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Role of graphs in the mathematization process in physics education

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Role of graphs in the mathematization process in physics education

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Abstract. Using mathematical concepts to describe physical phenomena lies at the heart of physics. Literature however shows that combining physics and mathematics is challenging for students on all levels and therefore, the study of students' ways to link both fields is a hot topic in Physics Education Research. In the process of mathematization graphs have a fundamental role not only in the context of scientific communication but also within the process of gaining knowledge. But there is evidence of serious difficulties of students in reading, understanding and constructing graphs. Therefore we bring together different approaches, employing mainly qualitative research methods, to understand how students deal with graphs related to the interplay of mathematics and physics, how the reasoning differs between mathematical and physical problems and how students connect graphical and algebraic representations to physical concepts. Especially we describe in detail - in different areas of physics - which difficulties with graphs students of secondary school experience and which strategies they employ. For this purpose we analyse the activities of 8th grade students in graphing, we study the learning of 9th grade students in an interdisciplinary teaching-learning sequence and we describe the development of physical-mathematical concepts through the use of graphs, generated with on-line sensors.

1. Introduction

The interplay of mathematics and physics has many appearances. Mathematics helps in describing, predicting and structuring physical relations, hence has a genuine communicative, a technical and a structural role [13]. For communication, physical processes or relations between physical quantities can be represented by several types of representations with different degrees of abstractness. There are pictures or verbal descriptions, tables (numerical, providing exact values), line graphs (graphical, indicating qualitative behaviour) or formula (algebraic, giving functional dependence). Each representation type carries its own information, serves a specific purpose and hence in the educational perspective activates specific reasoning. Therefore it is important that students learn to use, transform and change between these representations and draw conclusions from them. Especially the visual representation by graphs – above all line graphs – plays a special role because graphs often serve as a bridge between abstract algebraic and verbal or pictorial representations in displaying functional dependencies between physical quantities [12]. In this light graphs have a special role in physics and also in physics education not only for a synthetic data representation, but particularly because they give access to the "revelation of the complex" [17]. In spite of this importance, a vast literature related to many topics both in physics and other scientific disciplines evidenced the difficulties of students in all educational levels in constructing, reading and interpreting graphs [5]. As in this context often the mathematical abilities of students are doubted some studies tested how mathematical and physics ability are interrelated showing that the connection to physics adds to the observed problems [6]. It was shown that the physics context plays an important role and that the insightful connection of mathematical quantities and physics concepts presents a problem to students [14], suggesting that not only technical but above all also structural aspects seem important [13]. Therefore approaches are promoted to teach explicitly about graphs with different success, often with focus on kinematics. As the ultimate goal is to develop research-based interventions for different age groups a focus of the research lies on analysing the reasoning of students and their strategies in dealing with graphs in different areas of physics and on understanding these processes in detail.

2. Shedding light on the role of graphs

In learning physics students have to deal with graphs in many situations, e. g. graphs play an important role in solving problems in physics. In that context not only the interpretation but also the construction of graphs is an important competence to develop e. g. in evaluating experimental data or in relating different representational forms. E. g. when students carry out experiments the construction of a graph is often one part of the data analysis to investigate the relation between the measured quantities. In order to be able to provide support the teacher needs to know in depth the students' ways of thinking and their difficulties during the process of transferring data from a table, a formula or a verbal description into graphs. These were analysed in section 3. An explicit focus on the interplay of mathematics and physics is suggested by linear functions playing an important role in both subjects in the context of school, especially in the area of kinematics. Characteristic quantities are the slope and the y- or x-intercept of line graphs which are named and treated differently in mathematics and in physics. So the question arises if students taught in an interdisciplinary way cope better. We study students' understanding of concepts related to linear functions in kinematics and mathematics in the 9th grade considering graphs as well as formula. The students' explanations are being analysed and common errors are identified (section 4). Aside from the construction process and the use of linear functions arbitrary given graphs, e.g. gained by computer-aided measuring have to be interpreted and transferred into a meaningful physical interpretation. In an example presented here students should learn the physics concepts of optical diffraction and its mathematical expression from own graphing. Their learning path and the related strategies and difficulties have been analysed deeply in a qualitative study (section 5).

3. Lower secondary school students construct graphs in physics

To develop the graphing competency already in physics class of lower secondary school it is necessary to know the students' way of thinking and their difficulties during corresponding processes. This section presents various kinds of actions in constructing a graph in physics starting from different other representations. We will describe more deeply the students' approaches in the context of representational changes and finally arrive at conclusions for the selection and design of construction-tasks.

3.1. Framework: Students' actions constructing a graph

The construction of a graph can be divided into two main components: the construction of the frame of the graph and the insertion of the data (cf. [8]).

Construction of the frame: First students have to decide which quantities should be part of the graph. Either this is explicitly given or they have to choose between different given quantities depending on the purpose of the graph. After knowing which quantities shall be related to each other students have to decide how to allocate them to the axes. As a further step, students choose the proper quadrant(s) and draw the appropriate axes of the coordinate system. This requires knowledge about the magnitude of the quantities, which is also necessary for scaling the axes in a further step. If more than one relation is shown in a graph a legend or further labels have to be added.

Inserting the data: After building the frame the students insert single points. If it makes sense for the physical relation they will represent the relation with the help of a curve. Also a sketched trend would be part of this component of graph construction.

It can be seen that different components of the construction process can be distinguished from each other (see also table 1). In the context of biology, it was found that the two main components of the construction process are connected to each other: students who know how to construct the frame of a diagram are also able to insert data to a certain extent, and vice versa. This might also apply in the context of physics because no relation between the construction skills and the biological knowledge could be found in the study (cf. [8]).

Table 1. Actions of students constructing graphs (adapted from [8])

Construction of the frame	Inserting the data
• Choosing the relevant quantities	• Inserting single points
• Relating the variables to the axes and labelling them	• Representing the dependencie(s) or relation(s)
• Choosing the proper quadrant(s)	
• Scaling the axes	
• Adding a legend or further labels	

Construction of graphs as a change of representation
The construction of a graph can be seen as a change or rather transformation of representations: Some information is given in a source representation, for instance a table or a verbal description. Then this has to be transformed into a graph, the target representation.

During this process, different kinds of activities and translation processes can be applied. Students can (A) follow an algorithmic, stepwise approach or (B) they can use specific characteristics of the relation, e.g. the kind of dependency or (C) they can verify their approach and check the consistency between the source and the target representation. (cf. [3, 4]). All these three kinds of activities may be part of a technical translation or a structural transformation: Within a technical translation, students' reasons are formal ones, e. g. based on conventions or memories. They do not connect their thoughts to physics. Within a structural transformation students use a deeper mathematical or physical understanding of the representations and relations to create the target representation (cf. [3, 4]).

Both kinds of transformation approaches are important. During some actions, e. g. inserting points, it can be helpful and efficient to not think about the whole meaning of every element of the graph and the represented relation. However, in the connection with some other actions, e. g. interpolation, it is necessary to get a deeper understanding of the relation and to use this to create an appropriate graph.

3.2. Research questions
Unlike the interpretation of graphs in physics, the construction process has not yet been investigated in detail (cf. [9, 12]). Therefore, we (PER group of TU Dresden,) explore how students transform a table, a formula or a verbal description into a graph and answer the following research questions:

1. Which of the students' actions constructing a graph are (A) stepwise and algorithmic, (B) carried out by means of specific characteristics of the relation, or (C) verified by an retrospective perspective?
2. Do students use both technical and structural translation processes when they transform a source representation into a graph? How do the students reason?

3.3. Method
Because students start to develop their skills connected to graphs in physics already in lower secondary school, we explore the research questions at this early stage of physics learning. Thus, we have invited 17 pairs of students aged about 14 years to an explorative laboratory study in which they were asked to work on specifically designed tasks. These tasks were generated in the field of thermodynamics and contained (besides other changes of representations) the construction of a graph starting from a table, a formula or a verbal description (figure 1).
The pairs of students were asked to think aloud during solving the tasks on an interactive whiteboard. Their discussions and writings were recorded. After the teamwork, a short interview was conducted to ask for clarification of open questions. The recorded data was completely transcribed and analysed according to the method of qualitative content analysis [7]. Thereby, deductive and inductive categories were defined. Thematic categories describe the actions of the students (e.g. table 1). Evaluative

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categories judge the quality and specification of them (A stepwise realization, B use of characteristics, C verification of consistency; technical, structural). The data was coded according to both category systems. Compared to a second coder who coded about 30% of the material a good agreement even for the evaluative categories could be reached after revising the coding manual (63%, κ=0.6).

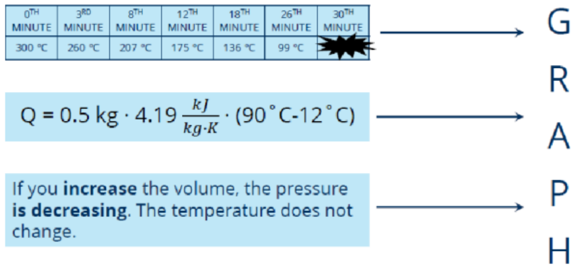

Figure 1. Three different kinds of source representations used in the laboratory study.

3.4. Results

The *construction of the frame* of the graph was separated into five different actions (table 1). Besides the action "adding a legend or further labels" all others occurred in at least two of the tasks (tables 2 and 3). When choosing the relevant quantities or the proper quadrant(s) and also when scaling the axes, the observed students mainly followed an algorithmic, stepwise approach (A). Only when they related the variables to the axes also a use of characteristics (B) was noticed. Here some students already connected their reasoning to an imagined curve ("because the temperature goes down we have to start at the top" putting temperature at the ordinate) or to the kind of dependency between the quantities ("You could switch it […]. Because it is proportional to each other. […] Because the graph would look the same." – This pair mistakenly believes that the graph of a direct proportional relationship is a bisecting line.) It could be seen rarely that students look back and think again about their actions to verify the construction of the frame. This mainly happened when they related the variables to the axes transforming a verbal description into a graph.

Table 2. Frequency of stepwise realization (A), use of characteristics (B) and verification of consistency (C) while constructing the frame of a graph starting from different source representations.

	Source representation		
Action	table	formula	verbal description
Choosing the relevant quantities	-	most of the time A	always A
Relating the variables to the axes	most of the time A, sometimes B	most of the time A	most of the time A, sometimes B and C
Choosing the proper quadrant(s)	most of the time A, sometimes B	most of the time A	always A
Scaling the axes	most of the time A	most of the time A	most of the time A

During the component *inserting the data* the students started to insert single points when they transformed a table into a graph and even sometimes when they transformed a formula into a graph. In the case of the latter they either calculated or estimated them connecting the formula to a situation (which was another subtask before). Within the task with a verbal source representation single points were not necessary; the students sketched a curve immediately.

As expected, the observed students followed an algorithm inserting single points (A). When it came to sketch a curve to represent the relation all three kinds of activities (A, B, C) occurred (table 3). Some students drew a linear line out of habit or connected point to point (A). Others assumed or discussed the kind of relationship to find an appropriate curve (B). To verify their drawn curves (C) students used different strategies. For instance, they checked if the monotony or the change of monotony is consistent (“No, the temperature is decreasing at a slower rate at the end” after inserting a linear line) or if the characteristics of the kind of relationship are fulfilled (“When you halve the pressure, like this, the volume, here, has doubled. Hence it would be correct.”). Some students also calculated further pairs of values with the help of the given formula to insert more points to revise their inserted curve.

Table 3. Frequency of stepwise realization (A), use of characteristics (B) and verification of consistency (C) while inserting data in a graph starting from different source representations.

	Source representation		
Action	table	formula	verbal description
Inserting single points	most of the time A	always A	-
Representing the dependencie(s) or relation(s)	most of the time A, B and C	most of the time A, B and C	most of the time A, B and C

3.5. Discussion and outlook

The construction of a graph in physics requires different actions which could be separated and observed in this study. Next to the insertion of the data also the construction of the frame of a graph should get attention. Here students have to decide on several aspects (e.g. the scaling) which finally leads to the (correct) appearance of the curve.

The observed students generally showed an algorithmic stepwise approach but depending on the different tasks also used characteristics of the relation or tried to verify their solutions within some of the actions. This shows that even some students at their 3rd year of school physics are able to use more advanced strategies like for instance taking a retrospective perspective. Very often they did not try to use any characteristics of the relation where it was not necessary and instead followed step-by-step procedures, e.g. while inserting single points. Therefore it could be helpful to trigger advanced approaches using a non-standard design for the construction-task, e.g. offering prepared coordinate axes with different quadrants or more than two quantities in the source representation to choose from or having the students to use the characteristics of a relation in representing the functional dependency appropriately. The students’ rationales connected to the different construction actions will provide more insights and will be the focus of the continuing analysis. It could already be seen in the data that both technical and structural translation processes appeared within all three tasks containing different source representations and within all three kinds of approaches (A, B, C).

These results are still subject to some restrictions, because the three different tasks containing three different source representations are belonging to a project which investigates changes of representations in general and thus they are not completely comparable. For instance the task involving verbal expression consists of three subtasks including three different kinds of verbal expressions. Thus, the amount of existing data for each task differs. Sometimes, students also used a formula as a transitional representation transforming a verbal expression into a graph (see also next section). This could influence some of the actions, for instance choosing the relevant quantities. Nevertheless the comparison of these construction tasks led to some consistent results and conclusions as presented here.

4. Linking concepts of linear relations in physics and mathematics

The research of the Leuven group is situated in the context of a newly designed integrated STEM curriculum for secondary education developed in the STEM@school project in Flanders (Belgium), which explicitly aims to better link concepts from all STEM fields.

4.1. Context of the Research and Research Questions

In the context of this project – together with Teacher Design Teams – teaching/learning materials were developed based on five key principles: integration of STEM content, problem-centered learning, inquiry-based learning, design-based learning, and cooperative learning [16]. The materials are structured around a central problem for which concepts and techniques from the different S-T-E-M fields are needed to design an appropriate solution. Through inquiry and design, the students cooperatively tackle this central challenge. About 30 test schools implemented the newly designed teaching/learning materials and took part in research on the effects of the new approach.

In one of the developed modules, designed for 9th grade, students are challenged to construct a programmable, autonomous car and program it to drive through a predefined sequence of traffic lights in a single motion, as fast as possible. To tackle the problem, students need ideas related to 1D kinematics, linear functions, programming, etc. In the context of that project, we studied student understanding of concepts related to linear relations both in the context of physics (uniform linear motion) and mathematics. We focus on linear relations because of the known difficulties students have with their abstract representations as an equation or a graph, the latter particularly in kinematics. Furthermore a performance gap has been shown to exist for student understanding of linear relations between mathematics with and without context [6]. The main research questions are:

1. How does grade 9 students' performance of linear function problems compare between kinematics and mathematics?
2. How does grade 9 students' performance of linear function problems compare between the STEM@school approach and the traditional approach?
3. How can we categorize grade 9 students' strategies when comparing and identifying velocity or initial position in linear $x(t)$ problems in kinematics and slope or y-intercept in $y(x)$ problems in mathematics?

4.2 Method

To answer the research questions, we designed a test consisting of open-ended questions, 12 in physics and 12 as isomorphic as possible questions in mathematics. Each item is formulated using an algebraic (formula) or a graphical representation. The problems ask to *determine* slope (velocity) or y-intercept (initial position) of a given linear relation based on a graph or formula, or to *compare* slope (velocity) and y-intercept (initial position) between two given relations based on a graph. Both positive and negative slopes but only positive $y(x)$-intercepts are included. Figure 2 shows an example of a 'determine' question on 'slope' in a graphical representation in physics and mathematics.

The test was administered in 2017 to 253 students. Student answers are coded as correct (1) if both the answer and the explanation are correct. If the answer is correct but the reasoning is missing, the answer is also scored correct. In the other cases, answers are coded incorrect (0). The answers are analysed using Generalized Estimating Equations (GEE), in which context (1D kinematics or mathematics), question type (compare given a graph, determine given a graph or determine given a formula), concept (intercept, slope), slope sign (+, -), gender and instructional approach (integrated STEM vs. traditional) are taken as independent variables. Main and interaction effects are studied.

To get insight in student reasoning, student explanations were studied in detail. A first categorization scheme describing student answers was build bottom-up from the data, by the first researcher. To achieve a sound categorization, the scheme was – based on a subset of the data – refined by a second researcher and then independently used to analyse a new subset of the data by the first researcher and a colleague.

4.3 Results

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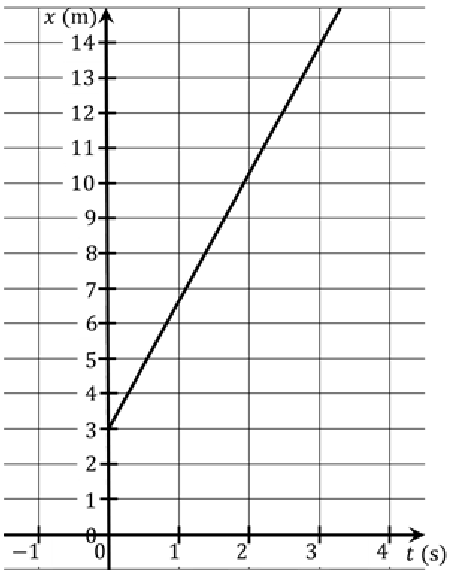
To answer the first two research questions the cohort of respondents is divided in a group (121 students) that followed the STEM@school curriculum and a control group (132 students) that followed traditional instruction. The GEE analysis shows that there is a main effect from the variables ‘context’, ‘question type’, ‘concept’, and ‘slope’, but not from ‘gender’, and not from the instructional approach. On average, mathematics questions are answered more accurately than physics questions, ‘compare’ questions are easier than ‘determine’ questions. Determine questions using a graph are easier than the ones using a formula. Questions on y-intercept are answered significantly more correct than questions on slope, and negative slopes are more difficult than positive ones. Besides these main effects, there are also significant interaction effects, but the instructional approach is not included in any significant 2-way or 3-way interaction effect. For a more detailed description we refer to [1].

K6

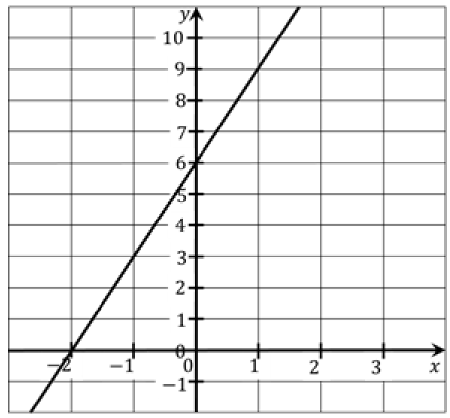
A train is moving along a straight track. The graph shows the position x of the train as function of the time t. The position is in meter, the time in seconds.



Determine the velocity of the train. Explain your answer.

M6

The figure shows the graph a function f.



Determine the slope. Explain your answer.

Figure 2: Example of two isomorphic items – K6 in kinematics and M6 in mathematics – asking to determine the velocity (slope) when given a graph with positive slope and positive y-intercept.

The detailed analysis of the written explanations gave rise to a categorization scheme describing different ways students reason to answer the questions. Table 4 shows the categorization scheme consisting of five shared categories for ‘y-intercept’ and ‘slope’, two which are unique to the ‘y-intercept’ and four which are unique to the ‘slope’. Firstly, the scheme achieved a good overall inter-rater reliability (Cohen’s Kappa) of 0.61 with item specific values often far higher.

Table 4. Concise categorization scheme for students’ reasoning

<table>
<tr><th>y-intercept / initial position</th><th>Slope / velocity</th></tr>
<tr><td colspan="2">Identification through the location of a coefficient in an equation.</td></tr>
<tr><td colspan="2">Identification of the root (x-intercept). (graphical or symbolical)</td></tr>
<tr><td colspan="2">Change representation: Construct an equation.</td></tr>
<tr><td colspan="2">Change representation: Construct a graph.</td></tr>
<tr><td colspan="2">Change representation: Construct a table.</td></tr>
<tr><td>Identification of the intersection with the vertical axis.</td><td>Reasoning with the ‘steepness’ of a graph.</td></tr>
<tr><td>Calculation of f(x). (often with x=0 or x=1)</td><td>Drawing a triangle on a graph</td></tr>
</table>

	Calculation/comparison of the ratio of differences.
	Calculation/comparison of the ratio of some numbers. (e.g.: coefficients in the equation, *y*-intercept over *x*-intercept etc.)

Some remarkable findings of our analysis are:

- A minority of students switches representation. A change of representation most often occurs in y-intercept questions in mathematics when confronted with a graph. In that case a fifth of all students constructs an equation, which is indicative of a higher reliance on, or a higher pressure to use equations.
- Quite a few students switch *x*- and *y*-intercept in the context of mathematics.
- Comparison and identification of negative slope(s) is more difficult in physics than in math
- In physics, students often base their motivation on dimension analysis, i.e. they look for 'meters over seconds'. They select the coefficients they think have these units or (incorrectly) manipulate the equation to end up with an expression for a quantity in meters and for a quantity in seconds and then take the ratio as shown in figure 3.

A train is moving on a straight track. The position (x) of the train a function of time (t) is given by $x=4+8t$. The position is expressed in meter, the time in seconds. Determine the velocity of the train. Explain your answer.

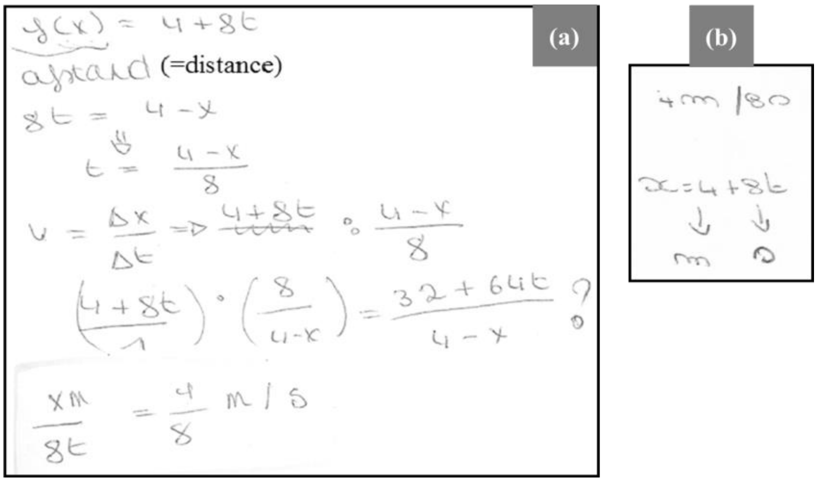


Figure 3. Question from the test and explanations from different two different students (a) and (b) presenting erroneous procedures as described in the text above.

4.4. Conclusion

We studied student understanding of the concepts *y*-intercept and slope in linear function problems with graphs and algebraic expressions in isomorphic kinematics and mathematics questions and looked at student accuracies for students in a traditional curriculum and in a newly developed curriculum that puts more emphasis on integration of different STEM fields. Results show that students' difficulties are concentrated in physics, algebraic expressions and negative slopes. Moreover, there was no significant main effect from the educational approach. The preceding qualitative analysis highlights intriguing difficulties and strategies. Student answers on isomorphic questions illustrated that for students there is a weak link between mathematics and 1D kinematics and that students have difficulty in transferring their mathematical understanding to kinematics.

5. The Role of graphs in IBL study of optical diffraction by secondary students

On-line sensors offer new learning opportunities in developing graphing competencies with focus on relating graphs to physics and constructing physics concepts. New strategies have been studied to remove graphing obstacles in learning environments [15, 18]. In the case of physical optics the use of on-line sensors allowed us to develop approaches based on the analysis of phenomenology as a premise for constructing a formalized interpretation of it [11]. Here, we discuss a study on the role of graphs in the students' process of analysing the phenomenology of optical diffraction and building the phenomenological laws describing it.

5.1. Research framework and context

Optical diffraction is the focus of a series of activities in an inquiry based laboratory (IBL) oriented to bridge from classical to modern physics for secondary school students [11]. A research based educational path suggests as first step a qualitative exploration of a diffraction pattern with the task to sketch the graph of light intensity vs position. The second step offers the students the opportunity to measure intensity vs position by means of USB computer on-line sensors system working as RTL [2].

In the context of educational laboratory of conceptual operative exploration (CLOE) [10], by means of tutorials, we studied how students develop competencies in the experimental analysis of diffraction phenomena and in the construction of phenomenological laws and the role of graphs in that process. In that context, the research focused on how to promote the experimental study of the optical diffraction according to an inquiry strategy, changing the guided approach adopted in previous activities. For that general aim, two preliminary pieces of information have to be collected: the kind and design of experiments suggested by students to characterize quantitatively optical diffraction in a single slit; and for that goal, the kind of analysis they suggest to perform on a diffraction distribution.

The sample included 316 Italian upper secondary students, 17-18 years old from 4 schools (three Scientific Lyceum and one Technical Institute of little towns in North-East Italy): The previous knowledge of students included only general issues about waves and Young experiment, but nothing about diffraction. Students' learning paths were monitored using an open inquiry based tutorial and free notes of researchers during activities, when they followed these three steps:

A) Qualitative observation on a white screen of the diffraction pattern produced by red-laser light diffracted by a single slit. Students were requested to draw the image on the screen, describe it, draw the corresponding intensity distribution vs transverse position they expect to observe performing a quantitative experiment, comment that graph.

B) Interacting dialogues in big groups with a mirroring method [2], discussion on the general features of the diffraction pattern, on the parameters affecting it and their role, observing how the pattern changes with changes of D - distance slit-screen; a - slit width; λ - laser color used.

C) After presenting the possibility to collect with a light sensor the intensity vs position [2], students were requested to design a quantitative experiment (Which parameters? Which variables?) and then which kind of data analysis they suggest to perform to characterize quantitatively the diffraction distribution. Students in groups of 3-4 performed some experiments in lab and analysed data (usually by tutoring of their school teacher).

5.2. Research questions

The present contribution focuses on the following research questions:

Concerning step A)

RQ1. Which typologies of graphs and descriptions can be recognized?

We will consider the conceptual aspects emphasized in the drawing/ description, the physical models underlying the representation, the mathematical aspects included.

Concerning step C)

RQ2. Which experiments do the students design, and which data and relations do they suggest to analyze or to extract from the graph?

In particular we were interested in understanding whether the students were able to distinguish between parameters and variables, if they suggested methods of analysis similar to those that a physicist might suggest, or which other variables or relationships they would look at.

5.3. Methodology of analysis

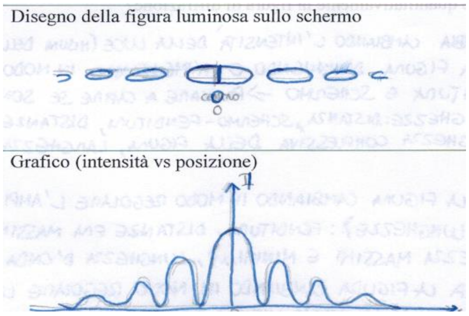
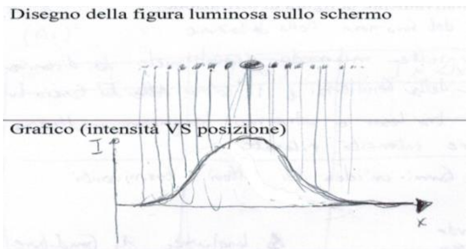
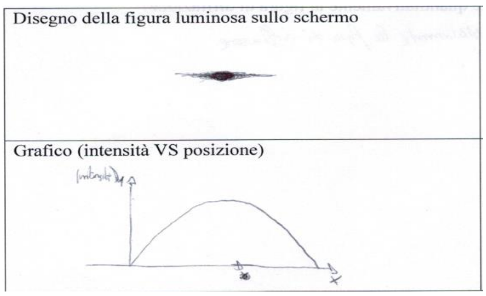
The tutorials filled by the students during the CLOE labs were analyzed by constructing inductively operative definitions of categories. For this typical students' answers were collected according to qualitative research criteria, distinguishing interpretative vs descriptive approaches, identifying models underlying the interpretation, conceptual references adopted and in particular concerning:

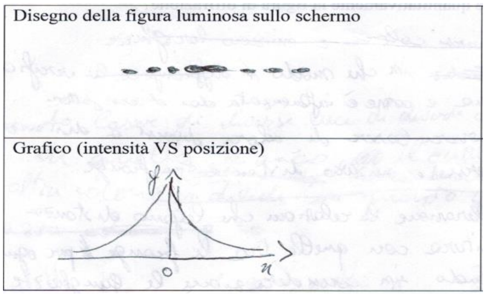
- step A) - Characteristics of diffraction pattern drawings; its description and included physical and math aspects;
- step C) - Design of an experiment/ data analysis (parameters vs variables, quantities to be collected, variables and relation to be analysed).

5.4. Results

Table 5 represents four categories of the typical representations and descriptions of the diffraction patterns done by students and the prefigured diffraction distributions they expected to observe on the computer screen before performing the experiment with on-line sensors. The majority of students (241 on 316 of the full sample - 76%) recognized that the diffraction figure consists of an alternation of bright spots and dark areas. Only for a third of students (cat A-34%) the spot pattern corresponded to a continuous intensity distribution with alternating minima and maxima, with decreasing intensity.

Table 5. Categories of drawings of diffraction pattern, intensity vs position distribution expected by students and corresponding descriptions, including number of students and percentage with respect of the full sample (N=316) per each category.

Cat	Drawing examples	Students' explanation/observation
Cat A n=108 34 %		«Light bands/points and dark bands/points» «Greater intensity at the center (intensity decreasing from the center)» «Symmetry» [«The band in the middle is wider» «The clear bands are wider» « Regular distances» «Horizontal figure with vertical slit»]
Cat B n=97 31 %		«Very long interspersed with absences of light, it is becoming increasingly weaker. It has a peak in the middle»
Cat C n=75 24 %		« Elliptic figure More bright in the center, less on the border, Parabolic curve tending downwards»

Cat D n=36 11 %		«light and dark fringes brighter than closer to the center»

The majority of the distributions (categories B-C-D, 208/316 - 66%,) represented only the envelope of the maxima. The difficulty of many students emerges to predict the features of such a complex function as the sinc^2, which well reproduces the experimental distribution in the typical measurement conditions (D >> a). These different difficulties can be linked to the habit of school mathematics to deal with algebraic or trigonometric functions separately and to consider a function such as the sinc(x) only as a fundamental limit with $x \rightarrow 0$. Another problematic issue at the intersection of mathematics and physics emerges in the use of elements of discontinuity in the representation of the expected distribution, as in the example illustrating category D) or other minority examples of distributions representing points or dotted lines or lines with cusps. This seems connected to an idea of a discontinuous distribution of the light intensity matching the pattern drawings. The context of diffraction can be very useful for expanding students' competencies on the use of functions, in the comprehension of the role of continuity/ discontinuity, in the behaviour of physical quantities.

When requested to suggest a quantitative experiment (step C), 156 students (49% of 316) answered. A small part (33/156) declared the purpose of the experiment. Often, the aim was to study how the figure changes by changing some parameters. In some cases, the stated goal was to "find the type of relationship" between intensity and position, that is an analytic expression of the distribution. This tendency emerged more frequently during the oral exchanges between students and subtended the idea that to find an explanation means to find a formula (here: $sinc^2(x)$). This could explain what emerged in other studies evidencing the students' resistance to overcome a geometrical behaviour of the light even after an in depth analysis of the optical diffraction phenomenology [11]. The remaining students listed the quantities they suggested to measure, often (55%) mixing parameters and variables, usually including the changes they suggest to perform ("*I change* ... [slit, laser, distance…] *and I measure/observe* …[distance, width, intensity ...]"). In graphing "intensity" the students remained in the first two actions of table 5.

Among the most quoted quantities are the "*differences"* in the positions of maxima and minima, width of the central maximum or, less frequently, the position of minima/maxima (37 - 24%), intensity (10 - 6%). Other quantities mentioned were also the width of the slit or its characteristics (28 - 18%) and the characteristics of the laser light used, such as wavelength and intensity (22-14%). The relation between D and ΔX (32-21%) was the most often cited by the students. The majority of students spontaneously did not look at the entire distribution, for example looking for the relation between position and order of interference, for fixed parameter values, but looked locally at the distances between successive minima / maxima and this in relation to the parameters they intend to change.

Performing the experiment and the analysis of an on-line graph, the large majority of students gained a complete vision of the phenomenology, distinguishing the role of parameters and of the physical quantities measured. Few students continued to emphasize the intensity of the central maximum without representing the decreasing maxima in the wings. In their analysis students feel the importance to analyze position of minima/maxima and relative intensity. Sometimes (20%), they connected the minima position vs order number to the trivial minima condition for determining λ.

5. 5. Discussion

Analysing first the drawings (at the end of step A) it emerged that the majority of students tend to represent only the envelope of the intensity, disregarding the periodic change in the intensity distribution and in particular the presence of minima. This seems connected on one side to the absence of a coherent physical model capable to explain the presence of minima [11], on the other side to the lack in their mathematical background of a complex function as the $sinc^2(x)$ describing a diffraction pattern. Another group of students showed the tendency of emphasizing the presence of minima with discontinuous curves. This seems connected to an idea of a discontinuous distribution of the light intensity, in accord with the students' pattern drawings. When asked to design an experiment, students tend to mix parameters (*D*-*a*-λ) and physical quantities (light intensity, position) to be collected. They spontaneously tend to analyse the distribution locally and it is necessary to fill a gap to bring them to consider global relationships between variables. Sharing the suggestions of all the students of a class it was possible to reach good results in the lab experiments and a satisfactory students' learning, without an instructional-guided path. Some students evidenced the idea that to find an explanation means to find a formula (the $sinc^2(x)$ that case). This probably is at the base of the resistance evidenced by some students to abandon a geometrical description of light behaviour also after discovering and analysing qualitatively and quantitatively the features of a diffraction pattern and the related distribution. This point will be studied in future researches.

6. Conclusion

Even if the three studies presented here differ by target and thematic context and use different methodologies of analysis they lead to specific results that deepen particular aspects such that some general conclusions seem to emerge and working hypotheses can be formulated for further research.

Students mainly use an algorithmic approach to graphs, positively applying procedures in "ritual" situations (positive quadrant, positive gradients, linear functions) demonstrating difficulty in transferring procedures known in mathematics in the context of physics and neglecting to review their work. This is seen both in using simple mathematical constructs related to linear functions (as in the case of simple kinematics problems), as well as in the management of more complex functions such as those involved in different contexts such as in thermal phenomena or in diffraction. On the other hand, when students experience challenging situations they tend to more reflection and review of their work. In some non-standard situations e. g. in transforming selected representations the studies show hints for the reliance on algebraic formulas as a transitional representation for a cohort of students when experiencing difficulties with graphs. These students effectively use formulas as a base representation, i.e. as a go-to representation with which the students perhaps feel more confident or more pressured to use. However, this observation was context and concept dependent. This might even lead to discard geometrical descriptions (third study). So, although graphs play an important role in mathematization, these three studies show that in tasks focusing on graphs, a significant number of students in the studied cohorts deflects to using formulas instead.

From these studies first hints concerning educational strategies emerge. It is important to offer different kinds of sufficiently complex constructing-tasks including graphs so that students can experience and train all kinds of actions and approaches beyond routines and straight-forward solutions.

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