Advances in Historical Studies
2013. Vol.2, No.3, 106-125
Published Online September 2013 in SciRes (http://www.scirp.org/journal/ahs)
Scientific Research
http://dx.doi.org/10.4236/ahs.2013.23015

On the Conceptual and Civilization Frames in René Descartes' *Physical Works*

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Received April 15th, 2013; revised May 20th, 2013; accepted May 30th, 2013

The paper try to provide a contribution to the scientific—historiographic debate concerning the relations between experiments, metaphysics and mathematics in Descartes' physics. The three works on which the analysis is focused are the *Principia philosophiae* and the two physical essays: *La Dioptrique* and *Les Météores.* The authors will highlight the profound methodological and epistemological differences characterizing, from one side, the *Principia* and, from the other side, the physical essays. Three significant examples will be dealt with: 1) the collision rules in the *Principia philosophiae*; 2) the refraction law in *La Dioptrique*; 3) the rainbow in *Les Météores*. In the final remarks these differences will be interpreted as depending upon the different role Descartes ascribed to the three books inside his whole work. The concepts of *intensity* and *gradation* of the physical quantities used by Descartes will provide an important interpretative means. In this paper, we compare the aprioristic approach to physics typical for Descartes' *Principia* with the experimental and mathematical one characterizing Descartes' *Essays*.

Keywords: Descartes; Newton; Collision Rules; Refraction Law; Rainbow; Intensity and Gradation of the Physical Quantities; Science and Society in the XVII Century

An Outline

On Science & Society. The social and civilization environment in which a scientist lives has profound influences on the way how his scientific results and methods are framed (e.g. see Schuhl). This is specifically true for the 17th century, the epoch of the scientific revolution and a century of deep social and political transformations. Nevertheless, we think influence of the social-political situation on the work of a scientist has to be deduced directly from the analysis of his scientific works. In other terms: an analysis of the society in a certain period can be useful to understand the general direction taken by the science in that period, but, in itself, it is not enough to understand the specific work and results of a certain scientist. This kind of general analysis risks to become a sort of an *a priori passe-partout* through which the scientific work is analysed and risks[1] to induce serious misunderstandings on the way in which a certain scientist presented the results of his researches. It is always necessary to begin a historical research—also a research concerning the relations between science and society in a determined period—from the alive, both theoretical and technical work of the scientists. If, in the analysis of the whole work of a scientist, the historian of science reveals some unclearness or internal inconsistencies or a lack of coherence between the methods used by this scientist in different works of his and if all these questions cannot be explained either with technical problems (for example the lack or the misunderstanding of certain mathematical methods) or with the general methodological and epistemological convictions of the scientist himself, then it is necessary to think of the general structure of the society in that period. Therefore technical analysis of the results and methods used by the scientist is a priori considered and then evaluated within civilization.

On Science. The case of René Descartes (1596-1650) is emblematic in this sense: in his essays *La Dioptrique* and *Les Méthéores* Descartes proposes—among other results—his theory of refraction and of rainbow. Every passage of these two works can be explained taking into account: 1) the level of the science in the 17th century; 2) Descartes' experiments and methods; 3) Descartes' use of mathematics; 4) Descartes philosophical convictions (Hattab, 2009). These books could be understood without taking into account the social non-scientific context in which Descartes lived. The situation as to the *Principia philosophiae* is different at all: we will see in the final remarks of this paper that many results and argumentative structures exposed by Descartes in his *Principia* can be explained taking into account Descartes' epistemological and philosophical convictions, but other parts of the book and some reasoning that appear tormented and unclear can be clarified only considering the particular social situation in which Descartes lived and operated. The sociological analysis becomes hence interesting and can represent a great means to understand the evolution of the scientific ideas only if it is based on the examination of the theoretical-technical results obtained by the scientists and explained in their works. The case Newton and his *civilization science* (Buchwald & Feingold, 2011) is as interesting as the

[1]On that see a good essay by Buchwald and Feingold (Buchwald & Feingold, 2011).

one of Descartes: the different social context in which Newton lived allowed him a major freedom than Descartes' (see final remarks of our paper). But in this case, too, the examination becomes interesting basing on Newton's physical and mathematical works. Only in this manner the sociological analysis of science becomes perspicuous and useful for history of science and scientific concepts.

Isaac Newton (1642-1727) explicitly claimed that a model of the solar system had to show the positions held by a planet (Jupiter in the specific case mentioned in the forthcoming quotation) in the course of time, and that this condition cannot be fulfilled following Descartesian physics. Consequently the physical system described by René Descartes (1596-1650) in his *Principia Philosophiae* (1644; see **Figures 1** and **2**) is not a good model of the universe. In his unpublished work *De Gravitatione et aequipondio fluidorum* (Ruffner), Newton criticized the model of the solar system proposed by Descartes in his *Pincipia philosophiae* as follows:

> And hence, about the place of Jupiter, which it kept the year before, and with equal reason, about the prior place of a moving body anywhere, according to the doctrine of Descartes [illeg] it is manifest that not even God himself (standing newly established with things) could accurately and in a geometrical sense describe [it], especially when, on account of the changed positions of bodies, it would no longer exist in the nature of things[2].

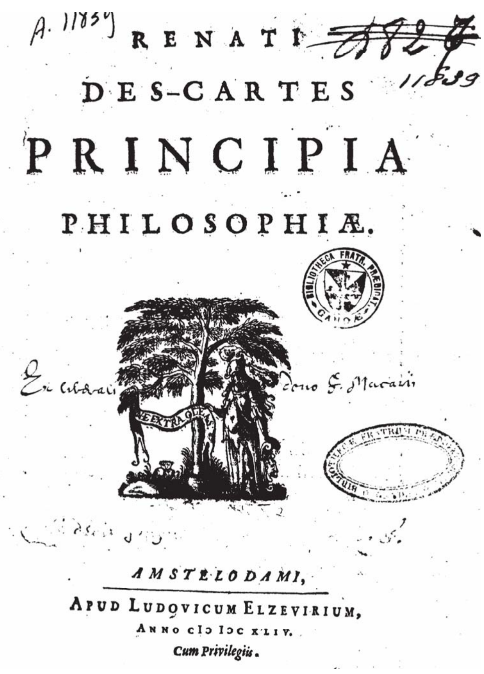
RENATI DES-CARTES PRINCIPIA PHILOSOPHIÆ.
AMSTELODAMI, APUD LUDOVICUM ELZEVIRIUM, ANNO CIϽ IϽC XLIV. Cum Privilegiis.

Figure 1.
The frontispiece of the first edition of Descartes' *Principia* (1644)[3].

[2]"Et proinde de loco Iovis quem ante annum habuit, parique ratione de præterito loco cujuslibet mobilis manifestum est juxta Cartesij [illeg] doctrinam, quòd ne quidem Deus ipse (stante rerum novato statu) possit accuratè et in sensu Geometrico describere, quippe cùm propter mutatas corporum positiones, non ampliùs in rerum naturâ existit" (Newton folios 9, Ms Add. 4003, Cambridge University Library, Cambridge, UK [retrieved via: http://www.newtonproject.sussex.ac.uk/view/texts/normalized/THEM00093]).

[3]Descartes 1897-1913, X-2.

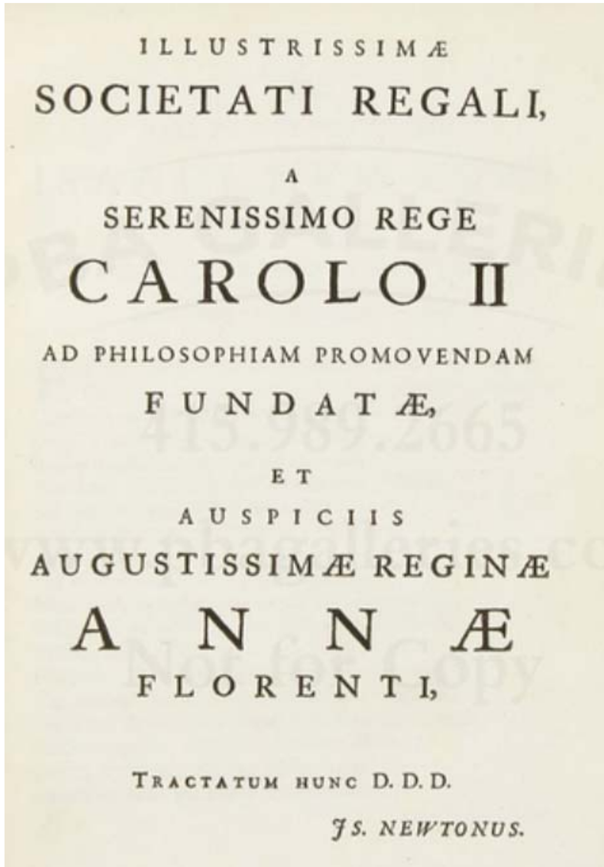
ILLUSTRISSIMÆ SOCIETATI REGALI, A SERENISSIMO REGE CAROLO II AD PHILOSOPHIAM PROMOVENDAM FUNDATÆ, ET AUSPICIIS AUGUSTISSIMÆ REGINÆ ANNÆ FLORENTI, TRACTATUM HUNC D. D. D. JS. NEWTONUS.

Figure 2.
The frontispiece of the second edition with *General Scholium* by Newton's *Principia* (1713)[4].

Albeit, from an epistemological point of view, it is difficult to exactly identify all characteristics a descriptive-explicative model of physical phenomena should keep, some of them cannot be ignored. Two of these characteristics are:

1) The coherence of the principles that are at the basis of the system, that is the principles must not be mutually contradictory.

2) The possibility to determine quantitative relations between the sizes of the system.

We note that in the classical physical studies, the possibility to express the position of a body as a function of the time, is necessarily a law of motion fundamental for quantitative relations. Generally speaking in order to express such law, it is necessary to determine a physical system in which the space-variable can be decomposed into three (dimensional) mutually perpendicular directions[5]. Then, for every motion, the position of the moving body can be expressed in function of the time i.e. $x = f(t)$, $y = g(t)$, $z = h(t)$. Thus, a law of motion can well interpret a classical physical phenomenon if a Descartesian system (time and each of the three directions) is provided. Hence, time and space have to be uniform quantities as far as they are the bases of the reference systems. A position of a body is a function of

[4]The English translation (1729) was by Andrew Motte (1696-1734) found in the second Latin edition (1713).

[5]That is in modern terms as *rectangular coordinate system* also called *Descartesian* or *Cartesian coordinate system* by three functions for coordinates.

time, but the space itself is not. According to Newton[6] the fundamental problem of Descartes' physics can be so summarized (see **Figure 3**):

The essays *La Dioptrique*, *Les Météores*, *La Géométrie* (see **Figures 4** and **5**) and numerous letters (Descartes, 1897-1913, I-II-III-IV-V) provide the idea of a completely different Descartes. He supplied substantially correct modelling of phenomena, as the refraction (*Ivi*, *La Dioptrique*, discours II, VI) with the consequent genial explanation of the rainbow and of other optical effects (*Ivi*, *Les Météores*, discours VIII, VI). Sometimes analogy brought him to incorrect explanations, as it is the case for the origin of the colours (*Les Météores*, discours VIII, VI). However, in these cases, too, a profound attempt to make the theory coherent with the facts is present. The idea to measure and to quantify the sizes constitute the conceptual and methodological basis of *La Dioptrique* and of *Les Météores* even if the transcription into mathematical terms is not always explicit. Particularly *La Géométrie* (*Ivi*, VI) deserves a separate series of considerations: despite mathematical problems are dealt with (hence not directly connected with the knowledge of the external world), the new modelling proposed by Descartes—the analytical geometry—will be fundamental for science, too, because of the idea to transcribe geometrical data into an analytical form. The Essays and some letters arouse hence a different impression from that given by the *Principia*.

In the *La Dioptrique* (and *Les Météores*) he was able to provide—plausible, even if non always exact—early models of the phenomena as refraction, rainbow and origin of the colours considering empirical data and framing them into a theoretical structure, as it will be clarified in the third section of our paper. Differently from this approach, in the *Principia*, as well known,

The hypotheses of Vortices is pressed with many difficulties. That every Planet by a radius drawn to the Sun may describe areas proportional to the times of description, the periodic times of the several parts of the Vortices should observe the duplicate proportion of their distances from the Sun. But that the periodic times of the Planets may obtain the sesquiplicate proportion of their distances from the Sun, the periodic times of the parts of the Vortex ought to be in sesquiplicate proportion of their distances. That the smaller Vortices may maintain their lesser revolutions about *Saturn*, *Jupiter*, and other Planets, and swim quietly and undisturbed in the greater Vortex of the Sun, the periodic times of the parts of the Sun's Vortex should be equal. But the rotation of the Sun and Planets about their axes, which ought to correspond with the motions of their Vortices, recede far from all these proportions. The motions of the Comets are exceedingly regular, are govern'd by the same laws with the motions of the Planets, and can by no means be accounted for by the hypotheses of Vortices. For Comets are carry'd with very eccentric motions through all parts of the heavens indifferently, with a freedom that is incompatible with the notion of a Vortex. [...]

a) If—as it is the case in Descartes—the space is identified with the *res extensa*, that is, if the separation between space and bodies moving in the space, is substantially denied, then the space has the same characteristics of the moving bodies and the position of the space itself becomes a function of time. Therefore it can happen that a point existing at the instant t_0, does not exist anymore at the instant $t_0 + \Delta t$, so that a system of coordinates in which the positions of the bodies can be given, cannot be established.
b) Newton writes that in Descartes' system not even a God could determine the position of a planet as a function of time and in *De gravitatione* he explains in detail the reasoning we have summarized in a modern language.
c) Thus, according to Newton, the description of the physical world ideated by Descartes in his *Principia Philosophiae* (hereafter *Principia*) does not satisfy the two characteristics needed for a model.
d) Besides these, there are further problems as the consequences of some laws expressed in the *Principia* and contradicted by the experience (as it is the case of the collision rules between two bodies) or the unscrupulous resort to analogy and the lack of clearness as to the relations between experience-experiment and theory.

Figure 3.
Newton's first paragraph on (implicitly) Descartes at the beginning of the *General Scholium*[7].

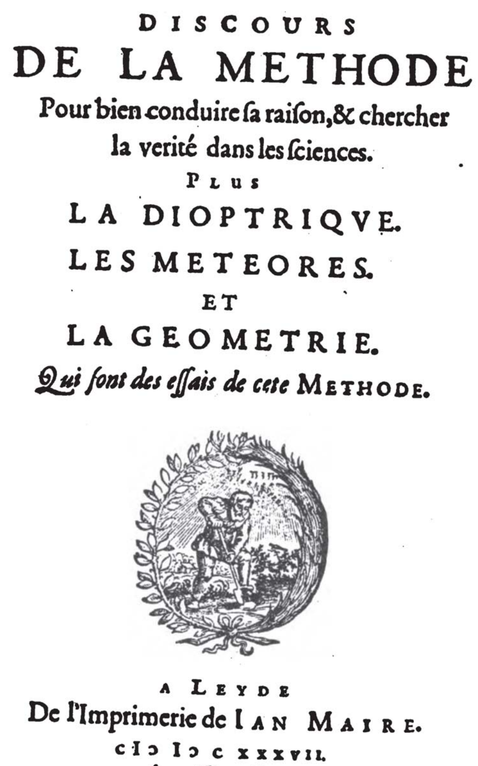
DISCOURS
DE LA METHODE
Pour bien conduire sa raison, & chercher
la verité dans les sciences.
PLUS
LA DIOPTRIQVE.
LES METEORES.
ET
LA GEOMETRIE.
Qui sont des essais de cete METHODE.
A LEYDE
De l'Imprimerie de IAN MAIRE.
CIↃ IↃ C XXXVII.
Auec Priuilege.

Figure 4.
The frontispiece of *Discours de la méthode* (1637)[8].

[6]It is well known that Newton spoke of absolute time and absolute space in the general *General Scholium* (Newton, [1713] 1729) where it does not begin with the introduction of the concepts of absolute space and absolute time, but with the prove that the vortices-theory of Descartes is untenable. Likely Newton introduced explicitly his concepts of absolute space and time as an epistemological answer to Descartes' theory. In this manner the initial part of the *General Scholium* can be interpreted as the physical refutation of Descartes' theory and the second part as the epistemological refutation. On historical-philosophical conceptualization around Newtonian colour theory and the new analytical theories one can see Panza (Panza, 2005, 2007), Blay (Blay, 1983, 1992, 2002), Rashed (Rashed) and on Newtonian Optik Hall (Hall, 1993; see also Halley, 1693). On Fresnel's optic one can also see Rosmorduc, Rosmorduc and Dutour (Rosmorduc J, Rosmorduc V, Dutour) interesting for our aims.

[7]Newton, [1713] 1729: p. 387. Recently on Newton a critic French edition is remarkable (Panza, 2004).

[8]Descartes 1897-1913, VI. *Discours de la Méthode* (*Ivi*: pp. 1-79). It includes *La Dioptrique* (*Ivi*: pp. 80-228), (*Ivi*, *Les Météores*: pp. 231-366), *La Géométrie* (*Ivi*: pp. 367-485). *Le Monde* (*Ivi*, XI-1: pp. 3-118). For the Latin edition (1644) of the *Principia* see *Ivi*, VIII-1; for the French translation (1647) see: *Ivi*, IX-2.

LA DIOPTRIQVE

Diſcours Premier.

DE LA LVMIERE.

Toute la conduite de noſtre vie depend de nos ſens, entre leſquels celuy de la veüe eſtant le plus vniuerſel & le plus noble, il n'y a point de doute que les inuentions qui ſeruent a augmenter ſa puiſſance, ne ſoyent des plus vtiles qui puiſſent eſtre. Et il eſt malaiſé d'en trouuer aucune qui l'augmente dauantage que celle de ces merueilleuſes lunettes qui, n'eſtant en vſage que depuis peu, nous ont deſia découuert de nouueaus aſtres dans le ciel, & d'autres nouueaus obiets deſſus la terre, en plus grand nombre que ne ſont ceus que nous y auions veus auparauant : en ſorte que, portant noſtre veüe beaucoup plus loin que n'auoit couſtume d'aller l'imagination de nos peres, elles ſemblent nous auoir ouuert le chemin, pour paruenir a vne connoiſſance de la Nature beaucoup plus grande & plus parfaite qu'ils ne l'ont eue. Mais, a la honte de nos ſciences, cete inuention, ſi vtile & ſi admirable, n'a premie-

Figure 5.
The first page of the *La dioptrique* (1694)[9].

Descartes tried to supply a global physical theory looking for its foundation in few basic notions without resorting to any quantification. He limited his speeches with qualitative and analogical arguments. Descartes does not seem to fully catch the difficulty and complexity of some problems as the nature of gravity and of magnetism (*Le Monde ou Traité de la lumière* (hereafter *Le Monde*), Descartes, 1897-1913, XI; *Id.*, *Principia*, IX-2, Part IV, § 20-27: pp. 133-183). The example of gravity is particularly significant: Descartes' mechanisitic conception brought him to think that the origin of gravity (to consider as a phenomenon taking place on the earth) depends on the effects of the quick movement of the particles ("particulae") of the second element around the earth (*Principia*, 1644, VIII, Part IV, § 20-21)[10] (see **Figures 6** and **7**).

The earth itself and the bodies on the earth are mostly composed of particles belonging to the third element. They are heavier than those of the second element surrounding the earth. The movement of the particles of the second element exerts a pressure on the bodies composed by particles of the third element so that they tend to the centre of the earth. In synthesis this is the mechanistic conception of gravity exposed by Descartes. A consequence of this conception is the theoretical impossibility to determine a relation between *mass* as physical measurable quantity and *quantity of matter* as (classical Descartesian) conception of internal part of an object (see **Figure 8**).

A consequence is that the explanation between what is the mass (physical measure) and what is the quantity of matter (mathematical interpretation) was not easily identifiable due

[9]Descartes, 1897-1913, VI.

[10]As to the theory of the particles composing the three elements (Descartes, 1897-1913 [*Principia*, 1644, III, § 48-53] VIII-1: pp. 102-107) and in particular the chapter 52 (Descartes 1897-1913, VIII-1: p. 105, line 11-30) titled *Tria esse huius mundi aspedabilis elementa*.

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omnino ſimilis exſiſtat. Quippe cùm globuli cœleſtes moventur in meatibus corporum terreſtrium liquidorum, particulas tertii elementi ſibi obvias aſſiduè loco expellunt, donec eas inter aliquas alias ita diſpoſuerint & ordinarint, ut non magis quàm iſtæ aliæ ipſorum motibus obſiſtant, vel, cùm ita diſponi non poſſunt, donec eas à reliquis ſegregarint. Sic videmus ex muſto fæces quaſdam, non modò ſurſum & deorſum (quod gravitati & levitati tribui poſſet), ſed etiam verſus vaſis latera expelli, vinumque poſtea defæcatum, quamvis adhuc ex variis particulis conſtans, eſſe pellucidum, & non denſius aut craſſius in imo quàm in ſummo apparere. Idemque de cæteris liquoribus puris eſt exiſtimandum.

XIX. *De Tertio effectu; quòd liquorum guttas reddat rotundas.*

Tertius effectus globulorum cœleſtium eſt, quòd aquæ aliorumve liquorum guttas in aëre, aliove liquore ab iis diverſo, pendentes, reddant rotundas, ut jam in Meteoris explicui. Cùm enim iſti globuli cœleſtes, longè alias habeant vias in aquæ guttâ quàm in aëre circumjacente, ſemperque quantum poſſunt ſecundùm lineas rectas, vel ad rectas quam-proximè accedentes, moveantur; manifeſtum eſt illos qui ſunt in aëre, objectu aquææ guttæ minùs impediri à motibus ſuis, ſecundùm lineas à rectis quamminimum deflectentes, continuandis, ſi ea ſit perfectè ſphærica, quàm ſi quamcunque aliam figuram ſortiatur. Si quæ enim ſit pars in ſuperficie iſtius guttæ, quæ ultra figuram ſphæricam promineat, majori vi globuli cœleſtes per aërem diſcurrentes, in illam impingent, quàm in cæteras, ideoque ipſam verſus centrum guttæ protrudent; ac ſi quæ pars ejus, ſuperficiei centro vicinior ſit quàm reliquæ, globuli cœleſtes in ipſa gutta contenti, majori vi eam à centro expellent; atque ita omnes ad guttam ſphæricam faciendam concurrent. Et cùm angulus contingentiæ, quo ſolo linea circularis à rectâ diſtat, omni angulo rectilineo ſit minor, & in nulla linea curva præterquam in circulari ſit ubique æqualis, certum eſt, lineam rectam nunquam poſſe magis æqualiter, & minùs in unoquoque ex ſuis punctis inflecti quàm cùm degenerat in circularem.

XX. *Explicatio ſecundæ actionis, quæ gravitas vocatur.*

Vis gravitatis, à tertia iſta globulorum cœleſtium actione non multùm differt; ut enim illi globuli per ſolum ſuum motum, quo ſine diſcrimine quaquaverſus feruntur, omnes cujuſque guttæ particulas, verſus ejus centrum æqualiter premunt, ſicque ipſam guttam faciunt rotundam; ita per eundem motum, totius molis terræ occurſu impediti, ne ſecundùm lineas rectas ferantur, omnes ejus partes

Figure 6.
Descartesian gravity and magnetism.

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partes verſus medium propellunt: atque in hoc gravitas corporum terreſtrium conſiſtit.

XXI. *Omnes Terræ partes, ſi ſolæ ſpectentur, non eſſe graves, ſed leves.*

Cujus natura ut perfectè intelligatur, notandum eſt primò, ſi omnia ſpatia circa Terram, quæ ab ipſius Terræ materiâ non occupantur, vacua eſſent, hoc eſt, ſi nihil continerent niſi corpus, quod motus aliorum corporum nullâ ratione impediret nec juvaret (ſic enim tantùm intelligi poteſt vacui nomen) & interim hæc terra circa ſuum axem, ſpatio viginti quatuor horarum proprio motu volveretur, fore ut illæ omnes ejus partes, quæ ſibi mutuò non eſſent valde firmiter alligatæ, hinc inde verſus cœlum diſſilirent: Eodem modo, quo videre licet dum turbo gyrat, ſi arena ſupra ipſum conjiciatur eam ſtatim ab illo recedere atque in omnes partes diſpergi; & ita Terrra non gravis, ſed contra potiùs levis eſſet dicenda.

XXII. *In quo conſiſtat levitas materiæ cœleſtis.*

Cùm autem nullum ſit tale vacuum, nec Terra proprio motu cieatur, ſed à materia cœleſti, eam ambiente, omneſque ejus poros pervadente, deferatur, ipſa habet rationem corporis quieſcentis; materia autem cœleſtis, quatenus tota conſentit in illum motum quo Terram defert, nullam habet vim gravitatis, nec levitatis; ſed quatenus ejus partes plus habent agitationis quàm in hoc impendant, ideoque ſemper terræ occurſu, à motibus ſuis ſecundùm lineas rectas perſequendis impediuntur, ſemper ab ea quantum poſſunt recedunt, & in hoc earum levitas conſiſtit.

XXIII. *Quomodo partes omnes terræ, ab iſtâ materiâ cœleſti deorſum pellantur, & ita fiant graves.*

Notandum deinde, vim quam habent ſingulæ partes materiæ cœleſtis ad recedendum à Terra, ſuum effectum ſortiri non poſſe, niſi, dum illæ aſcendunt, aliquas partes terreſtres in quarum locum ſuccedunt, infra ſe deprimant & propellant. Cùm enim omnia ſpatia quæ ſunt circa Terram, vel à particulis corporum terreſtrium, vel à materia cœleſti occupentur; atque omnes globuli hujus materiæ cœleſtis, æqualem habeant propenſionem ad ſe ab eâ removendos, nullam ſinguli habent vim, ad alios ſui ſimiles loco pellendos; ſed cùm talis propenſio non ſit tanta in particulis corporum terreſtrium, quoties aliquas ex ipſis ſupra ſe habent, omnino in eas vim iſtam ſuam debent exercere. Atque ita gravitas cujuſque corporis terreſtris, non propriè efficitur ab omni materiâ cœleſti illud circumfluente, ſed præcisè tantùm ab eâ ipſius parte, quæ, ſi corpus iſtud deſcendat, in ejus locum immediatè aſcendit, ac proinde quæ eſt illi magnitudine planè æqualis. Sit exempli causâ, B corpus terreſtre

Figure 7.
Descartesian gravity and magnetism[11].

[11]**Figure 6**: Descartes 1897-1913 [*Principia*, 1644, VIII-1, Part IV, § 20-21] IX-2: pp. 210-211 [Full Latin version: *Ivi*, VIII-1: pp. 1-348].

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in corpore B; atque in hoc uno ejus gravitatem conſiſtere.

XXV. *Ejus quantitatem non reſpondere quantitati materiæ cujuſque corporis.* Utque nihil omittatur, advertendum etiam eſt, per materiam cœleſtem non hîc intelligi ſolos globulos ſecundi elementi, ſed etiam materiam primi iis admiſtam, & ad ipſam quoque eſſe referendas illas particulas terreſtres, quæ curſum ejus ſequutæ, cæteris celeriùs moventur; quales ſunt eæ omnes quæ aërem componunt. Advertendum præterea, materiam primi elementi, cæteris paribus, majorem vim habere ad corpora terreſtria deorſum pellenda, quàm globulos ſecundi, quia plus habet agitationis; & hos majorem, quàm particulas terreſtres aëris quas ſecum movent, ob ſimilem rationem. Unde fit, ut ex ſolâ gravitate non facilè poſſit æſtimari, quantum in quoque corpore materiæ terreſtris contineatur. Et fieri poteſt, ut quamvis, exempli cauſsâ, maſſa auri vicies plus ponderet, quàm moles aquæ ipſi æqualis, non tamen quadruplo vel quintuplo plus materiæ terreſtris contineat: tum, quia tantundem ab utraque ſubducendum eſt, propter aërem in quo ponderantur; tum etiam, quia in ipsâ aquâ, ut & in omnibus aliis liquidis corporibus, propter ſuarum particularum motum, ineſt levitas, reſpectu corporum durorum.

XXVI. *Cur corpora non gravitent in locis ſuis naturalibus.* Conſiderandum etiam, in omni motu eſſe circulum corporum quæ ſimul moventur, ut jam ſuprà oſtenſum eſt, nullumque corpus à gravitate ſuâ deorſum ferri, niſi eodem temporis momento, aliud corpus magnitudine ipſi æquale, ac minùs habens gravitatis, ſurſum feratur. Unde fit, ut in vaſe, quantumvis profundo & lato, inferiores aquæ alteriuſve liquoris guttæ, à ſuperioribus non premantur;

nec

It can happen i.e. that, albeit a mass of gold is twenty times heavier than a quantity of water of the same size, it does not contain twenty times the quantity of matter contained in that mass of water, but only four or five times […][12]

Figure 8.
Some Descartes' arguments on matter concept[13].

their difficulties of transcription into quantitative physical terms. The mechanistic and *a priori* conviction of Descartes brought hence him to the impossibility to have a well defined conception of space and of mass[14]. This is a substantial, not only formal difference. In fact, the scientific framework of the treatises can deceive. For example, in *Principia*, Newton wrote eight definitions and the three *laws* (or *axioms*) at the beginning (see **Figure 9**).

Therefore one can get the impression he started from these to explain the phenomena analysed in the three books of *Principia*. Actually, the two introductory sections (*definitions* and *axioms*) give an Euclidean order to the text that is different from the way in which Newton reached to determine the nature of the phenomena. The definitions and the laws were enucleated on the basis of the phenomena, not before a detailed examination

[12]"Et fieri potest, ut quamvis, exempli caussa, massa auri vicies plus ponderet, quam moles aquae ipsi aequalis, non tamen quadruple vel quintuplo plus materiae terrestris contineat […]" (Descartes [*Principia*, 1644, Part IV: p. 202] VIII-1: p. 213, line 16). The translation is ours.

[13]*Ibidem*.

[14]The concept of mass from physical and mathematical standpoint was a hard concept until 19th century for new theories i.e. like chemistry and thermodynamics, machines theory (Pisano, 2010, 2011). For example Lazare Carnot (1753-1823) explicitly was ambiguous (Gillispie & Pisano 2013: p. 377) on the concept of force (Carnot, 1803: p. xj, p. 47) and mass assuming both of Descartesian and Newtonian assumptions (Carnot, 1803: p. 6). Ernst Mach (1838-1916) wrote interesting speeches on that (Mach, ([1896] 1986): pp. 368-369) tried to formulate an operative interpretation of mass using the third principle of mechanics (Mach, 1888, [1896] 1986)).

LAWS. *of Natural Philoſophy.* 19

Axioms or Laws of Motion.

LAW I.

Every body perſeveres in its ſtate of reſt, or of uniform motion in a right line, unleſs it is compelled to change that ſtate by forces impreſs'd thereon.

PRojectiles perſevere in their motions, ſo far as they are not retarded by the reſiſtance of the air, or impell'd downwards by the force of gravity. A top, whoſe parts by their coheſion are perpetually drawn aſide from rectilinear motions, does not ceaſe its rotation, otherwiſe than as it is retarded by the air. The greater bodies of the Planets and Comets, meeting with leſs reſiſtance in more free ſpaces, preſerve their motions both progreſſive and circular for a much longer time.

LAW II.

The alteration of motion is ever proportional to the motive force impreſs'd; and is made in the direction of the right line in which that force is impreſs'd.

If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force be impreſs'd altogether and

C 2 at

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at once, or gradually and ſucceſſively. And this motion (being always directed the ſame way with the generating force) if the body moved before, is added to or ſubducted from the former motion, according as they directly conſpire with or are directly contrary to each other; or obliquely joyned, when they are oblique, ſo as to produce a new motion compounded from the determination of both.

LAW III.

To every Action there is always oppoſed an equal Reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

Whatever draws or preſſes another is as much drawn or preſſed by that other. If you preſs a ſtone with your finger, the finger is alſo preſſed by the ſtone. If a horſe draws a ſtone tyed to a rope, the horſe (if I may ſo ſay) will be equally drawn back towards the ſtone: For the diſtended rope, by the ſame endeavour to relax or unbend it ſelf, will draw the horſe as much towards the ſtone, as it does the ſtone towards the horſe, and will obſtruct the progreſs of the one as much as it advances that of the other. If a body impinge upon another, and by its force change the motion of the other; that body alſo (becauſe of the equality of the mutual preſſure) will undergo an equal change, in its own motion, towards the contrary part. The changes made by theſe actions are equal, not in the velocities, but in the motions of bodies; that is to ſay, if the bodies are not hinder'd by any other impediments. For becauſe the motions

4 are

Figure 9.
Newton's laws[15].

[15]"Axioms; or Laws of Motion. Law I. *Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon*; Law II: *The alteration of motion is ever proportional to the motive force impressed*; *and is made in the direction of the right line in which that force is impressed*; Law III: *To every action there is always opposed an equal reaction*: *or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts*." (Newton, [1686-7] 1803, I: pp. 19-20; *Italic style* and capital letters belong to the author). (Newton, [1686-7] 1803, I: p 2; author's *italic* style and Capital letters). On forces and their geometrical interpretation one can see De Gandt (De Gandt, 1995).

and comprehension of the phenomena themselves. Instead in Descartes' *Principia* the laws, and above all the ideas concerning the constitution of matter, were thought almost independently from phenomena and, afterwards, applied to them.

On the *Principia Philosophiae*

In this section, we will deal with the cases in which the physical laws established by Descartes in his *Principia* are self-contradictory and contradicted by the experience itself, particularly on the collision rules theory.

Some Historiography on Descartes' Collision Rules

The historiography concerning Descartes' collision rules is conspicuous. Here we analyze only those studies directly connected with the logic of our reasoning[16].

Ernst Cassirer (1874-1945) stresses that the collision rules are self-contradictory, even if he does not enter into details. Consequently such rules do not provide a unified picture and, hence, a model of the phenomenon. Cassirer ascribes this situation to the fact that Descartes

> […] leaves the continuous and patient development of his deductive-mathematical premises and passes directly to explain concrete particular phenomena that are very complex[17].

Nevertheless, it is necessary to point out that in other cases of complex phenomena, as the refraction and the rainbow, Descartes is faithful to the mathematical approach of his own work.

Pierre Boutroux (1880-1922) after having eulogized Descartes for the introduction of the inertia principle and the conservation of the quantity of motion principle, writes as to the collision: "Unfortunately, Descartes makes a very serious mistake, that is surprising from his part"[18]. The mistake consists in the fact that Descartes did not catch the vectorial nature of the quantity of motion. The mistakes in the collision rules are due, according to Boutroux, to this misconception.

René Dugas (1897-1957) claims there is more than one reason why Descartes did not succeed in the explanation of the collision: a) lack of distinction between elastic and inelastic collisions (Dugas, [1954] 1987: pp 150-151); see also 1954; b) existence of dissymetries with regard to the reasons that can produce, increase or diminish the quantity of motion of a body; c) lack of comprehension of the vectorial nature of velocity (*Ivi*). Dugas adds that the experience is anyway necessary for a correct formulation of the collision rule (*Ivi*). Furthermore he underlines that from Descartes' correspondence, it is possible to deduce he had carried out some experiments, but that, between experimental results and principles, he had chosen the principles. Therefore Dugas ascribes the failure of Descartes' collision rules to an unclear comprehension of the basic principles connoting the motion quantity (theoretical reason) and to the lack of serious experiments on this subject (empirical reason).

Pierre Costabel (1912-1989), after having analysed the collision rules in Descartes claims:

> It has been said and repeated the Descartesian collision rules are only an outline. Nevertheless, it has been stressed that the principles, of which such rules would be an outline, were already acquired in Descartes' thought. Actually we believe that things work in the opposite manner. These rules are only an outline because they are the expression of a thought that is still researching[19].

By the way, still Costabel's opinion that Descartes proposed only some outlined rules in the work he considered the result of his most mature thought in physics appears disputable.

Recently, Stephen Gaukroger discusses that Descartes' physics is based upon modelling drawn from statics and tries to explain the genesis itself of the collision rules on this basis. He proposes an interesting examination of the fourth rule (Gaukroger, 2000: pp. 60-80).

Peter McLaughlin, analyses the Descartesian concept of *determination* of a motion. He also frames the Descartesian rules inside a context deriving from statics and, in this way, he tries to provide an explanation of such rules (McLaughlin, 2000: pp. 81-112).

Beyond the principles explicitly formulated in his works, likely Descartes also resorted to some principles of minimum exposed in some of his letters. Gary C. Hatfield mentions a letter on 17 February 1645 to Clerselier[20] (1614-1684) in which Descartes wrote:

> […] when two bodies in incompatible modes collide, some change in these modes must truly occur, so as to render them compatible, but that this change is always the least possible […][21].

McLaughlin also points out that Descartes resorts to a "principle of minimal modal change" (McLaughlin, 2000: p. 99). He also tries to interpret the meaning of *mode*. In particular, he remarks that *determination* and *velocity* of a motion are two different *modes*. It is then maybe possible to think that the collision rules are conceived so that the modal change is the less possible. In this manner, for example, in the rule 4, the change of the *determination* of the body B represents a *modal change* less than the one existing if the body C, too, would move, because, in this case, two *modes* would change: *determination* and *velocity*.

The existence of *principles of minimum* in Descartes' corpus is reasonable by what he wrote in the fifth *discours* of the *Les Météores* concerning with form of the clouds under the action of irregular winds:

> [...] figure which can least [assume the form and] prevent

[16]With regard to the various factors on which historiography of science depends, see: Kragh, 1987; Pisano & Gaudiello, 2009a, 2009b; Kokowski, 2012; Poincaré, [1923] 1970, [1935] 1968; Rossi; Taton, 1965, 1966; Westfall, 1971.

[17]"[…] er den stetigen Gang und den geduldigen Ausbau seiner deduktivmathematischen Voraussetzungen verläßt, um unvermittelt zu der Erklärung verwickelter konkreter Sonderphänomene". (Cassirer, [1906] 1922: p. 479). The translation is ours.

[18]"Malheureusement, Descartes commet une erreur très grave et qui est bien surprenante da sa part". (Boutroux, 1921: p. 677). The translation is ours.

[19]"On a dit et redit que les règles cartésiennes du choc ne sont qu'une esquisse, mais on l'a fait en sous-entendant que les principes dont elles seraient l'esquisse étaient déjà fermes dans la pensée de Descartes. La réalité nous parait différente. Ces règles ne sont qu'une esquisse parce qu'elles sont l'expression d'une pensée en état de recherche". (Costabel, [1967] 1982: pp. 141-[152]158). The translation is ours. See also Costabel, 1960.

[20]Clerselier is an important figure in the scientific frameworks of Descartes. He edited and translated many Descartes' works i.e. *Correspondences* (1657, 1659, 1667), *Le Monde* (1667) and *Principes* (1681).

[21]"[…] *lors que deux cors se rencontrent, qui ont en eux des modes incomepatibles, il se doit véritablement faire quelque changement en ces modes, pour les rendre compatibles, mais* (*que*) *ce changement est tousiours le moindre qui puisse être* […]." (Descartes, 1897-1913, IV: p. 185, line 13). Author's *italic*. See also Hatfield, 1979, p. 133.

[opposes less resistance to] their movement [...][22]

The plurality of approaches through which many distinguished scholars and historians tried to explain the reasons that led Descartes to formulate collision laws that are self-contradictory and not confirmed by experience, proves that this question is not clear at all. Therefore, every explanation has, at least partially, a conjectural and hypothetical character.

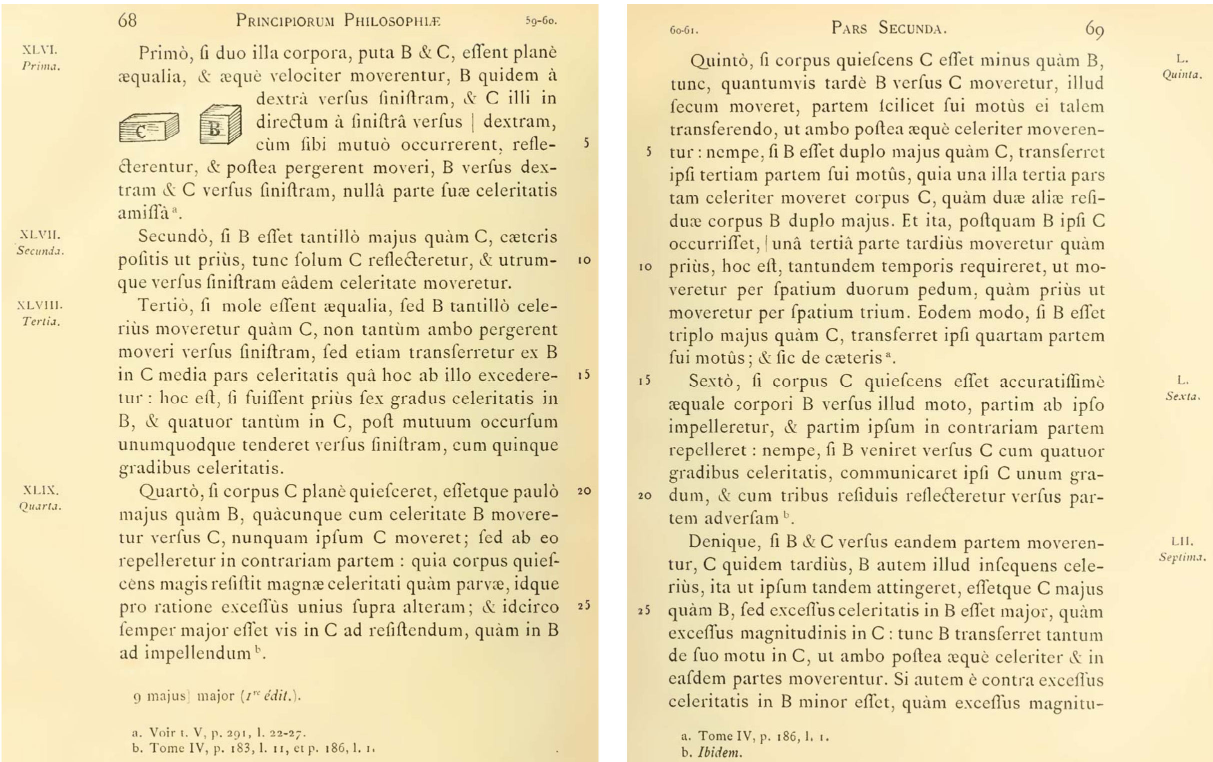
Collision Dynamics and Physics-Mathematics Arguments

The study of the collision rules between two bodies is a subject on which the literature was relatively abundant in Descartes' age (McLaughlin, 2000: pp. 81-112). Descartes established seven rules (Descartes, 1897-1913, *Principia*, VIII-1: pp. 69-69; see **Figure 10**).

Descartes did not make distinction between elastic and inelastic collision. However, considering the structure of his reasoning above exposed, he was referring to elastic collisions on a surface without friction. It is known that only the first one of these previous rules is correct. A part from this, *do these rules have an inner coherence—for inner coherence we mean the property according to which no contradictory conclusions can be draw from the principles*? Let us see an example in order to consider two bodies B and C, the first one of mass 5, the second one 7.

Let C be at rest and let B move with velocity v. After the collision, C remains at rest, while B inverts the direction of its motion (by using rule 4).

Let us now imagine that B increases its mass with continuity and that C decreases it with continuity, in a way that the sum of the sizes remain invariable. When B has size 7 and C size 5,


68 PRINCIPIORUM PHILOSOPHIÆ 59-60.

XLVI. *Prima.* Primò, ſi duo illa corpora, puta B & C, eſſent planè æqualia, & æquè velociter moverentur, B quidem à dextrâ verſus ſiniſtram, & C illi in directum à ſiniſtrâ verſus | dextram, cùm ſibi mutuò occurrerent, reflecterentur, & poſtea pergerent moveri, B verſus dextram & C verſus ſiniſtram, nullâ parte ſuæ celeritatis amiſſâ [a].

XLVII. *Secunda.* Secundò, ſi B eſſet tantillò majus quàm C, cæteris poſitis ut priùs, tunc ſolum C reflecteretur, & utrumque verſus ſiniſtram eâdem celeritate moveretur.

XLVIII. *Tertia.* Tertiò, ſi mole eſſent æqualia, ſed B tantillò celeriùs moveretur quàm C, non tantùm ambo pergerent moveri verſus ſiniſtram, ſed etiam transferretur ex B in C media pars celeritatis quâ hoc ab illo excederetur : hoc eſt, ſi fuiſſent priùs ſex gradus celeritatis in B, & quatuor tantùm in C, poſt mutuum occurſum unumquodque tenderet verſus ſiniſtram, cum quinque gradibus celeritatis.

XLIX. *Quarta.* Quartò, ſi corpus C planè quieſceret, eſſetque paulò majus quàm B, quàcunque cum celeritate B moveretur verſus C, nunquam ipſum C moveret; ſed ab eo repelleretur in contrariam partem : quia corpus quieſcens magis reſiſtit magnæ celeritati quàm parvæ, idque pro ratione exceſſùs unius ſupra alteram; & idcirco ſemper major eſſet vis in C ad reſiſtendum, quàm in B ad impellendum [b].

9 majus] major (*1re édit.*).

a. Voir t. V, p. 291, l. 22-27.
b. Tome IV, p. 183, l. 11, et p. 186, l. 1.

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L. *Quinta.* Quintò, ſi corpus quieſcens C eſſet minus quàm B, tunc, quantumvis tardè B verſus C moveretur, illud ſecum moveret, partem ſcilicet ſui motùs ei talem transferendo, ut ambo poſtea æquè celeriter moverentur : nempe, ſi B eſſet duplo majus quàm C, transferret ipſi tertiam partem ſui motûs, quia una illa tertia pars tam celeriter moveret corpus C, quàm duæ aliæ reſiduæ corpus B duplo majus. Et ita, poſtquam B ipſi C occurriſſet, | unâ tertiâ parte tardiùs moveretur quàm priùs, hoc eſt, tantundem temporis requireret, ut moveretur per ſpatium duorum pedum, quàm priùs ut moveretur per ſpatium trium. Eodem modo, ſi B eſſet triplo majus quàm C, transferret ipſi quartam partem ſui motûs; & ſic de cæteris [a].

LI. *Sexta.* Sextò, ſi corpus C quieſcens eſſet accuratiſſimè æquale corpori B verſus illud moto, partim ab ipſo impelleretur, & partim ipſum in contrariam partem repelleret : nempe, ſi B veniret verſus C cum quatuor gradibus celeritatis, communicaret ipſi C unum gradum, & cum tribus reſiduis reflecteretur verſus partem adverſam [b].

LII. *Septima.* Denique, ſi B & C verſus eandem partem moverentur, C quidem tardiùs, B autem illud inſequens celeriùs, ita ut ipſum tandem attingeret, eſſetque C majus quàm B, ſed exceſſus celeritatis in B eſſet major, quàm exceſſus magnitudinis in C : tunc B transferret tantum de ſuo motu in C, ut ambo poſtea æquè celeriter & in eaſdem partes moverentur. Si autem è contra exceſſus celeritatis in B minor eſſet, quàm exceſſus magnitu-

a. Tome IV, p. 186, l. 1.
b. *Ibidem*.

1) If two bodies B and C, whose mass[23] is equal, go one against the other with the same speed, then, after the collision, they bounce back in the starting direction with unmodified speed; 2) In the same situation, but with B greater than C, the body C, after the collision, bounces back in the original direction and the two bodies proceed unified in that direction; 3) If B and C have the same size, but B is quicker than C, then, after the collision, C bounces back and, *mutatis mutandis*, the situation is the same as in the rule 2; 4) If C is bigger than B and C is at rest, whatever the speed of B is after the collision, C remains at rest and B bounces back in the direction from which it was coming; 5) If C is smaller than B and C is at rest, when B collides with C, the two bodies proceed unified, according to the principle of conservation of the quantity of movement; 6) If C is at rest and B and C have the same size, and if B hits C, after the collision, C will move in the same direction and verse as B, while B itself bounces back; 7) If B and C move in the same verse and C is bigger and slower than B and the excess of the velocity of B is greater than the excess of the size of C, then B transfers part of its movement to C, so that the two bodies move with the same velocity in the same verse. The rule also considers the symmetric case in which the excess of speed of B is less than the excess of size of C.

Figure 10.
Collision rules arguments[24].

[22]"[...] la figure qui peut le moins empêcher leur mouvement […]". (Descartes, 1897-1913, VI: p. 286, lines 28-29). The translation is ours.

[23]The word used by Descartes for *mass* is—in general—"mole" (Descartes, 1897-1913, VIII: p. 68, line 9) when he speaks of the third rule of the collision and other occurrences. In the *Principia* Descartes uses the word "corpus" plus an adjective ("major" or "minor"). For example, at the beginning of the fifth collision rule, we read: "Quinto, si corpus quiescens C esset minus quam B […]" (Descartes, 1897-1913 Principia VIII: p. 69, line 1). We have translated these words with *mass* because other translations would be even worse and without refereeing to modern concept of the mass. On the history of the concept of mass, at first glance one cans see Jammer (Jammer, 1961).

[24]Descartes, 1897-1913, VIII: pp. 68-69.

after the collision, B pushes C and the two bodies prosecute their motion unified (rule 5).

Thus, because of the continuity principle, a physical state necessarily exists in which, after the collision, B remains at rest and this must happen when B is as great as C.

The conclusion of this reasoning, deduced by applying the rules 4 and 5, is that when a body B of size m and speed v strokes a body C—that is at rest—of the same size, B remains at rest and C prosecutes in the same direction and verse as B and with speed v.

However, this results contradicts the rule 6.

Beyond the lack of inner coherence, there is also the problem that the Descartesian rules of the collision contradict the daily experience concerning the collisions themselves in an evident manner. In the Principia, *Descartes*—who was aware of this—underlined (Descartes, 1897-1913, IX-2, §53) that his rules are referred to ideal situations that can be hardly experimented, after having concluded the paragraph 52 claiming that "these rules [of collision] are so evident that no empirical confirmation is necessary"[25]. In this part of his scientific framework the relation between experience and modelling-theory would provide that i.e. single events of a phenomenon are determined by circumstances that are contingent in respect to an *a priori* theoretical model. Thus this kind of approach is typically deductive and aiming to test a final theoretical reasoning. As a matter of fact, the eventual inconsistency between theory and experience depends either on such circumstances or on the inadequacy of the model to represent the phenomenon to which it was applied[26].

The lack of an agreement between experiences-data and modelling-theory is typical of a scientific theory, especially physics and chemistry. For example, a unit of measurement is effectively a standardised quantity of a physical (and chemical) property, used as a factor to express occurring quantities of that property. Therefore, any value of a physical quantity is expressed as a comparison to a unit of that quantity. In the physics mathematics[27] domain one generally precedes by means of calculations, therefore the units of measurement are not a priority in terms of a solution to an analytical problem (Pisano, 2013; Lindsay, Margenau, & Margenau). In this sense, the physical (and chemical) nature of the quantities is not a priority[28]. One may discuss the role played by a certain science in history (e.g., physics), focusing solely on the historical period, the kind of mathematics adopted and the relations between experiments and theory in the analysed historical period (Pisano, 2011). For our aim, the most important aspect is the role played by the relationship between physics and mathematics adopted in a scientific theory in order to describe mathematical laws—e.g., the second Newtonian mathematical law of motion or, in the case of Descartes's *Principia*, the lack of such mathematical structure and its conesquences (Nagel, 1961, 1997). On the other hands, the *time* is a crucial physical magnitude in mechanics (Truesdell, 1968) but in the aforementioned, the *time* (and *space*) is also a mathematical magnitude since it is a mathematical variable in variations (later derivatives) operations aimed to interpret a certain phenomenon. Most importantly, if we lose the mathematical significances of time and space magnitudes, we would lose the entire mechanical paradigm. Nevertheless, the approaches to conceive and define foundational *mechanical-physical quantities* and their *mathematical quantities* and interpretations change both within a physics mathematics domain and a physical one (Duhem). One could think of mathematical solutions to Lagrange's energy equations (Lagrange, 1778, 1973; Panza, 2003) rather than the crucial role played by collisions and geometric motion in Lazare Carnot's algebraic mechanics or Faraday's experimental science (Faraday, 1839-1855; Heilbron; Pisano, 2013) with respect to André-Marie Ampère (1775-1836) mechanical approach in the electric current domain and finally the physic mathematics choices in James Clerk Maxwell (1831-1879) electromagnetic theory (Maxwell, 1873; Pisano, 2013). Physical science makes use of experimental apparatuses to observe and measure physical magnitudes. During and after an experiment, this apparatus may be illustrated and/or and designed. Generally, this procedure is not employed in pure mathematical studies. Thus, one can claim that experiments and their illustrations can be strictly characterized by physical principles and magnitudes to be measured. A modelling of results of the experimenttal apparatus allows for the broadening of the hypotheses and the establishment of certain theses. If one avoids study-modelling experimental results, one may generate an analytical scientific theory since there is no interest in the nature of physical magnitudes and their measurements.

In the Descartesian rules on the collisions this eventual lack mostly concerns: a) Descartes, in physics, guessed the importance of conservation principles, but, in the collision rules, he was not able to exploit this fundamental and correct idea in a suitable manner; b) a lack of an adequate mathematical interpretation which could be helpful for the fully comprehension of a phenomenon; and c) the impossibility to operate adequate measure since the lack of fully knowledge of the concept of physical quantities for some substances (i.e., one can think of the concept of velocity, rather mass, or temperature, heat etc.)

[25]"Nec ista egent probatione, quia per se manifesta". (Descartes, 1897-1913, [*Principia*, VIII]: p. 70, line 12). The translation is ours.

[26]On that Thomas Samuel Kuhn (1922-1996) proposed (Kuhn, [1962] 1970) that some contradictions between facts and theory are simply ignored by the scientists until a *dominant paradigm* provides exhaustive explanations of the majority of phenomena in which, in a certain period, the scientific community is interested. In particular see *Anomaly and the Emergence of Scientific Discoveries* (Kuhn, [1962] 1970, Chap. VI: pp. 52-65) and the *The Response to Crisis* (Kuhn, [1962] 1970, Chap. VIII: pp. 77-91, in particular pp. 80-82; see also Osler, 2000). Besides that, Paul Feyerabend (1924-1994) has shown—basically through an analysis of Galileo's work—that some experiences are often neglected and that the *critics of experience* is constituted by *ad hoc* argumentations ideated by the scientists to achieve his/her theoretical purposes (Feyerabend, 1975, chaps. 5-8; see also 1991). On Galileo, recently one can see: Festa, 1995, Pisano, 2009a, 2009b,

[27]One of us stressed the relationship between physics and mathematics in the history of science by means of many studies. Among physicists, mathematicians, historians and philosophers who are credited with study of mathematical physical quantities by means of experiments, modelling, properties, existences, structures etc. one can strictly focus on how physics and mathematics work in a unique discipline physics mathematics (or, if one prefers, mathematics physics). Thus, it is not a mathematical application in physics and vice-versa but rather a new (for example in the 19th century) way to consider this science: a new discipline physics mathematics and not mathematical physics, where the change in the kind of infinity in mathematics produces a change in both significant physical processes and interpretations of *physical quantit*ies (Pisano, 2013: pp. 39-58; see also 2011: pp. 457-472).

[28]For instance, one can see an analogous situation concerning heat and temperature concepts in the analytical theory of heat (Fourier, 1807, 1822) with respect to Sadi Carnot's thermodynamic theory (Pisano, 2010, 2011; Gillispie & Pisano, 2013; Pisano, 2010). I briefly note that physics considers the indispensable agreement between theoretical data and observations—experimental data (including the properties of magnitudes) to establish a physical theory. Generally, such arguments are not considered rigorous by physics mathematics.

and the lack of experiments on collision; even considering the whole described picture, it is hard to conceive how it is possible. Thus, a conclusion would be that, in many cases, the lack of a mathematical quantitative treatment prevented Descartes to realize his procedures were not always correct.

Quantification of Physical Reasonings

In his physical works Descartes frequently takes position against the *essentialism* typical of the *Scholastic* because he thinks that such an approach cannot help in anyway to understand the physical phenomena. Particularly he points out the necessity to give a clear and quantitative form to the principles and to the problems themselves of physics. However, Descartes' physical conception is not free from essentialist aspects. For example

> [...] a consequence of his first law of motion, Descartes insists that the quantity conserved in collisions equals the combined sum of the products of size and speed of each impacting body. Although a difficult concept, the "size" of a body roughly corresponds to its volume, with surface area playing an indirect role as well. This conserved quantity, which Descartes refers to indiscriminately as "motion" or "quantity of motion", is historically significant in that it marks one of the first attempts to locate an invariant or unchanging feature of bodily interactions[29].

Descartes seemed to have understood—in the second *discours* of the *La Dioptrique* on the decomposition of the motion and of the *determination* of a motion—that velocity has a direction[30] besides a modulus (Descartes, 1897-1913, VI: pp. 93-105). While, in the *Principia* no quantitative specifications with regard to the role that the modulus and the direction[31] of velocity should get in the physical phenomena is provided.

In the end, concerning his quantitative physical reasonings we can mainly claim:

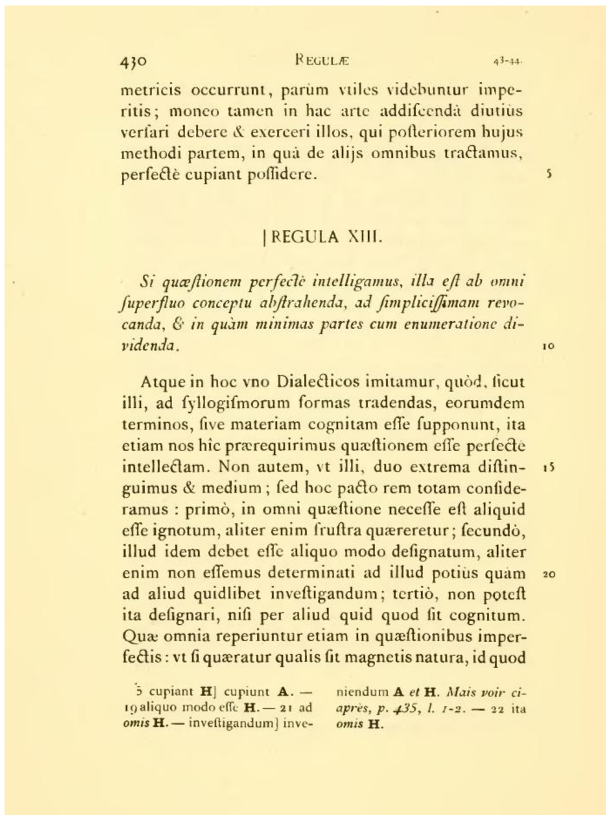
1) Important concepts and rules—as quantity of motion, and significant rules, the seven collision laws—are introduced by Descartes (*Principia*) in a manner that they could not be expressed in an adequate mathematical terms.

2) The three physical parts of the *Principia* (the second, the third and the fourth ones) are inscribed into a physical conception that, in many regards, is still linked to the *Scholastic* ontologism.

Relying upon his researches on the fluid dynamics exposed in the second book of his *Principia*, Newton argued that the vortices, of which Descartes imagined the universe was composed, cannot be stable (Newton, [1713] 1729, II, *General Scholium*: pp. 387-388). This means the physics of Descartes' *Principia* does not satisfy to the minimal requests for it to be translated into quantitative terms in a quantitative model. If Descartes had tried this operation, likely, he would have noticed the physical and factual inconsistencies to which his principles brought. For example, he could have seen that his collision rules were self-contradictory. Coherently with what he himself had claimed in various passages of *Regulae ad directionem ingenii* (Descartes, 1897-1913, X: pp. 351-488; see **Figure 11**) and of *Discours de la méthode* (Descartes, 1897-1913, VI: pp. 1-78). As a matter of fact, in the 13th *regula* Descartes claims (Descartes, 1897-1913, X: pp. 430-438) that every problem has to be divided and analysed into a series of enumerated parts whose knowledge is absolutely certain.

According Descartes, for a phenomenon[33], one should carry out a large series of experiments and after a profound analysis, to consider only those who are really suitable to comprehend such a phenomenon and to exclude the others. Moreover, this reasoning is also valid as to single parts of an experiment. Thus Descartes should have specified the experiments on which his collision rules were based. This would have been important to interpret into mathematical terms their results and in order to make sure the concepts which he used were perspicuous. Nevertheless, as above seen, he acted in a completely different way as to the collision rules (this was also the case for concepts as *force*, *pressure*, *power*). Thus, he would have realized how difficult a satisfactory introduction of a conceptual structure suitable to explain the physical phenomena is. In other words,

> [Descartes] expresses so well the basic idea of a mathematical physics, but he fails to specify how he want to make sure physics susceptible to mathematical treatment. For sure, he completely underestimated the difficulty of this task: it is clear when with a candor typically of scholastic he propose as example to represent by means of a


430 REGULÆ 43-44.

metricis occurrunt, parùm vtiles videbuntur imperitis; moneo tamen in hac arte addiſcendâ diutiùs verſari debere & exerceri illos, qui poſteriorem hujus methodi partem, in quâ de alijs omnibus tractamus, perfectè cupiant poſſidere.

| REGULA XIII.

Si quæſtionem perfectè intelligamus, illa eſt ab omni ſuperfluo conceptu abſtrahenda, ad ſimpliciſſimam revocanda, & in quàm minimas partes cum enumeratione dividenda.

Atque in hoc vno Dialecticos imitamur, quòd, ſicut illi, ad ſyllogiſmorum formas tradendas, eorumdem terminos, ſive materiam cognitam eſſe ſupponunt, ita etiam nos hîc prærequirimus quæſtionem eſſe perfectè intellectam. Non autem, vt illi, duo extrema diſtinguimus & medium; ſed hoc pacto rem totam conſideramus : primò, in omni quæſtione neceſſe eſt aliquid eſſe ignotum, aliter enim fruſtra quæreretur; ſecundò, illud idem debet eſſe aliquo modo deſignatum, aliter enim non eſſemus determinati ad illud potiùs quam ad aliud quidlibet inveſtigandum; tertiò, non poteſt ita deſignari, niſi per aliud quid quod ſit cognitum. Quæ omnia reperiuntur etiam in quæſtionibus imperfectis : vt ſi quæratur qualis ſit magnetis natura, id quod

5 cupiant **H**] cupiunt **A**. — 19 aliquo modo eſſe **H**. — 21 ad *omis* **H**. — inveſtigandum] inveniendum **A** *et* **H**. *Mais voir ci-après, p. 435, l. 1-2.* — 22 ita *omis* **H**.

Figure 11.
Regulae ad directionem ingenii[32].

[29]Slowik, § 4. Author's quotations.
[30]This does not mean that he had completely caught the concept of vector. Nevertheless his conception might be indented as *orientation* (*direction* and *versus*).
[31]This happens i.e. more than once, at the beginning of the second *discours* of the *La Dioptrique*. The word recurs by Descartes to indicate a direction is "costé" (i.e. see: Descartes 1897-1913, [*La Dioptrique*], VI: pp. 94-95).
[32]Descartes, 1897-1913, X: p. 430 [pp. 351-488]).
[33]Descartes, particularly quotes Gilbert's experiments with the magnets (Descartes, 1897-1913, X: pp. 430-438).

> line the degree of whiteness, without making allusion to the difficulties inherent in the measurement of a qualitative intensity[34].

In the *Principia*, an eloquent mechanistic conception (Dijksterhuis 1961) is provided: "[...] every modification of the matter as well as the diversity of all its forms depends on motion [...]"[35]. The interplay between *gradation* and *intensity* of the considered quantities plays an essential role especially when he enumerates some of the fundamental properties of the particles composing the three elements (Descartes, 1897-1913, X, III part, § 52 and § 53: pp. 105-107). The intensity of the velocity of these particles decreases with a continuous *gradation* from the first to the third element: the first one is composed of very small fluted particles whose motion is extremely quick; the second one by small circular particles, that anyway are a little bit bigger and less quick of those composing the first element; finally, the third element is constituted by the biggest and slowest particles. The form of the particles and the different intensity of their speed is the cause of being the first element the luminous, the second the transparent and the third the opaque. Given these presuppositions, one could expect a transcription of all these physical relations into quantitative terms, also considering that many scientists (Galileo is the most famous example) had already given a mathematical form to their physics. Actually, in the *Principia* there is no mathematization of the physical relations (Panza, 2006).

On the Two *Essays*, *La Dioptrique* and *Les Météores*

The two *Essays* were written as appendices to the *Discours de la méthode* to illustrate concrete applications of the theoretical precepts previously exposed. However, *La Dioptrique* and the *Les Météores*, do not give the impression to follow pre-established methodological precepts (Braunstein), rather they show the lively work of the scientist and because of this they are so interesting. The language used by Descartes is the French because these texts also had a practical scope (construction of lenses and telescopes) and therefore they had to be understood by the artisans who, in general, were not confident with the Latin. The purposes of the *La Dioptrique* and of the *Les Météores* are clear. It is possible to identify four conceptual centres, whose treatment is based on rather diversified methodological approaches (see **Figure 12**):

Since the second conceptual centre is the most significant from a historical-scientific point of view and it is the one in which Descartes follows explicitly a quantitative approach, we will address two themes treated there: 1) the law of refraction; 2) the rainbow. We will see that the approach is completely different from that connoting the *Principia*.

Reflexions on the Law of Refraction

In the second *discours* of the *La Dioptrique* Descartes (see **Figures 13(a)** and **(b)**) determines the law of refraction.

The main purpose of the *Dioptrique* was the improvement of optical instruments. To this end, Descartes derived the sine law of refraction by analogy with the inflection of the motion of a tennis ball upon entering water[40].

Let us imagine (**Figures 13(a)** and **(b)**) that a ball *K* is thrown from *A* to *B* and that it meets the surface of the cloth *CBE* in *B*. If one supposes that *K* has a sufficient power to break the cloth, then the ball will continue its movement beyond the cloth, losing a certain fraction of its velocity—let us suppose half of the initial velocity—Descartes claims that, if the *determination* (Descartes, 1897-1913, VI: p. 97, line 14 and following pages) of the movement is decomposed into two components, the one parallel and the other perpendicular to the cloth, only the perpendicular component will be modified by the encounter with the cloth, while the parallel will not be. If now the three perpendiculars *AC*, *HB*, *FE* to *CBE* are drawn, so that $HF = 2AH$, the ball will reach the point *I* of the circumference of radius *AB* in a time which is the double to the one needing to cover the part *AB*. A questions arises: *how is it possible to determine the point I*? The ball maintains its *determination* to proceed in the horizontal direction, therefore it will cover a double space in a double time in the direction parallel to the cloth. Thus, the point *I* is the one whose projection *BE* on the cloth *CBE* is the double of *CB*. Descartes argues that if a means is posed at the place of the cloth, that, as the water, opposes a major resistance to the motion of the ball than the air (supposed to be over the water), then the law determining the change of the ball motion in the passage from one mean to the other one, is the same as the law determining the passage of the ball between two portions of the same means separated by the cloth.

Let us now suppose (**Figure 13(b)**) that the ball, once reached *B* (let *t* be the time needed for the passage from *A* to *B*) does not miss its velocity[41], but rather receives a push so that the velocity increases by 1/3. In this case, if we carry out a construction analogous to the previous one, the ball will reach the point *I* in a time equal to $2/3t$, so that the projection *BE* on the separating surface is equal to $2/3CB$. This depends on being the velocity along the horizontal *determination* unmodified. Descartes claims that the action of light has the same behaviour as the motion of the ball (Buchwald). Hence, if a light ray starts from a less refracting means and reaches a more refracting means (according to Descartes, this happens, for example, in the passage from the air to the water) the component of motion *determination* parallel to the separating surface will remain unmodified, whereas, according to the nature of the two means, the perpendicular component will be modified. Therefore as in the following **Figure 14**:

The ratio between segments as *KM* and *NL* is invariant, namely

$$\frac{KM}{NL} = \frac{AH}{gI} = \ldots$$

[34]Dijksterhuis, [1950] 1977: p. 71, see also pp. 60-82. The translation is ours. Still interesting is Dijksterhuis, 1961.

[35]Descartes, 1897-1913, VIII: pp. 52-53. The translation is ours.

[40]Darrigol 2012, Chap. 2. (Author's italic). For our aim, in this highlightly book an interesting account concerning Newton's optic is presented (*Ivi*, Chap. 3).

[41]In this context Descartes often speaks of *force de son mouvement* (a kind of motion force: Descartes, 1897-1913 VI: p. 100, lines 1-2) and also uses the word *vitesse* (velocity). In our specific case, the translation of *force de movement* with velocity does not look to betray Descartes' thought. *Force de movement* looks a concept similar to quantity of motion, but since the mass is an invariant in the interaction described by Descartes, the translation velocity looks appropriate. In this case, too, the lack of scientific concepts and of a language universally codified makes these notions similar to various post-Newtonian concepts in the history, but not perfectly identifiable with them.

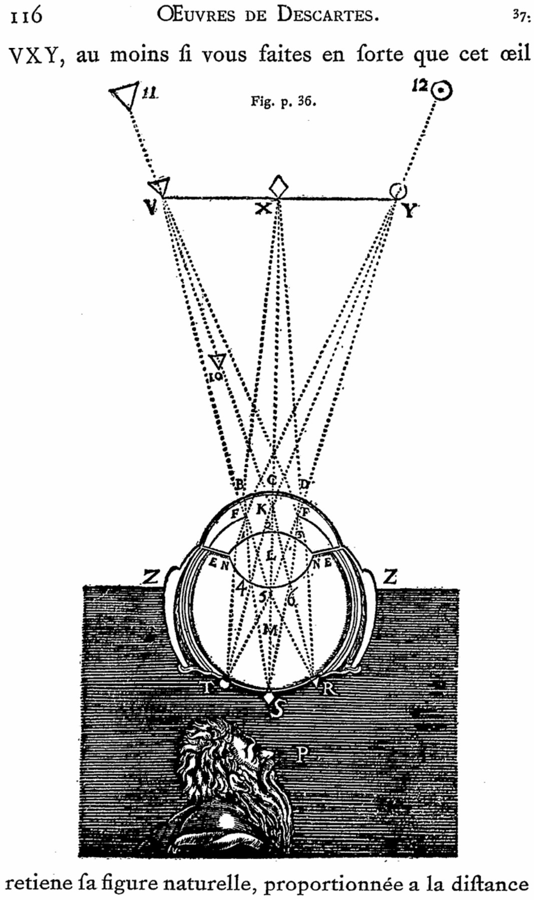


Figure 12.
The mechanism of the vision according to Descartes[37].

1) The first *discours* of the *La Dioptrique* poses, in substance, the bases for the prosecution of the treatment. Descartes carries out some considerations on the nature of the light (in part they are used and developed in the successive *discours*), but specifies that in the Essays he will deal with the problems how the light is spread rather than what is its nature[36]. Given these aims, Descartes explains he will limit to illustrate the easiest manner to conceive the light in relation to the phenomena he has to clarify. He will rely upon experiences and hypotheses, as the astronomers do in order to describe the motions in the skies. Therefore the first *discours* of the *La Dioptrique* represents the true methodological introduction to the two "physical" Essays rather than the *Discours de la méthode*; 2) The second conceptual core, which is the broadest and the most important, includes the *discours* II-IX of the *La Dioptrique* and the *discours* VIII-X of the *Les Météores*. Here Descartes faces, in the *La Dioptrique*, the theme of the refraction, of the form of the eye and of the vision-mechanism, of the properties of the lenses and of the most suitable form the lenses must have to reach their purpose (correction of sigh-defects, magnification of the objects, and so on). In the *discours* VIII, IX and X of the *Les Météores* Descartes exposes the theory of the rainbow and of the parhelions. The treatise is developed in a quantitative form. The author resorts to the experiments and reaches to explanations of phenomena that, even if not correct or based upon correct presuppositions, provide anyway a substantially perspicuous picture of the phenomena; 3) The third conceptual core includes the 10th *discours* of the *La Dioptrique*. This core could be defined the *practical* one because Descartes proposes projects of machineries to construct optical instruments with as most perfect as possible lenses. This is an interesting document of history of scientific technology (even if almost no one of the projected machines was built); 4) Finally, the initial seven *discours* of the *Les Météores*, that concern subjects as the nature of the winds, the clouds formation, the causes of the precipitations, etc., have—in comparison to the other parts of the Essays—a style which is nearer to the one used by Descartes in his *Principia*. Actually, subjects dealt with in an original manner are not missing, as it is the case in the sixth *discours* with regard to the form assumed by the snowflakes.

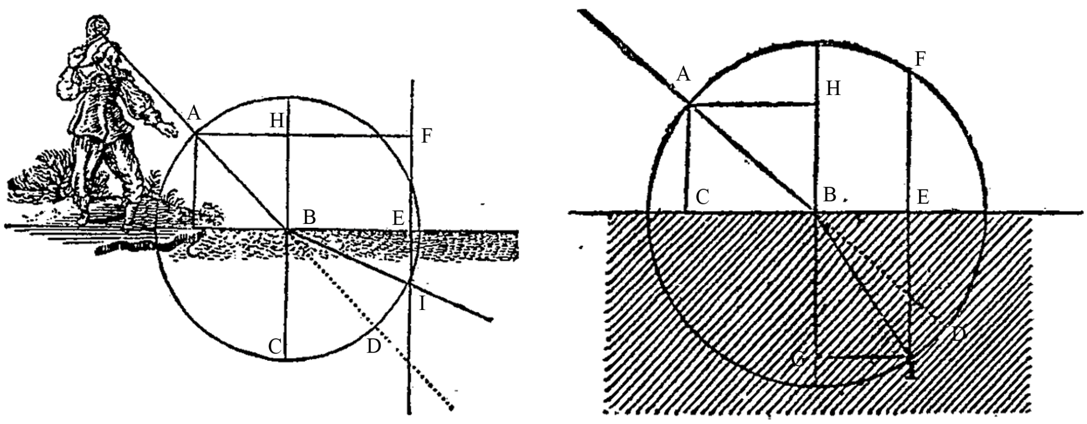


Figure 13.
(a) Analogy between the movement of a ball and a light ray[38]; (b) The ball in B receives a push. Analysis of the consequences[39].

[36]The reference text as to Descartesian ideas on light is *Le Monde ou le Traité de la Lumière*, published posthumous in 1664. The chapters 13 and 14 are those specifically dedicated to light. Numerous paragraphs of the third part of *Principia* concern the nature of light inside the context of Descartes' theory of matter (Descartes, 1897-1913, VIII) Descartes tries to explain what light is, what its effects are, how the stars irradiate. The literature on Descartesian theory of light is huge and it is impossible to provide even general indications. Recently for a very good and definitive history of optic see Olivier Darrigol (Darrigol, 2012, and all references cited). The Essays constitute the second volume of the Italian translation of Descartes' scientific works (Descartes, 1983). The editor Lojacono added an adequate list of references and suggesting (Descartes, 1983: pp. 95-110; see also: Sabra, 1967; Tiemersma, 1988; Malet, 1990; Armogathe, 2000; Schuster, 2000; Shapiro, 1974; Schuster, 2013).
[37]Descartes, 1897-1913 [*La Dioptrique*, V *discours*], VI; p. 116.
[38]Descartes, 1897-1913 [*La Dioptrique*, I *discours*] VI; p. 91.
[39]Descartes, 1897-1913 [*La Dioptrique*, II *discours*] VI; p. 100.

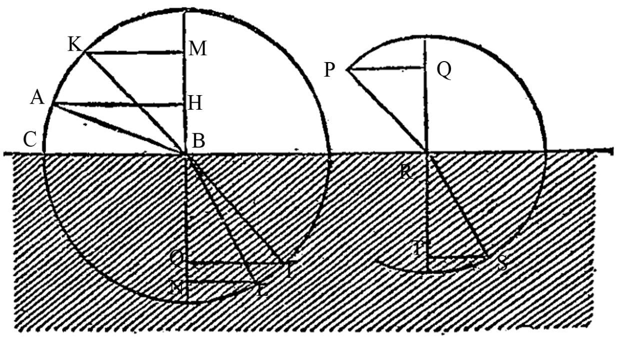


Figure 14.
Determination of the motion and its components[42].

These previous segments are the *sinus* of the incidence and of the refraction angles respectively. The law of refraction is, thus, formulated like this: *the ratio between the sinus of the two angles is a constant and depends on the refraction index of the two means* (Descartes, 1897-1913, VI: pp. 101-102).

The argumentation proposed by Descartes does not claim to be a demonstration of the refraction law, rather an explanation that makes it deductively plausible. Nevertheless, there are many questions concerning the picture proposed by Descartes:

1) What is exactly the *determination* of a motion?

2) Can the analogy of the ball that perforates the cloth (see **Figure 13**) be legitimately extended to the light rays?

3) Does the nature of light have an influence on the refraction law?

These questions are, as a matter of fact, doubts on the legitimacy of Descartesian argumentation. On the other hand, Newton represents a conceptual and linguistic line of separation for physics because the concepts used in pre-Newtonian physics were—in general—not defined. One holds on the common use of the words, or one oscillates between the common use and forms of specification that were not always univocal or even mutually coherent[43]. Therefore no surprise if Descartes did not define the concept of *determination*. In any case, the *determination* does not look *tout court* identifiable with the direction of a movement because Descartes uses the word *direction*, too, when he speaks of movement. The concept of *determination* has been studied for a long time by Descartes' scholars (many in: Mclaughlin, 2000) because undoubtedly it is difficult to enucleate. It is maybe possible to think that Descartes intended by *determination* the tendency of a body to reach the points of its actual direction. These point are really reached or would be reached if no impediment subsists. In the example of the second *discours* of the *La Dioptrique*, the ball has a *determination* towards *D*, however this point is not reached because of the impediment of the cloth. The *determination* would hence be a tendencies inherent to the motion of the body, while the direction is a geometrical line. This is an interpretation because the concept of *determination* remains, in any way, problematic.

As to the analogy between motion of the ball and action of light, Descartes assumes it without any further discussion and justification. He underlines anyway that this analogy is not complete because the ball is deviated far from the normal to the surface of the cloth by the cloth itself, while if a light ray passes from a less dense to a more dens means, the ray approaches the normal, as well known. This brought Descartes to the wrong conclusion that, given two means with different density, light, in its movement, encounters less resistance in the more dense means. However, according to Descartes the light propagates instantaneously in every means. Therefore one cannot claim that, according to Descartes, the light speed is major in a dense means rather than in a less dense means. Rather Descartes explains that since light is "[…] an action by a very subtle matter that fulfils the pores of the other bodies"[44]. Such action is hindered by more "soft" bodies, as air, rather than by less "soft" bodies, as water. Hence, as light encounters less resistance to spread in the water rather than in the air, this is the reason why in the passage air/water light approaches the normal.

The conceptual equipment used by Descartes to determine the refraction law is hence tied to notions that are not always well defined (as the one of *determination* of a motion), to analogies and to wrong ideas; despite this the formulation of the law is correct[45]. This induces us to think that the whole equipment exposed to the reader in the second *discours* of the *Dioptrique* is not directly connected to the way in which Descartes discovered the refraction law. Rather it looks to have the aim to convince the reader and to frame optics inside the mechanistic project Descartes had already in his mind when he wrote the Essays. In a brilliant and profound paper Schuster underlines that:

> Descartes was willing to try to ride out likely accusations that the premises are empirically implausible, dynamically ad hoc, and in some interpretations, logically inconsistent, because the premises provided elegant and more or less convincing rationalisations for the geometrical moves in his demonstration[46].

The premises were confused and wrong, but the model was elegant and worked. Schuster produces convincing evidences in favour of the thesis that the refraction law was ideated by Descartes through an itinerary based upon his studies of geometrical optics. If this is true, the law was deduced independently of dynamic considerations added by Descartes in a second time.

Conceptual Streams behind Descartes' Law of Refraction

The physical-geometrical core of Descartesian argumentation can hence be connected to the idea of decomposing a

[42]Descartes, 1897-1913 [*La Dioptrique*, II *discours*], VI: p. 101.

[43]In this sense, a classical example is the concept of force. Many scholars used it in the 16th and 17th centuries. There is an abundant and interesting literature on this subject, that allowed—in great part—to clarify how different authors used this term. In this case, too, before Newton had given his definition of force, this word did not have a univocal meaning.

[44]"[…] une action reçue en une matière très subtile, qui remplit les pores des autres corps […]" (Descartes, 1897-1913,VI: p. 103, lines 13-14). The translation is ours.

[45]We do not enter here into either the problem concerning the relations between Descartes and the other authors who, substantially, had understood refraction law, as Willebrord Snel van Royen called Snellius (1580-1626), Claude Mydorge (1585-1647) and Thomas Harriot (1560-1621) or the fundamental role Johannes Kepler (1571-1630) had in this studies on refraction (Pisano and Bussotti 2012; see also Malet 1990). The notes to the first and second *discours* of the *Dioptrique* (Descartes 1983) are thorough in this regard. See also Schuster 2000.

[46]Schuster, 2000: p. 271. Particularly Schuster has recently published an important contribution to the mechanistic Descartes' conception (Schuster 2013).

movement in two mutually perpendicular components. Descartes could hence imagine to decompose the motion of a light ray into these two components, without introducing the mechanical analogy of the ball or the concept of *determination*. His convictions on the nature of light induced him to introduce these notions. There was no necessity connected to the physical-geometrical argumentation to do that because the argumentation itself would have lost nothing of its validity without the mechanical analogy of the ball.

The further element to take into account is that Descartes led many experiments concerning the refraction and optics in general. In order to prove this, three examples are indicative: a) at the beginning of the third *discours* of the *La Dioptrique*, the experiment on the way in which eye forms the imagines (Descartes, 1897-1913, VI: pp. 105-106. The problem of the vision is further specified in the fifth and sixth *discours*, of the same work, pp. 114-147); b) in the tenth *discours* of the same text the affirmation that, in order to establish the most suitable form for a hyperbolic lens and the best position for its focus "[…] experience will teach better than my reasoning"[47]. Thus Descartes realized that the "[…] exact proportions are not so necessary that they cannot be changed a little bit"[48]. Hence, in this case, geometry is a guide for the form of the lens, but it does not determine such form in an absolute and univocal manner; c) the experiments with an ampoule full of water (VIII *discours* of the *Les Météores*) to comprehend the rainbow phenomenon. The experiments in optics are hence a fundamental aspect of Descartes' works. The genesis of the discovery of refraction law can perhaps summarized this way:

1) Descartes knew the tradition of geometrical optics studies;

2) He had realized—and this is a great, even if not exclusive, merit of his—that the physical phenomena can be understood only if quantities that remain invariable are determined;

3) He had carried out a plurality of experiments. All these facts brought him to intuit and to formulate the refraction law in a correct way. The other argumentations we have seen, were introduced because of philosophical convictions and to make the law plausible, but they do not play a role in the discovery of the law and—it is necessary to add—they are extraneous to the nature of the phenomenon.

The situation for the case here analysed is far different from that of the *Principia*: the refraction law is a paradigmatic example of a reasoning in which the mathematical apparatus is poor, but an easy formalization of Descartes' reasoning shows its consistency and correctness. In fact, it is enough:

1) To decompose the motion of light in a vectorial form[49], according to the parallel and perpendicular components to the incidence surface.

2) To use a symbolic notation to indicate the angles.

3) To introduce the concept of incidence and refraction angle. All this is clear in Descartes' treatise, even if the reasoning is not completely symbolized. Therefore a coherent quantification is possible, while it was not the case with the collision rules introduced in the *Principia*.

The *La Dioptrique* shows that a mathematical structure exists at the basis of Descartesian reasoning: let us consider the VIII *discours*, where Descartes exposes the focal properties of the parabolic and hyperbolic lenses. Furthermore, in the last part of the second book of his *Geometry*, Descartes extends the study of reflection and refraction to the oval lenses (Descartes, 1897-1913, VI: pp. 424-441). The treatment is, in this case, highly formalized and completely expressed in mathematical terms. Descartes carried out experiences and experiments. He mathematized the results and proposed explicative models based on the quantification of the phenomena. Because of this, the role of Descartes in the scientific turning point of the 17th century is relevant. The concepts of *gradation* and intensity represent an interesting instrument through which Descartes' ideas on refraction can be interpreted. First of all, *gradation* is the basis of refraction itself: if the different transparent materials had not different refraction indices, the phenomenon itself would not exist. Therefore gradation of the refraction indices represents the basis of this optical phenomenon. Since every material has its own index, it is possible to construct a graduated scale: the refraction indices represent the intensity with which every material refracts light. Descartes, by discovering the exact form of refraction law, made the intuitive idea that the materials have different refraction powers perspicuous. In this manner he ideally established a scale of *gradation*, even thought Descartes ideas that most dense materials also are the most refracting is wrong.

The Rainbow

The eight and most important *discours* of the *Les Météores* is dedicated to the rainbow (Maitte, 1981, 2006; Ronchi & Armogathe, 2000).

The way in which Descartes faces the rainbow problem presents an excellent epistemological model for the genesis of the scientific discovery. It is also indicative of the non univocal manner in which Descartes addressed the problems of physical arguments. The most significant aspects are three:

1) The use of experience to catch the properties of the phenomena;

2) Scientific quantification of the reasoning to obtain perspicuous results;

3) Elements of Descartes' mechanistic conceptions that influenced his rainbow theory.

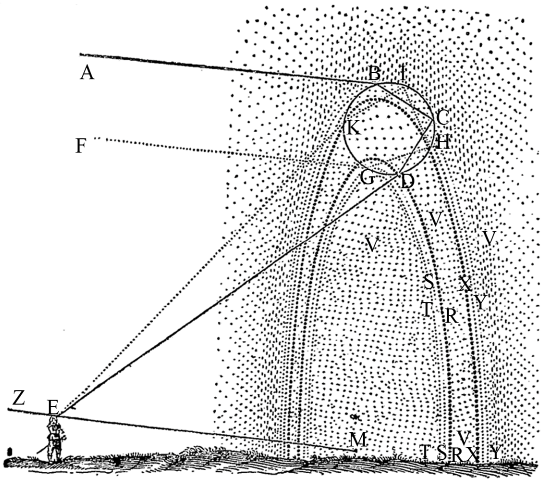
From the beginning, Descartes resorts to experience in an appropriate manner (Descartes, 1897-1913, [VIII *discours*], VI: pp. 325-327). In an initial phase of his work, his purpose is to realize, in a qualitative manner, what the invariants characterizing the rainbow phenomenon are: since rainbow is visible not only in the sky, but also, for example, in the fountains in which the water is illuminated by sun rays under particular conditions depending on the way in which the sun rays hit the drops of water in respect to the observer, Descartes reasoned this way (see **Figure 15**, on the right our paraphrase).

This means the rainbow is not necessarily connected to atmospheric events as the rains. Furthermore Descartes observes that the size of the water drops does not have any influence on the phenomenon. He remarks that if the ampoule is raised and suspended by a machinery, which is not described in the text, but that can be easily imagined, the conclusion is the following: a sun ray hits the ampoule (that is the drop of water) in *B*, it is refracted by the water in *C*. From here it is reflected in *D*, from

[47]"[…] l'experience enseignera mieux que mes raisons" (Descartes, 1897-1913, VI: p. 202, lines 9-10). The translation is ours.

[48]"[…] proportions ne sont pas absolument nécessaires, qu'elles ne puissent beaucoup être changées". (Descartes, 1897-1913, VI: pp. 201-202). The translation is ours.

[49]Even if, probably, Descartes did not catch the concept of vector in its generality, he himself proposed to decompose the *determination* of a motion into two mutually perpendicular components, as we have seen.



[…] this arc [the rainbow] can appear not only in the sky, but also in the air near us, every time there are many drops of water illuminated by the sun, as the experience shows in some fountains. Therefore I established easily that the rainbow depends only on the way in which the light rays act on the drops and on the inclination with which the rays reach our eyes from the drops[50]. If *AB* or *ZM* indicate the direction Sun-eye, when the angle *DEM* is about 42 degrees, then a brilliant red appears in the part *D*. Such colour continues to be present whatever is the movement of the ampoule, as long as the angle *DEM* remains 42 degrees. As soon as this angle is increased, even of a very small quantity, the red disappears. While, if the size of the angle is reduced, the red pencil of light is divided into less brilliant pencils in which the other colours of the rainbow appear. If the size is further diminished, every colour disappears. However, when the angle *KEM* is 52 degrees, the zone *K* is illuminated by a red, that is less brilliant than the one present in *D* when $D\hat{E}M$ is 42 degrees. If the angle *KEM* is made broader, the other colours appear in zones as *Y*. These colours have a minor intensity than the red in *K*. If the size of the angle is either slightly diminished or made it much bigger, every colour disappear. It is likely that at this stage, Descartes had already understood the role of reflection and refraction in rainbow genesis. However, to have a confirmation he carries out the following experiment: he poses an obscure and opaque body in one of the points of the lines *AB*, *BC*, *CD* e *DE*. He remarks that the red colour disappears. While, if the whole ampoule is covered, excluded the points *A*, *B* e *D*, and no obstacle disturbs the action of the rays *ABCDE*, the red continues to be present.

Figure 15.
Explanation of the rainbow in the *Les Météores*[51].

where it is refracted to the observer in *E*. Consequently the red appearing in *D* is given by two refractions and one reflection. The red in *K* of the second rainbow is given by one refraction of the ray in *G*, followed by one reflection in *H*, a further reflection in *I* and a refraction in *K* until the ray reaches *E*. Since there are two reflections and two refractions, the red is less intense. In this way, the general nature of the phenomenon is explained. Two questions are still to be answered:

1) Why does the rainbow appear when the angles DEM and KEM are respectively 42 and 52 degrees?

2) What is the cause of the rainbow colours?

Descartes answers the first question through the following very acute reasoning: let the drop of water be represented by the circumference (see **Figure 16**).

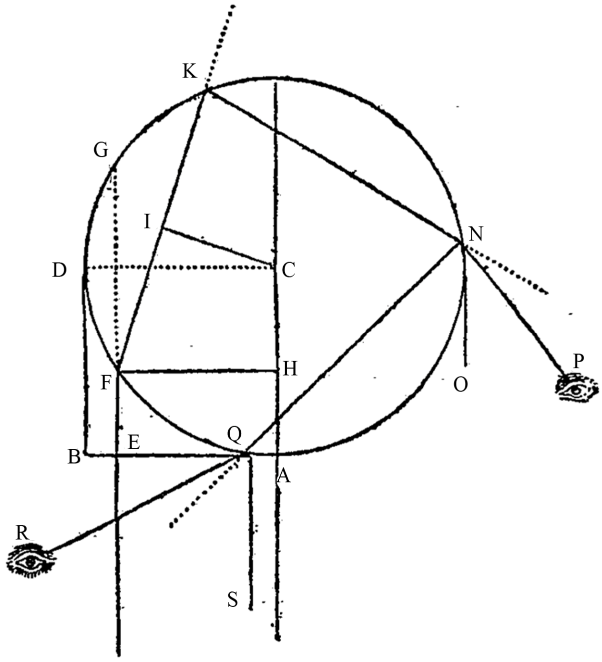
In the picture traced by Descartes there are many elements that characterize a great part of the scientific discoveries:

1) The use of the experience to achieve a global qualitative vision of the studied phenomenon.

2) The resort to the experiment having in mind not only the questions, but also a series of possible answers.

3) The quantification of the data and resort to the demonstration to explain the phenomena in a perspicuous way.

The concepts of *gradation* and *intensity* represent once again



Let *F* be the point of the drop in which the solar ray strikes. Let this ray be refracted in *K*, from *K* reflected to *N* and from here refracted towards the eye in *P* or reflected to *Q* and from *Q* refracted towards the eye in *R*. This figure is hence a model of the first and of the second rainbow. Traced the perpendicular *CI* to *FK* from the centre *C* of the circumference, it results that $\frac{HF}{CI}$ is the ratio between the refraction indices of the air and of the water. In fact, let us trace the radius *CF* and the tangent at the circumference in *F*, in the triangle *FCC'*, the sine of the angle *C'FC* is *CC'* = *FH*, but *C'FC* is equal at the incidence angle and *CFI* is the angle of refraction. Since the refraction index water-air is known, the ratio of *FH* to the radius *CD* is known. It is therefore possible to determine the ratio of *IC* with these two quantities. Thus, it is possible to establish the size of the arcs *FG* and *FK* hence to calculate the angle *ONP*. If the position of the point *F* in which the solar ray strikes the drop varies, the angle *ONP* will vary in a way that can be calculated. The calculation shows that, when *F* varies—it is enough to limit the analysis at the quarter of circumference *AD*—the rays that come out with an angle *ONP* of about 40 degrees are more numerous than the rays that come out with other angles. This explains why the first rainbow is visible when the angle *DEM* in figure 9 is about 42 degrees. In an analogous way, it is possible to prove that most part of the angles *SQR* are about 52 degrees. This explains the second rainbow.

Figure 16.
Geometrical model of the rainbow in the *Les Météores*[52].

[50]"[...] cet arc ne peut pas seulement paroistre dans le ciel, mais aussy en l'air proche de nous, toutes fois & quantes qu'il s'y trouve plusieurs gouttes d'eau esclairées par le soleil, ainsi que l'expérience fait voir en quelques fontaines, il m'a esté aysé de iuger qu'il ne procède que de la façon que les rayons de la lumière agissent contre ces gouttes, & de là tendent vers nos yeux". (Descartes, 1897-1913 [VIII *discours*], VI: p. 325, line 10). The translation is ours.

[51]Descartes, 1897-1913 [VIII *discours*], VI: p. 326.

[52]Descartes, 1897-1913 [VIII *discours*], VI: p. 337.

a lens through which the physical phenomenon can be observed and the work by Descartes interpreted: *gradation* and *intensity* are inherent (**Figure 15**) to the angles between the lines *EM*, *ED* and *EM*, *EK*. When the degrees of these two angles vary, the two arcs of the rainbow either subsist with the different colours or disappear at all. The intensity of the colours is a function of these angles, too. The **Figure 16** can be interpreted as a model that provides a geometrical representation of the *gradation* of the angles. The reasoning and the calculation demonstrate why those particular sizes (42 and 52 degrees) are the *critical* ones for the rainbow. Inside a context that, for many aspects can be defined of a *modern physical context*, the angular *gradation* justifies the existence of the rainbow. This *gradation* is connected to the intensity of chromatic *gradation* by an elegant functional link.

For sure Descartes aims to explain the nature of the colours. Thus, he provides an interesting answer, in part based on experiments carried out with an optical prism and in part on his mechanistic convictions. Namely, in his theory of the colours, he takes into account the empirical data, but tries to explain them by means of presuppositions tied to the way in which he conceives the nature of light. If, as to refraction law, the analogy of the balls was not the central core of the argumentation, here the idea that the action of light is transmitted by the particles of the subtle matter is essential for the explanation.

Descartes constructs a mechanic model in which he imagines that a subtle matter composed of little spheres having a determined velocity of translation is present. The reciprocal collisions among these particles and/or the collisions with some other body can modify this velocity and also induce a rotational motion in each single particle. He claims the colours depend on the motions of the particles that constitute the subtle matter and that transmit the action of light. At different velocities of rotation correspond different colours. This idea is brilliant, but (differently from what had happened in the rest of the *discours* on the rainbow) no relation between modelling and physical phenomenon is shown; such relation is only supposed. Furthermore a direct connection between the experiment with prism and the supposed explanation of this experimental result is missing. There is no demonstration. Thus Descartes replaced the facts of the chromatic world with a set of other facts relative to the motion of the supposed particles, but these motions are as difficult to be explained as the colours themselves. The modelling proposed is not easier than the phenomenon because it contains the same number of elements: simply Descartes replaces the facts of a certain *world* with the facts of another world. There is no precise assumption that explains why the world of the particles is, from an epistemological and physical point of view, easier than the chromatic world and justifies hence why the world of the particles should provide an explanation for the chromatic world. Because of this, the model is not explicative. In a sense, the reasoning could be inverted until reaching a supposed explanation of the motion of the particles by means of the colours and not vice versa. The way of reasoning proposed by Descartes in this case is more similar to that of the *Principia* than to the one connoting the rest of the *discours* on the rainbow, even if the form in which the subject is exposed and the fascination of Descartesian speculations give an appearance of plausibility to Descartes' theory of colours. Again, in this case, *gradation* and intensity are also cardinal concepts. When the *gradation* of the velocities and of the motions of the spheres of the subtle matter vary, a corresponding variation of the chromatic scale exists. Therefore the two concepts of *gradation* and *intensity* can provide a good perspective description through which to analyse Descartes' *physical works.*

Conclusion

Notes on Science & Society Civilization

Usually a discussion concerning history of science and technique/technology is presented such as a discipline within the history of science for understanding eventual relationship between science and the development of art crafts produced by non–recognized scientists in a certain historical time. The relationship between science and science and society and consequent civilizing by science is centred on the possibility that the society effetely developed a fundamental organization in capacity to *absorb* science and produce technologies (i.e., water and electrical supply, transportation systems etc.). Thus, *a development civilization was necessary parallel to development of the science within society*? *Is effetely happened that*? *Did Descartesian and Newtonian physical works develop as a response to the needs of society*? Alexandre Koyré (1892-1964) strongly remarked the history of science and the role played by mathematics between Newton and Descartes (Koyré, 1965, Chapter III) in the history of scientific thought. Through the intuition that the fundamentals of scientific theories contain two basic choices, Koyré' intellectual matrix (Pisano & Gaudiello, 2009, 200b) has been cleared up.

> The new science, we are told sometimes, is the science of craftsman and engineer, of the working, enterprising and calculating tradesman, in fact, the science of rising bourgeois classes of modern society. There is certainly some truth in this descriptions and explanations […]. I do not see what the *scientia activa* has ever had to do with the development of the calculus, nor the rise of the bourgeoisie with that of the Copernican, or Keplerian, astronomy theories. […] I am convinced that the rise and the growth of experimental science is not the source but, on the contrary, the result of the new *theoretical*, that is, the new *metaphysical* approach to nature that forms the content of the scientific revolution of the seventeenth century, a content which we have to understand before we can attempt an explanation (whatever this may be) of its historical occurrence[53].

> [...] I shall therefore characterize this revolution [the birth of the modern science] by two closely connected and even complementary features: (a) the destruction of the cosmos and therefore the disappearance from science—at least in principle, if not always in fact—of all considerations based on this concept, and (b) the geometrization of space, that is, the substitution of the homogeneous and abstract—however now considered as real—dimension space of the Euclidean geometry for the concrete and differentiated place-continuum of pre-Galilean Physics and Astronomy[54].

According to the Russian historian[55] we can consider that: 1) the history of scientific thought has never been entirely separated by philosophical thought. 2) the most important scientific revolutions have always been determined by a replacement of

[53]Koyré, 1965: pp. 5-6.

philosophical speculations. Thus i.e., the history of scientific thought (i.e. for physical Descartesian and Newtonian sciences) has not developed by *vacuum*, but it moves in a set of ideas, foundational principles, or axiomatic evidences.

Final Remarks on Descartes' *Physical Works*

Particularly in this paper we have highlighted the two different ways in which 3) Descartes developed his physical research-frames in his *Physical Works*: a) typical of an essentialist and aprioristic way of thinking, b) based on experiments and mathematization. The differences between, the physical *Essays Dioptrique* and *Météores*, and the *Principia*, concern both the content and the methodological aspect. These differences regard the way in which the scientific work is addressed. The approach of the *Essays* can be epistemologically interpreted:

1) Descartes presents his experimental and theoretical work as a scientist.

2) He realizes that science and technique have deep connections. Therefore he had the idea to address the essays basically to the artisans.

3) Descartes had close relations with export artisans as Ferrier. He fully understood the role that science could play in the construction of machines.

4) Descartes thought machines were fundamental for the future of mankind.

5) The mechanistic convictions of Descartes are apparent from many passages of the two *Essays*, even if these conceptions do not play a fundamental role for the discoveries exposed in the *La Dioptrique* and in the *Les Météores*.

The *Essays* present hence Descartes as a producer of the scientific work. On the basis of his whole scientific work, likely Descartes was one of the first scientists to have the idea that, in a physical theory every phenomenon must be explained on the basis of a precise law. However, the perspectives of the scientific work were not completely rosy:

1) In Descartes' epoch it was already clear that experiments were the basis of physics and they would have been still more in the future;

2) The costs for the research were increasing more and more. From here the necessity of financial supports;

3) In Descartes' time, ecclesiastical censorship continued to represent a problem;

4) Descartes was profoundly surprised by Galileo's conviction. He wrote in a letter to Mersenne in November 1633:

> In fact, I cannot imagine that he, who is Italian and well-liked to the Pope himself—as far as I know—was considered a criminal for no other reason but he wanted to establish the earth movement. I am aware that this conception was censured by some cardinals. However—as far as I remember—I had heard that it had continued to be taught in Rome itself[56].

Therefore in the first half of the XVII century the social and political situation was difficult for the scientists. In fact, new discoveries were emerging with a rhythm far more rapid than in the previous centuries, but, at the same time, the financial supports to develop research depended on the power holders and not on the scientists themselves. The aim of the power holders was of course to use the science for their scopes. Furthermore the ecclesiastic censorship was strong. If we take into account this picture, it is perhaps possible to understand the approach of the *Principia*. This book is the expression of personal Descartes' physical and metaphysical ideas (as the mechanistic conception) and of his desire to become—despite his declaration in a contrary sense—a sort of new scientific authority. From here the encyclopaedic character of the book arises, whose intention is to face all problems of physics. However, at the same time, the *Principia* can be interpreted as a book profoundly influenced by the social situation we have rapidly outlined. This situation is also connected to the most convenient way to present science. Descartes wanted to make science acceptable to the Church and to the power people with whom he would have had contacts. In this manner, for example, the paragraphs III, 16-19 of the *Principia* can be explained, where the validity of the systems of Ptolemy (II century AD), Nicolaus Copernicus (1473-1543) and Tycho Brahe (1546-1601) is denied. Descartes shows that, in his own system, the earth has to be considered, as a matter of fact, at rest. This assertion can be interpreted as an insurance for Catholic Church. The insistence on the fact that, in determining physical laws, deduction is far more important than experiments, is one of the most evident ideas expressed in the *Principia*. This idea is in part coherent with what Descartes really thought, but in part it has the aim to show that experiment—namely a *fine* enquire on the world that can modify the world itself—with its potentially *subversive* value, has a secondary importance. In other terms: science is not dangerous from a social point of view and can hence be accepted by the power holders. As a matter of fact, we have seen that, when Descartes produces science, he cannot renounce to experiment. The *Principia* can hence been interpreted as the text that concludes a phase of the scientific revolution, namely the phase in which the social role of the scientists was not yet clear. The scientists were not yet, in every aspect, institutional figures, as they became in the second half of the 17th century[57]. Because of all these reasons, the *Principia* are a text that presents a relevant historiographic interest. It is not always easy to distinguish what Descartes wrote to justify the scientific work and to make it acceptable to whom could valuate this work potentially dangerous from a social point of view from what he wrote for a real conviction. Finally, main comparisons between Descartes's and Newton's conceptions we carried out summarized by the following table as relevant dissimilarities between these two scientists, with regard to the subjects deal with in our paper (see **Table 1**).

[54]Koyré, 1965: pp 6-7. In the following explaining of Alexandre Koyré' choice for the history of science: "*The destruction of the cosmos*" that is a replacement of the finite world, as it had been hierarchically classified by Aristotle, with the infinite universe. "*The geometrization of space*": that is a replacement of Aristotle' physical (concrete) space with the abstract space of the Euclidean geometry. (Pisano & Gaudiello, 2009a, 2009b).

[55]A conference (1954, Boston) of *American Association for the Advancement of Science*. Cfr.: *The scientific Monthly*, 1955; Koyré, 1971.

[56]"Car ie ne me suis pû imaginer, que luy qui est Italien, & mesme bien voulu du Pape, ainsi que I'entens, ait pû estre criminalizé pour autre chose, sinon qu'il aura fans doute voulu establir le mouvement de la Terre, lequel ie sçay bien auoir elle autresfois censuré par quelques Cardinaux; |mais ie pensois auoir oüy dire, que depuis on ne laissoit pas de l'einseigner publiquement, mesme dans Rome" (Descartes, 1897-1913, I: p. 271, lines 2-9). The translation is ours. Moreover, we remark that a possible problem with ecclesiastical censorship induced him to avoid the publication of *Le Monde*, as he wrote in the same letter (Descartes, 1897-1913, I).

[57]By concerning the social and political situation in 17th century and the relations with science, one can see the following works: Heilibron, 1979; Dear, 1995, 1987; Kokowski, 2004; Gorokhov, 2011.

Table 1.
Descartes and Newton's main arguing.

Descartes	Newton
General Conceptions	
A) In the *Principia Philosophiae* Descartes exposed his mechanistic conception. An attempting to unify the whole physical world on the bases of few principles and rule is present; the treatment was not mathematized and accurate definitions are missing. The geometric model proposed provided weak indications about the positions of the bodies in function of the time. Important laws are introduced as the inertia law and the law of the conservation of movement. The main reference are the *Principia Philosophiae* (Descartes, 1897-1913, Inertia law, VIII-1, II part, § XXXVII: pp. 62-63; the conservation of movement: *Ivi*, § XXXIX and § XL: pp. 63-65). Nevertheless, since a mathematical and definitional apparatus is missing, it is difficult—and probably in part wrong—to interpret Descartesian conception of *inertia* and *quantity of motion* as in the Newtonian and post-Newtonian physics. B) In the Essays, *La Dioptrique* and *Les Methéores* Descartes deals with specific problems connected to reflection and refraction. Here the proposed models are mathematical or, at least, can be easily acceptably mathematized; demonstrations are presented. The refraction-law is expressed (Descartes, II Discours of the *La Dioptrique* in: Descartes, 1897-1913, VI: pp. 93-105).	A) According to Newton, a mechanical model has to foresee the positions of the bodies through mathematical relations between the space variable and the time variable. In general, a physical model must supply precise laws and deductions expressible in a mathematical form (for example in the Author's Preface, Newton writes: [...] and then from these forces, by other propositions, which are also mathematical, we deduce the motions of the Planets, the Comets, the Moon and the Sea" (Newton, [1713] 1729, I, Preface: p. A2; see also *Ivi*, II, *General Scholium*: p. 392) Definitions (8), *axioms* or *laws* (3) are exposed in the initial section of the *Principia* (Newton, [1713] 1729, I, Definitions: pp 1-18). *Axioms* or *laws* plus their corollaries: pp 19-40). For the first time a physicist feels the need to provide definitions of the quantity he is dealing with. B) Generally speaking the structure of *Principia* looks Euclidean, but, Newton introduced his apparently abstract formulations in order to explain, from a unitary point of view, physical phenomena.
Connection theory-experience	
A) In the *Principia Philosophiae* there is an insufficient connection between theory and experience. In some cases—as in the one of the movements of the planets—there are intrinsic difficulties to connect theory and experience because no provisional model is supplied, but only a descriptive one. In other cases, as the collision rules, experience is not coherent with theoretical rules (Descartes, 1897-1913, VIII-1, second part, §§ XLVI-LII: pp. 68-70). Descartes ignore the experience, claiming that many conditions can influence the experiments and the experience (*Ivi*, § LIII: p 70). But a *critics of experience* is lacking. B) In the *Essays* the experience and the experiments play a fundamental role. Descartes analyzed many empirical details and explains them through the theory. The experience and the experiments guided him to develop his theory (See i.e., how Descartes focused on the experience of the rainbow colours to explain this phenomenon in *Les Météores* (Descartes, 1897-1913, VI: pp. 325-344).	A) The experience and the experiments are the bases of Newton's physics. He is explicit in the *Optiks*, where a plurality of experiments are presented and the theory is clearly constructed to provide a model to the phenomena deriving from experiments (Newton, [1704] 1730: p. 1). A profound *critics of experience* is *implicitly* presented because Newton specifies the experimental conditions and the effects that could perturb the results of the experiments (Newton, [1713] 1729, II: pp. 202-205). B) The *Philosophiae Naturalis Principia Mathematica* (Newton, [1713] 1729, et editions) too, have their source of inspirations in the phenomena (no really experimental) and in the attempt to explain and foresee the phenomena; i.e, the second section of the third book of the *Philosophiae Naturalis Principia Mathematica* is titled *The Phaenomena or Appearances* (Newton, [1713] 1729, III: pp. 206-212)
Fundamental concepts: space, time, mass	
A) In the *Principia Philosophiae*, the space cannot be distinguished from *res extensa*. It is always relative (Descartes, 1897-1913, VIII-I, II part, Question X: p. 45).The time is relative (Descartes, 1897-1913, VIII-I, I part, questions LVI-LVII: pp. 26-27). B) It is known that a scientific (i.e., by magnitudes) distinction between mass and weight was problematic at that time.	A) The absolute time exists (before than in the famous general *General Scholium*, Newton spoke of absolute space and time in the *General Scholium* posed as a conclusion to his Definitions. (Newton, [1713] 1729, I: pp. 9-18). Both were introduced also as an answer to the problems present in Descartes' physics explained by Newton himself (Newton, [1713] 1729, II, *General Scholium*: pp. 387-393). B) The mass is clearly distinguished by the weight (*Ivi*, definition I: p. 1). Even if Newton's concept of mass can be criticized for well known reasons, the concept of mass was (within his physical mathematical system) reasonable at that time.
Problems connected to the cultural environment	
A) Descartes had to face a series of problems that in Newton's land and time wee far less serious: 1) Catholic censure of Copernican theory; 2) role of the scientist still not well defined. Power holders could think that scientists were dangerous for the social order; 3) scarce financial support. Since the *Principia* are Descartes' world-system and the most conspicuous manifest of his way of thinking, he tried and intermediation between his ideas and possible dangerous consequences. B) This is one of the reasons why the *Principia* are such a tormented text. In the Essay (*La Dioptrique*, *Les Météores*, *La Géométrie*) dealing with more specific arguments, he did not have these problems and the treatment was clearer and more coherent.	A) Newton did not deal with the problems addressed by Descartes because of: 1) Different period: end of the 17^{th} begin of the 18^{th} century. 2) Different land: England where the Catholic censure was not effective. 3) Newton represents a scientist who is perfectly integrated in the system; rapidly the scientists were becoming persons with public roles and well defined social positions. B) Newton was completely free to publish his works without the problems faced by Descartes. C) Comment: Newton's theoretical conceptions are not *directly* influenced by the social and political environment. He was free, as to the subjects he dealt with, and he had non problems with censure. The influence of the social and political environment was *indirect* as far as it allowed Newton to develop freely his researches.

When a thought does not have clear and delineated scopes, it likely results tormented, often self-contradictory and difficult to frame into an organic picture. Under this point of view, *Le* is different from the *Principia*: in this original and pleasant brief treatise Descartes exposes his mechanistic conceptions and his theory of light, based upon them. But in this case, the scope is clear—independently of the correctness of the basic ideas expressed in *Le Monde*—and the argumentation is linear. Finally, even if, as Scott and Koyré claimed

> Thus Descartes' hypothesis at least has the merit of explaining the nature of weight without recourse to any occult force acting across space. More than that, it is easy to detect in it a groping after a universal law; the mechanism by which a body falls to the earth is in the last resort the same as that which keeps the planets in the solar vortex […][58]

> [...] thought of course unsuccessful, attempt at a rational cosmology, an identification of celestial and terrestrial physics, and therefore the first appearance in skies of centrifugal forces [...][59]

Descartes had in physics the merit to have tried a unique explanation for the gravity on the Earth and for the orbital movements of the planets (unification of terrestrial and celestial physics), this has happened with a form more coherent with a mentality typical of an aprioristic conception of the physics rather than the one connoting an observative-experiment-quantitative approach.

Acknowledgements

We want to express our gratitude to anonymous referees for precious comments and helpful suggestions.

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[58]Scott, [1952] 1987, p. 184.

[59]Koyré, 1965, p. 65. See also *Id*., 1934, 1957, 1961, 1966.

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