
Vi è processo che permette trasformazione di energia in cinetica, a partire da un'altra forma. Una produzione di energia cinetica richiede, in meccanica classica, una forza che compia un lavoro. Qui *non* c'è tale forza. **E' forse la variazione di massa che rende possibile la variazione di energia?**

Non vale la legge di conservazione dell'energia (supposizione storicamente rilevante)

MA tutti i risultati sperimentali ottenuti fino allo studio del decadimento, ed anche dopo, sono in accordo con la conservazione dell'energia. A partire dagli attriti, che compensano la perdita di energia meccanica totale

12. Approssimazioni per basse velocità

a. Controlli numerici: confronto fra l'espressione esatta e quella approssimata per velocità crescenti (ad es. 30 km/s, c/10, c/2).
b. Metodo algebrico esatto per termini di ordine inferiore a 4 in v.



Vogliamo mostrare che $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}=1+\frac{v^2}{2c^2}+O(v^4)$. [1]

Si ha $\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}=\frac{\sqrt{1+\left(\frac{v}{c}\right)^2}}{\sqrt{1-O(v^4)}}\cong\sqrt{1+\left(\frac{v}{c}\right)^2}$. **Sostituendo al primo membro della [1] e quadrando l'equazione si ottiene**

$1+\left(\frac{v}{c}\right)^2\cong\left(1+\frac{v^2}{2c^2}\right)^2=1+\frac{v^2}{c^2}+O(v^4)$ **identità! (a meno di termini del quarto ordine in v)**

13. Limite newtoniano

$$\Delta T=\Delta E(\gamma-1)=\Delta E\left(1+\frac{1}{2}\frac{v^2}{c^2}+O(v^4)-1\right)\cong\frac{1}{2}\Delta E\frac{v^2}{c^2}$$

$$\Delta T=\Delta\left(\frac{1}{2}mv^2\right)=\frac{1}{2}(\Delta m)v^2.$$

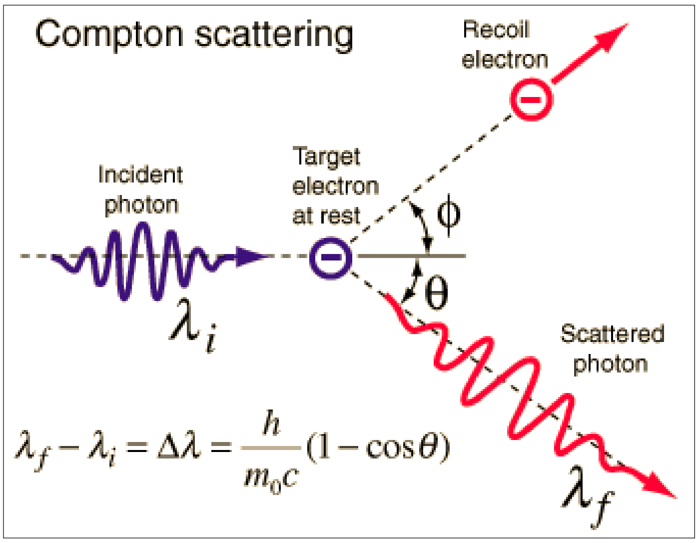
Di conseguenza otteniamo $\Delta m=\Delta E/c^2$.

*«Se un corpo emette l'energia L sotto forma di radiazione, la sua massa diminuisce di L/V^2 […] La massa di un corpo è una misura del suo contenuto di energia; variando l'energia di una quantità L la massa varia nello stesso senso di L/(9*10^{20}), se l'energia è misurata in erg e la massa in grammi».*

14. **Calcolo degli effetti inerziali di un fotone che urta le pareti di una scatola**

    Vi sono vari modi in cui un fotone può interagire con la materia. Oltre all'effetto fotoelettrico che già conosci, vi sono l'effetto Compton, l'assorbimento/emissione atomico e la produzione di coppie. Concentriamoci sul primo: il suo scopritore, Arthur Compton, fu essenzialmente in grado di spiegare le osservazioni della diffusione anelastica di un fotone da parte di un elettrone. Dovette utilizzare la conservazione dell'energia e della quantità di moto (momento lineare) quanto-relativistiche[133]. Ci interessa qui considerare il caso di una scatola sottoposta ad un'accelerazione verticale costante, a causa di una forza esterna (ad esempio un parallelepipedo in caduta libera nel vuoto). All'interno della scatola vi sia un fotone che si muove nella direzione del moto e viene riflesso prima sulla parete superiore, poi su quella inferiore.

15. **Effetto Compton**



16. Per arrivare ad un modello in accordo con i dati *a qualsiasi lunghezza d'onda* è stato necessario scrivere, in accordo con la teoria della relatività, la quantità di moto del fotone (ora "particella") come $p = h\nu/c = h/\lambda$ [dualismo

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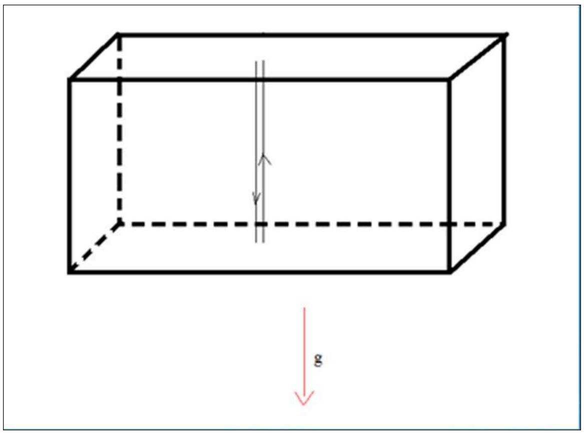
[133] Tale diffusione è infatti modellizzabile come urto fra due oggetti con proprietà corpuscolari, nel quale l'energia si *conserva*. Perciò si può dire che, in tal senso, l'urto è elastico.

onda/corpuscolo → onde di materia].

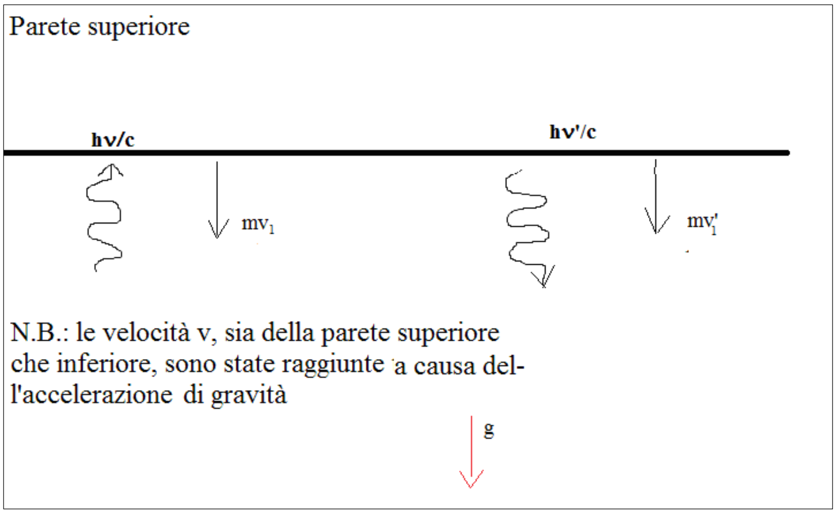
17.



18. Iniziamo anche noi con i due principi di conservazione, nella forma non relativistica (le velocità che la scatola raggiunge sono sempre trascurabili rispetto a *c*).

$$\begin{cases} -\dfrac{h\nu}{c} + mv = \dfrac{h\nu'}{c} + mv' \\ h\nu + \dfrac{1}{2}mv^2 = h\nu' + \dfrac{1}{2}mv'^2 \end{cases}$$

19.



N.B.: le velocità v, sia della parete superiore che inferiore, sono state raggiunte a causa dell'accelerazione di gravità

20.

$$\begin{cases} mv\left(c+\dfrac{v}{2}\right)=2h\nu'+mv'\left(c+\dfrac{v'}{2}\right) \\ -2h\nu+mv\left(c-\dfrac{v}{2}\right)=mv'\left(c-\dfrac{v'}{2}\right) \end{cases}$$

$$\frac{2h\nu'}{2h\nu}=\frac{m\left[v\left(c+\dfrac{v}{2}\right)-v'\left(c+\dfrac{v'}{2}\right)\right]}{m\left[v\left(c-\dfrac{v}{2}\right)-v'\left(c-\dfrac{v'}{2}\right)\right]}.$$

Applicando il teorema di De l'Hôpital, considerato che le due velocità differiscono di poco (l'impulso dato da un fotone è trascurabile rispetto al moto dovuto a "g") e quindi possiamo far tendere l'una all'altra, si ha

$$\lim_{v'\to v}\frac{m\left[v\left(c+\dfrac{v}{2}\right)-v'\left(c+\dfrac{v'}{2}\right)\right]}{m\left[v\left(c-\dfrac{v}{2}\right)-v'\left(c-\dfrac{v'}{2}\right)\right]}=\lim_{v'\to v}\frac{-c-v'}{-c+v}=\frac{c+v}{c-v}$$

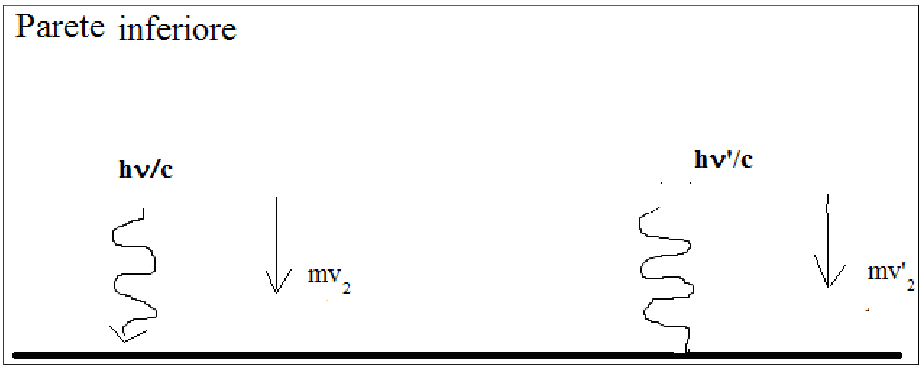
21. Variazione quantità di moto del fotone:

$$(\Delta p)_{ph}=\frac{h\nu'}{c}-\left(-\frac{h\nu}{c}\right)=\frac{h\nu}{c}\left(1+\frac{c+v}{c-v}\right)=\frac{2h\nu}{c-v}$$

Consideriamo ora la parete <u>inferiore</u>: con un ragionamento analogo si ottiene

$$\nu'=\nu\frac{c-v}{c+v}\Rightarrow(\Delta p)_{ph}=-\frac{h\nu'}{c}-\left(\frac{h\nu}{c}\right)=-\frac{h\nu}{c}\left(1+\frac{c-v}{c+v}\right)=-\frac{2h\nu}{c+v}$$

22. Ci interessa relazionare l'**impulso** δp trasferito alla **scatola dal fotone** (uguale ed opposta a quella del fotone) all'**accelerazione** costante **con cui si muove la scatola.**



$$(\delta p)_{sup}=-\frac{2h\nu_1}{c-v_1}\qquad(\delta p)_{inf}=\frac{2h\nu_2}{c+v_2}$$

Teniamo conto che $\boxed{\nu_2 = \nu_1' = \nu_1 \frac{c+v_1}{c-v_1}.}$ $\boxed{v_2 = v_1 + a\frac{dt}{2}}$

$$\boxed{\begin{aligned}\delta p &= (\delta p)_{sup} + (\delta p)_{inf} = 2h\left(\frac{\nu_1}{c-v_1} - \frac{\nu_2}{c+v_2}\right) = -2h\left(\frac{\nu_1}{c-v_1} - \nu_1\frac{c+v_1}{c-v_1}\frac{1}{c+v_1+a\frac{\Delta t}{2}}\right) = \\ &= -\frac{2h\nu_1}{c-v_1}\left(1 - \frac{c+v_1}{c+v_1+a\frac{\Delta t}{2}}\right) = -\frac{2h\nu_1}{c-v_1}\left(\frac{a\Delta t/2}{c+v_1+a\Delta t/2}\right) \underset{a\frac{\Delta t}{2} << v_1+c}{\cong} -\frac{2h\nu_1}{c^2-v_1^2}a\frac{\Delta t}{2} \underset{v_1<<c}{\cong} -\frac{2h\nu_1}{c^2}a\frac{dt}{2}.\end{aligned}}$$

23. Perciò

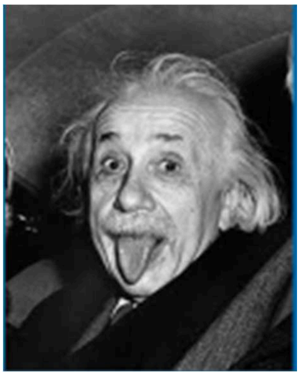
$$\boxed{\frac{\delta p}{dt} = F_{ph} \cong -\frac{h\nu_1}{c^2}a = -\frac{E_{ph}}{c^2}a}$$

- Senza fotone $\boxed{F_{ext} = Ma}$

- Con il fotone $\boxed{F'_{ext} + F_{ph} = Ma \quad \Rightarrow \quad F'_{ext} = \left(M + \frac{E_{ph}}{c^2}\right)a}$

- ***2013 summer school (Udine) (4-vector rationale)***

1. **Origine e significato dell'equivalenza massa-energia** (Emanuele Pugliese, Lorenzo Santi)



2. **Relatività: i 2 postulati**

   1. *Principio di relatività,* nelle forme che seguono:

   - P.R. teorico: « *tutte* le leggi fisiche (non solo le leggi della meccanica) hanno la stessa forma in tutti i SdR inerziali »;
   - P.R. sperimentale: « esperimenti (di qualsiasi natura) condotti nelle stesse condizioni in diversi SdR inerziali danno gli stessi risultati. »

   [I postulato]

   2. *Invarianza della velocità della luce nel vuoto: «c* è l'unica velocità il cui valore non dipende dal SdR in cui si misura, né dalla direzione di propagazione».

   [II postulato]

3. **L'orologio a luce**

   Consideriamo 2 *osservatori* in due diversi riferimento, il primo su un ipotetico treno in viaggio a velocità costante *v* rispetto alla stazione, e il secondo sul marciapiede della stazione. Dal primo l'orologio a luce sarà sempre visto fermo, dal secondo in moto con velocità *v*. Percorrerà quindi uno spazio orizzontale Δx in un tempo Δt.
   http://webphysics.davidson.edu/physlet_resources/special_relativity/default.html

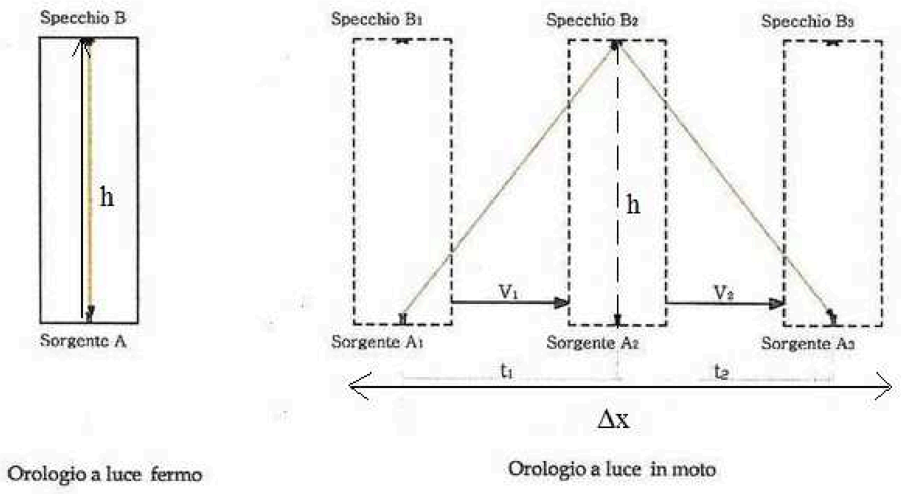


Figura 8.1 rappresentazione dell'esperimento per la spiegazione della dilatazione del tempo con un orologio a luce secondo l'ipotesi relativistica

4. L'orologio a luce

Quanto impiegherà il raggio di luce, visto da terra, a compiere andata e ritorno?

$$c\Delta t = 2\sqrt{h^2+\left(\frac{\Delta x}{2}\right)^2} = 2\sqrt{\left(\frac{\Delta \tau}{2}c\right)^2+\left(\frac{\Delta x}{2}\right)^2} = \sqrt{(c\Delta \tau)^2+(\Delta x)^2}\,.$$

L'intervallo temporale fra due eventi che avvengono nella stessa posizione è detto intervallo di ***tempo proprio*** Δτ. Il SdR in cui tali eventi accadono nella stessa posizione è detto ***riferimento proprio*.**

$$\frac{\Delta x}{\Delta t} = -v \quad \Rightarrow \quad (c\Delta t)^2 = (c\Delta\tau)^2 + v^2(\Delta t)^2$$

$$(\Delta\tau)^2 = \frac{c^2-v^2}{c^2}(\Delta t)^2 = \left[1-\frac{v^2}{c^2}\right](\Delta t)^2 \Rightarrow \Delta t = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\Delta\tau$$

$$\Delta\tau = \frac{\Delta t}{\gamma} \quad \Leftrightarrow \quad \Delta t = \gamma\,\Delta\tau$$

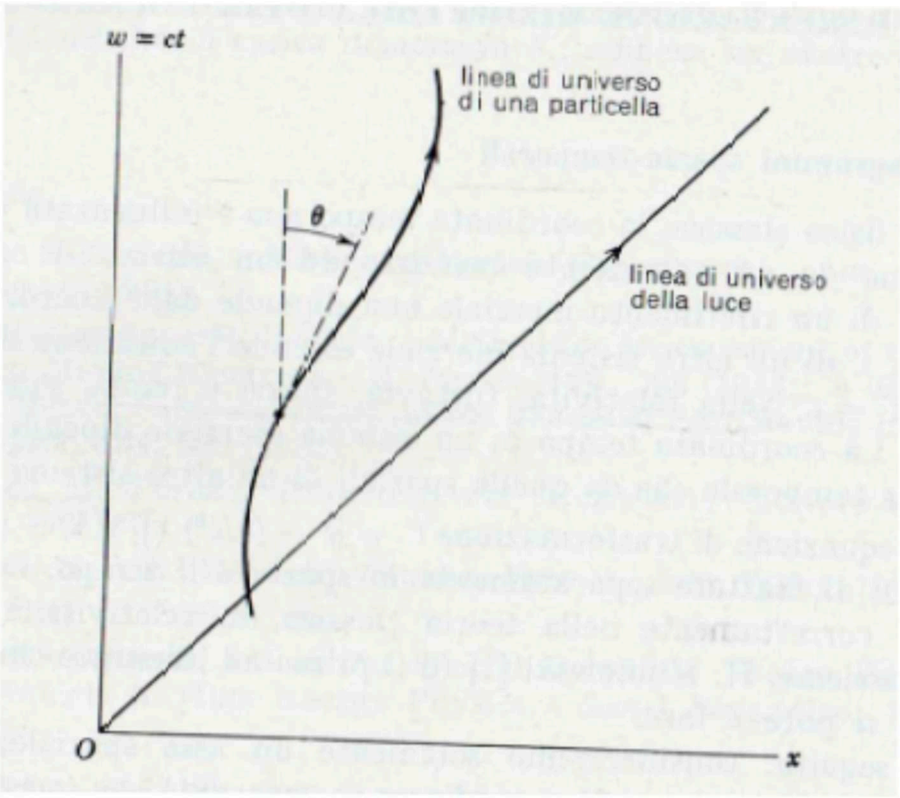
L'espressione sopra esprime l'effetto di ***dilatazione delle durate*** (o *dilatazione dei tempi*).

5. L’orologio a luce

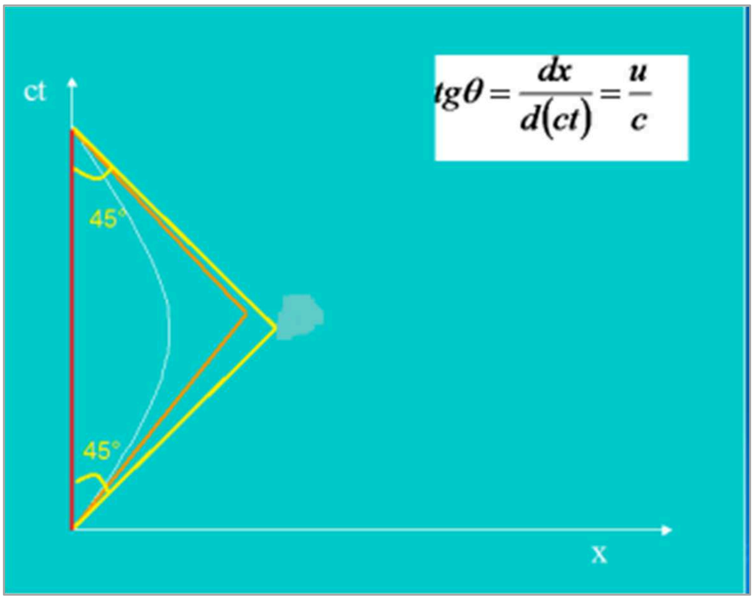
- L’istante e la posizione (il tempo e lo spazio rispettivamente) in cui avvengono i fenomeni, pur rimanendo distinti, sono uniti fra loro dalla formulazione di Einstein: viene teorizzata l’esistenza di uno *spazio-tempo*. Quest’ultimo è stato interpretato geometricamente nel 1908 dal matematico Hermann Minkowski.
- Dai calcoli svolti nell’esperimento dell’orologio a luce risulta

$$\boxed{(c\Delta\tau)^2 = (c\Delta t')^2 - (\Delta x')^2}$$

6. Spazio-tempo di Minkowski



7. Spazio-tempo di Minkowski



Quale delle linee d’universo nel grafico sopra descrive la traiettoria con minor tempo proprio, cioè con minor “lunghezza” ?

8. Quadrispostamento

- Nello spazio euclideo lo spostamento è rappresentato, in coordinate cartesiane, dal vettore $(\Delta x, \Delta y, \Delta z)$.
- Il suo modulo, distanza fra due punti legati dal moto di un oggetto fisico, non dipende in meccanica classica dal particolare riferimento in cui è calcolato: è un *assoluto*. Analogamente si può introdurre un *quadrivettore nello spazio-tempo di Minkowski*: $(c\Delta t, \Delta x, \Delta y, \Delta z)$.

9. Quadrivelocità

- Per ottenere la velocità si dividono le componenti dallo spostamento per l’intervallo temporale (invariante in meccanica classica), poi si prende limite $\Delta t \to 0$.
  - Analogamente, per ottenere il quadrivettore velocità a partire dal quadrivettore spostamento bisogna dividerlo per uno *scalare invariante*, che in relatività non può essere l’intervallo di tempo coordinato Δt.

10. **Quadrivelocità**

- L'unico candidato è l'intervallo di tempo proprio $\Delta\tau$, o $\Delta s = c\,\Delta\tau$.
- **Quadrivelocità:**

$$\lim_{\Delta\tau\to 0}\left[c\frac{\Delta t}{\Delta\tau},\frac{\Delta x}{\Delta\tau},\frac{\Delta y}{\Delta\tau},\frac{\Delta z}{\Delta\tau}\right]$$

- (Analogamente alla velocità classica)

$$\lim_{\Delta t\to 0}\left[\frac{\Delta x}{\Delta t},\frac{\Delta y}{\Delta t},\frac{\Delta z}{\Delta t}\right]$$

11. **Quadrimpulso**

- La quantità di moto in meccanica classica è data dal prodotto massa × velocità.
- Per ottenere il quadrivettore analogo relativistico (di carattere non più cinematico, ma dinamico) bisogna moltiplicare la quadrivelocità per un *invariante dinamico*.
- ***NEW*** **Ipotizziamo** che in relatività introdurre la massa inerziale newtoniana, invariante nel passaggio fra riferimenti.
- ***Il quadrimomento*** è dato allora da

$$\lim_{\Delta\tau\to 0} m\left[c\frac{\Delta t}{\Delta\tau},\frac{\Delta x}{\Delta\tau},\frac{\Delta y}{\Delta\tau},\frac{\Delta z}{\Delta\tau}\right]=\lim_{\Delta\tau\to 0} m\left[c\frac{\Delta t}{\Delta\tau},\frac{\Delta x}{\Delta t}\frac{\Delta t}{\Delta\tau},\frac{\Delta y}{\Delta t}\frac{\Delta t}{\Delta\tau},\frac{\Delta z}{\Delta t}\frac{\Delta t}{\Delta\tau}\right]=$$

$$=\lim_{\Delta t\to 0}\left[\gamma mc, m\frac{\Delta x}{\Delta t}\gamma, m\frac{\Delta y}{\Delta t}\gamma, m\frac{\Delta z}{\Delta t}\gamma\right]=\left(\gamma mc,\gamma mv_x,\gamma mv_y,\gamma mv_z\right).$$

12.

Facciamo il limite newtoniano della prima componente del quadrimomento: $\gamma mc^2 \underset{\frac{v}{c}\to 0}{=} mc^2+\frac{1}{2}mv^2$, a meno di termini del IV ordine in v.
Per corrispondenza con la meccanica classica poniamo $K_{rel}\equiv(\gamma-1)mc^2$, da giustificare sperimentalmente.
Possiamo a questo punto dare alla componente temporale del quadrimomento il significato di energia totale relativistica: $E=E_0+K_{rel}=\gamma mc^2$.

In tal modo abbiamo attuato l'identificazione

$$\boxed{E_0 = mc^2}$$

13. Sviluppi in serie

Metodo algebrico per termini di ordine inferiore a 4 in u: potenze della velocità inferiori alla quarta.

$$\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}=\frac{\sqrt{1+\frac{u^2}{c^2}}}{\sqrt{1-\frac{u^4}{c^4}}}=\frac{\sqrt{1+\frac{u^2}{c^2}}}{\sqrt{1-O(u^4)}}\cong\sqrt{1+\frac{u^2}{c^2}}$$ **(trascurando termini di quarto ordine o superiore)**

Se ne facciamo il quadrato otteniamo $1+\frac{u^2}{c^2}$.

D'altra parte $\left(1+\frac{1}{2}\frac{u^2}{c^2}\right)^2=1+\frac{u^2}{c^2}+O(u^4)\cong 1+\frac{u^2}{c^2}$

Abbiamo dimostrato che $\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\cong 1+\frac{1}{2}\frac{u^2}{c^2}$ **(trascurando termini di quarto ordine in *u* o superiore)**

14. *$E_0=mc^2$ – Equivalenza massa-energia*

La somma di tutti i contributi all'energia di un corpo, escluso quello cinetico, è data quindi dalla massa del corpo stesso, a meno di un fattore c^2. **L'energia interna di un corpo, in tutte le sue forme, è equivalente alla sua massa.**

Alternativamente, **la massa è uguale a tutta l'energia di un corpo quando è in quiete (energia interna o "energia a riposo").** Numericamente, c'è il fattore c^2 fra le due quantità, ma in opportune unità di misura può essere reso adimensionale e pari a 1.

➢ *Cremona (energetic phenomenological rationale)*

1. **Richiami di dinamica classica ed elettrostatica**

   Lavoro ed energia cinetica; lavoro elettrico e accelerazione di particelle cariche. Emanuele Pugliese – URDF Udine.

2. **Lavoro ed energia cinetica**

- Se applichiamo ad un corpo rigido una forza F costante (= non dipendente dalla *posizione)* che produca uno spostamento Δx del suo C.M. *nella direzione della forza* definiamo il prodotto $F\ \Delta x$ come il **lavoro compiuto dalla forza sul corpo.**
- Sappiamo che in generale il lavoro è una misura quantitativa dell'energia trasferita ad un corpo (rigido) per mezzo di una forza esterna.
- Quando il compimento di lavoro su un corpo (sistema fisico) genera esclusivamente un cambiamento del suo stato di moto*, allora diciamo che il corpo ha acquisito energia cinetica. *Lo stato di moto cambia quando varia la velocità.

3. **Teorema dell'energia cinetica**

   Inserendo la II legge della dinamica $F = ma = m\dfrac{\Delta v}{\Delta t}$ nella definizione di lavoro si ottiene – poiché l'accelerazione è costante –

$$L = F\Delta x = m\,\Delta v\,\frac{\Delta x}{\Delta t} = m\left(v_f - v_i\right) v_m \Rightarrow$$

$$L = m\left(v_f - v_i\right)\frac{\left(v_f + v_i\right)}{2} = \frac{1}{2}m\left(v^2{}_f - v^2{}_i\right)$$

4. $$L = \frac{1}{2}mv^2{}_f - \frac{1}{2}mv^2{}_i = \Delta K$$

5. **Lavoro elettrico e potenziale**

- Definiamo d.d.p. elettrostatico il lavoro elettrico per unità di carica.
- Lavoro svolto per portare una particella di prova* di carica q_0 da un punto iniziale A con potenziale elettrostatico V_A a B con potenziale V_B (il campo elettrico è conservativo) è dato da $L_e = q_0\left(V_A - V_B\right) = -q_0\ \Delta V$
- Una particella che "cade" da un punto di potenziale V_A a uno con $V_B < V_A$ acquisisce energia cinetica → viene accelerata.
- Se $V_B > V_A$ la sua energia cinetica diminuisce → è frenata.

$$\Delta K = -q_0\ \Delta V \qquad \begin{array}{l} \text{Se } \Delta V < 0 \Rightarrow \Delta K > 0 \\ \text{Se } \Delta V > 0 \Rightarrow \Delta K < 0 \end{array}$$

---

*Particella la cui carica sia sufficientemente piccola da interagire con il campo elettrico presente nello spazio in modo trascurabile.

6. **Acceleratori elettrostatici**

- Principio di funzionamento degli acceleratori e.s.: viene mantenuta costante una d.d.p. che permette di aumentare l'energia cinetica di un fascio di particelle cariche, portandole ad alte velocità.
- Nel Van de Graaff le d.d.p. raggiunte sono di parecchi MV.
- Per questo motivo è spesso usato come unità di misura dell'energia di particelle cariche l'eV. 1 eV è l'energia acquisita da un elettrone che attraversa la d.d.p. di 1 V.
- Quanti eV acquista un protone che attraversa 1V ?

6. **Esercizi: soluzione esatta in minor tempo**

- Quanti J corrispondono a 1 eV?
- Due superfici conduttrici parallele, piane, distanziate di $d = 1.00$ cm hanno una differenza di potenziale $\Delta V = 625$ V. Un protone viene proiettato da un piatto verso l'altro. Qual è la velocità iniziale del protone se esso si ferma proprio sulla superficie del secondo piatto?
- Una particella alfa (composta da due protoni e due neutroni) viene accelerata attraverso una d.d.p. di 1.0 MV in un acceleratore di Van de Graaff (a) Che energia cinetica acquisisce? (b) Un protone che energia cinetica acquisirebbe nelle stesse circostanze? (c) Quale particella acquisterebbe la maggiore velocità partendo da ferma? [*Lo studente svolga i calcoli in eV e multipli*]

7. **Domande**

- Gli elettroni tendono a spostarsi nelle regioni ad alto potenziale o a basso potenziale? Esplicita il ragionamento utilizzato per rispondere.
- Abbiamo fatto riferimento all'energia cinetica. Quali altre forme di energia conosci?
- Considera un corpo che si muove sotto l'azione di forze conservative (forza gravitazionale, elettrica, elastica,…) in un sistema isolato. Qual è la *proprietà fondamentale* dell'energia meccanica totale posseduta dal corpo?
- Se le forze agenti sono anche non conservative (es. attriti) *esiste una grandezza fisica che possieda la stessa proprietà? Quale?* Spiega.

8. **Una nuova dinamica**

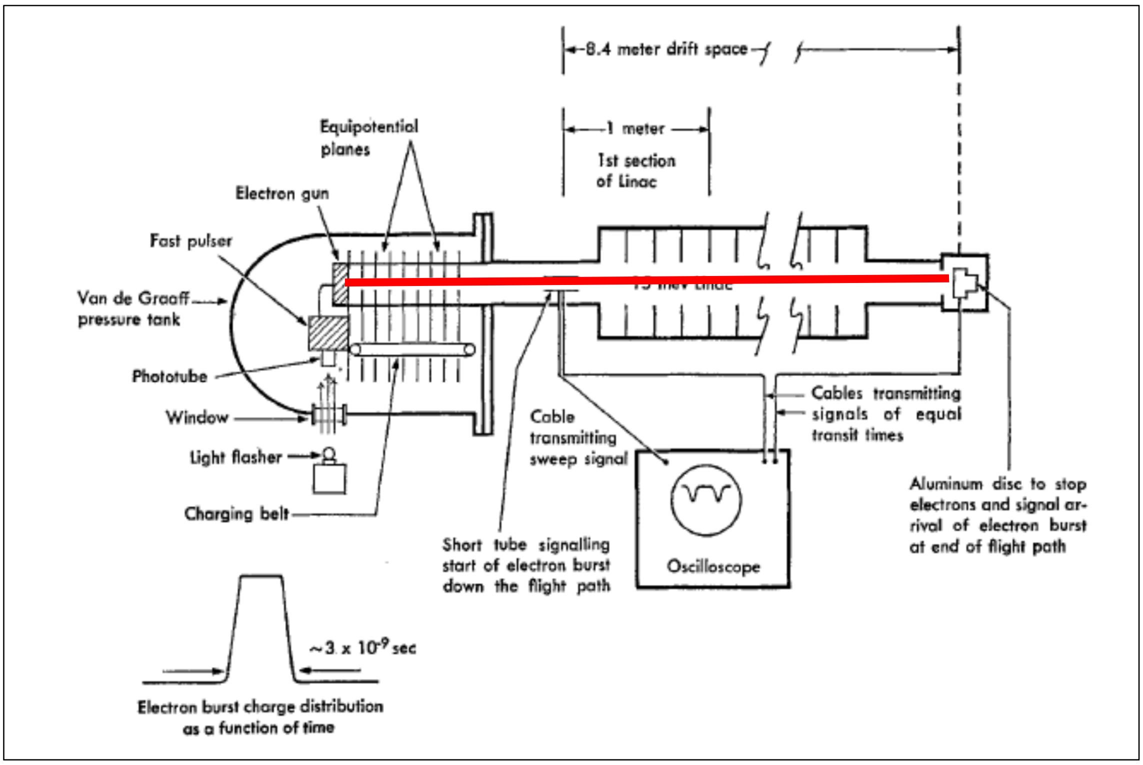
Emanuele Pugliese, Lorenzo Santi – URDF Udine (contributi di Silvio Bergia).

9. **Velocità ed energia cinetica di elettroni relativistici (W. Bertozzi, 1964)**

Abbiamo visto che dall'equazione fondamentale della dinamica classica inserita nella definizione di lavoro segue il teorema dell'energia cinetica. Il lavoro compiuto da un generatore di Van de Graaf è di tipo *elettrico* → per un e^- inizialmente a riposo accelerato da una d.d.p. ΔV ci aspettiamo che valga la relazione

$$e\,\Delta V = \frac{1}{2} m \mathrm{v}^2 \Rightarrow \mathrm{v}^2 = \frac{2e}{m}\Delta V$$

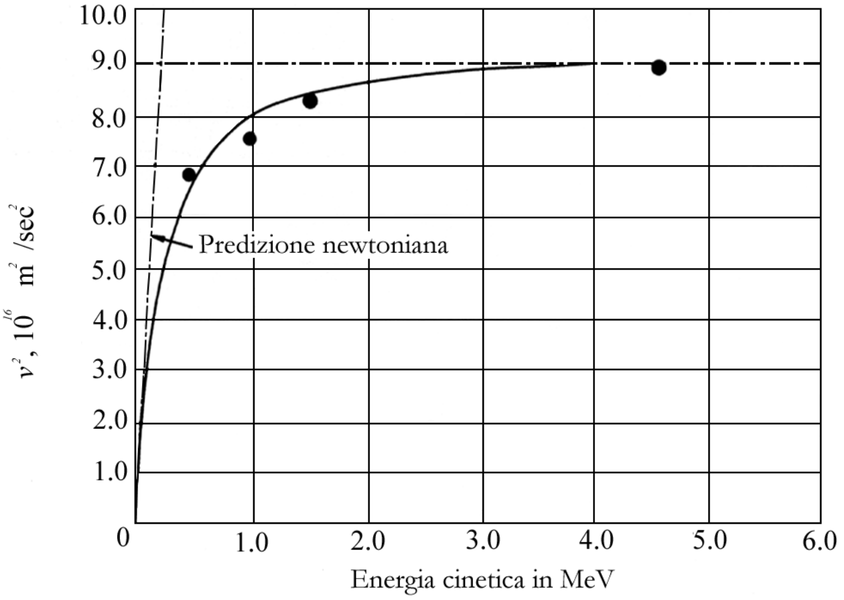
10. **Velocità ed energia cinetica di elettroni relativistici (W. Bertozzi, 1964)**



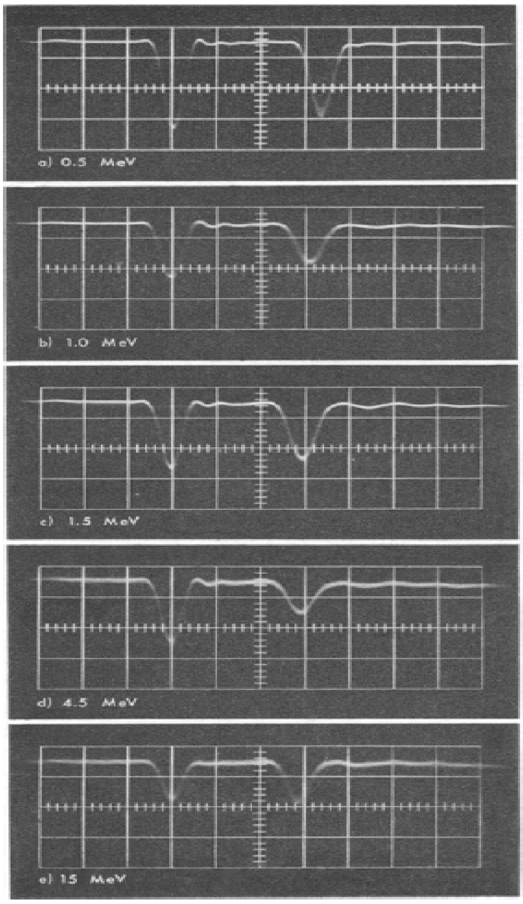
Picture adapted from Bertozzi (1964).

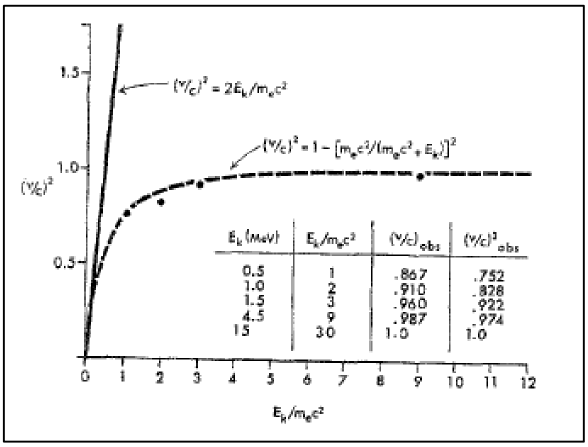
11. **Velocità ed energia cinetica di elettroni relativistici (W. Bertozzi, 1964)**

La previsione classica è un aumento indefinito della velocità al crescere della ΔV mantenuta nel Van de Graaf secondo una legge di potenza: $v^2 \propto \Delta V$. Sperimentalmente si rileva invece che, per quanto si aumenti la ddp, la velocità non raggiunge (né supera) mai *c*.



12. Velocità ed energia cinetica di elettroni relativistici (W. Bertozzi, 1964)





E_k (MeV)	E_k/m_ec^2	$(v/c)_{obs}$	$(v/c)^2_{obs}$
0.5	1	.867	.752
1.0	2	.910	.828
1.5	3	.960	.922
4.5	9	.987	.974
15	30	1.0	1.0

Perciò non vale $v^2 \propto \Delta V$, che è conseguenza diretta della legge fondamentale della dinamica → **l'intera dinamica newtoniana è invalidata**.

Pictures adapted from Bertozzi (1964).

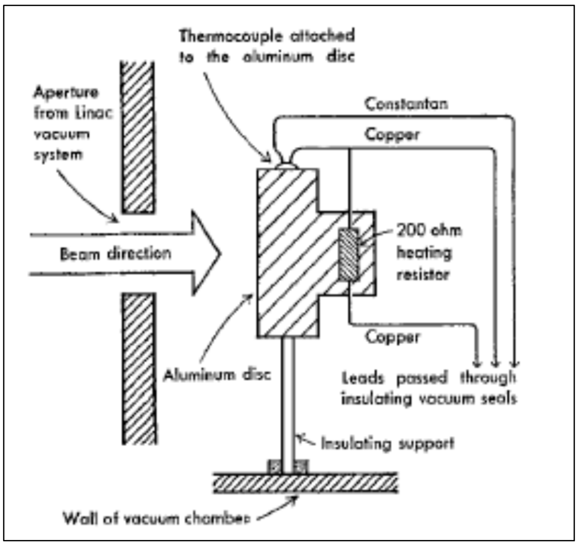
13. Misura calorimetrica (W. Bertozzi, 1964)

Con una termocoppia si misura la variazione di temperatura ΔT del bersaglio.

Legge fondamentale della calorimetria: $\Delta E_{int} = M\ c\ \Delta T$.

Per il principio di *conservazione dell'energia* il lavoro compiuto sull'elettrone deve valere $L = \Delta E_{int}$. Poiché il lavoro ha cambiato lo stato di moto degli elettroni assumiamo che $L = \Delta K = \Delta E_{int}$: il teorema dell' energia cinetica vale anche alle alte velocità.

Poiché la meccanica newtoniana non è rispettata, sarà necessario cambiare l'espressione dell'energia cinetica.



Picture by Bertozzi (1964).

14. P.R. einsteiniano

- Le **sonde spaziali**, quando la loro traiettoria è lontana da masse rilevanti, possono essere considerati SI. La *stessa fisica* valida nei laboratori terrestri* funziona anche su di esse. Questo può essere testimoniato da esperimenti eseguiti dagli astronauti.
- Anche **sulla Luna** l'acceleraz di gravità non dipende da m.
- **Stelle:** per spiegarne l'evoluzione osservata si è costruito un modello basato sulla fisica valida in un lab. terrestre. **Questo modello funziona: è in accordo con le osservazioni astronomiche!** Altre osservazioni ci hanno permesso di trovare che le stelle **si muovono a velocità ≥ 100 km/s** rispetto ai lab. sulla Terra. Questo non è scontato: nei SdR comoventi a tali corpi valgono per ogni intervallo temporale infinitesimo *dt* le stesse leggi fisiche che valgono nel SdR del lab e delle sonde! Gli esperimenti condotti nelle stesse condizioni danno gli stessi risultati.

- **Galassie:** può essere fatta la stessa assunzione, che è corroborata da risultati sperimentali (ad es. redshift cosmologico di linee spettrali di elementi noti).

*Trascurando effetti non-inerziali, che possono a loro volta essere accuratamente spiegati da un osservatore inerziale esterno.

15. **La velocità limite dev'essere invariante**

Immaginiamo di compiere **un esperimento di accelerazione di elettroni ottenuta mantenendo una d.d.p. costante** in un lab in moto rispetto alla Terra con velocità v, mantenendo l'asse dell'acceleratore orientato positivamente nella direzione del moto. Acceleriamo un elettrone fino alla velocità $c - \varepsilon$ (con $\varepsilon < v$).

Con che velocità si muove l'elettrone rispetto alla Terra?

Se valesse la regola classica per la somma dovrebbe raggiungere

$$\boldsymbol{c - \varepsilon + v > c} \ ,$$

Superando la velocità limite. Qualcosa non va nelle hp formulate.

Non sembra infatti fisicamente e logicamente accettabile che usando come propellente elevate d.d.p. in grado di fornire alte energie cinetiche non si riesca a portare un elettrone alla velocità c … e utilizzando un propellente chimico per produrre una piccola accelerazione del catodo emittente si riesca a fargliela superare !!

16. **Invarianza di *c***

La conclusione di Einstein: non esiste un sistema di riferimento privilegiato rispetto al quale, e solo rispetto al quale, la luce si propaga nel vuoto alla velocità c. Quale che sia lo stato di moto di un osservatore rispetto a tale ipotetico riferimento inerziale privilegiato ***egli ne riscontrerà lo stesso valore***. Non c'è un sistema di riferimento privilegiato, un sistema ***assoluto***. Tutti i sistemi di riferimento (inerziali) sono equivalenti. Il ***principio di relatività*** valido in meccanica classica non lo era per l'elettromagnetismo di fine Ottocento.

La teoria einsteiniana ne sanciva la validità anche per l'elettromagnetismo, in breve **per tutti i fenomeni**. A questo si deve il suo nome.

17. **Domanda**

Quali sono i principali risultati dell'esperimento di Bertozzi? Indicali.

18. **<u>1905, Einstein: la relatività ristretta</u>**

C'era qualcosa che non funzionava nella visione ottocentesca riguardo alla propagazione della luce. Al proposito, nella sua autobiografia scientifica, pubblicata nel 1984, Einstein scrisse:

"A poco a poco incominciai a disperare della possibilità di scoprire le vere leggi attraverso tentativi basati su fatti noti. Quanto più a lungo e disperatamente provavo, tanto più mi convincevo che solo la scoperta di un principio formale universale avrebbe potuto portarci a risultati sicuri.

Dopo dieci anni di riflessione, un siffatto principio risultò da un paradosso nel quale m'ero imbattuto all'età di 16 anni: se io potessi seguire un raggio di luce a velocità *c* (la velocità della luce nel vuoto), il raggio di luce mi apparirebbe come un campo elettromagnetico oscillante nello spazio, in stato di quiete. Ma nulla del genere sembra poter sussistere sulla base dell'esperienza o delle equazioni […]" dell'elettromagnetismo.

19. **<u>1905, Einstein: la relatività ristretta</u>**

"E` chiaro che in questo paradosso è già contenuto il germe della relatività particolare."

Se viaggiassi alla stessa velocità di un'onda piana monocromatica che si propaga nel vuoto vedrei un profilo sinusoidale statico.

La luce è però radiazione e.m. che necessariamente si propaga a c, come richiesto dalla teoria e mostrato dalle onde elettromagnetiche rivelate in laboratorio nel 1887 da Hertz.

Poiché l'ipotesi che esista un SdR in quiete con la luce porta a una contraddizione con la teoria e l'esperimento, tale ipotesi dev'essere irrealizzabile: **deve essere impossibile (per un qualsiasi corpo materiale) viaggiare alla velocità della luce nel vuoto.**

Questa velocità deve essere dunque una **velocità limite.**

20. **<u>I due postulati</u>**

«L'idea teorica […] non nasce al di fuori ed indipendentemente dall'esperienza; né può derivare dall'esperienza per puro procedimento logico. È il prodotto di un atto creativo. Una volta che l'idea teorica sia acquisita, è bene seguirla finché non si dimostra insostenibile» [Einstein, *Scientific American,* 1950]

- P.R. teorico: **tutte** le leggi fisiche hanno la stessa forma in tutti i sistemi di riferimento inerziali (SI).

- P.R. sperimentale: esperimenti **di qualsiasi natura** condotti nelle stesse condizioni in SI diversi danno gli stessi risultati.
- Invarianza della velocità della luce: *c* non dipende dal SI in cui è misurata, né dalla direzione della propagazione della luce.

21. Composizione delle velocità

Se abbiamo in un laboratorio in moto rispetto a noi alla velocità v qualcosa che viaggia nella stessa direzione e verso alla velocità *u*' secondo la fisica di tutti i giorni (fis. classica) la sua velocità *u* rispetto a noi sarà

$$\boxed{u = \mathrm{v} + u'}$$

$$\begin{aligned} x &= x' + \mathrm{v}t \\ t &= t' \end{aligned}$$

Allora, se quel qualcosa viaggia alla velocità limite *c*, dovrebbe avere velocità $u = \mathrm{v} + c > c$ rispetto a noi! Bisogna cambiare la legge di composizione delle velocità. Essa, nella cinematica classica, si deriva dalle leggi di trasformazione delle coordinate:

Se vogliamo cambiare la legge di composizione delle velocità in modo da garantire l'invarianza di *c* dobbiamo quindi cambiare la forma delle trasformazioni galileiane. Un esperimento mentale ci dà al proposito un risultato importante.

22. Orologio a luce

Consideriamo due *osservatori* in differenti *SI,* il primo (A) su un treno in corsa con velocità uniforme ***v*** rispetto alla stazione, il secondo (B) fermo sul marciapiede. A vedrà sempre l'orologio a luce fermo, B lo vedrà in moto

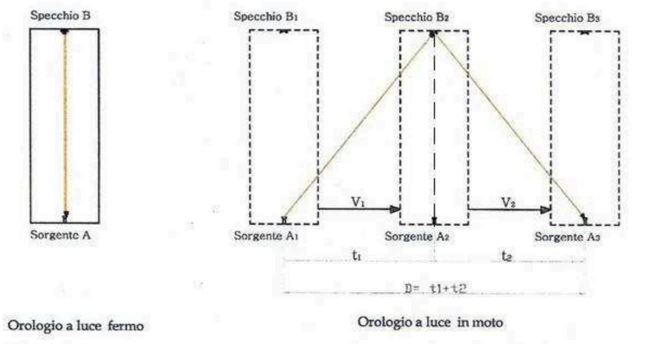


Figura 8.1 rappresentazione dell'esperimento per la spiegazione della dilatazione del tempo con un orologio a luce secondo l'ipotesi relativistica

Si pone per convenzione che il tempo di percorrenza della luce in un percorso "one-way" valga la metà di quello (misurabile) di andata e ritorno.

uniforme con velocità v. Nel SI di B l'orologio a luce percorrerà quindi una distanza orizzontale Δx in un intervallo Δt.

23. **Orologio a luce**

Calcoliamo quanto dura il percorso di andata e ritorno (valutato da B)

$$c\Delta t = 2\sqrt{h^2+\left(\frac{\Delta x}{2}\right)^2} = 2\sqrt{\left(\frac{\Delta \tau}{2}c\right)^2+\left(\frac{\Delta x}{2}\right)^2} = \sqrt{(c\Delta\tau)^2+(\Delta x)^2}\,.$$

L'intervallo temporale fra due eventi che avvengono nella stessa posizione è detto ***intervallo di tempo proprio*** $\Delta\tau$. Il SI in cui tali eventi accadono nella stessa posizione è detto **riferimento proprio.**

Dilatazione delle durate

$$\frac{\Delta x}{\Delta t} = -v \quad \Rightarrow \quad (c\Delta t)^2 = (c\Delta\tau)^2 + v^2(\Delta t)^2$$

$$(\Delta\tau)^2 = \frac{c^2-v^2}{c^2}(\Delta t)^2 = \left[1-\frac{v^2}{c^2}\right](\Delta t)^2 \Rightarrow \Delta t = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\Delta\tau$$

$$\Delta\tau = \frac{\Delta t}{\gamma} \quad \Leftrightarrow \quad \Delta t = \gamma\Delta\tau$$

- L'istante e la posizione (il tempo e lo spazio rispettivamente) in cui avvengono i fenomeni, pur rimanendo distinti, sono uniti fra loro dalla formulazione di Einstein: viene teorizzata l'esistenza di uno *spaziotempo*.
- Dai calcoli svolti nell'esperimento dell'orologio a luce risulta

$$\boxed{(c\Delta\tau)^2 = (c\Delta t')^2 - (\Delta x')^2}$$

- Se gli intervalli temporali (durate) sono misurati da diversi SI essi *variano*: danno risultati diversi, proprio come gli intervalli spaziali (lunghezze).
- Ma qualunque sia il SI considerato, il **tempo proprio**, differenza dei quadrati dei due, ha sempre lo stesso valore (II invariante relativistico).
- Qual è il primo invariante relativistico?

24. **Stimoli per la riflessione sull'effetto di dilatazione dei tempi**

➢ Abbiamo dotato l'osservatore sul treno e quello sulla banchina di due orologi identici.
  - I battiti del cuore del secondo osservatore risultano realmente rallentati secondo il primo?
  - Il secondo osservatore sente i propri battiti rallentati?

  Spiega.

➢ Abbiamo dotato un manovratore di ponti e un passeggero su un treno di due orologi identici.
  - L'intervallo di tempo fra l'apertura e la chiusura di un ponte mobile è differente per il passeggero del treno rispetto a chi lo manovra? Spiega.

25. **Teorema energia cinetica…**

Riscrivo il teorema nella forma più generale seguente, utilizzando la variazione di q.d.m. e la velocità (media e istantanea)

$$F = \frac{\Delta p}{\Delta t} \Rightarrow L = F\Delta x = \frac{\Delta p}{\Delta t}\Delta x = \Delta p \frac{\Delta x}{\Delta t} = \Delta p\, u_m$$

$$L = \Delta p\, u_m = \Delta K$$

Passando ai differenziali

$$dp\left[\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\right] = dK$$

$$u\, dp = dK$$

La matematica suggerisce che occorre modificare l'espressione di p.

26. **E la quantità di moto?**

La conservazione della quantità di moto ***p*** in meccanica classica è fondamentale: possiamo fare l'ipotesi che abbia un corrispettivo anche in ambito relativistico.

Di conseguenza la quantità di moto p di un corpo animato da moto vario, **misurata rispetto al SI del laboratorio,** dipenderà in maniera diversa dalla velocità istantanea rispetto alla q.d.m. classica: $p \neq mv$.

27. **Urti elastici**

- Gli urti elastici possono essere studiati senza conoscere i dettagli dell'interazione tramite la conservazione della q.d.m. e dell'energia cinetica: a fianco il caso 2-D.

$$\boxed{\begin{cases} m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x} \\ m_1 v_{1y} + m_2 v_{2y} = m_1 v'_{1y} + m_2 v'_{2y} \\ \frac{1}{2} m_1 v^2_1 + \frac{1}{2} m_2 v^2_2 = \frac{1}{2} m_1 v'^2_1 + \frac{1}{2} m_2 v'^2_2 \end{cases}}$$

- Nel caso proiettile-bersaglio ($v_2=0$) **unidimensionale** le espressioni per le velocità finali si trovano facilmente essere quelle a destra.
- Se le masse sono anche uguali fra loro, il primo oggetto si ferma e il secondo continua con la stessa velocità che aveva il primo.
- Se invece l'urto proiettile bersaglio avviene in 2-D, le direzioni delle particelle uscenti formano **sempre un angolo retto** fra loro.

$$\boxed{\begin{cases} v_1' = \dfrac{(m_1 - m_2)}{(m_1 + m_2)} v_1 \\ v_2' = \dfrac{2m_1}{(m_1 + m_2)} v_1 \end{cases} \Rightarrow \begin{cases} v_1' = 0 \\ v_2' = v_1 \end{cases} \text{(m uguali)}}$$

28. **Seconda parte**

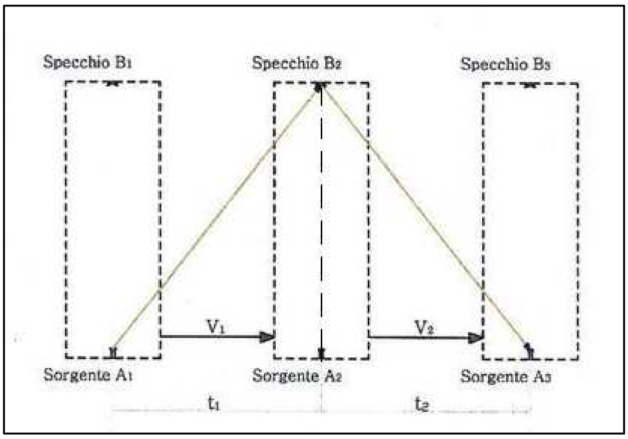
    Riassunto I parte e deduzione K relativistica

29. **I due postulati**

1. Assumiamo il Principio di Relatività
- P.R. (teorico): tutte le leggi fisiche hanno la stessa forma in tutti i sistemi di riferimento inerziali (SI).
- P.R. (sperimentale): esperimenti di qualsiasi natura condotti nelle stesse condizioni in SI diversi danno gli stessi risultati.
2. P.R. + rilevazione sperimentale di una velocità limite ⇒ In ogni SI si misura la stessa velocità limite…
- Invarianza della velocità della luce: *c* non dipende dal SI in cui è misurata, né dalla direzione della propagazione della luce.

… e bisogna cambiare la legge di composizione delle velocità

30. L’intervallo temporale fra due eventi che avvengono nella stessa posizione è detto ***intervallo di tempo proprio*** $\Delta\tau$. Il SI in cui tali eventi accadono nella stessa posizione è detto **riferimento solidale**.



$$\frac{\Delta x}{\Delta t} = -v \quad \Rightarrow \quad (c\Delta t)^2 = (c\Delta\tau)^2 + v^2(\Delta t)^2$$

$$(\Delta\tau)^2 = \frac{c^2 - v^2}{c^2}(\Delta t)^2 = \left[1 - \frac{v^2}{c^2}\right](\Delta t)^2 \Rightarrow \Delta t = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\Delta\tau$$

$$\Delta\tau = \frac{\Delta t}{\gamma} \quad \Leftrightarrow \quad \Delta t = \gamma\Delta\tau$$

31. L’istante e la posizione in cui avvengono i fenomeni, pur rimanendo distinti, sono uniti fra loro dalla formulazione di Einstein: viene teorizzata l’esistenza di uno *spaziotempo* fatto di *eventi* (= istante + posizione).

    Dai calcoli svolti nell’esperimento dell’orologio a luce risulta

    Se gli intervalli temporali (durate) sono misurati da diversi SI essi danno risultati diversi: *variano*.

    Anche gli intervalli spaziali (lunghezze) misurati da diversi SI danno risultati diversi…

    Ma il **tempo proprio $\Delta\tau$** (differenza tra il quadrato dell’intervallo temporale e di quello spaziale) misurato da diversi SI dà sempre lo stesso valore: è un *invariante relativistico*.

32. **Teorema dell'energia cinetica**

Riscrivo il teorema in una forma più generale, utilizzando la variazione di q.d.m. e la velocità (media e istantanea)

$$F = \frac{\Delta p}{\Delta t} \Rightarrow L = F\Delta x = \frac{\Delta p}{\Delta t}\Delta x = \Delta p \frac{\Delta x}{\Delta t} = \Delta p\, u_m$$

$$L = \Delta p\, u_m = \Delta K$$

Passando ai differenziali

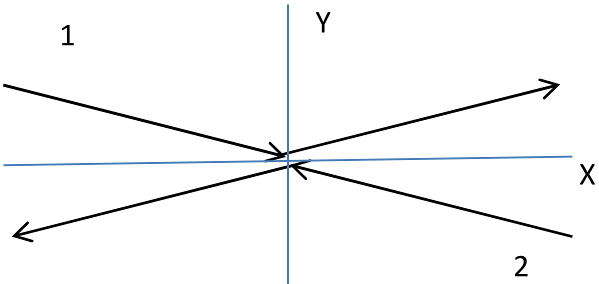
$$dp\left[\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\right] = dK$$

$$u\, dp = dK$$

K non ha la forma classica (esp. Bertozzi) → la matematica suggerisce che occorre modificare anche l'espressione di *p* (conservata in relatività).

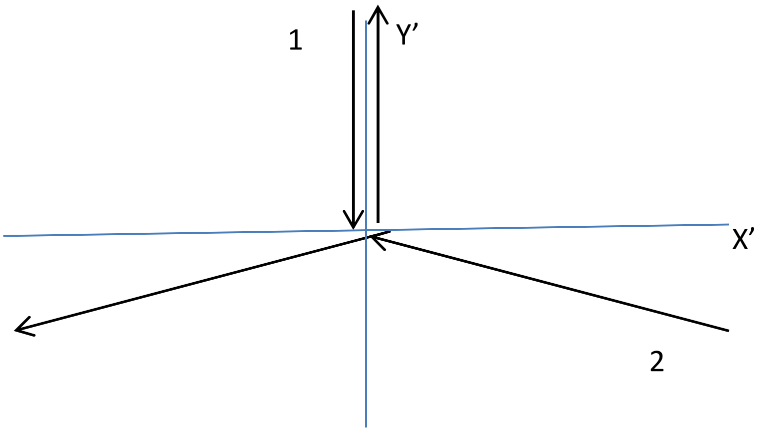
33. **Urto di due particelle nel caso relativistico**

Riferimento CM. Le velocità e gli impulsi cambiano solo di direzione, non in modulo. $p_1 = p_2 = p'_1 = p'_2$; $u_1 = u_2 = u'_1 = u'_2$



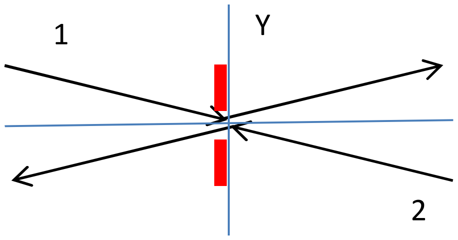
- Hp: (1) la quantità di moto si conserva negli urti relativistici analizzati da *qualunque* SI; (2) urto radente.
- Simmetria completa tra 1 e 2.

Riferimento lab. v_1 e v'_1 sono lungo y.

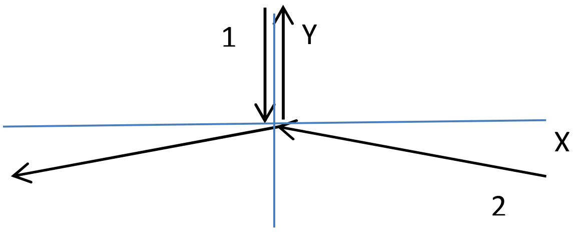


Part. 1 non relativistica: $p_1 = m v_1$; $p_1' = m v'_1$.

34. Urto di due particelle nel caso relativistico



C.M.



LAB

Invarianti:

- **spostamenti ortogonali** alla direzione del moto del SI lab rispetto al CM;
- **intervallo $\Delta\tau$** fra due posizioni della particella 2.

C.M.: Spazi (lungo y) uguali e tempi coordinati uguali → $v_{y1} = v_{y2}$

LAB.: Tempi propri uguali e spostamenti (lungo y) uguali fra 1 e 2, ma *tempi coordinati diversi* per *dilatazione tempi* → $v'_1 = v'_{2y} / (1 - (v'_2/c)^2)^{1/2}$.

Conservazione q.d.m. + collinearità tra v e p → $p'_2 = m\, v'_2 / (1-(v'_2/c)^2)^{1/2}$.

$$p = m\, v \Big/ \sqrt{1 - v^2 / c^2}$$

35. Quesito

Scrivi una ragione per la quale la q.d.m. non può essere scritta alle alte velocità nella forma classica p = m v.

36. Energia cinetica

Abbiamo ottenuto l'espressione matematica per la q.d.m. a qualunque velocità. Assumiamo la validità di

$$p\,(u) = m\, u \Big/ \sqrt{1 - u^2 / c^2} = \gamma(u) m\, u$$

Quale sarà la forma per la *variazione di K di un corpo che viene portato nel SI del lab da fermo alla velocità u* ?

37. **Energia cinetica**

$$dK = u\,dp = m\,u\;d\left(\gamma u\right) = mu\left\{\left(d\gamma\right)u + \gamma du\right\}\,(*)$$

$$\text{Poiché }\frac{1}{\gamma^2} = 1 - \frac{u^2}{c^2} \Rightarrow -\frac{2}{\gamma^3}\frac{d\gamma}{du} = -2\frac{u}{c^2} \Rightarrow \frac{d\gamma}{du} = \gamma^3\frac{u}{c^2}$$

$$d\gamma = \gamma^3\frac{u}{c^2}du \Rightarrow du = \frac{1}{\gamma^3}\frac{c^2}{u}d\gamma \text{ posso sostituire il secondo differenziale}$$

$$(*)\; dK = mu\left\{u\,d\gamma + \gamma\frac{1}{\gamma^3}\frac{c^2}{u}d\gamma\right\} = m\,u\,d\gamma\left\{u + \frac{c^2}{u} - u\right\} = m\,c^2 d\gamma$$

$$\text{Se } dK = m\,c^2 d\gamma \text{ allora } \Delta K = m\,c^2\Delta\gamma = m\,c^2\left(\gamma(u) - \gamma(0)\right) = m\,c^2\left(\gamma - 1\right)$$

L'energia "cinetica" dev'essere nulla per def. quando l'oggetto è fermo :

$$K(0) \equiv 0.$$

$$\text{Quindi } \Delta K = K(u) - K(0) = K$$

$$K = m\,c^2\left(\gamma - 1\right)$$

38. **Limite newtoniano di K**

Vediamo a che cosa si riduce questa relazione per velocità molto inferiori a quelle della luce (limite classico). Dobbiamo approssimare la funzione

$$\gamma\,(u) = 1\,/\,(1 - (u/c)^2\,)^{1/2}$$

Utilizzeremo a tal fine la formula di approssimazione delle potenze di un binomio (anche con esponente frazionario)

$$\left(1 - X\right)^N \underset{X<<1}{=} 1 - NX + O\left(X^2\right)$$

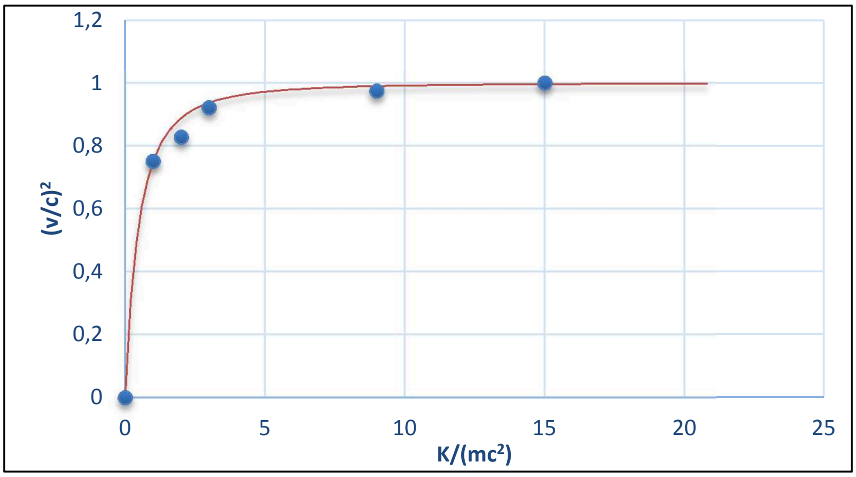
Nel nostro caso N = -1/2 e quindi

$$\gamma = \left(1 - \left(\frac{u}{c}\right)^2\right)^{-1/2} \underset{\left(\frac{u}{c}\right)^2<<1}{\cong} 1 - \left\{-\frac{1}{2}\left(\frac{u}{c}\right)^2\right\}$$

L'espressione per K diventa $K = m\,c^2\,(\gamma - 1) \cong m\,c^2\,(1 + \tfrac{1}{2}\,(u/c)^2 - 1) = \tfrac{1}{2}\,mu^2$, **coincidente con l'espressione classica.**

39. **Interpolazione dati di W. Bertozzi**

Un **controllo** dell’espressione ricavata per K si può fare con i dati dell’esperimento di Bertozzi. Si può facilmente ricavare l’espressione relativistica per il quadrato della velocità in funzione dell’energia cinetica invertendo la formula trovata per K: $v^2/c^2 = 1 - [\, m_e c^2 / (m_e c^2 + K) \,]^2$.



(previsione di Einstein: curva rossa). **Essa riproduce bene l’andamento dei dati.**

$K = m c^2 (\gamma - 1)$ è la nuova espressione
per l'energia cinetica

40. **Domande**

L’energia cinetica è:

- La quantità espressa dall’equazione $K = \frac{1}{2} m v^2$;
- La forma di energia associata allo stato di moto dell’e^-;
- Un termine essenziale per soddisfare il principio di conservazione dell’energia;
- Una combinazione delle precedenti (precisa quale).

41. **Il fotone**

Emanuele Pugliese, Lorenzo Santi – URDF Udine

42. **Interpretazione di Einstein dell’effetto fotoelettrico**

Esistono «particelle»* di luce: i **fotoni**! La luce è composta da quantità indivisibili di energia (*quanti di luce*) **localizzati** in punti dello spazio che si muovono **senza suddividersi** e che possono essere prodotti e assorbiti solo come **entità complete** (in «pacchetti»). Essi interagiscono in modo discreto con la materia e in tal modo vengono rivelati. Hanno $m = 0$.

Esistono **altre proprietà**, oltre all’energia, che possono essere associate ai fotoni?

* Particelle “speciali”, non come il protone o l’elettrone. I fotoni mostrano sia una natura corpuscolare sia ondulatoria, a seconda dell’esperimento in cui vengono osservati.

43. La pressione di radiazione

Alla fine dell’Ottocento venne formulata da Maxwell l’ipotesi la luce esercitasse una «spinta» sui materiali su cui essa incide.

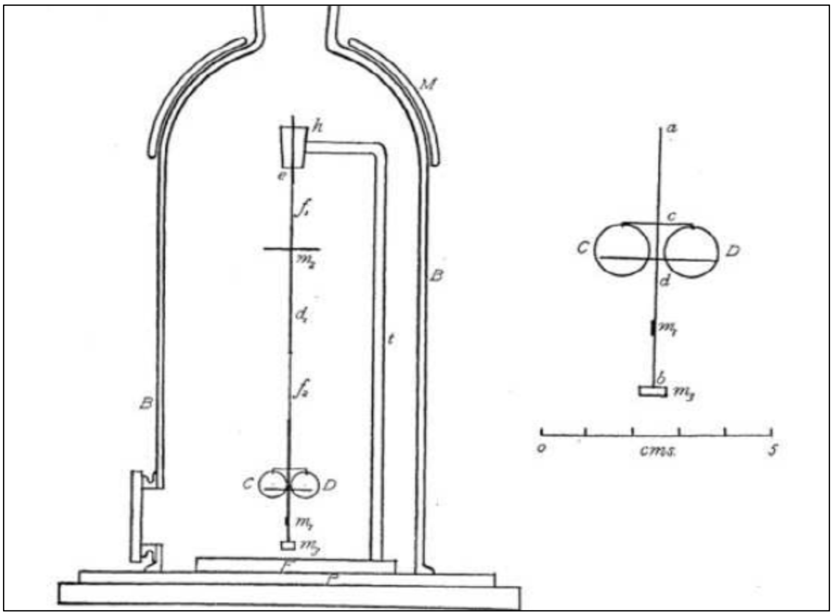
- L’hp, formulata a partire dal suo studio della luce come radiazione elettromagnetica, prevedeva che **la luce scambiasse quantità di moto con la materia** oltre che energia.
- La supposta espressione della q.d.m. da associare alla luce era

$$p = E / c$$

ove E è l’energia luminosa incidente.

44. Indagini sperimentali

Lebedev (1901) e Nichols (1903) effettuarono degli esperimenti per controllare tale ipotesi. Utilizzarono una bilancia di torsione equipaggiata con due specchi. Su uno degli specchi viene inviato un fascio di luce, che provoca la rotazione dell’equipaggio mobile.



Nichols and Hull (1903)

45. Indagini sperimentali

I 2 esperimenti hanno mostrato (con diversi gradi di precisione):

- La forza F esercitata dalla luce sullo specchio (impulso trasmesso per unità di tempo) è proporzionale alla potenza P (energia trasmessa per unità di tempo) associata al fascio luminoso;
- Tale costante di proporzionalità risultava essere, entro gli errori, pari a **2/c**. Quindi

$$\mathbf{F = P * (2 / c) \qquad [legge\ empirica]}$$

- $F = \Delta p / \Delta t$; $P = E / \Delta t \Rightarrow \Delta p = 2\,p$ (è una riflessione) $= E * (2 / c)$
- Alla luce incidente allora deve venire associato un impulso

$$\mathbf{p = E / c}$$

$$\mathbf{E = pc}$$

E è l'energia trasportata dalla luce. Tale risultato deve essere esteso anche ai **singoli fotoni** che compongono il fascio.

46. Domande sul fotone

"La luce scambia la sua energia con la materia in modo continuo o discreto? Da che cosa la scienza moderna pensa sia composta la luce?"

"Perché risulta che i fotoni trasportano q.d.m?"

47. Massa ed energia

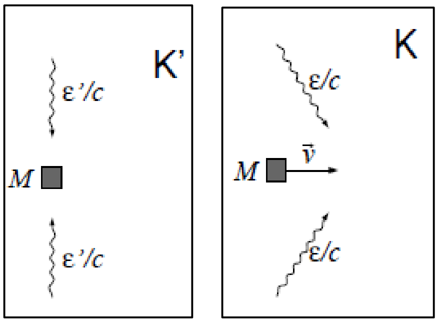
Emanuele Pugliese – URDF Udine

48. L'assorbimento di fotoni: un esperimento mentale

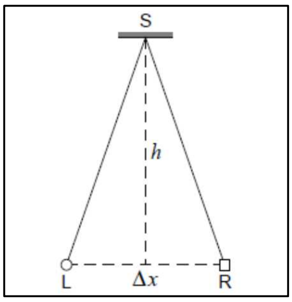
Due fotoni vengono emessi con quantità di moto uguali e opposte da una sorgente in quiete in K'. In questo SI la quantità di moto del corpo è nulla PRIMA e DOPO l'assorbimento.

In un SI K in moto con velocità v rispetto al primo l'energia dei fotoni è diversa (la indichiamo con ε' per distinguerla da ε).

La quantità di moto del corpo PRIMA è



$P = M\,\gamma v + 2\varepsilon/c\ (v/c)$.



49. **<u>L’assorbimento di fotoni: un esperimento mentale</u>**

La quantità di moto DOPO è $P' = M\,\gamma_f v_f$

MA ….

Nel primo sistema di riferimento il corpo è fermo prima e dopo, e quindi la velocità v_f nell’altro sistema di riferimento sarà pari a v: **il sistema fisico assorbe energia senza variare la sua velocità.**

Quindi DOPO $M\,\gamma\,v = M\,\gamma\,v + 2\varepsilon\,v/c^2$

ASSURDO!

50. **<u>L’assorbimento di fotoni: un esperimento mentale</u>**

L’unico modo per risolvere l’assurdo è supporre che la massa DOPO l’assorbimento sia diversa da quella PRIMA.

Conservazione q.d.m.: $M\,\gamma_f\,v_f = M\,\gamma\,v + 2\varepsilon\,v/c^2$

Manteniamo l’hp $v_f = v \Rightarrow \gamma_f = \gamma$

$$M'\,\gamma v = M\,\gamma v + 2\varepsilon\,v/c^2$$

$$M' = M + 2\varepsilon/(\gamma c^2)$$

Ma 2 ε è l’energia assorbita dal corpo: ΔE.

Perciò possiamo affermare se **un corpo assorbe energia senza variare la sua velocità, la sua massa varia di**

$$\mathbf{\Delta M = \Delta E / (\gamma\, c^2)}$$

Nel SI in cui il corpo è fermo $\gamma = 1 \Rightarrow$ un input di energia ΔE in un corpo a riposo nel lab. **può** far aumentare la sua massa di

$$\mathbf{\Delta M = \Delta E / c^2}$$

mantenendolo a riposo nel SI del lab.

51. **<u>Domanda</u>**

Prima dell’assorbimento, il fotone ha massa?

Con l’assorbimento il sistema (fotone + scatola) ha aumentato la sua massa? Perché?

52. **Che cos'è l'energia posseduta da un sistema ?**

Se consideriamo un oggetto di massa iniziale **trascurabile** ($m<\varepsilon$, ε arbitrariamente piccolo) **l'input di energia necessaria perché esso acquisti una massa *m* è dato da** $\Delta E = (\Delta m)\, c^2 = (m - 0)\, c^2 = m\, c^2$

$$\Rightarrow \mathbf{m\, c^2 \equiv E_0}$$

può essere vista come **l'energia necessaria per la creazione di una particella di massa *m* a riposo.** $m = E_0 / c^2$ è la massa che l'oggetto ha assunto per il fatto che abbiamo fornito energia E_0 al vuoto → creazione di particelle negli acceleratori: basta che

- ci sia energia a disposizione $\geq m\, c^2$
- che sia rispettata la conservazione q.d.m.

- **La massa misura l'energia complessiva posseduta da un sistema o particella a riposo: $E_0 = m\, c^2$.**

53. **Energia totale**

E_0 è detta **energia a riposo.** Ma finora abbiamo ragionato in un solo SI; prendiamo ora in considerazione l'energia dovuta al moto del SI solidale all'oggetto rispetto al SI del lab: essa è **l'energia cinetica del corpo** nel SI del lab.

Per ottenere **l'energia totale** di un sistema dovrò quindi sommare il contributo di energia a riposo a quello cinetico:

$$\mathbf{E = E_0 + K}$$

La massa misurata nel SI solidale è la stessa che si misura nel lab perché per misurare *m* ci si riferisce sempre al riferimento co-movente (solidale). In esso si eseguono misure a basse velocità tramite cui si trova il valore di *m* utilizzando la $F = ma$.

La massa è quindi un invariante relativistico: la sua misura dà lo stesso risultato per tutti i SI. Perciò possiamo sommare:

$$\mathbf{E = m\, c^2 + m\, c^2 (\gamma - 1) = m\, c^2 + \gamma\, m\, c^2 - m\, c^2 = \gamma\, m\, c^2}$$

$$\mathbf{E = \gamma\, m\, c^2} \text{ (energia totale relativistica).}$$

54. **Domande**

"Se compio lavoro su un corpo esteso in modo da accelerarlo, la sua massa varia? E nel caso di una particella?"

Come si possono interpretare i comportamenti dei nuclidi osservati? (1) è distrutta massa; (2) la massa si trasforma in qualcos'altro; (3) la massa rappresenta, solo nello stato iniziale ma non in quello finale, qualcos'altro che si conserva; (4) la massa totale non è data dalla somma delle masse dei costituenti. Motiva le tue scelte.

- ***Ancona (energetic phenomenological rationale)***

1. **Richiami di dinamica classica ed elettrostatica**

   Lavoro ed energia cinetica; lavoro elettrico e accelerazione di particelle cariche.

2. **Lavoro ed energia cinetica**
   - Se applichiamo ad un corpo una forza *F* costante durante uno suo spostamento *Δx nella direzione della forza* definiamo il prodotto come il **lavoro compiuto dalla forza sul corpo F Δx.**
   - Il lavoro è interpretabile come una quantità di energia trasferita ad un corpo per mezzo di una forza esterna.
   - Quando il compimento di lavoro su un corpo (sistema fisico) genera esclusivamente un cambiamento del suo stato di moto*, allora diciamo che il corpo ha acquisito energia cinetica.

*Lo stato di moto cambia quando varia la velocità.

3. **Teorema dell'energia cinetica**

   Inserendo la II legge della dinamica $F = ma = m\frac{\Delta v}{\Delta t}$ nella definizione di lavoro si ottiene – poiché l'accelerazione è costante –

$$L = F\Delta x = m\,\Delta v\,\frac{\Delta x}{\Delta t} = m\left(v_f - v_i\right)v_m \Rightarrow$$

$$L = m\left(v_f - v_i\right)\frac{\left(v_f + v_i\right)}{2} = \frac{1}{2}m\left(v^2{}_f - v^2{}_i\right)$$

$$L = \frac{1}{2}mv^2{}_f - \frac{1}{2}mv^2{}_i = \Delta K$$

4. **Lavoro elettrico e potenziale**

- Definiamo differenza di potenziale elettrico il lavoro elettrico per unità di carica.
- Lavoro svolto per portare una particella di prova di carica *q₀* da un punto A con potenziale V_A a B con potenziale V_B è dato da

$$L_e = q_0\left(V_A - V_B\right) = -q_0\,\Delta V$$

$$\Delta K = -q_0\,\Delta V$$

Con $q_0 > 0$

Se $\Delta V < 0 \Rightarrow \Delta K > 0$

Se $\Delta V > 0 \Rightarrow \Delta K < 0$

- Una carica positiva che “cade” da un punto di potenziale V_A a uno con $V_B<V_A$ acquisisce energia cinetica → viene accelerata.
- Se $V_B>V_A$ la sua energia cinetica diminuisce → è frenata.

5. **Acceleratori elettrostatici**

- Principio di funzionamento degli acceleratori e.s.: viene mantenuta costante una d.d.p. che permette di aumentare l’energia cinetica di un fascio di particelle cariche, portandole ad alte velocità.
- Nel Van de Graaff le d.d.p. raggiunte sono di parecchi MV.
- Per questo motivo è spesso usato come unità di misura dell’energia di particelle cariche l’eV. 1 eV è l’energia acquisita da una carica elementare che attraversa la d.d.p. di 1 V (carica elementare $e = 1.6\ 10^{-19}$ C).

6. **Esercizi: soluzione esatta in minor tempo**

- Quanti J corrispondono a 1 eV?
- Due superfici conduttrici parallele, piane, distanziate di $d = 1.00$ cm hanno una differenza di potenziale $\Delta V = 625$ V. Un protone viene proiettato da un piatto verso l’altro. Qual è la velocità iniziale del protone se esso si ferma proprio sulla superficie del secondo piatto?
- Una particella alfa (composta da due protoni e due neutroni) viene accelerata attraverso una d.d.p. di 1.0 MV in un acceleratore di Van de Graaff (a) Che energia cinetica acquisisce? (b) Un protone che energia cinetica acquisirebbe nelle stesse circostanze? (c) Quale particella acquisterebbe la maggiore velocità partendo da ferma? [*Lo studente svolga i calcoli in eV e multipli*]

7. **Domande**

- Gli elettroni tendono a spostarsi nelle regioni ad alto potenziale o a basso potenziale? Esplicita il ragionamento utilizzato per rispondere.
- Abbiamo fatto riferimento all’energia cinetica. Quali altre forme di energia conosci?
- Considera un corpo che si muove sotto l’azione di forze conservative (forza gravitazionale, elettrica, elastica,…) in un sistema isolato. Qual è la *proprietà fondamentale* dell’energia meccanica totale posseduta dal corpo?
- Se le forze agenti sono anche non conservative (es. attriti) *esiste una grandezza fisica che possieda la stessa proprietà? Quale?* Spiega.

8. **Una nuova dinamica**
9. **Velocità ed energia cinetica di elettroni relativistici (W. Bertozzi, 1964)**

   Ricordiamo

   - Teorema dell'energia cinetica

   $$L = \Delta K \qquad K = \tfrac{1}{2} m v^2$$

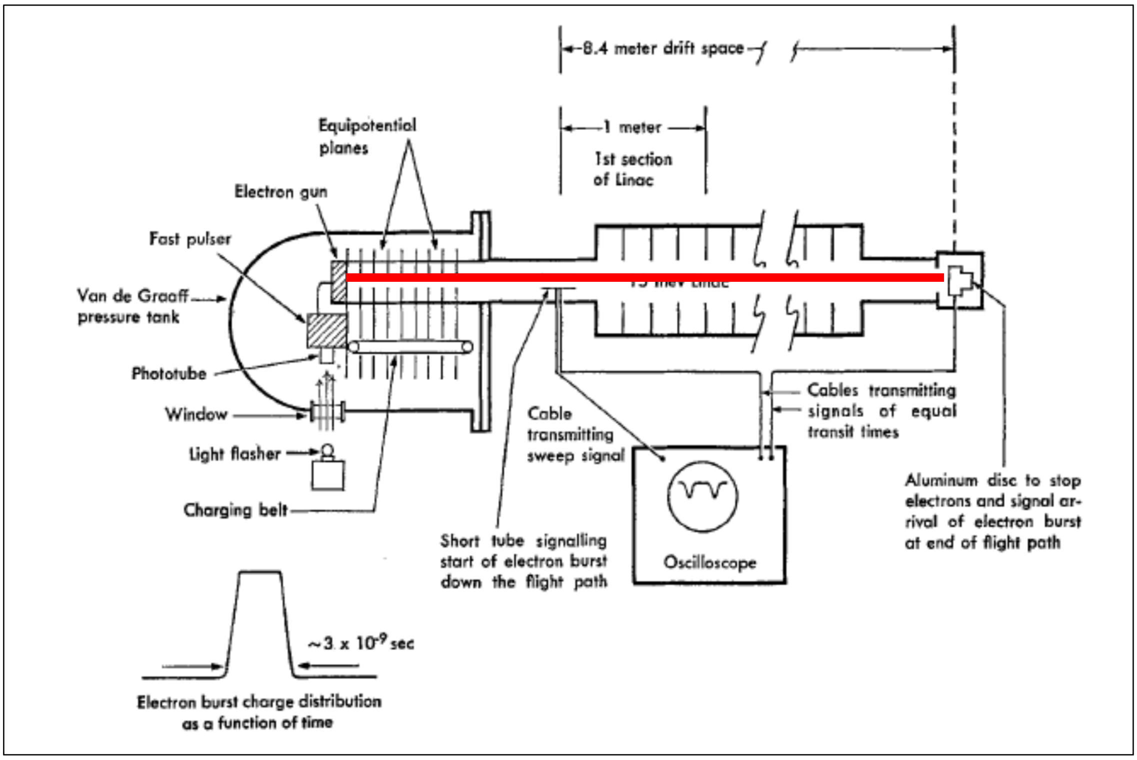
   - Il lavoro compiuto da un generatore di Van de Graaf è di tipo *elettrico*

   $$L = e\,\Delta V$$

   Quindi per una particella di carica *e* e massa *m* su cui si compia lavoro elettrico

   $$e\,\Delta V = \frac{1}{2} m v^2 \Rightarrow v^2 = \frac{2e}{m}\Delta V$$

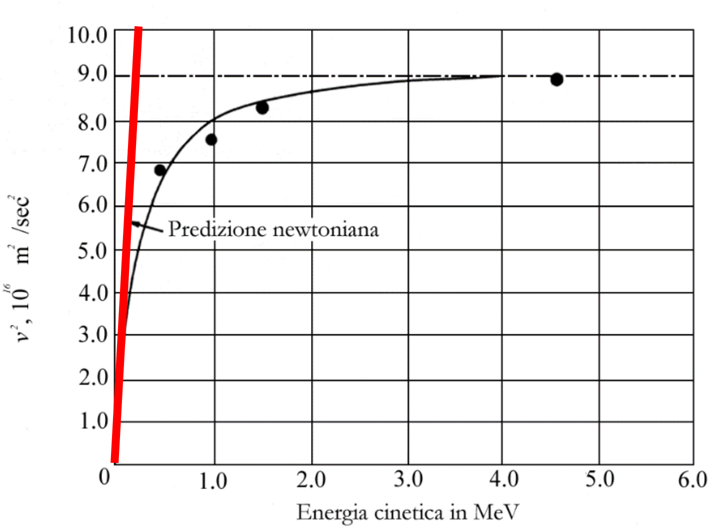
10. **Velocità ed energia cinetica di elettroni relativistici (W. Bertozzi, 1964**



11. **Velocità ed energia cinetica di elettroni relativistici (W. Bertozzi, 1964)**

   La previsione classica è un aumento indefinito della velocità al crescere della ΔV mantenuta nel Van de Graaf secondo una legge di potenza: $v^2 \propto \Delta V$. Sperimentalmente si rileva invece che, per quanto si aumenti la ddp, la velocità non raggiunge (né supera) mai *c*.

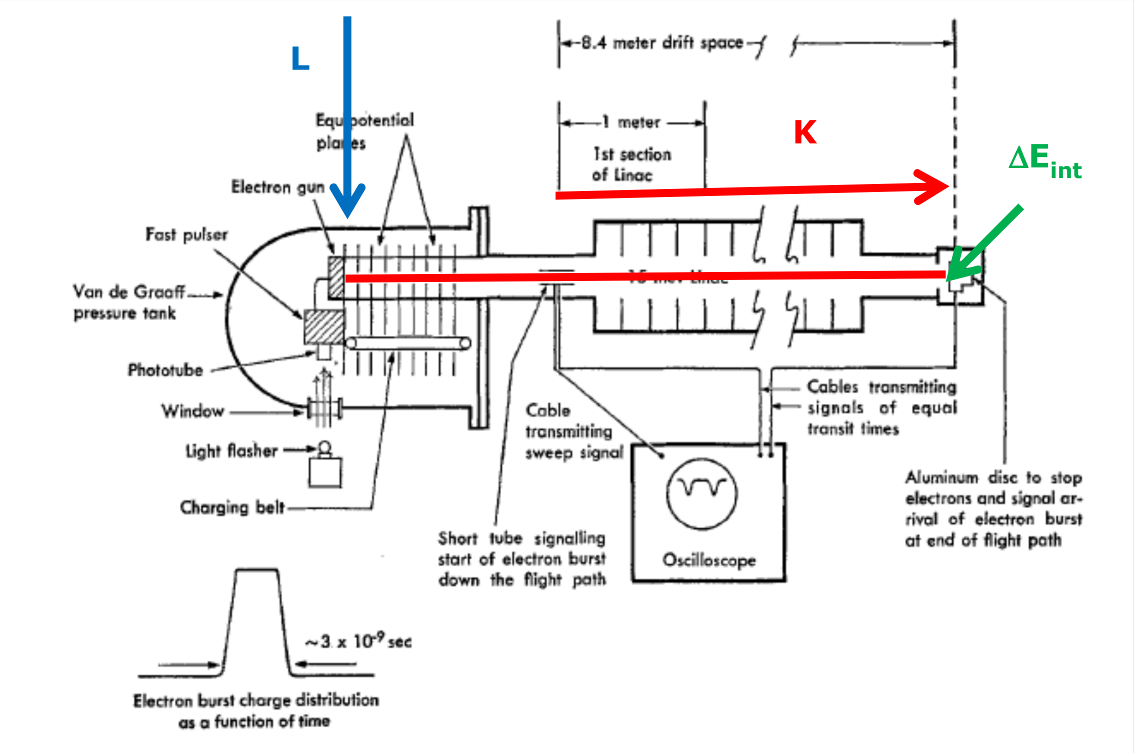
12. Velocità ed energia cinetica di elettroni relativistici (W. Bertozzi, 1964)



Cosa non funziona?

- Teorema dell'energia cinetica $L = \Delta K$
- Espressione del lavoro elettrico $L = e\,\Delta V$
- Espressione dell'energia cinetica $K = \frac{1}{2} m v^2$

13. Velocità ed energia cinetica di elettroni relativistici (W. Bertozzi, 1964)



14. Misura calorimetrica (W. Bertozzi, 1964)

Con una termocoppia si misura la variazione di temperatura ΔT del bersaglio.

Legge fondamentale della calorimetria: $\Delta E_{int} = M\ c\ \Delta T$.

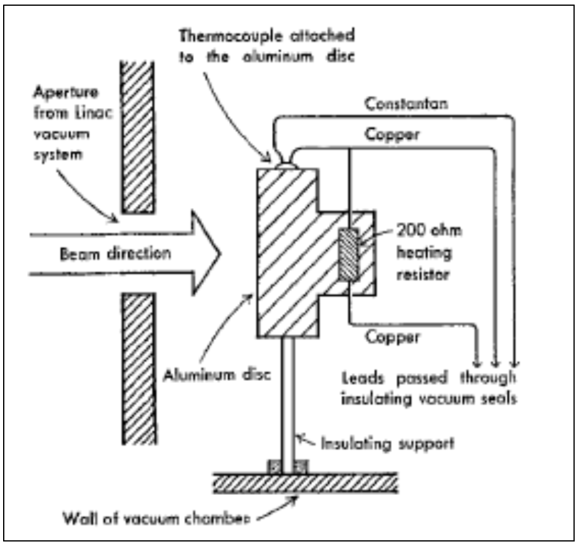
Sperimentale si trova che l'energia rilasciata dall'elettrone quando viene fermato corrisponde al valore del lavoro calcolato con $L = e\ \Delta V$.

Come è arrivata lì quell'energia?

ENERGIA CINETICA

$\Delta K = \Delta E_{int}$

$L = \Delta E_{int} = \Delta K$: il teorema dell' energia cinetica vale anche alle alte velocità.



Picture by Bertozzi (1964).

15. Velocità ed energia cinetica di elettroni relativistici (W. Bertozzi, 1964)

Cosa non funziona?

- Teorema dell'energia cinetica	$L = \Delta K$	OK
- Espressione del lavoro elettrico	$L = e\ \Delta V$	OK
- Espressione dell'energia cinetica	$K = ½\ m\ v^2$	←

16. Riassumendo…

- Esiste una velocità invalicabile per gli elettroni accelerati (c)
- L'espressione dell'energia cinetica non è più valida ad alte velocità (prossime a c)

17. Riassumendo…

Questi risultati non sono limitati alla situazione specifica, ma sono generalizzabili mediante il principio di relatività e portano conseguenze molto profonde:

❑ Principio relatività teorico: **tutte** le leggi fisiche hanno la stessa forma in tutti i sistemi di riferimento inerziali (SI).

☐ Principio relatività sperimentale: esperimenti **di qualsiasi natura** condotti nelle stesse condizioni in SI diversi danno gli stessi risultati.

18. **La velocità limite dev’essere invariante**

Immaginiamo di compiere, con lo stesso apparato, l’esperimento di Bertozzi su un veicolo spaziale, in moto rispetto alla Terra con velocità v, mantenendo l’asse dell’acceleratore orientato positivamente nella direzione del moto. Acceleriamo un elettrone fino alla velocità $c - \varepsilon$ (con $\varepsilon < v$).

Con che velocità si muove l’elettrone rispetto alla Terra?

Se valesse la regola classica per la somma dovrebbe raggiungere

$$c - \varepsilon + v > c\ ,$$

superando la velocità limite.

19. **Invarianza di *c***

La conclusione di Einstein: non esiste un sistema di riferimento privilegiato rispetto al quale, e solo rispetto al quale, la luce si propaga nel vuoto alla velocità c. In **qualunque** sistema di riferimento si misurerà lo stesso valore c. Non c’è un sistema di riferimento privilegiato, un sistema ***assoluto***. Tutti i sistemi di riferimento (inerziali) sono equivalenti.

20. **Domanda**

Quali sono i principali risultati dell’esperimento di Bertozzi e dalle considerazioni successive? Indicali.

21. **I postulati della relatività ristretta**

- Principio relatività teorico: **tutte** le leggi fisiche hanno la stessa forma in tutti i sistemi di riferimento inerziali (SI).
- Principio relatività sperimentale: esperimenti **di qualsiasi natura** condotti nelle stesse condizioni in SI diversi danno gli stessi risultati.
- Invarianza della velocità della luce: c non dipende dal SI in cui è misurata, né dalla direzione della propagazione della luce.

22. **Composizione delle velocità**

I risultati precedenti mostrano che la legge galileiana di composizione delle velocità $u = \mathrm{v} + u'$ non funziona.

Però essa deriva dalle leggi di trasformazione delle coordinate tra sistemi di riferimento diversi della meccanica classica:

$$x = x' + vt \qquad \boxed{t = t'}$$

Se vogliamo cambiare la legge di composizione delle velocità in modo da garantire l'invarianza di *c* dobbiamo quindi cambiare la forma delle trasformazioni galileiane. Un esperimento mentale ci dà al proposito un risultato importante.

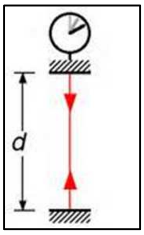
23. **<u>I postulati della Relatività ristretta e le conseguenze cinematiche</u>**

    - La dilatazione relativistica dei tempi

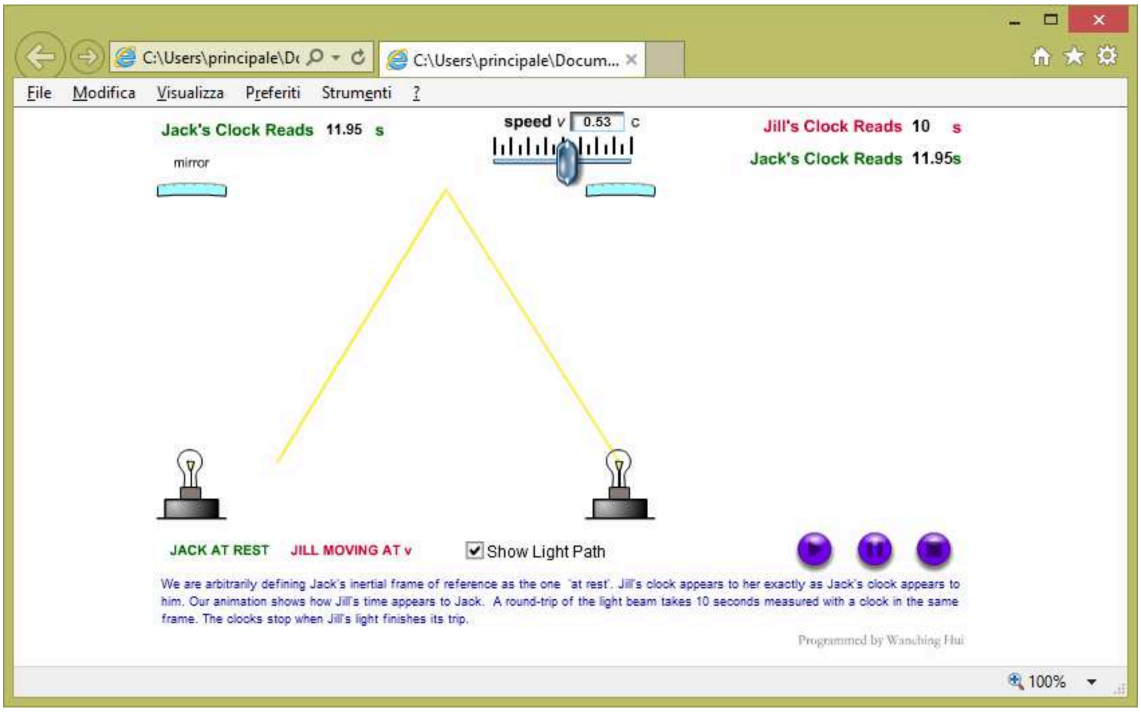
    Cos'è un orologio

    Un orologio fatto con un raggio di luce riflesso tra due specchi

24. **<u>I postulati della Relatività ristretta e le conseguenze cinematiche</u>**



25. Esperimento mentale: l'orologio a luce [(shockwave)]() Due orologi a luce, entrambi esaminati nel sistema di riferimento «fermo».



Jack's Clock Reads 11.95 s

speed v 0.53 c

Jill's Clock Reads 10 s

Jack's Clock Reads 11.95s

We are arbitrarily defining Jack's inertial frame of reference as the one 'at rest'. Jill's clock appears to her exactly as Jack's clock appears to him. Our animation shows how Jill's time appears to Jack. A round-trip of the light beam takes 10 seconds measured with a clock in the same frame. The clocks stop when Jill's light finishes its trip.

26. L’orologio a luce e la dilatazione dei tempi

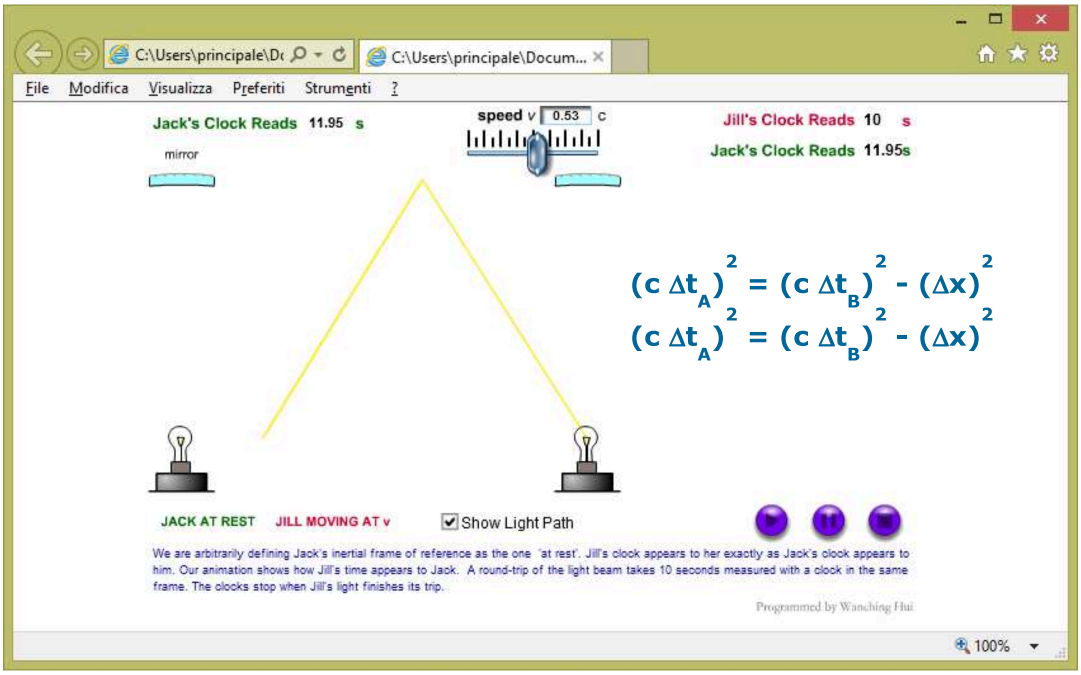
$(c\,\Delta t_B/2)^2 = (V\,\Delta t_B/2)^2 + (c\,\Delta t_A/2)^2$

$\Delta t_B^2\,(c^2 - V^2) = c^2\,\Delta t_A^2$

$\Delta t_B = \Delta t_A /(1 - (V/c)^2)^{1/2}$

$\Delta t_B = \Delta t_A\,\gamma \qquad \gamma = 1/(1 - (V/c)^2)^{1/2}$

27. ANCORA…



$$(c\,\Delta t_A)^2 = (c\,\Delta t_B)^2 - (\Delta x)^2$$
$$(c\,\Delta t_A)^2 = (c\,\Delta t_B)^2 - (\Delta x)^2$$

We are arbitrarily defining Jack's inertial frame of reference as the one 'at rest'. Jill's clock appears to her exactly as Jack's clock appears to him. Our animation shows how Jill's time appears to Jack. A round-trip of the light beam takes 10 seconds measured with a clock in the same frame. The clocks stop when Jill's light finishes its trip.

Programmed by Wanching Hui

28. Spaziotempo e tempo proprio

Δt_A è quindi calcolabile universalmente (da qualsiasi osservatore) e la relazione $(c\,\Delta\tau)^2 = (c\,\Delta t_B)^2 - (\Delta x)^2$

Fornisce un valore univoco per qualsiasi sistema di riferimento (invariante) →

$\Delta\tau$ (intervallo di) tempo proprio

Poi, il tempo misurato direttamente in un certo sistema di riferimento, diventerà $\Delta t_B = \Delta\tau\,\gamma$.

29. Stimoli per la riflessione sull’effetto di dilatazione dei tempi

1. Abbiamo dotato l’osservatore sul treno e quello sulla banchina di due orologi identici.

- I battiti del cuore del secondo osservatore risultano realmente rallentati secondo il primo?
- Il secondo osservatore sente i propri battiti rallentati?

Spiega.

2. Abbiamo dotato un manovratore di ponti e un passeggero su un treno di due

orologi identici.

- L’intervallo di tempo fra l’apertura e la chiusura di un ponte mobile è differente per il passeggero del treno rispetto a chi lo manovra? Spiega.

30. **Vediamo quali sono le conseguenze**

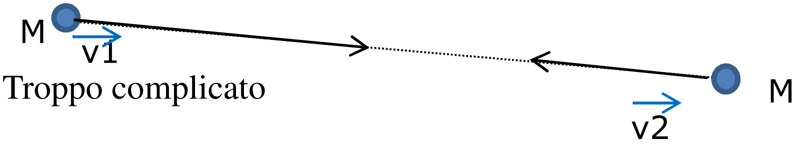
    - Riscrivo il teorema dell’energia cinetica nell’ambito classico, utilizzando la variazione di quantità di moto e la velocità (media e istantanea)

$$\boxed{\begin{array}{c} F=\frac{\Delta p}{\Delta t}\Rightarrow L=F\,\Delta x=\frac{\Delta p}{\Delta t}\Delta x=\Delta p\frac{\Delta x}{\Delta t}=\Delta p\, u_m \\ L=\Delta p\, u_m=\Delta K \end{array}}$$

    - La matematica suggerisce che se l’espressione classica di K non vale più ad alte velocità, occorre modificare anche l’espressione di p ($p \neq m\,v$).

31. **Quantità di moto = urti!**

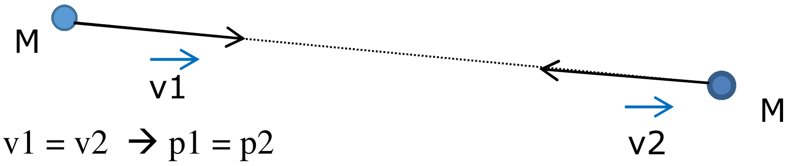
Il caso più semplice: due particelle identiche



Troppo complicato

32. **Quantità di moto = urti!**

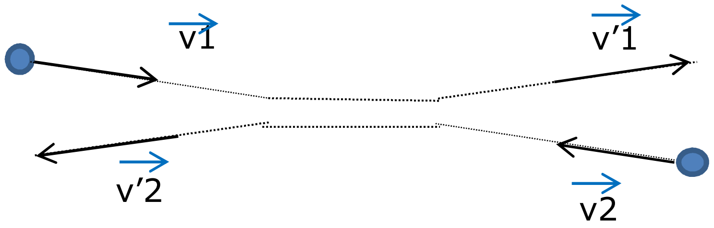
Ancora più semplice: due particelle identiche considerate nel sistema di riferimento del C.M.



v1 = v2 → p1 = p2

33. **Quantità di moto = urti!**

Dopo l’urto

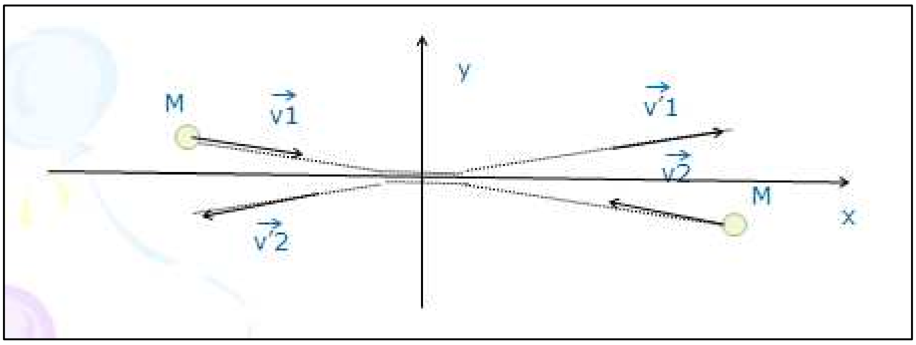


Se urto elastico (conservazione dell’energia cinetica)

v1 = v’1 = v2 = v’2

34. **Quantità di moto = urti!**

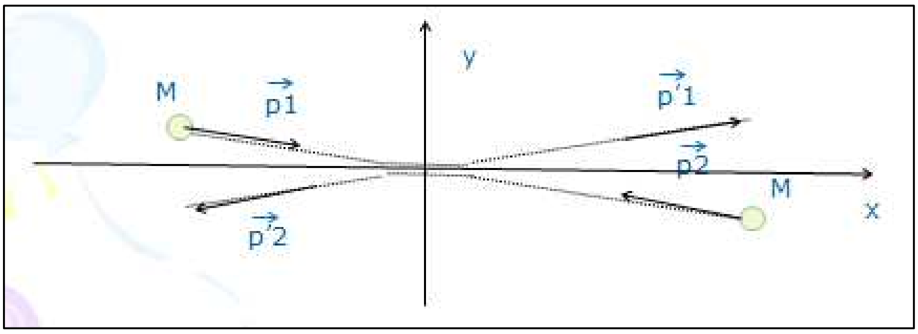
Urto radente (piccola deflessione)



Piccola componente y di v1 (e v’1).

35. **Quantità di moto = urti!**

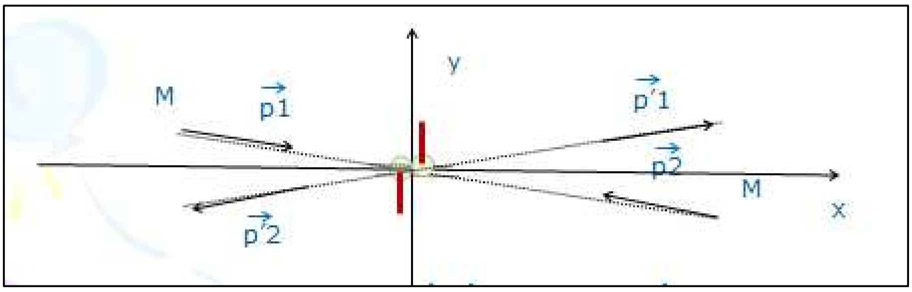
Tutto ciò deve valere anche per la quantità di moto



Piccola componente y di p1 (e p’1), e eguale opposta a p2 (p’2).

36. **Quantità di moto = urti!**

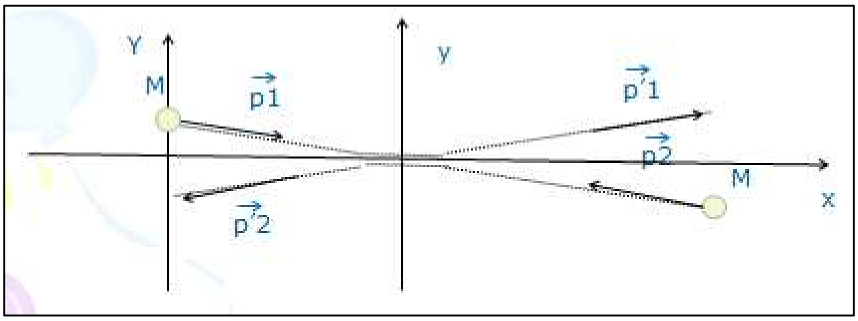
Vediamo che succede dopo l’urto



Spostamento eguali lungo y che avvengono in tempi uguali (ed anche in tempi propri $\Delta\tau$ uguali!)

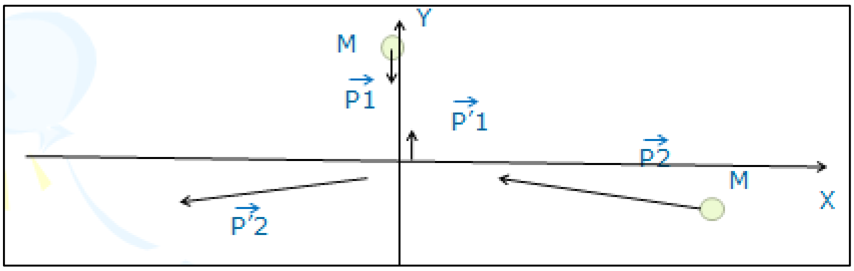
37. **Quantità di moto = urti!**

Cambiamo sistema di riferimento, ci mettiamo in uno che si muova lungo l’orizzontale come la particella 1



38. **Quantità di moto = urti!**

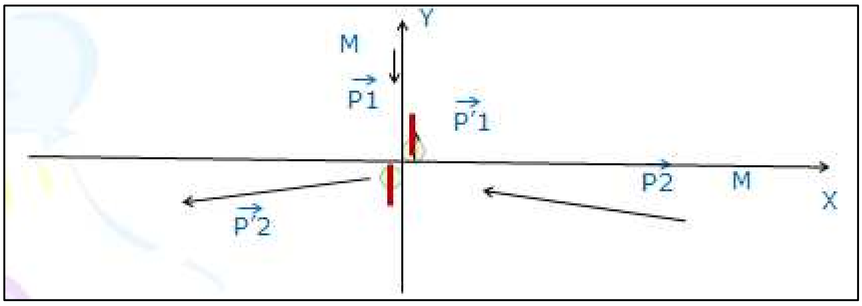
Anche dopo l'urto il nuovo sistema di riferimento si muoverà orizzontalmente come la particella 1 (e quindi P'1 lungo Y)



Piccola componente y di P1 (e P'1) e eguale opposta a p2 (p'2)

39. **Quantità di moto = urti!**

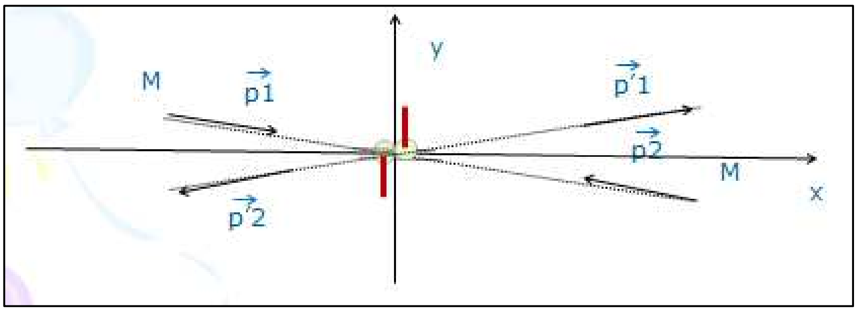
Vediamo cosa succede dopo l'urto



Stiamo considerando lo stesso spostamento ΔY di prima.

40. **Quantità di moto = urti!**

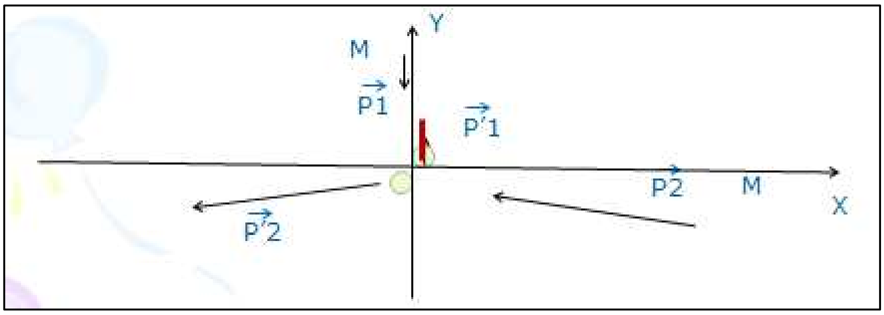
Ma ΔY avviene veramente nello stesso tempo?



Nel C.M. i tempi erano uguali (ed anche i tempi propri)

41. Quantità di moto = urti!

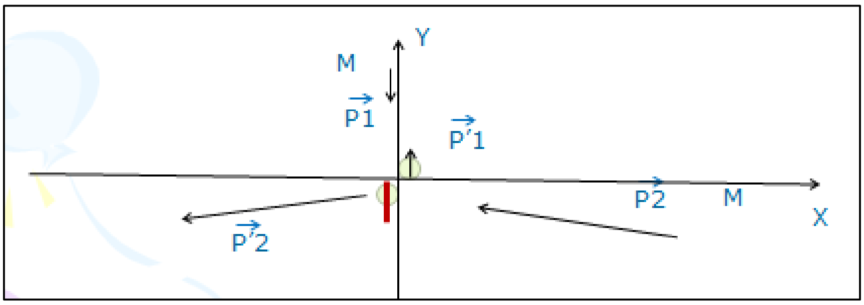
Nel sistema di riferimento in moto la particella 1 si muove lungo ΔY nel tempo Δt1



Ma v1 è piccola (non relativistica!!!). Quindi Δt1 e Δτ coincidono

42. Quantità di moto = urti!

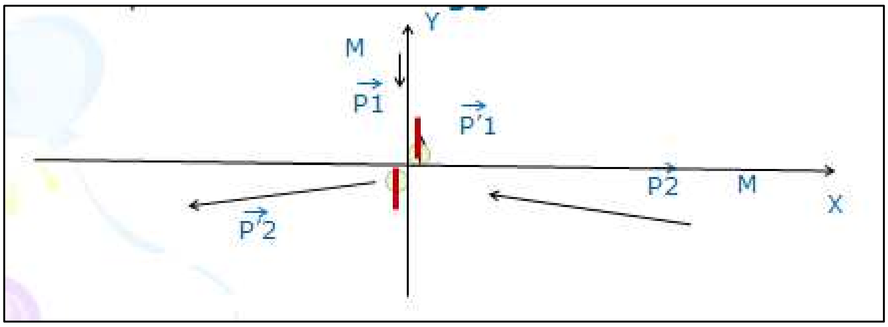
La particella 2 invece si muove veloce, e copre ΔY in un tempo Δt2



Ma lo spostamento ΔY deve sempre avvenire in un tempo proprio Δτ (lungo Y non cambia niente)

43. Quantità di moto = urti!

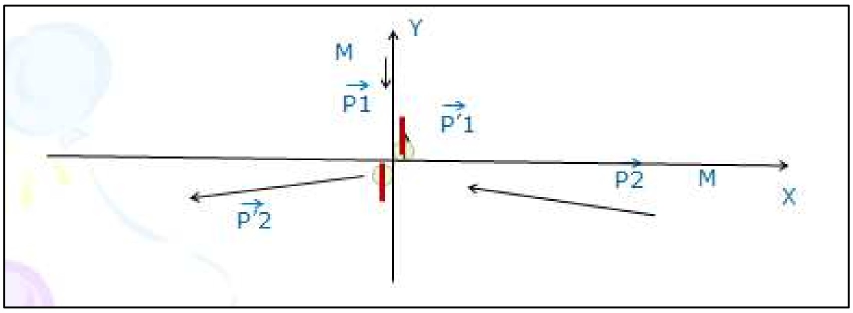
Ma la particella è relativistica, quindi al tempo proprio Δτ del suo spostamento corrisponde un Δt2 maggiore



Quindi Δt2 > Δτ = Δt1!

44. Quantità di moto = urti!

Usando la formula della dilatazione dei tempi $\Delta t2 = \Delta\tau\ \gamma(v2) = \Delta t1\ \gamma(v2)$



45. Quantità di moto = urti!

Da questi tempi impiegati a percorrere lo spazio ΔY

$$\Delta t2 = \Delta\tau\ \gamma(v2) = \Delta t1\ \gamma(v2)$$

Si ottiene questa relazione tra le componenti Y delle velocità

$$V2Y = \Delta Y / \Delta t2 = \Delta Y / (\Delta t1\ \gamma(v2))$$

$$V1Y = \Delta Y / \Delta t1$$

E quindi, confrontando

$$V1Y = V2Y\ \gamma(v2)$$

46. Quantità di moto = urti!

Passiamo alle quantità di moto. Sappiamo che P'1 = P'2Y (conservazione quantità di moto).

Ma la particella 1 non è relativistica, per cui possiamo scrivere con buona approssimazione

$$P'1 = M\ V'1$$

Ne risulta

$$P'2Y = P'1 = M\ V'1$$

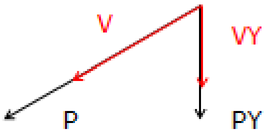
Ricordando il risultato per le velocità

$$P'2Y = M\ V2Y\ \gamma(V2)$$

47. Quantità di moto = urti!

P’2Y = M v2Y γ(v2)

Vale per la componente Y di **P**’2, ma i vettori **v**’2 e **P**’2 sono allineati, per cui devono valere anche per i moduli (similitudine di triangoli).



Ne segue (eliminiamo anche gli indici inutili)

$$P = M\ v\ \gamma(v)$$

48. Quesito

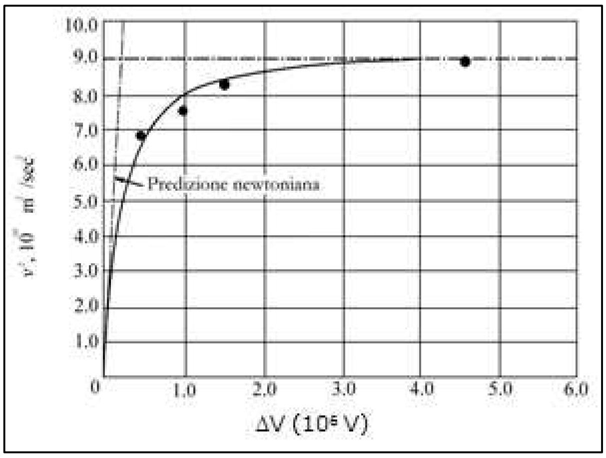
Scrivi una ragione per la quale la quantità di moto non può essere scritta alle alte velocità nella forma classica p = m v.

49. La «dinamica» relativistica

Il risultato ottenuto per la quantità di moto

$$P = M\ v\ \gamma \text{ ove } \quad \gamma = 1/(1-(v/c)^2)^{1/2}$$

suggerisce che nella dinamica relativistica una importante funzione della velocità è la γ. Vediamo cosa implica ciò per l’energia cinetica K dell’elettrone nell’esperimento di Bertozzi



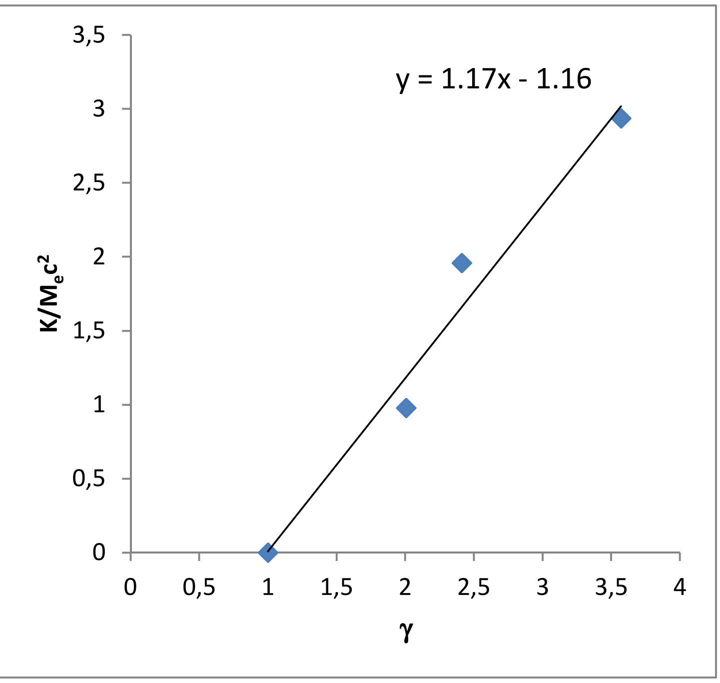
50. La «dinamica» relativistica

Qui a fianco sono riportati i valori di $K/M_e c^2$ in funzione di γ (è aggiunta la coppia di valori (1,0), valida di principio per un elettrone fermo).

I dati (entro l'errore sperimentale) sembrano indicare una relazione del tipo

$$K = A\,M\,c^2\,(\gamma - 1)$$

ove A è una qualche costante molto vicina ad 1



51. La «dinamica» relativistica

- Il limite classico dell'espressione della energia cinetica

Vediamo cosa implica questa relazione per basse velocità, nel limite classico. Dobbiamo approssimare la relazione

$$\gamma = 1/(1-(v/c)^2)^{1/2}$$

Utilizzeremo a tal fine una regola di approssimazione delle potenze (anche frazionarie) di un binomio

$$(1 - X)^N \cong 1 - NX$$

Approssimazione tanto migliore quanto X<<1. Nel nostro caso è N = -1/2 e quindi $\gamma = 1/(1-(v/c)^2)^{1/2} \cong 1-(-\,½)\,(v/c)^2$

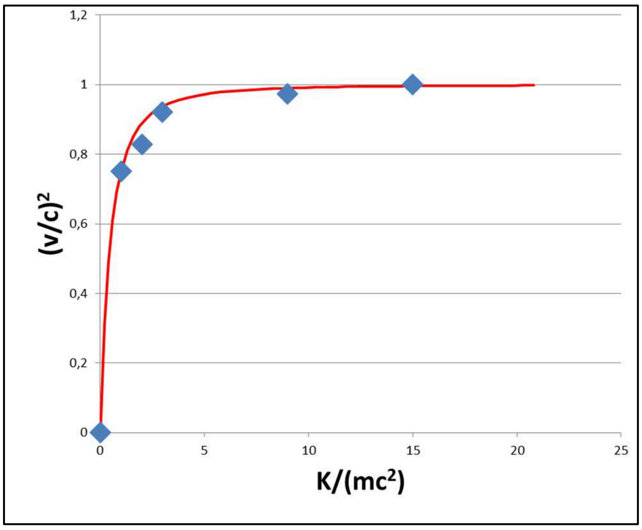
L'espressione per K diventa

$$K = A\,M\,c^2\,(\gamma - 1) \cong A\,M\,c^2\,(\,1 + ½\,(v/c)^2 - 1) = ½\,A\,M\,v^2$$

Cioè coincidente con l'espressione classica, se A = 1.

$$K = M\,c^2\,(\gamma - 1) \qquad (\cong ½\ M\,v^2)$$

52. Interpolazione dati di W. Bertozzi



Un **controllo** dell'espressione ricavata per K si può fare con i dati dell'esperimento di Bertozzi.
Si può facilmente ricavare l'espressione relativistica per il quadrato della velocità in funzione dell'energia cinetica invertendo la formula trovata per K:
$v^2/c^2 = 1- [\ m_e\, c^2 / (m_e\, c^2 + K)\ ]^2$ (previsione di Einstein: curva rossa)
Essa riproduce bene l'andamento dei dati.

53. Domande

L'energia cinetica è:

- La quantità espressa dall'equazione $K = \frac{1}{2}\, m\, v^2$;
- La forma di energia associata allo stato di moto dell'e^-;
- Un termine essenziale per soddisfare il principio di conservazione dell'energia;
- Una combinazione delle precedenti (precisa quale).

54. Il fotone

55. Il fotone come «oggetto relativistico ideale»

- Introduzione del fotone (effetto fotoelettrico)

Effetto tipico nell'interazione a bassa energia con materiali metallici.
Emissione di fotoelettroni.
L'effetto non si manifesta per luce con frequenza troppo bassa, non importa quanto sia intenso il fascio incidente.

56. Il fotone come «oggetto relativistico ideale»

Introduzione del fotone (effetto fotoelettrico)
Quello che si osserva sperimentalmente

- I fotoelettroni non vengono emessi a meno che la luce incidente non abbia una frequenza superiore ad un certo minimo (dipendente dal materiale)
- L'energia cinetica dei fotoelettroni emessi è lineare con la frequenza della luce incidente
- Questi effetti sono indipendenti dalla intensità luminosa incidente

57. **Interpretazione di Einstein dell'effetto fotoelettrico**

Esistono «particelle» di luce: i **fotoni**!

La luce è composta da quantità indivisibili di energia (*quanti di luce*) e che possono essere prodotti e assorbiti solo come per singole **entità complete**.

→ Essi interagiscono in modo discreto con la materia e in tal modo vengono rivelati.

→ L'energia trasportata dal singolo fotone dipende dalla sua frequenza $E = h\nu$.

58. **Interpretazione di Einstein dell'effetto fotoelettrico**

- I fotoelettroni non vengono emessi a meno che la luce incidente non abbia una frequenza superiore ad un certo minimo (dipendente dal materiale) → per far emettere un fotoelettrone occorre comunicargli una energia minima W (il lavoro di estrazione) → il fotone che lo fa espellere deve comunicargli una energia almeno pari a W

  $E = h\,\nu > W \rightarrow \quad \nu > \nu 0 = W/h$

- L'energia cinetica dei fotoelettroni emessi è lineare con la frequenza della luce incidente

- Questi effetti sono indipendenti dalla intensità luminosa incidente

59. **Interpretazione di Einstein dell'effetto fotoelettrico**

L'energia cinetica dei fotoelettroni emessi è lineare con la frequenza della luce incidente → una volta espulso, il fotoelettrone ha a disposizione per l'energia cinetica, quanto rimane dell'energia del fotone dopo l'estrazione

$$K_{max} = h\,\nu - W \rightarrow \quad \nu > \nu_0 = W/h$$

60. Interpretazione di Einstein dell’effetto fotoelettrico

Questi effetti sono indipendenti dalla intensità luminosa incidente → un singolo fotone interagisce con un singolo elettrone e quindi non po’ comunicargli più dell’energia posseduta, non importa quanti fotoni ci sono nel fascio luminoso.

61. Interpretazione di Einstein dell’effetto fotoelettrico

Il fotone quindi è un oggetto singolo (nelle interazioni), che trasporta una fissa energia (dipendente dalla sua frequenza).
Ci sono altre proprietà che lo fanno assomigliare ad una particella?

→ Quantità di moto!

62. La pressione di radiazione

Alla fine dell’Ottocento venne formulata da Maxwell **l’ipotesi** la luce esercitasse una «spinta» sui materiali su cui essa incide, a partire dai suoi studi delle radiazioni elettromagnetiche.

- la luce scambia quantità di moto con la materia oltre che energia.
- La quantità di moto da associare alla luce eè

$$p = E / c$$

ove E è l’energia luminosa incidente.

63. La pressione di radiazione e la «equazione di stato» del fotone (E = p c)

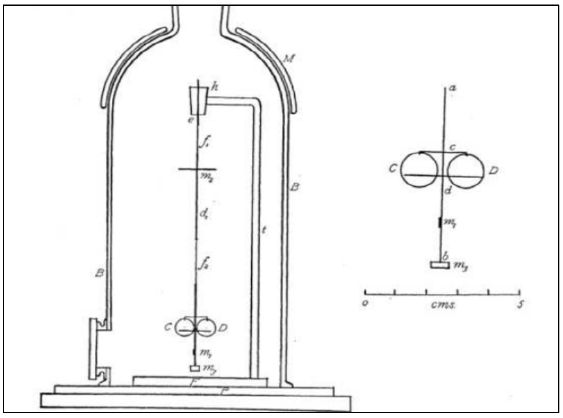
Lebedev (1901) e Nichols (1903) effettuarono degli esperimenti per controllare tale ipotesi.

Radiometro di Nichols

Bilancia di torsione

equipaggiata con due specchi.

Su uno degli specchi viene inviato
un fascio di luce, che provoca la rotazione dell’equipaggio mobile.



64. La pressione di radiazione e la «equazione di stato» del fotone (E = p c)

I 2 esperimenti hanno mostrato (con diversi gradi di precisione)

- La forza F esercitata dalla luce sullo specchio (impulso trasmesso per unità di tempo) è proporzionale alla potenza P (energia trasmessa per unità di tempo) associata al fascio luminoso;
- Tale costante di proporzionalità risultava essere, entro gli errori, pari a **2/c**. Quindi

$$F = P * (2 / c) \qquad \textbf{[legge empirica]}$$

- $F = \Delta p / \Delta t$; $P = E / \Delta t \Rightarrow \Delta p = 2\, p = E * (2 / c)$
- Il fattore 2 deriva dalla riflessione della luce, che raddoppia l'effetto
- Alla luce incidente allora deve venire associato un impulso

$$p = E / c$$

65. Un riassunto

- Il fotone rappresenta l'entità unitaria con cui la luce interagisce con la materia
- Esso trasporta energia E e quantità di moto p, legate tra di loro da

$$E = p\, c$$

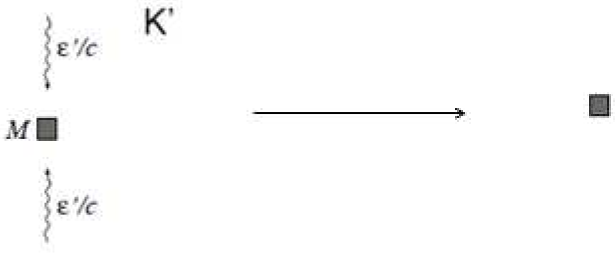
- Il fotone può essere interpretato come una particella priva di massa

66. Domande sul fotone

- "La luce scambia la sua energia con la materia in modo continuo o discreto? Da che cosa è fatta la luce?"
- "Perché risulta che i fotoni trasportano quantità di moto?"

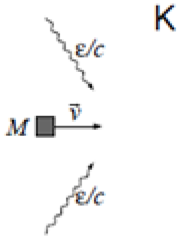
67. L'assorbimento di fotoni: un esperimento mentale

Due fotoni incidono collineari su un corpo in quiete. In questo sistema di riferimento PRIMA e DOPO la quantità di moto del corpo è nulla.



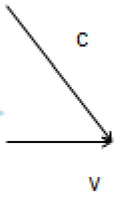
68. L’assorbimento di fotoni: un esperimento mentale

In un sistema di riferimento in moto verso SINISTRA



La quantità di moto del sistema + fotoni PRIMA dell’assorbimento è

$$P = M\,\gamma\,v + 2p_{fot}\boxed{(v/c)}$$
$$= M\,\gamma\,v + 2\varepsilon/c\,(v/c)$$



69. L’assorbimento di fotoni: un esperimento mentale

Dopo l’assorbimento, il corpo è da solo e quindi la sua quantità di moto sarà

$$P' = M\,\gamma\,v'$$

Nel primo sistema di riferimento il corpo è fermo prima e dopo, e quindi la velocità finale v’ nell’altro sistema di riferimento sarà pari a v: **il sistema fisico assorbe energia senza variare la sua velocità.**

Poiché nell’assorbimento si deve conservare la quantità di moto

$$M\,\gamma\,v = P' = P = M\,\gamma\,v + 2\varepsilon\,v/c^2$$

ASSURDO

L’unico modo per risolvere l’assurdo è supporre che la massa DOPO l’assorbimento sia diversa da quella PRIMA

$$M'\,\gamma\,v = P' = P = M\,\gamma\,v + 2\varepsilon\,v/c^2$$

$$M' = M + 2\varepsilon/(\gamma c^2)$$

70. **L'assorbimento di fotoni: un esperimento mentale**

$$M' = M + 2\varepsilon/(\gamma c^2)$$

Ma 2 ε è l'energia assorbita dal corpo: ΔE.

Perciò possiamo affermare **se un corpo assorbe energia senza variare la sua velocità, la sua massa varia di**

$$\mathbf{\Delta M = \Delta E/(\gamma\, c^2)}$$

Questa relazione deve valere per qualsiasi velocità v, in particolare anche per v=0. cioè $\gamma = 1 \Rightarrow$ un input di energia ΔE in un corpo a riposo **può** far aumentare la sua massa di

$$\mathbf{\Delta M = \Delta E/ c^2}$$

mantenendolo a riposo nel SI del lab.

71. **Domanda**

- Prima dell'assorbimento, il fotone ha massa?
- Con l'assorbimento il sistema (fotone + scatola) ha aumentato la sua massa? Perché?

72. **Che cos'è l'energia posseduta da un sistema ?**

Se consideriamo un oggetto di massa iniziale **trascurabile** ($m < \varepsilon$, ε arbitrariamente piccolo) **l'input di energia necessaria perché esso acquisti una massa *m* è dato da**

$$\Delta E = (\Delta m)\, c^2 = (m - 0)\, c^2 = m\, c^2$$

$$\Rightarrow \mathbf{m\, c^2 \equiv E_0}$$

può essere vista come **l'energia necessaria per la creazione di una particella di massa *m* a riposo.**

$m = E_0 / c^2$ è la massa che l'oggetto ha assunto per il solo fatto che abbiamo fornito un'energia E_0 al vuoto → creazione di particelle negli acceleratori: basta che ci sia energia a disposizione $\geq m\, c^2$

- **La massa misura l'energia complessiva posseduta da un sistema o particella a riposo: $E_0 = m\, c^2$.**

73. Energia totale

- E_0 è detta **energia a riposo.** Ma finora abbiamo ragionato in un solo SI; prendiamo ora in considerazione l'energia dovuta al moto del SI solidale all'oggetto rispetto al SI del lab: essa è **l'energia cinetica del corpo** nel SI del lab.
- Per ottenere **l'energia totale** di un sistema dovrò quindi sommare il contributo di energia a riposo a quello cinetico:

$$\mathbf{E = E_0 + K}$$

La massa misurata nel SI solidale è la stessa che si misura nel lab perché per misurare *m* ci si riferisce sempre al riferimento co-movente (solidale). In esso si eseguono misure a basse velocità tramite cui si trova il valore di *m* utilizzando la F = ma.

La massa è quindi un invariante relativistico: la sua misura dà lo stesso risultato per tutti i SI. Perciò possiamo sommare:

$$\mathbf{E = m\,c^2 + m\,c^2\,(\gamma - 1) = m\,c^2 + \gamma\,m\,c^2 - m\,c^2 = \gamma\,m\,c^2}$$

$$\mathbf{E = \gamma\,m\,c^2} \text{ (energia totale relativistica)}$$

74. Domande

- "Se compio lavoro su un corpo esteso in modo da accelerarlo, la sua massa varia? E nel caso di una particella?"
- Come si possono interpretare i comportamenti dei nuclidi osservati? (1) è distrutta massa; (2) la massa si trasforma in qualcos'altro; (3) la massa rappresenta, solo nello stato iniziale ma non in quello finale, qualcos'altro che si conserva; (4) la massa totale non è data dalla somma delle masse dei costituenti. Motiva le tue scelte.

Appendix 5 – Calculi for the photon in a box (thought experiment)

The aim of this thought experiment is to show that a photon inside a box, although a massless particle, influences the inertial behaviour of the system "photon + box" observed from outside. This can be deduced by analyzing two successive photon-electron collisions modelled as Compton scattering. The electrons in the scattering are thought as bound to an atom of the upper or lower wall of a box. So these collisions occur when the photon bounces against the opposite mirrored walls of the box.

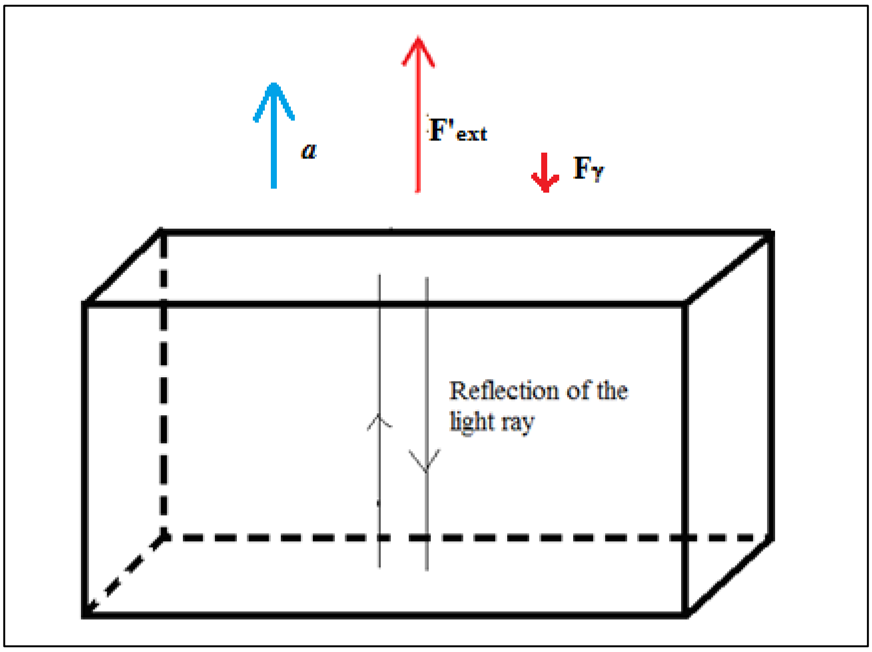


Figure 6.1. The box undergoing a constant external force with the reflecting light ray inside. The photon "force" is actually much smaller than in this picture.

At first, let us suppose the box of mass M is moving at **a constant speed *v*** with respect to the LabIF. The photon linear momentum is given by the relativistic expression $p = E/c$. Although the momentum variation of the box for a back-and-forth path has to be zero, in a *single collision* momentum will slightly change. This variation is being calculated. Since the box travels at non-relativistic speed we shall use *classical expressions* for its kinetic energy and momentum, the *Newtonian limit* for calculating the transmitted momentum (impulse), and *Newton's II law* for interpreting the final results.

Let us begin with linear momentum and energy conservation principles.

$$\boxed{\begin{cases}\frac{1}{2}Mv^2+E=\frac{1}{2}Mv'^2+E' \\ Mv+\frac{E}{c}=Mv'-\frac{E'}{c}\end{cases}}$$

Here *v* e *v'* indicate the speeds before and after the wall-photon collision respectively. They are very slightly different: inertial effects of photon cause energy and velocity variations very much smaller than those due to macroscopic massive objects.

Let us substitute *E'* in order to obtain the momentum variation as a function of the photon initial energy *E*:

$$\boxed{\begin{array}{l}\begin{cases}E'=E+\frac{1}{2}M\left(v^2-v'^2\right) \\ M\left(v-v'\right)=-\left[2E+\frac{1}{2}M\left(v^2-v'^2\right)\right]\Big/c\end{cases} \Rightarrow M\left(v-v'\right)\left\{1+\frac{1}{2c}\left(v+v'\right)\right\}=-\frac{2E}{c} \\ M\left(v'-v\right)\left\{1+\frac{1}{2c}\left(v+v'\right)\right\}=\frac{2E}{c} \Rightarrow \Delta p\cong\frac{2E}{c}\Big/\left(1+\frac{v}{c}\right)\end{array}}$$

The last passage draws on $v \approx v'$. In a stationary motion, the momentum transferred from the photon to the box is (in the non-relativistic limit)

$$\boxed{\Delta p \underset{\frac{v}{c}\to 0}{=} \frac{2E}{c}\left(1-\frac{v}{c}+O\left(v^2\right)\right)=I(v)}$$

If the motion of the box is *accelerated* by an external force, its speed will increase. Consider a successive stationary state in which the box travels at *v+dv*. The infinitesimal impulse variation is given by

$$\delta I=\frac{2E}{c}\frac{d}{dv}\left(1-\frac{v}{c}+O\left(\frac{v^2}{c^2}\right)\right)dv\cong-\frac{2E}{c^2}a\,\frac{dt}{2}.$$

This is an approximation for low speeds, where the two stationary states are at *v* (photon on the lower wall) and *v+dv* (photon on the upper wall); *dt* is considered as the time light takes for going back-and-forth: this explains the factor ½. Eventually, let us find the net very small "force" F_γ of the photon on the box:

$$\delta I = F_\gamma\, dt = -\frac{E}{c^2} a\, dt \Rightarrow \boxed{\overrightarrow{F_\gamma} = -\frac{E}{c^2}\vec{a}}$$

Suppose $\vec{F}_{ext}$ is the external force needed to keep the acceleration of the *empty* box constant ($\vec{F}_{ext} = M\vec{a}$). The force needed to keep the *same* acceleration constant *with the photon inside* will then be $\overrightarrow{F'}_{ext} + \vec{F}_\gamma = M\vec{a}$, $\vec{F'}_{ext}$ being the new external force on the box (see figure 6.1), which will have to compensate for the additional "force" F_γ too. So the external force which gives rise to the same acceleration *a* will have to be greater if the photon is in the box: the system has an overall *greater inertia* (***the photon hinders acceleration***). In scalar equations:

$$F'_{ext} - \frac{E}{c^2} a = Ma \Rightarrow \boxed{F'_{ext} = \left(M + \frac{E}{c^2}\right) a = (M + \delta m)\, a}$$

where $\delta m \equiv E/c^2$.

In conclusion, the thought experiment may be utilized at school to *supply a model* of the inertial behaviour of a system whose elements carry energy, and thus furnish a qualitative and (approximate) quantitative idea of the inertia of energy. It must be remarked that the two terms in the last equation are *very much different in magnitude*, because of the magnitude order of photon energies. So students have to be warned against the interpretation of this result as a physically significant in a strict sense.