





Safety Performance Functions for road intersections in the Friuli Venezia Giulia Region

Nicola Baldo^{1*}, Matteo Miani¹

¹Polytechnic Department of Engineering and Architecture, University of Udine, Via del Cotonificio 114, Udine 33100, Italy

Abstract

The aim of this paper was developing Safety Performance Functions (SPFs) of four different types of road intersections: unsignalized three-leg intersection, unsignalized four-leg intersection, signalized three-leg intersections, which are under the jurisdiction of Friuli Venezia Giulia Strade S.p.A., were collected and analyzed. To model the empirical relation between crash frequency and traffic volume, a feed-forward artificial neural network (ANN), characterized by the hyperbolic tangent transfer function and one hidden layer, was used. SPFs of the analyzed road intersections have shown a slope reduction of the tangent line when annual average daily traffic increased on the main leg, namely, the road capacity is not able to meet the increase in the volume of traffic. As a result, this leads to a saturation that induces slowdowns with a lower probability of running into an accident.

Keywords: Safety Performance Function; Artificial Neural Network; Road Safety.

1. Introduction

Road safety is a topic of great interest, both in the technical-scientific and socioeconomic fields. In fact, economic development is related to the availability of transport but a rapid increase in the number of vehicles can cause a serious social problem due to the increase in the number of accidents or the worsening of their severity. The European Commission, since the publication of the White Paper in 2001, has started a path of improvement of road safety, divided into a series of ten-year programs, with the aim of halving the number of victims on the road. Considering the period 2010-2016, in the 28 Member States of the European Union (EU28) the number of victims on the roads decreased by 18.6% (Istat, 2018). In Italy, the reduction in mortality over the same period has been slightly more pronounced than the average European trend (-20.2%). Nowadays, road accidents are one of the top 10 causes of mortality in the world (World Health Organization, 2018).

However, to reduce as much as possible the social cost related to road traffic accidents, the road designer has to assess the safety level of an infrastructure and consequently

^{*} Corresponding author: Nicola Baldo (nicola.baldo@uniud.it)

optimize the design choices according to road agencies priorities and economic resources available. The safety of a road infrastructure is closely linked to the number of accidents that occur on it and the severity of the consequences that come from it. The correct assessment of the safety level of a road infrastructure is extremely difficult, due to the obvious unpredictability of strategic variables, such as behavioural and environmental variables, which make the modeling of the incidental phenomenon extremely complex. To overcome this problem, the most common approach involves a safety classification related to the number and severity of accidents. However, this approach, affected by a considerable degree of uncertainty, is limited to the identification of the so-called "black spot" of the road network, i.e. sites characterized by a high accident rate. On the other hand, it is necessary to have a methodology that goes beyond the mere identification of black spots and that allows a quantitative analysis, of a predictive nature, of the incidental phenomenon to be performed, in order to formulate in advance a reliable assessment of the safety of a road infrastructure. In this context, Safety Performance Functions (SPFs) are an effective practical solution: the SPFs seek to identify an empirical relation between the occurrence of the generic crash event and the multiplicity of factors that make up the road environment, including in particular the annual volume of average daily traffic (AADT, i.e. the average number of vehicles travelling through a given road section over a 24-hour day) expressed in vehicles per day. In other words, the SPFs are regression equations or mathematical models having predictive purposes that are developed, starting from a sample of data representing the incidental phenomenon on a specific site type, using different statistical or computational techniques. However, the complexity of relationships sometimes requires analytical techniques different from the traditional ones. In fact, the classical statistical regression techniques are based on the priori assumption of a probability distribution, typically Poissonian or negative-binomial (Chang, 2005), to describe the crash frequency on a specific site. This implicitly imposes a relationship between dependent and independent variables that, if violated, could lead to incorrect estimates of the crash frequency.

A recent advanced computational approach for analysing complex nonlinear relationships is the artificial neural network system (ANNs). The ANNs are information processing systems that numerically simulate the functioning of the biological nervous system, which have shown an excellent ability to understand phenomena of a different nature. The peculiar characteristic of this computational approach is the ability to identify robust correlations between data sets regardless of the nature of the problem. To this end, the ANNs make use of a "learning" mechanism, hence the term "machine learning" frequently used in literature.

With the aim of providing road designers with a rational tool to support decisions, the main interest of this study is the development of predictive models of road accidents, calibrated and validated for the intersections (unsignalized three-leg - 3ST, unsignalized four-leg - 4ST, signalized three-leg - 3SG, signalized four-leg - 4SG) of the competence network of the company FVG Strade S.p.A., using the ANN technique.

2. Literature review

The target of developing reliable predictive models led researchers to study and adopt different techniques for analysing road accident data, which include multivariate statistical regression techniques up to the most recent and advanced computational approaches. The system of artificial neural networks is emerging as a predominant technique in the development of predictive models because the many variables and the complexity of the non-linear relationships that characterize the incidental phenomenon require flexible methods that do not consider the intrinsic nature of the problem. In fact, although the road accident data typically shows a tendency to over-dispersion that makes negative binomial modeling an appropriate technique for analysing crash frequency (Chang, 2005), a priori assumption of a probabilistic distribution of the traffic accidental data implicitly imposes the relationship type of the predictive model. Learning from past experiences to deal with new and unexpected situations allows ANNs to achieve better performance in road accident prediction than traditional statistical regression techniques. De Luca (2015) analyzed the road accident data over the period 2000-2005 on a road located in southern Italy through a multivariate analysis (MVA) and a multilayer ANN, reaching two predictive models of the same phenomenon. Comparing the two models, it emerged that the total sum of the residual is smaller for the predictive model identified by ANN. Therefore, the ANN model was more reliable in predicting future road accident. Jadaan et al. (2014) showed that multilayer networks produce more accurate estimates of the number of accidents on Jordan's urban highways than the multiple linear regression technique even when the dataset is limited. Moreover, artificial neural networks have also been used to evaluate the influence of each explanatory variables on the predicted accident parameter, through the computation of an influence index based on the weights of the trained network connections (Chiou, 2006). Among the studies that have used ANN modeling approach to predict multiple vehicle collisions at intersections, the experience of Mussone et al. (1999) is of particular interest. By modeling the occurrence of road accidents in some intersections of the city of Milan (Italy), they showed a higher hazard in terms of running over of pedestrian in unsignalized intersections, particularly at nighttime, and noted that the use of variables with non-homogenous distribution and high levels of correlation makes linear model unreliable, with higher error. Instead, Abdelwahab and Abdel-Aty (2001) used a multilayer perceptron (MLP) neural network as a modeling approach for two-vehicle accident severity at signalized intersections and its relating factors (such as human factors, roadway conditions, weather conditions and etc.). The results of this study, using data for the Central Florida area, show that urban intersections are less dangerous in terms of driver injury severity than rural ones and female users are more likely to suffer a serious injury compared to the male ones.

The aim of this paper is the development of SPFs for four different types of road intersections (3ST, 4ST, 3SG, 4SG) through the use of a feed-forward multilayer neural network. To define road traffic variables related to crash frequency (i.e. the average number of road accidents occurred in a reference period at a specific site), the intersection vehicle collisions models and the analysis of the influence of each explanatory variable on the prediction reported in Chapter 12 (Harwood et al., 2007) of the Highway Safety Manual (HSM), international benchmark for the road safety analysis, were considered in the present study. These predictive models were developed through negative binomial regression on road accident data, collected from large-scale studies on United States. Specifically, they have identified a strong degree of correlation between the crash frequency (N) and the annual average daily traffic volumes on the main ($AADT_{maj}$) and secondary ($AADT_{min}$) legs. The correlation can be expressed as follows, regardless of the type of intersection:

$$N = N_{mv} + N_{sv} \tag{1}$$

$$N = e^{a_1 + b_1 \ln(AADT_{maj}) + c_1 \ln(AADT_{min})} + e^{a_2 + b_2 \ln(AADT_{maj}) + c_2 \ln(AADT_{min})}$$
(2)

where a_1 , b_1 , c_1 are regression coefficients (Table 1) for multiple vehicle collisions frequency (N_{mv}) , and a_2 , b_2 , c_2 are regression coefficients (Table 1) for single vehicle collisions frequency (N_{sv}) .

Intersection type	a ₁ (Intercept)	$b_1 (AADT_{maj})$	$c_1 (AADT_{min})$	a ₂ (Intercept)	$b_2 (AADT_{maj})$	$c_2 (AADT_{min})$
3ST	-13.36	1.11	0.41	-6.81	0.16	0.51
3SG	-12.13	1.11	0.26	-9.02	0.42	0.40
4ST	-8.90	0.82	0.25	-5.33	0.33	0.12
4SG	-10.99	1.07	0.23	-10.21	0.68	0.27

Table 1: Regression coefficients of HSM models for vehicle collisions at intersections.

The assumption of the same explanatory variables allowed a valid comparison between the predictive models described in the HSM and the models developed in the present study with the ANN approach, to be carried out. Obviously, the HSM models are valid only for the US context because they refer to base condition (road design, pavement characteristics, driving behavior, etc.) that cannot be reproduced in the FVG Region. This inevitably implies a quantitative mismatch between the crash frequency predicted by the two models at the same traffic conditions on the intersection legs.

3. Artificial neural network approach

The artificial neural network system is a computational approach, adopted in various research fields for pattern recognition, identification, classification, time series analysis and function fitting. The fundamental element of a network's architecture is the artificial neuron. Inspired by the functioning of the biological one, this computational unit carries out a weighted sum of the input "signals" and modulates its response due to an "activation" function (also called transfer function). In the biological neuron, this task is performed by the soma (the central body of the cell that contains the nucleus) that continues or interrupts the transmission of an impulse, based on the comparison with a threshold value; if the weighted sum of the input signals exceeds the threshold value, the neuron will activate and an "action potential" is produced. Within an artificial network, neurons are interconnected and operate in parallel. The weights of the connections determine the network function and are defined through a "training" process. In supervised learning, weights are progressively adjusted to minimize the difference between experimental targets and network output, using backpropagation algorithms which involves performing computations backward through the network (Figure 1). In this way, the artificial network "learns" to recognize the implicit relationship between input and target, and can provide a solution to new input data, not presented in the initial training dataset.

To develop predictive models of intersection related collision, in this research a twolayer feedforward network was used, with a tan-sigmoid transfer function ("tansig") in the hidden layer and a linear one ("purelin") in the output layer (Figure 2). In feedforward networks, information flows only in one direction (i.e. no recurring cycles are performed within the network) and neurons are organized into interconnected layers; neurons in the same layer do not communicate with each other. A detailed description of the structure of feedforward networks, the computational process performed by the neuron and backpropagation algorithms, can be found in Baldo et al. (2018, 2019).

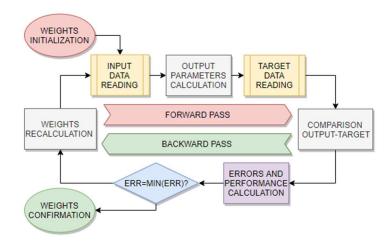


Figure 1: The iterative cycle performed by backpropagation algorithm.

The backpropagation algorithm used to train the network and improve its generalization (i.e. the ability to accurately predict the dependent variable for new datasets), is the Bayesian Regularization. This algorithm, which updates the weight values according to the Levenberg-Marquardt optimization, is particularly recommended when modeling a limited and over-dispersed dataset, as generally happens when the road accidental phenomenon is studied. In fact, the regularization process involves modifying the performance function, through a parameter related to the variance of the distribution of network weights, in order to make the network response more regular and less subject to overfitting problems. For a detailed description of the Bayesian Regularization algorithm, see Foresee and Hagan (1997).

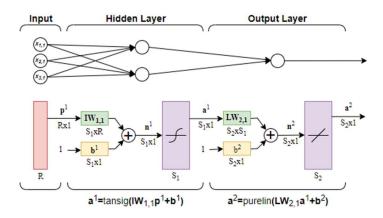


Figure 2: Representation of a two-layer feedforward neural network.

The performance function used to optimize network training is the mean square error (mse), i.e. the average squared difference between the experimental target (t) and the network output (a). It is defined as follows:

$$mse = \frac{1}{N} \sum_{i=1}^{N} (e_i)^2 = \frac{1}{N} \sum_{i=1}^{N} (t_i - a_i)^2$$
(3)

where N is the number of samples in the dataset. However, to validate the performance it is necessary to calculate the correlation coefficient (R) of training and testing datasets. This coefficient measures the linear relationship between two variables and is expressed by the ratio between the covariance of the two considered variables (t and a) and the product of the respective mean square deviations:

$$R = \frac{Cov(t, a)}{\sqrt{Var(t)Var(a)}} = \frac{\sum_{i}(t_{i} - \bar{t})(a_{i} - \bar{a})}{\sqrt{\sum_{j}(t_{j} - \bar{t})^{2}\sum_{k}(a_{k} - \bar{a})^{2}}} \quad i = 1 \dots N$$
(4)

where \bar{t} and \bar{a} are the mean value of targets and of network outputs respectively. In general, the closer the value of R is to the unity, the stronger results the linear relation between t and a, so confirming that the training has been completed successfully (if $R \cong 1$ for the training dataset) and that the degree of generalization achieved can be considered optimal (if $R \cong 1$ for the testing dataset). The creation, training and testing of the artificial neural network for studying the road accidental phenomenon was performed using MATLAB, and in particular the Neural Network Fitting Tool.

4. Data collection

The Regional Road Safety Monitoring Center (CRMSS FVG), established in 2004, gathers all information related to road accidents detected by the police forces operating in the Friuli-Venezia Giulia Region. The tool chosen for the management of the CRMSS FVG is the MITRIS system, a software composed of a centralized database, a WebGIS server application and computational statistics modules. The MITRIS Client allows the computerization of road accident data to be performed, including location referencing on regional cartography and production of analysis reports. All users of the service can directly access the WebGIS information system, through a browser and an Internet connection, for viewing, querying and managing road accident data. The main accident information provided by the MITRIS database are: date, time, first road and possible second road (if the crash occurred at a road intersection), number of vehicles involved, number of injured, number of deaths, type of accident, road surface conditions. In addition to accident data, an accurate analysis of the road accident phenomenon requires data on the degree of user exposure, i.e. parameters that define the actual use of a road infrastructure. The average annual daily traffic volume (AADT), is an international benchmark for assessing the risk exposure. Traffic volumes on the FVG regional road network are processed using the VISUM software. These elaborations are made on traffic surveys, carried out in 2005 on the entire regional area, and on traffic measurements automatically retrieved by twelve permanent detection stations, installed in 2011 on the network of competence of FVG Strade S.p.A., that acquire data h24. The VISUM results enrich the MITRIS database and can be accessed by querying the main road from the WebGIS server application.

The present study analyses road accidents that occurred between 2006 and 2016 in 28 intersections (signalized or not), that the company FVG Strade S.p.A. turned into roundabouts (starting from 2008) with the aim of improving road safety. Obviously post operam accidents are not considered for the purpose of this study. Having identified a

study site within the MITRIS Client, it is necessary to select road accidents that have occurred near the site itself. The dimensions of an intersection must extend beyond its physical boundaries, including an area of influence that may contain some accidents related to the presence of the road junction. In this study, all road accidents within a radius of 150 meters from the center of the road intersection are associated with the site concerned. This radius is considered appropriate, according to the types of roads treated and the visibility distance possessed by users on these roads. Traffic volumes on legs of each intersection are obtained from the VISUM elaborations and assumed constant over the years.

5. Network results and models comparison

The two-layer feedforward network used in the study is characterized by 2 neurons with a tan-sigmoid transfer function ("tansig") in the hidden layer and one neuron with a linear transfer function ("purelin") in the output layer. The dependent variable is the crash frequency N (expressed in crash per year), while independent variables are the intersection type $x_{1,1}$ (this is a categorical variable with value 1 for 3ST intersections, 2 for 3SG, 3 for 4ST and 4 for 4SG), the volume of annual average daily traffic on the main leg $x_{2,1}$ (expressed in vehicle per day) and the volume of annual average daily traffic on the secondary leg $x_{3,1}$ (expressed in vehicle per day). The network was trained using 75% of the dataset and tested on the remaining 25%. This configuration of the network has allowed the difference between target and output in training (R = 0.851 and mse = 0.146) to be achieved. Referring to the computational process performed by the network presented in Figure 2, the model equation is as follows:

$$y = purelin\{LW2_1 \cdot [tansig(IW1_1 \cdot x + b1)] + b2\}$$
(5)

$$y = \left\{ \sum_{i=1}^{2} w_{1,i}^{0} \left[\frac{2}{1 + e^{-2\left(\sum_{j=1}^{3} w_{i,j} x_{j}\right) - 2b\mathbf{1}_{i,1}}} - 1 \right] \right\} + b2$$
(6)

where $IW1_1$ is the weight matrix connected to the normalized input vector **x**, $LW2_1$ the weight matrix of the output layer connected to the hidden layer, **b1** the bias vector of the hidden layer, *b2* the bias of the output layer and *y* the normalized network output. The estimated value of this parameters is shown in Table 2.

Table 2: Parameters of the SPF for vehicle collisions at FVG's intersections.

IW1_1	IW1_1	IW1_1	LW2_1	LW2_1	b1	b2
(1° column)	(2° column)	(3° column)	(1° column)	(2° column)	(1° column)	(scalar)
$w_{1,1} = 0.3014$	$w_{1,2} = 1.0959$	$w_{1,3} = 0.3916$	$w_{1,1}^0 = 1.2180$	$w_{1,2}^0 = -0.2357$	$b1_{1,1} = -0.7831$	0.1640
$w_{2,1} = 0.1033$	$w_{2,2}$ = -0.2152	$w_{2,3} = 0.1130$			$b1_{2,1} = -0.1390$	

Normalization allows the network to be trained more efficiently and consists in mapping the row maximum and minimum values of the input and target vectors to [-1 1]. Equations (7) and (8) respectively show the pre-processing of the input vector components and the post-processing of the target, while Table 3 reports the values of parameters therein.

$$x_{j} = x1_step1. gain_{j,1}(x_{j,1} - x1_step1. xoffset_{j,1}) + x1_step1. ymin_{j,1}$$
(7)

$$N = \frac{y - y1_step1.ymin}{v1 step1.gain} + y1_step1.xoffset$$
(8)

Input parameters	x _{1,1} (Intersection type)	$\begin{array}{c} x_{2,1} \\ (AADT_{maj}) \end{array}$	$x_{3,1}$ (AADT _{min})	Target Parameters	N (Crash frequency)
x1_step1.xoffset	1	0	1,091	y1_step1.xoffset	0
x1_step1.gain	0.6667	0.0001	0.0002	yl_step1.gain	0.6842
x1_step1.ymin	-1	-1	-1	yl_step1.ymin	-1

Table 3: Normalization parameters for inputs and target.

Figures 3 to 6 show the comparison between the crash frequency model for FVG's intersections, estimated using the artificial neural network approach, and the SPFs of the US context (Harwood et al., 2007), explained in Equation (1); results have been computed for $AADT_{min}$ values within the range 2,000 – 10,000 vehicle per day.

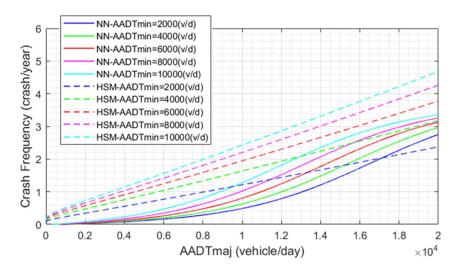


Figure 3: Graphical comparison of SPFs for vehicle collisions on three-leg unsignalized intersections (3ST).

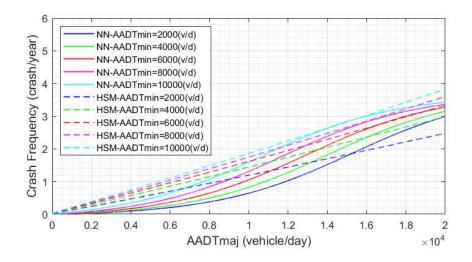


Figure 4: Graphical comparison of SPFs for vehicle collisions on three-leg signalized intersections (3SG).

Overall, the crash frequency estimated with ANN is lower than the predicted one by HSM models, regardless of the intersection type. Furthermore, the crash rate is affected by the traffic volume and the intersection capacity: the developed functions are characterized by an increasing trend with traffic flow, but the slope of the tangent line progressively decreases as the volume of annual average daily traffic on the main leg is increased. It is assumed that this result may depend on the achievement of the intersection saturation level, associated with a volume-to-capacity ratio greater than or equal to 1. As a result, the intersection capacity is not able to meet the increase in the volume of traffic and this leads to a condition of congestion characterized by a stationary crash rate (Zafar and Wilson, 2011, Lord et al., 2005). This phenomenon is not considered by the HSM models which predict an increasing crash frequency as the traffic flow on the main leg is increased.

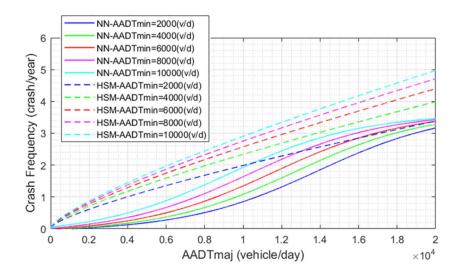


Figure 5: Graphical comparison of SPFs for vehicle collisions on four-leg unsignalized intersections (4ST).

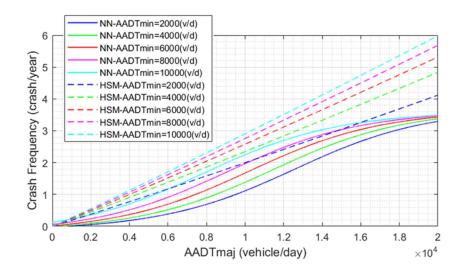


Figure 6: Graphical comparison of SPFs for vehicle collisions on four-leg signalized intersections (4SG).

Considering the comparison between Figure 3 and 4, it can be observed that ANN curves are characterized by similar trends, as it was expected given the same 3ST geometric configuration. However, for an AADT value equal to 10,000 vehicle/day, unsignalized intersections has shown a crash frequency rate lower then signalized ones. Similar considerations can be stated comparing 4ST and 4SG, as can be observed in Figure 5 and 6. The HSM models outline the same result for the 4ST-4SG comparison, for an AADT value greater than 10,000 vehicle/day. This result probably depends on the level of attention payed by the driver to the traffic environment, which may vary depending on the complexity and traffic regulation system of the intersection (Mussone et al., 1999).

Intersection type	N – HSM (crash/year)	N – ANN (crash/year)	Variation %
3ST	1.4 - 2.9	0.8 - 1.8	45% - 37%
3SG	1.4 - 2.2	1.0 - 2.1	29% - 4%
4ST	2.3 - 3.3	1.3 - 2.4	43% - 27%
4SG	2.4 - 3.5	1.6 - 2.7	32% - 23%

Table 4: Expected average crash frequency for $AADT_{maj}$ equal to 12,000 vehicles/day.

Table 4 shows the minimum and maximum variations of the expected average crash frequency, predicted by HSM and ANN models for the four types of at-grade intersection, assuming an $AADT_{mai}$ equal to 12,000 vehicles per day.

6. Conclusion

The aim of this research was developing road accident predictive models of four different types of intersections: signalized and unsignalized, with 3 or 4 legs. Road accidental and traffic data of 28 intersections being run by FVG Strade S.p.A, were collected. The artificial neural network approach has allowed the analytical expression of the models to be identified, which in the international literature are called SPFs. The implemented feedforward neural network is composed of 2 layers and receives 3 inputs:

the intersection type and the traffic volumes on the main and secondary legs. The hidden layer is composed of 2 neurons with tan-sigmoid transfer function and the output layer has 1 neuron with linear transfer function. The training was performed with the Bayesian Regularization algorithm. A comparison between the predictive models of the FVG regional context and the US one (HSM) was performed. Results show a significant difference both quantitatively and qualitatively. Because of this, the development of road accident models for the regional context, which consider the local base conditions, is justified. These models will allow the road designer to identify and analyse the critical issues of the regional road network in term of accident rates. However, accident modification factors (AMFs) must be estimated for the regional context in order to quantify the effectiveness of mitigation measures and to correct the prediction of road accidental models. Furthermore, in order to improve ANN models prediction reliability, in a future development of the research, it should be taken into account the approaching speed to the intersection, especially on rural environment. In particular, the quantitative assessment of effects on accident rates produced by the transformation of an existing intersection into a roundabout will help the road designer to identify priorities for action.

References

- National Institute of Statistic (Istat) and Automobile Club of Italy (2017) "Road Accident, Year 2017: Annual Report for 2017", *Annual Report 2017*, Roma, IT.
- World Health Organization (2018) "Global Health Estimates 2016: Deaths by Cause, Age, Sex, by Country and by Region, 2000-2016", *Working Papers Series*, Genève, CH.
- Chang, L.Y. (2005) "Analysis of freeway accident frequencies: Negative binomial regression versus artificial neural network", *Safety Science* 43 (8), pp. 541-557.
- De Luca, M. (2015) "A comparison between prediction power of artificial neural networks and multivariate analysis in road safety management", *Transport* 32 (4), pp. 379-385.
- Jadaan, K.S., Al-Fayyad, M., Gammoh, H.F. (2014) "Prediction of Road Traffic Accidents in Jordan using Artificial Neural Network (ANN)", *Journal of Traffic and Logistics Engineering* 2 (2), pp. 92-94.
- Chiou, Y.C. (2006) "An artificial neural network-based expert system for the appraisal of two-car crash accidents", *Accident Analysis and Prevention* 38 (4), pp. 777-785.
- Mussone, L., Ferrari, A., Oneta, M. (1999) "An analysis of urban collisions using an artificial intelligence model", *Accident Analysis and Prevention* 31 (6), pp. 705-718.
- Abdelwahab, H.T., Abdel-Aty, M.A. (2001) "Development of Artificial Neural Network Models to Predict Driver Injury Severity in Traffic Accidents at Signalized Intersection", *Transportation Research Record Journal of the Transportation Research Board* 1746 (1), pp. 6-13.
- Harwood, D.W., Bauer, K.M., Richard, K.R., Gilmore, D.K., Graham, J.L., Potts, I.B., Torbic, D.J., Hauer, E. (2007) "Methodology to Predict the Safety Performance of Urban and Suburban Arterials", *Final Report for NCHRP Project 17-26*, National Cooperative Highway Research Program Web-Only Document 129: Phases I and II, NCHRP, TRB, Washington, DC.
- Baldo, N., Manthos, E., Pasetto, M. (2018) "Analysis of the mechanical behaviour of asphalt concretes using artificial neural networks", *Advances in Civil Engineering* 2018, Article number 1650945.

- Baldo, N., Manthos, E., Miani, M. (2019) "Stiffness modulus and marshall parameters of hot mix asphalts: Laboratory data modeling by artificial neural networks characterized by cross-validation", *Applied Sciences (Switzerland)* 9 (17), Article number 3502.
- Foresee, F.D., Hagan, M.T. (1997) "Gauss-Newton approximation to Bayesian learning", Proceedings from the *1997 IEEE International Conference on Neural Networks*, Houston, TX, 3, pp. 1930–1935, Article number 614194.
- Zafar, R., Wilson, D.J. (2011) "Traffic Crashes and Traffic flow Conditions on Auckland's SH1 Southern Motorway", Proceedings from the *IPENZ Transportation Group Conference*, Auckland, New Zealand.
- Lord, D., Manar, A., Vizioli, A. (2005) "Modeling crash-flow-density and crash-flow-V/C ratio relationships for rural and urban freeway segments", *Accident Analysis and Prevention* 37, pp.185-199.