When the news of the publication of this book started to spread among set theorists, the first reaction was wonderment (and for the most pessimistic people, doubt): here there was a book that promised to bring a person from the basic notions of set theory to some of the most celebrated results in inner model theory, in just a little more than 300 pages. The task seemed arduous: inner model theory is notorious among set theorists as one of the most complex branches, built on very refined and labyrinthine constructions and, unfortunately, severely lacking a complete and accessible introduction in literature.

But the promise is maintained: the thirteen chapters do offer a steady, clear path from the axioms of set theory to the Martin-Steel Theorem, passing through the Solovay Model and Jensen's Covering Lemma, in a self-contained way, without resorting to external sources. It is decidedly a pleasurable experience to follow all the threads, sometimes spanning decades, and see everything in the end working out and painting a surprising picture. Of course, this cannot be done without some sacrifices.

The first three chapters lay down the basics: the initial approach (Chapter 1) is not abstract, but revolves around $\mathbb{R}$, an object every student is familiar with, introducing Zermelo-Fraenkel and Gödel-Bernays theories only in Chapter 2. It is a toughtful touch for making the book more approachable to people who are not familiar with mathematical logic. Chapter 3 is a short chapter on ordinals and recursion constructions.

In Chapter 4 it starts to be clear that, because of the aim of the book, some non-conventional choices are made: after an introduction on cardinal combinatorics (stationary sets included), large cardinals are immediately presented: inaccessible, Mahlo, ineffable, weakly compact, measurable, strong, supercompact and even Reinhardt, without much consideration on the metamathematical problems that the use of large cardinals involve. Generally speaking, the strongest a large cardinal axiom is, the more wary people are of it, as it also increases the probability that it is inconsistent, but among inner model theorists it is common to use any large cardinal that is needed. In a certain sense, it is a philosophical choice that the author, and the book, make. Chapter 5 introduces the constructible universe $L[E]$, where E is a set or a class, the $J_\alpha[E]$ hierarchy (and J-structures), HOD, and proves classical results on L like GCH and $\Diamond$. Chapter 6 deals with forcing, and no details have been skipped. Among the different ways that the Cohen forcing is usually introduced, the author has chosen the one that is probably more difficult in calculations, but that expresses better the inner workings of such a forcing, and that is easier to generalize to higher cardinals. Of course, also the Levy collapse is defined and there are many applications of the two forcing notions. Chapter 7 is about descriptive set theory, but does not really lingers on it, the objective is to introduce the projective hierarchy, the regularity properties that are central in the AD analysis, and the Boundedness Lemma.

From Chapter 8 the big results are layed down. In Chapter 8 the first direction of the Solovay-Shelah Theorem is proven, due to Solovay, i.e., that the consistency of an inaccessible cardinal implies the consistency of "every set of reals is Lebesgue measurable". It is also the occasion to define the universally

Baire sets, an ubiquitous concept in inner model theory. Chapter 9 is the opposite direction of the Theorem, due to Shelah, proven via an elegant argument via the rapidness of the Raisonnier filter (this is the only part that necessitates some previous knowledge of measure theory). Chapter 10 is heavy duty, where no big results are introduced but there are all the techniques needed: iterations of ultrapowers, Prikry forcing, uniform Σ_1 Skolem functions, mice, $0^\sharp$, extenders (short and long), iteration trees and Woodin cardinals. The first section of Chapter 11 is an exercise in patience, as all the necessary results on fine structure theory are proven so that in the second section, finally, one can prove Jensen's Covering Lemma. In the third section is clarified its relationship with the combinatorial principle $\square$. Chapter 12 starts with a classical introduction of Determinacy, then continuing with the proofs that ω_1 and ω_2 are measurable under AD, and Harrington's Theorem, that says that analytic determinacy is equivalent to the existence of $x^\sharp$ for every x real. The gran finale is the proof of Martin-Steel Theorem, i.e., projective determinacy from Woodin cardinals, in Chapter 12. This is probably the most pleasant surprise of the book: such a proof is not the original one, but an easier one that gets rid of the weakly homogeneously Suslin trees and works directly on the iteration trees. The only reference to this proof is the diploma thesis (in German) of Windßus K., in 1993. There are no other publications under this name: I have been told that it refers to Karen Windßus, a student that worked out the details of such proof in her thesis and then left academy. In any case, while this proof was probably already known by the specialists, it is considerable that now it is accessible to a wider audience.

In fact, this can be said to most of the book. Not only inner model theory suffers from a lack of publications, but the whole set theory has very few textbooks. This limits the possibilities for a student, but not only for them: textbooks are also used by researchers, both at initial stages and advanced, to know more about a topic, maybe because they are curious of some famous result or because they want to enlarge their field of research. So a lack of textbooks is detrimental to the whole growth of a subject, and for this reason a book like this is not only useful, it is necessary for set theory. It contains great results, some of the most celebrated in set theory, it finally has a comprehensive exposition of many essential tools, and it is accessible to anybody that wants to know more, therefore also to the researcher that does not known how to navigate set theory literature, whether because at the beginning of the career or because coming from different subjects (or even different areas of set theory).

Another important thing is that the book is incredibly modern. There are few asides from the straight path to the Martin-Steel Theorems, but they are central to current set theory (for example tree combinatorics). Also, the path was chosen carefully so to hit many staples of set theory research. The sharp is a good example of that: it is usually introduced in textbooks as an EM-blueprint, or, in other words, as the theory of L provided that there is the right class of indiscernibles. The author prefers to define it directly as a mouse. It is admittedly a more difficult definition, as it involves iteration and fine structure, but in current research it is exactly the notion that is needed. When inner model theory researchers think of a sharp, this is what they think, not the EM-blueprint (that is introduced almost as a corollary), therefore that is the notion that now a student needs to know, even if it necessitates more effort.

Of course to bring the reader straight to the Martin-Steel Theorem in so few pages sacrifices must be made. There are many, many important subjects that are missing in this book, not only from different parts of set theory (that is to be expected), but also that are connected to the exposition and would be natural to add. For example, many descriptive set theorists would probably raise their eyebrows knowing that the Borel hierarchy is not defined, but the author goes straight from open to Borel and projective. Other examples are Cantor Theorem on the equivalence of dense countable orders with no endpoints, Cantor normal form, lifting of elementary embeddings, Vopenka Theorem, Mathias forcing and other forms of Prikry forcing, the Wadge hierarchy and the homogeneously Suslin trees. The way the author dealt with this, is to put everything as an exercise. There is in fact an abundance of exercises, sometimes they are calculations that were left off the proofs to make them more readable, other times they are entire new subjects introduced as an aside. Other than the ones listed above, in the exercises one can found Ackermann set theory, Magidor's definition of supercompactness, Kripke-Platek theory and admissible cardinals, p-points and q-points, and cardinal characteristics. In this respect it is a relatively complete book, but naturally an exercise, even with some hints, cannot possibly be a substitute of a clear exposition. Even less pleasing is when in a proof it is used in an essential way a concept that it is introduced only in the exercises (and it is not a rare occurence). This is a bit frustrating, as something that was presented marginally in the book, and therefore skimmed or even skipped altogether, suddenly becomes important.

But probably the worst problem is the other side of what makes it also good: it is too fast. Pedagogically, it would be very difficult for a student to follow it. Many concepts are introduced immediately in their most general form, therefore not giving enough intuition for a student to understand what is going one. For example, in the construction of L, the natural hierarchy of the L_α's is never introduced, starting instead with $J_\alpha[E]$ as a hierarchy of $L[E]$, where E can be a proper class. This initial difficulty propagates to all the chapter, as the proofs are of course more complicated for $L[E]$, that does not even necessary satisfy AC. Other examples could be the many large cardinals dropped just after an introduction to cardinal combinatorics, the extenders defined with their definition (and in the general long form), and not slowly starting from an embedding, or diamond introduced directly as $\diamondsuit_\kappa(R)$. Therefore for a student to be left alone with the book would be a challenging task. The description of the concepts is also sometimes arid, so it can become difficult to understand the big picture. (Also, of less importance, the copy I read is the first edition, therefore there is a not negligible amount of typos. This adds another layer of challenge to the student. Hopefully there will be a cleaner second edition.) Anyway, the author indicates that the book is suitable for advance students and researchers, so for people who can take a challenge.

In conclusion, this is the perfect book for someone who already has a familiarity with set theory, and wants to know more about this specific part. It is also a good introduction for somebody who comes from outside set theory, but is curious about it. As a university text, it could be a great reference for a teacher that wants to address her students towards inner model theory: the teacher can then follow the path indicated by the book, and then integrate and expand the points that are too terse, solving the pedagogical problem. It is a good resource for a reading course, but probably the teacher would have to intervene and help in some points the reader.