NOTES ON MECHANICS AND MATHEMATICS IN TORRICELLI AS PHYSICS MATHEMATICS RELATIONSHIPS IN THE HISTORY OF SCIENCE

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Abstract

In ancient Greece, the term "mechanics" was used when referring to machines and devices in general and intended to mean the study of simple machines (winch, lever, pulley, wedge, screw and inclined plane) with reference to motive powers and displacements of bodies. Historically, works considering these arguments were referred to as Mechanics (from Aristotle, Heron, Pappus to Galileo). None of the treatises entitled Mechanics avoided theoretical considerations on its object, particularly on the lever law. Moreover, there were treatises which exhausted their role in proving this law; important among them are the book on the balance by Euclid and On the Equilibrium of Planes by Archimedes. The Greek conception of mechanics is revived in the Renaissance, with a synthesis of Archimedean and Aristotelian routes. This is best represented by Mechanicorum liber by Guidobaldo dal Monte who reconsiders Mechanics by Pappus Alexandrinus, maintaining that the original purpose was to reduce simple machines to the lever. During the Renaissance, mechanics was a theoretical science and it was mathematical, although its object had a physical nature and had social utility. Texts in the Latin and Arabic Middle Ages diverted from the Greek and Renaissance texts mainly because they divide mechanics into two parts. In particular, al-Farabi (ca. 870-950) differentiates between mechanics in the science of weights and that in the science of devices. The science of weights refers to the movement and equilibrium of weights suspended from a balance and aims to formulate principles. The science of devices refers to applications of mathematics to practical use and to machine construction. In the Latin world, a process similar to that registered in the Arabic world occurred. Even here a science of movement of weights was constituted, namely Scientia de ponderibus. Besides this there was a branch of learning called mechanics, sometimes considered an activity of craftsmen, other times of engineers (Scientia de ingeniis). In the Latin Middle Ages various treatises on the Scientia de ponderibus circulated. Some were Latin translations from Greek or Arabic, a few were written directly in Latin. Among them, the most important are the treatises attributed to Jordanus De Nemore, Elementa Jordani super demonstratione ponderum (version E), Liber Jordani de ponderibus (cum commento) (version P), Liber Jordani de Nemore de ratione ponderis (version R). They were the object of comments up to the 16th century. The distribution of the original manuscript is not well known; what is certain is that Liber Jordani de Nemore de ratione ponderis (version R), finished in Tartaglia's (1499-1557) hands, was published posthumously in 1565 by Curtio Troiano as Iordani Opvsculum de Ponderositate. In order to show a mechanical tradition dating back to Archimedes' science, at least till

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the 40s of the 17th century, we present Archimede's influence on Torricelli's mechanics upon the centre of gravity (Opera geometrica).

Key words: *Mechanics, Scientia de Ponderibus, Archimedes, Torricelli, Relationship physics and mathematics in the history of science.*

Introduction

In our historical perspective, the influence of *On the Equilibrium of Planes* (Heath, 2002) by Archimedes (fl. 287 BC-212 BC) for Renaissance scholars is important; there are two principal traditions. The main humanistic tradition, very careful to philological aspects, followed by Moerbeke, (fl. 1215-1286), Regiomontanus (1436-1476) and Commandinus (1509-1575) and the pure-main mathematical tradition followed by Maurolico (1494-1575), Luca Valerio (1553-1618), Galileo and Torricelli (1608-1647).

Archimedes' approach to geometry (*On the Equilibrium of Planes*) is different from the Euclidean one (Capecchi and Pisano 2007a). The object is different, because he mainly deals with metric[1] a quite new matter; the aim is different, more oriented to solve practical problems, and mainly the theory organization is different, because Archimedes does not develop axiomatically the whole theory, but sometimes he uses an approach for problems, characterized by *reductio ad absurdum* (like, Galileo and Torricelli will do afterwards). Furthermore, the epistemological status of the principles is different because the Archimedean principles are not always self-evident as those of the Euclidean tradition and may have empirical nature. Some of an Archimedean supposition / proposition has a clear methodological aim, and though they may express the daily feeling of the common man, they have a less cogent evidence-character in comparison to those of the Euclidean geometry. Anyway, whatever is the origin of Archimede's axioms, they were used by him to set *rational criteria* for determining centres of gravity. As a matter of fact, Archimedes' work contains physical concepts formalised on a mathematical basis. As far as we know, for the first time in history, mathematics was successfully applied to physics, in particular to statics. For, Archimedes studied the rule governing the law of the lever and also found the centre of gravity of various geometrical plane figures (*Book* I, *On the Equilibrium of Planes*). By his *Suppositio* (principles), Archimedes is able to prove *Propositio* (theorems) (Heath, 2002, pp. 189-202) useful to *calculate* the centre of gravity of composed bodies. In particular, the sum of all the components may require the adoption of the method of exhaustion. The ingenuity of Archimedes is not limited to this: not only he applied mathematics to physics, but also physics to mathematics. For, the so called *method* is exactly based on the use of the physical concept of barycentre and on the principle of the lever to solve problems belonging to pure mathematics.

It is known that Archimedes was one of the most influential mathematicians at the beginning of the so called scientific revolution because the algebraic technique which was developing in Europe from the 16th century was neither wide nor perspicuous enough to offer a solid base for science. Therefore the scientists carried out two operations: 1) developed some techniques - as that of the logarithms - and improved trigonometry; 2) resorted to the only mathematical discipline which was believed rigorous and whose technique was well consolidated: geometry. This means Euclid, if the problems needed the construction of relatively easy figures, but Archimedes and Apollonius if the problems were more complicated. Furthermore, Archimedes

1 Alongside, but quite different from Euclidean *Stoicheiosis* tradition of mathematics (rigour and logical structure of mathematical) *Elements,* a metric approach by Greek mathematicians (mainly Democritus, and then Eudoxus) was provided more and less in the same period. The latter stressed the relationships between geometrical measurements (solids, shapes, areas) and theorems-formulas. On our side, Archimedes was one of the mathematicians who adopetded the metric approach in *Measurement of a Circle*, *Quadrature of the Parabola*, *On the Sphere and Cylinder*, *On the Conoids and Spheroids*, *On Spiral*, *On the Method*, and relevant is *On the Equilibrium of Planes* (Pisano, 2009a, 2009b, 2009c, 2011; Pisano and Cepecchi, 2014).

had also developed statics as a physical-mathematical discipline and, through the method of exhaustion, had provided an approach which, in the beginning phases of the scientific revolution, could offer something similar to the infinitesimal methods. This approach converged or was overcome (this depends on the interpretations to which a scholar adheres) by Cavalieri's *indivisibili* and, in the second half of the 17th century, by calculus. It is hence evident why scientists as Kepler and Galilei, who were not particularly confident in algebra, resorted to geometry and why Archimedes was one of their main reference points.

The case of another scientist is particularly interesting: that of Torricelli (1608-1647). He lived and worked not many years after Kepler and Galilei, but the scientific atmosphere was different: in the same years in which Torricelli was active, scholars as Cavalieri, Descartes and Fermat (whose contributions were well known by Torricelli) - only to mention the most famous - worked. Thanks to the contributions of these scientists and mathematicians, in the second half of the 30s and in the 40s of the 17th century, the mathematical technique was substantially improved. However, Archimedes remained a reference point for Torricelli and this is interesting because Torricelli was not an old-fashion scientist; he was a modern and open-minded one. Therefore the valuation of Archimedes' influence on Torricelli, which is the subject of our paper, is particularly significant to understand what the influence of the great Syracusan and, more in general, of Greek geometry and science on modern science was and how this influence changed in the course of the 17th century. We are not going to develop here a complete historiographical thesis on such a complex subject, but to supply some historical-conceptual elements useful for a wider research, concentrating on Torricelli.

On Torricelli and Archimedes

The investigation into Archimedes' influence on Torricelli has a particular relevance because of its depth. It also allows us to understand in which sense Archimedes' influence was still relevant for most scholars of the seventeenth century. Besides there being a general influence on the geometrization of physics, Torricelli was particularly influenced by Archimedes with regard to mathematics of indivisibles. Indeed, it is Torricelli's attitude to confront geometric matter both with the methods of the ancients, in particular the exhaustion method, and with the indivisibles, so attempting to compare the two, as is clearly seen in his letters with Cavalieri (Torricelli, 1919-1944; see mainly vol. 3). Torricelli, in particular, solved twenty one different ways the squaring a parabola (Heath, 2002; *Quadrature of the parabola*, Propositio 17 and 24, p. 246; p. 251), eleven times with exhaustion, ten with indivisibles. The *reductio ad absurdum* proof is always present. It is known that Archimedes solved this problem in two different manners: 1) by means of his *mechanical method*; 2) by the use of exhaustion.

The influence of Archimedes on Torricelli is detectable also considering the two general approaches of both scientists because Torricelli considered the axioms in a way similar to Archimedes' and different from Euclid's. That is the axioms could have their origin not only in evidence, but also in experience. Additionally, they could be *local axioms*, namely unproved propositions valid in a given theory, whose validity could be non-universal, differently from Euclid's axioms which were, at that time, believed universally valid. Furthermore, Torricelli exploited all Archimedes' techniques in an original manner. Thence, the knowledge of Archimedes' contribution is also fundamental to an historical study of Torricelli's mechanics. Archimedes was the first scientist to set *rational criteria* for determining centres of gravity of bodies and his work contains physical concepts formalised on a mathematical basis. In *Book I* of the *On the Equilibrium of Planes* (Heath 2002) Archimedes, besides studying the rule governing the law of the lever, also finds the centres of gravity of various geometrical plane figures (Heath, 2002, Clagett 1964-1984; Heiberg, 1881). Archimedes' typical method of arguing in mechanics was by the use of the reduction *ad absurdum*. Torricelli in his study on the centres of gravity resumes the same approach.

With regard to Torricelli's works, we studied mainly his mechanical theory (Capecchi and Pisano, 2007a, 2007b; Pisano, 2009) in the *Opera geometrica*[2] (Torricelli 1644). We focused in detail on his discourses upon centres of gravity (Pisano 2007) where he enunciated his famous principle: *It is impossible for the centre of gravity of two joined bodies in a state of equilibrium to sink due to any possible movement of the bodies*. Torricelli in his theory on the centre of gravity, following Archimedes' approach, uses

a) *Reductio ad absurdum* as a particular instrument for mathematical proof.
b) Geometrical representation of physical bodies: weightless beams and reference in geometrical form to the law of the lever.
c) Empirical evidence to establish principles.

We focused mostly upon the exposition of studies contained in *Liber primis. De motu gravium naturaliter descendentium,* where Torricelli's principle is exposed. In Galileo's theory on dynamics, Torricelli presents problems, which, according to him, remain unsolved. His main concern is to prove a supposition by Galileo, which states: velocity degrees for a body are directly proportional to the inclination of the plane over which it moves (Galilei, 1890-1909, VIII, p. 205).

Torricelli seems to suggest that this supposition may be *proved* beginning with a "theorem" according to which "the momentum of equal bodies on planes unequally inclined are to each other as the perpendicular lines of equal parts of the same planes" (Torricelli 1644, *De motu gravium naturaliter descendentium et proiectorum*, p. 99). Moreover, Torricelli also assumes that this theorem has not yet been demonstrated. For, in the first edition of Galileo's *Discorsi* in 1638, there is no proof of the "theorem". It was added only in 1656 to the *Opere di Galileo Galilei linceo*, (Galilei, 1656). However Torricelli knew it, as is clear in some letters from Torricelli to Galileo regarding the "theorem"; Torricelli, 1919-1944, III, p. 48, p. 51, p. 55, p. 58, p. 61. Torricelli frequently declares and explains his Archimedean background.

> Inter omnia opera Mathematicas disciplinas pertinentia, iure optimo Principem sibi locum vindicare videntur Archimedis; quae quidem ipso subtilitatis miraculo terrent animos (Torricelli, 1644, *Proemium*, p. 7).

As we remarked above, Archimedes, in the *Quadratura parabolae*, first obtains results using the mechanical approach and then reconsiders the discourse with the classical methods of geometry to confirm the correctness of his results in a rigorous way (Heath, 2002). Similarly, Torricelli, with the compelling idea of duplicating the procedure, devotes many pages to prove certain theorems on the "parabolic segment", by following, the geometry used in antiquity (Torricelli, 1644), *Quadratura parabolae pluris modis per duplicem positionem more antiquorum absoluta*, p. 17-54) and then proving the validity of the thesis also with the "indivisibilium" (Heath 2002, *Quadratura parabolae,* p. 253-252; p. 55-84; Torricelli 1644, *De solido acuto hyperbolico problema alterum*, p. 93-135). In this respect, it is interesting to note that he underlines the "concordantia" (Torricelli 1644, *De solido acuto hyperbolico problema alterum*, p. 103) of methods and reasonings (Torricelli 1644, *Quadratura parabolae per novam indivisibilium Geometriam pluribus modis absoluta*, p. 55).

The main idea was not only the desire to give the reader results and methods, but also to say that the indivisibles technique was not completely unknown to the ancient Greek scholars. Besides, Torricelli seems to hold onto the idea that the method of demonstration of the ancients,

2 Recently oen of us (RP) organized an international symposium (6th-ESHS Congress at Barcelona, 4–6 Sept. 2014) and an edited book for *the 370th Anniversary of Torricelli's Opera Geometrica (1644): Statics, Mathematical and Geometrical Conceptual Streams*, with Jean Dhombres and Patricia Radelet de Grave P (Springer, pre-print).

such as the Archimedes' method, was intentionally kept secret. He states that the ancient geometers worked according to a method "in invenzione" suitable "ad occultandum artis arcanum" (Torricelli, 1644, *Quadratura parabolae per novam indivisibilium Geometriam pluribus modis absoluta*, p. 55). However, the Archimedean influence in Torricelli goes further. The well known books *De sphaera et solidis sphaeralibus* (Torricelli, 1644, *Liber primus*, p. 3-46) present an enlargement of the Archimedean proofs of books I-II of *On the sphere and cylinder* (Heath, 2002, p. 1-90). For, we read:

> […] In quibus Archimedis Doctrina de sphaera & cylindro denuo componitur, latius promovetur, et omni specie Solidorum, quae vel circa, vel intra, Sphaeram, ex conversione polygonorum regularium gigni possint, universalius Propagatur (Torricelli, 1644, *De sphaera et solidis sphaeralibus*, p. 2).

In other parts, Torricelli has faced problems not yet solved by Archimedes, or by the other mathematicians of antiquity. With the same style as Archimedes, he does not try to arrive at the first principles of the theory and does not limit himself to a single way of demonstrating a theory. (Torricelli, 1644, *De solido hyperbolico acuto problema secundum,* p. 116).

On Torricelli's Archimedean Proofs

We note that the exposition of the mechanical argumentation present in Archimedes' *Method* was not known at Torricelli's time because Johan Heiberg only discovered it in 1906 (Heath, 1912). Therefore, in Archimedes' writing, there were lines of reasoning which, because a lack of justification, were labelled as mysterious by most scholars. Thus, in such instances, it was necessary to assure the reader of the validity of the thesis and also to convince him about the strictness of Archimedes' approaches, particularly exhaustion reasoning and *reductio ad absurdum*, by proving his results with some other technique. It is well known from the *Method* (Heiberg 1912) that Archimedes studied a given problem whose solution he anticipated by means of crucial propositions, based on the concept of gravity centre and on the equilibrium of the lever, which were then proved by the *reductio ad absurdum*, using, in many cases the exhaustion method. Indeed Archimedes' himself did not attribute the same amount of certainty to his *Method* of proof, as he attributes to classical mathematical proofs. His reasoning on *Quadratura parabolae* (Heath 2002, Proposition 24, p. 251) is exemplary. Addressing Eratosthenes (276-196 B.C.), Archimedes wrote at the beginning of his *Method*

> Seeing moreover in you, as I say, an earnest student, a man of considerable eminence in philosophy, and an admirer [of mathematical inquiry], I thought fit to write out for you and explain in detail in the same book the peculiarity of a certain method, by which it will be possible for you to get a start to enable you to investigate some of the problems in mathematics by means of mechanics. This procedure is, I am persuaded, no less useful even for the proof of the theorems themselves; for certain things first became clear to me by a mechanical method, although they had to be demonstrated by geometry afterwards because their investigation by the said method did not furnish an actual demonstration. (Heath, 1912, p. 13):

One of the characteristics of Torricelli's proofs was the syntactic return to the demonstration approach followed by the ancient Greeks, with the explicit description of the technique of reasoning actually used. Besides the well-known *ad absurdum* there was also a strong use of the proportions with the classical techniques of the *permutando* and the *ex aequo*. In *De proportionibus liber* he defined them explicitly (Torricelli 1919-1944, *De Proportionibus liber*, p 314).

Torricelli - as well as Galilei and Kepler - seems to neglect algebra of his time and adheres to the language of proportions. He dedicated the mentioned *De Proportionibus liber* (Torricelli 1919-1944, pp. 295-327) book to this language, where he only deals with the theory of proportions to be used in geometry. In such a way he avoids the use of the *plus* or *minus*, whose logical status was still uncertain. He replaced the *plus* and *minus* by the *composing* (Torricelli 1919-1944, p. 316) and *dividing* (Idem, p. 313). Such an approach allows him to work always with the ratio of segments. By following the ancients to sum up segments, he imagines them as aligned and then translated and connected, making use of terms like “simul”, “et” or “cum” (Torricelli 1919-1944, Prop. XV, p. 318).

We notice that proofs by means of indivisibles are not *reductio ad absurdum*. Instead, in nearly all other proofs Torricelli uses the technique typical of proportions, *dividendo, permutando* and *ex aequo*. The correct use of proportions was an important step in the construction of modern science and in particular in its initial phases. The complex historical process called *rediscovery of the ancients* concerns many aspects; one of them is the rediscovery of the classical Greek geometry with his heritage of results and methods. In this process the use of proportions and the full requisition of a concept, which is only apparently simple - the one of similarity -, played a fundamental role. Galileo himself, in *Discorsi e dimostrazioni intorno a due nuove scienze* (1638), used a series of reasoning whose mathematical basis-technique is represented by the proportions associated with the concept of similarity. Actually, the development of the methods and techniques of the ancient geometers were so important in the 17th century that they remained a conspicuous part of the mathematical apparatus used by the physicists until Newton. For, let us think that in Newton’s *Principia* the new infinitesimal notions are treated in a geometrical manner. Only after Newton, the mathematical technique used in physics became almost purely analytical. Thus, the case-study we have presented, concerning Torricelli, is a significant example of the use the *moderns* made of *ancients’* mathematical methods, drawning their technique, but adapting it to the needs of the new science. In particular, we focused on the conceptual aspects of Archimedes’ and Torricelli’s studies of the centre of gravity theory based on previous investigations on Archimedes’ *On the Equilibrium of Planes* and Torricelli’s *Opera geometrica* (Capecchi and Pisano 2007a, 2007b). In the present work we have outlined some of the fundamental concepts common to the two scholars: the logical organization and the paradigmatic discontinuity with respect to the Euclidean technique. Indeed the Archimedes’ theory (mechanical and geometrical) does not appear to follow a unique pattern. It maintains two kinds of organization, one problematic, the other axiomatic deductive. If one would use Kuhnian concepts and language, one could claim that the breaking of the Euclidean paradigm by Archimedes offers, with the limitation implicit in the concept of paradigm, a typical example in this sense: we pass from a normal science composed of axioms and self-evidence to a new science where to prove also means to find a field of applicability of a new theory, the *centrobarica*, in our specific case (Pisano and Capecchi, 2014).

Conclusion

We have provided some ideas about the influence Archimedes exerted on Torricelli for the following reasons:

1) From the point of view of the history of science and mathematics, the late 30s and the 40s of the 17th century represent a particular period because the mathematical technique was improving in comparison to that available in the initial period of the century, but the infinitesimal techniques of calculus, which will allow the scientists to solve many problems connected to the calculations of areas, volumes, barycentre, in a methodical manner and to deal with the instantaneous quantities, were not yet available. In this phase the paradigm of calculus was not

yet born. The scholars had understood that *finitaries techniques* were not enough to address the problems posed by the new science and the new mathematics. By *finitary technique*, we indicate a technique by means of which it is possible to prove the equivalence of two planes of solid figures through the equidecomposition in a finite number of parts. Thence, they referred to the only available model: Archimedes. In this context, the exhaustion method is particularly significant because of two reasons: a) many problems were solved by this method, as we have seen with regard to Torricelli; b) the exhaustion method and its potentially infinite procedures were a source of inspiration for techniques which were slightly different from the exhaustion method itself. For, Cavalieri, the inventor of the indivisibles methods wrote to Galilei in 1621, that is at the beginning of his mathematical researches:

> […] vado dimostrando alcune proposizioni d'Archimede diversamente da lui, et in particolare la quadratura della parabola, divers'ancora da quello di V.S; […]. (Letter from Cavalieri to Galilei, 1621, 15 December. Galilei 1890-1909, XIII, p 81).

On the other hand, Archimedes had been the inspirer of Kepler and Galilei themselves. For example the infinitaries techniques used by Kepler, expecially in the *Nova Stereometria doliorum*, has the Syracusan scientists as a reference point. When calculus became the standard paradigm of the infinitaries procedures (and this happened about in the 80s of the 17th century), the direct references to Archimedes became rarer and more indirect. Torricelli's researches represent one of the ripest results achieved without resorting to calculus. Archimedes was fundamental for Torricelli, because of this his influence on the Italian scientist is so significant from a historical-scientific standpoint.

2) From an epistemological point of view: as we have underlined, Archimedes represents, for some aspects, a mathematical-scientific paradigm different from Euclid's. We do not enter here in the discussion if Kuhn's ideas on scientific paradigms can be accepted; we think, however, that they catch some interesting aspects of the development of science, even if we are convinced that a general theory of such a development cannot exist. So, if we interpret the *normal science à la* Kuhn as the Euclidean geometry, namely a discipline based upon a system of axioms which rely on self-evident, then Archimedes offers a different scientific paradigm. The new science, whose necessity was not only to develop pure mathematics, but also to apply mathematics to the external world and to technique, adopted a paradigm *à la Archimedes* rather than *à la Euclid*. This means - as already stressed - that the basic principles derived not only from self-evidence, but were also drawn from experience. This is the case with Torricelli, but it also happens within in Tartaglia's statics (*Libro VII* and *Libro VIII, Quesiti et Invenzione diverse*), and in Galilei's works. In this sense, Galilei's contributions to architecture and engineering are particularly significant (*Mechanics* and *fortifications*). Other aspects of the scientific revolution connected to the rediscovery of the ancients could be highlighted. For example:

a) the reacquisition of the concepts themselves of mathematical demonstration and of mathematical rigour were almost completely lost till the 16th century and they were reconquered, with difficulty, through a process which - as to mathematical disciplines different from calculus - reached the 17th century with the fundamental contributions of Fermat (see Bussotti 2006, in particular chapter 1 and chapter 2) and Descartes. In this phase the Greek scholars, and above all Archimedes, represented a model of rigour and creativity which was an important guide for the scientists living in the 16th and 17century. As to calculus and more modern branches of mathematics, the rigourization-process was completed only in the 19th century.
b) the role that Archimedean tradition had in the birth of new early mathematized sciences in the Renaissance; included machines/fortifications designs by architects and

engineers at that time. The following questions are particularly interesting with regard to practical science, technique, architecture and engineering: what is the cultural background of a common Renaissance and early modern age scholar? What were the beliefs and generalized pseudo-science? The doctrine of *imitatio naturae*. What mathematical theories were available in the 16th and 17 centuries? Is a machine-art crafts independent from science? Could the artisans and the machines-builders conceive and construct a running machine (including the calculation of mechanical advantage) without knowledge of science? When science plays with machines? What kind of modelling? What was the role of perpetual motion in the designs? The role of scale, friction, and velocities, profile of the machineries-machines?

We do not have room here to deal with these questions which are, however partially connected to the rediscovery of the ancients. Our aim has been to provide a contribution to this subject, focusing on such an important scientist and mathematician as Torricelli because the Archimedean inspiration and the inventiveness typical of many scientists lived in the 17th century allow us to compose an interesting historical-scientific and epistemological picture.

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