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Review article

Kinematic and dynamic modeling of mechanical systems towards Digital Twins

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ABSTRACT

The development of Digital Twins has become a central topic in digital transformation, offering new possibilities for the prediction, control, and optimization of physical systems. In a general sense, a Digital Twin features real-time data exchange between a precise replica of a physical object in the virtual world, and vice versa. The virtual replica of the physical entity is usually referred to as Digital Model. A faithful representation can however affect the computational effort required by the model and consequently the data exchange in real-time. This is why, in recent years, the evolution of kinematic and dynamic models of mechanical systems in Digital Twins is essential but challenging, particularly due to the trade-off between model fidelity and computational feasibility required for real-time integration. This paper presents a systematic literature review of the methods and practices employed in modeling such systems for Digital Twin applications, focusing on tools and simulation environments used, as well as communication and computational challenges. As a final contribution, a theoretical framework is proposed as a guidance in the development of fully integrated Digital Twins for mechanical systems.

Contents

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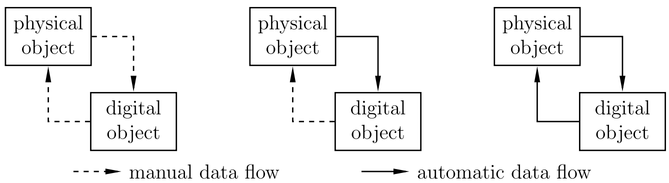

Fig. 1. Graphical representations of DM (left), DS (middle) and DT (right).

1. Introduction

Creating Digital Twins (DTs) to represent physical systems and taking advantages of their benefits for prediction, control, and optimization is a growing practice within the global scientific community [1]. Consequently, technologies supporting this digital transformation are seeing increasing development and enhancement, marking a significant ongoing trend. Broadly said, a DT represents an exact copy of a physical entity, characterized by a real-time information exchange with its physical twin and vice versa [2]. The automatization of real-time information exchange represents the most accurate difference that lays between a Digital Model (DM), a Digital Shadow (DS) and a DT. A clear classification of these three approaches for digital transformation is introduced by [3], differentiating them regarding the level of integration between the physical and digital counterpart. Fig. 1 displays this concept graphically.

Each of the shown approaches is characterized by the presence of a digital representation, called digital object, and of a physical counterpart, which can physically exist or may be in development. In a DM, the data exchange between the real world and the physical one happens manually. A DS relies on an autonomous data acquisition, but still needs the human presence for transferring information, e.g. simulation results or production line measures for optimization purposes, from the virtual world to the real one. Finally, the DT features a fully integrated and automatic data flow, from the real world to the virtual one and vice versa. To do so, three ingredients become fundamental [4]:

- simulation, since the digital counterpart should encompass all the behavioral characteristics of the physical system at any instant of time.
- synchronization, since the digital counterpart should reflect the configuration of the physical system in real-time;
- optimization, since the digital counterpart should enable the physical system to operate at its optimal point thanks to the data exchange;

As previously stated, the concept of DT represents one of the most popularly discussed trends addressed by researchers dealing with the topic of digital transformation. The creation of digital counterparts of physical entities, e.g. manufacturing plants, complex mechatronic systems, etc., brings together a plentiful set of benefits, ranging from prediction speed increase, cost reduction, performance enhancement and many more others [5]. The advantages brought by the enrichment of tools for working in the virtual world, together with their increased accessibility, user-friendliness and, above all, computational power, are currently driving various fields of research in this direction [6]. In fact, it is evident that DT-related technologies are widely and intensively exploited in a number of different fields, ranging from logistics and production, mechatronics and robotics, and so on [7].

It can be stated that the DT represents the proper evolution of the DM, passing through the intermediate step of the DS [8]. A DT can indeed complete the digital transformation of a physical entity, making available a digital counterpart that can self-optimize and reflect the intrinsic features of its physical twin. It is however important to specify that the creation of a fully furnished DT cannot overlook the peculiarities of its related DM and DS, which can be considered as essential steps for a faithful representation of the physical system in exam. As a result, it can be stated that the first and important step for the creation of a DT is constituted by the model of the physical entity to be digitally represented, faithfully mirroring its peculiarities [5].

Accurate models of mechanical systems can be both kinematic, to study how bodies move in space, and dynamic, to derive how systems evolve in time under peculiar force and torque conditions that produce motion, as well as focusing on the dynamic interactions of bodies with other bodies, bodies with fluids, etc. While kinematic and dynamic modeling techniques for mechanical systems have firmly established over the years, the development of these models within Digital Twins architectures has gained significant attention and become a trending subject [6]. In fact, what is fundamental to the definition and creation of a DT is how well, efficiently, and lightly the model of the physical counterpart works [8].

Specifically applied to mechanical and mechatronic systems, kinematic and dynamic models become crucial for the future creation of the digital counterpart. On one hand, they must include the intrinsic characteristics of the physical entity to be mirrored, e.g. geometrical, structural, etc. [9]. On the other hand, the model should ensure the real-time data exchange possibility that a DT requires. As a result, striking a balance between the model accuracy and the required computational power to allow synchronization is of great importance in DT architectures. While some contributions try to explain in detail the importance of this, in the majority of cases this is extremely simplified or taken for granted. Moreover, a comprehensive review of how scientists can approach kinematic and dynamic modeling for an effective and successful creation of a DT, together with challenges and limitations, is still missing. The specifications and details provided by these publications often apply only to the specific case or problem being examined, but lack of a generalized contribution potentially applicable to different scenarios. In addition, the search for previous literature reviews on methods and approaches in kinematic and dynamic modeling for DT did not yield any results. Therefore, a systematic literature review of methods and approaches to kinematic and dynamic modeling for the subsequent creation of DTs has been developed.

In the conducted literature survey, the aim was to answer the main research question (RQ):

- **RQ:** what are common and best practices to kinematically and dynamically model mechanical systems towards their evolution into Digital Twins?

This paper is organized as follows: in Section 2 the literature selection strategy is described, together with a descriptive analysis of the conducted literature review; in Section 3, a review of existing techniques for DT-oriented modeling is presented, focusing on how the model has been specifically formalized by scholars and which peculiar software or application(s) have been utilized. Section 4 presents an in-depth analysis of the level of integration of the proposed models, while in Section 5 a detailed description and comparison of the evolution of such models into their reference DTs is presented. Furthermore, a theoretical framework highlighting crucial steps in the creation process of a fully-furnished Digital Twin for a generic mechanical system is proposed. Finally, in

Section 6, conclusions are drawn and future directions for research are illustrated.

2. Search and description of literature

2.1. Literature selection strategy

Purpose of this section is to describe the selection methodology utilized for the literature survey presented in this work. With the scope of analyzing methods and approaches in kinematic and dynamic modeling of mechanical systems towards DT, a Systematic Literature Research (SLR) methodology has been followed. In fact, this methodology represents a consolidated approach to identify, synthesize and analyze evidence in the literature on a peculiar topic, obtaining an easily replicable and objective result [10].

The collection of papers has been obtained by adopting a combined approach between SLR and Forward and Backward Review (FBR) methodology. In the early stage, the SLR was conducted to collect the most relevant contributions on the topic of kinematic and dynamic modeling of mechanical systems towards the future creation of DTs. For completeness' sake, the former stage included the application of FBR, which consists of two main phases:

a) forward research, aiming at finding relevant works among the one that cited at least one of the articles resulting from the SLR;
b) backward research, with the objective of looking for relevant papers among the references of the ones found in the SLR. By doing so, the pool of relevant publications considered in the review is extended to works that were not considered in the SLR.

Regarding the SLR conducted in the first stage, Scopus was the database chosen for this literature review. In first instance, an experimental set of keywords was used to perform an exploratory study of the works available in literature specifically related to kinematic modeling towards DT. Subsequently, another set of keywords was used to assess papers related to dynamic modeling towards DT. The finally used search strings are as follows:

a) (TITLE-ABS-KEY(kinematic AND modeling) and TITLE-ABS-KEY-(digital AND twin))
b) (TITLE-ABS-KEY(dynamic AND modeling) and TITLE-ABS-KEY(digital AND twin))

Scopus search was conducted with a field restriction limited to contributions in the field of engineering, and time restriction limited to December 2024, period in which the SLR has been performed. The search was also focused on three categories of publications: journal papers, conference proceedings or review articles. Papers written in different languages than English were excluded a priori. The SLR led to the identification of an early number of contributions equal to 792. Upon these publications, a two-stages screening approach has later been adopted:

1. Papers' title, abstract and keywords have been revised. A semaphoric-like categorization was then adopted, based on the assignment of a relevance grade to each paper (red = not relevant; yellow = potentially interesting but less relevant; green = relevant).
2. Relevant papers (green) have been read in their totality, excluding non-open access contributions.

Fig. 2 summarizes the entire literature selection strategy above explained.

As previously stated, the second stage of this literature survey included the adoption of FBR approach, including both phases of forward and backward research. This stage led to a restricted collection of papers with respect to the amount identified in the first stage but was however important to be performed. To conclude, 104 papers are considered in this review, given by the sum of papers selected during the SLR process and those identified along the FBR approach. The full list of papers is categorized in according to the type of content in Table 1 and the model level of integration in Table 2.

In the next section a descriptive analysis about the pool of contributions included in this review is provided.

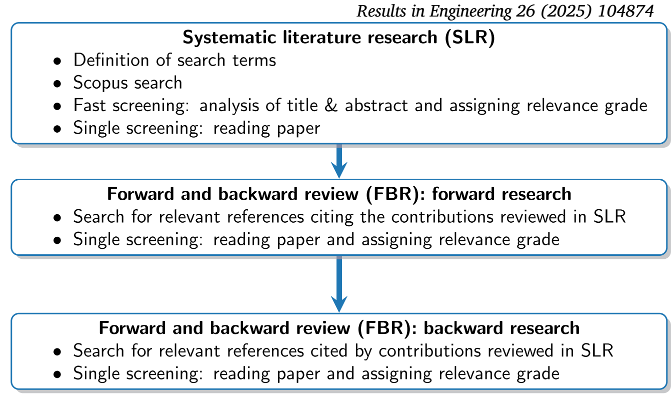

Fig. 2. Combined approach of SLR and FBR.

Table 1
Publications categorization based on the type of content.

Type of content	Citation IDs
Framework	[11–13]
Case study [14–91]	
Framework + Case study	[92–106]
Review	[107–110,48,111]

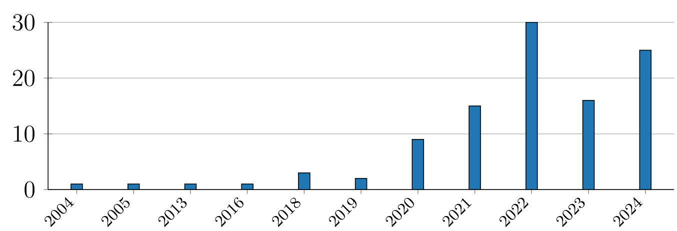

Fig. 3. Number of publications per year.

2.2. Descriptive analysis

To start with, the publications included in this literature survey have been categorized with time characterization. As supposed and proved through the analysis of previews literature reviews, the discussion around DT development and implementation is still growing over the recent years.

The timely distribution of the publications collected during the SLR and FBR for this literature review proves this fact once again. Fig. 3 graphically shows the number of papers that have been published until December 2024. Years with no remarkable contribution to the topic of this review have been left out. The increasing number of publications per year in this field suggests also that interest in DT topic is following a positive trend over the years and new research branches are increasingly gaining the interest of scholars to further develop the potentialities of this technology.

To conclude this subsection, it is worth mentioning that 78 out of 104 papers studied problems related to dynamics, while 10 papers limited just to the study of system kinematics and kinetostatics. Regarding content typologies, the majority of revised contributions presented a practical and specific case study, see Table 1. Only few contributions also proposed a new architecture or framework to model a system and

Table 2
Publications categorization based on the level of integration.

Level of integration	Citation IDs
Digital Model (DM)	[14,15,92,17,11,93,19–21,12,23,24,26,28–30,96,97,34,36,98,44,45,47,49,51,101,52–55,58,103,62,66,68,70,104,73–75,77–82,105,83,85–87,106,89–91]
Digital Shadow (DS)	[94,18,95,25,27,31,33,35,37,99,38,40,43,13,112,50,60,61,63–65,67,69,71,72,84,113,88,114]
Digital Twin (DT)	[16,22,32,100,39,41,42,46,56,102,59,76]

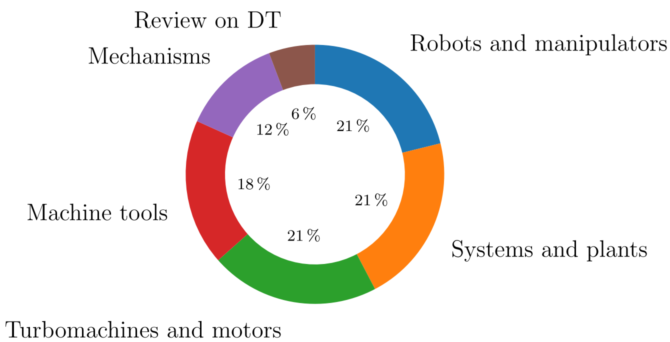

Fig. 4. Distribution of application fields of analyzed systems and machines.

create its related DM, DS or DT; some of them directly applied the architecture or framework to a practical case study. Fig. 4 shows the application fields identified for the sake of systems and machines' characterization.

3. Modeling techniques

The accurate representation of mechanical systems in DT frameworks requires the selection of appropriate modeling techniques, which can vary based on system complexity, intended analysis, and computational requirements. First of all, mechanical systems' models can generally be classified into lumped-parameter and distributed-parameter models, depending on how system variables are treated in space and time. Lumped-parameter models and distributed-parameter models can be used to describe the same mechanical system, but differ in their semantic and modeling approach [115]. While lumped-parameters models utilize a network of aggregated components to depict the configuration of engineering systems, such as masses, springs, and dampers, distributed-parameters models explicitly incorporate the geometric and material characteristics of engineering systems [116]. This means that lumped parameters create a simplified representation of a physical system, that assumes all components are concentrated at a single point [117]. On the contrary, with distributed parameters the behavior of a physical system is distributed spatially and cannot be considered as localized into discrete entities [118].

Remarkable examples for lumped-parameters models are present in the selected literature. In [46], for instance, authors exploit a lumped-parameter approach to evaluate the equivalent masses of the parts of a rolling mill. In this way, they obtain a simplified dynamical model, which fully encompasses the physical properties of the rolling mill for the further evaluation of its vibration modes in the virtual environment. Another example is provided by [47], whose mechanical system is represented by a turntable ladder. A lumped-parameter model is created, dividing the main chassis in four parts, each simplified by a spring-mass-damper system. Thus, they end up with the complete set of physical parameters to be fed into the equations of motion of the ladder, which they derive following the Lagrangian approach. Also in [72], a dynamic model is based on lumped parameters: the motion of a biped robot with a double pendulum's motion is approximated, defining its configuration thanks to generalized coordinates. In [75], the rotational behavior of an aeroengine disk is studied via a single degree-of-freedom (DOF) system, deriving its equation of motion using Lagrange's method. Likewise, [84] exploit a non-linear lumped-parameter dynamic model with nine degrees of freedom to analyze the behavior of moving components of a gearbox for agricultural applications. To conclude, a simple but clean example is provided by [90], in which a scissor transmission mechanism is modeled with masses concentrated at specific connecting points and the driving forces are estimated in recursive matrix form. Last but not least example is represented by [107], in which the virtual model of the machine tool feed drive in exam is established based on lumped and nonlinear parameter models.

Regarding distribute-parameters models, remarkable examples in literature are few, due to the notable limitations of this approach. Most distributed parameter models have disadvantages in their use and application resulting from: simplifications of the system, inadequate calibration data, or poorly understood system geometry and boundary conditions [119]. In truth, among the revised contributions, only one work explicitly used such an approach [37]. What needs to be specified, however, is that the problem addressed by this work is very specific, focused on the DT development of a Fresnel solar collector, whose temperature varies in time and space.

As previously said, accurate models of mechanical systems can be kinematic, to study how bodies move in space and time, dynamic, to derive how systems evolve in time under peculiar force and torque conditions that produce motion, as well as focusing on the dynamic interactions of bodies with other bodies (multibody dynamics), bodies with fluids (fluid dynamics), etc. Due to the increased complexity of the industry sector in recent times, it is indeed evident that also machines and mechatronic systems are increasingly becoming more sophisticated [120]. Thanks to the evolution of 3D modeling software and specific digital tools, the first step to characterize a system behavior in the virtual world is referred to as Digital Model, usually created thanks to simulation [121].

Simulation comes nowadays along with the benefits of digitalization, e.g. system variables trend prediction, system parameters optimization, etc. [122]. This is also where the trending subject towards the evolution of digital models into DTs comes in. The creation of a model instantly connected with real data puts into direct communication the real world to the digital environment, enabling the flow of information characterizing a mechanical system to be constantly updated [123]. This trend mirrors indeed the willingness to create interconnected systems in the context of Industry 5.0.

As a result, what emerged from the poll of contributions revised for this work was that the adoption of simulation methods represents the leading approach to create Digital Models, not only from a visualization perspective, but rather to have a physics-based working virtual replica. Generally speaking, systems modeled with lumped parameters are usually solved by a set of idealized and ordinary differential equations (ODEs), that relate to variables changing over time. In contrast, systems modeled with distributed parameters are subject to partial differential equations (PDEs), that consider variables varying in time and space.

To accurately capture the behavior of mechanical systems in Digital Twin frameworks, different simulation techniques are employed depending on the nature of the system and the type of interactions involved. Multibody Analysis (MBA) is widely used for modeling rigid and flexible body dynamics, particularly in mechanical assemblies and robotic systems. Finite Element Method (FEM) is essential for studying structural deformations and material behavior, ensuring accurate stress and strain predictions. Finally, Computational Fluid Dynamics (CFD) plays a crucial role in analyzing fluid-structure interactions, making it indispensable for systems where aerodynamics, lubrication, or thermal effects must be considered. The following subsections will explore these methodologies, highlighting their significance in developing accurate, physics-based Digital Models. A direct relation to the type of systems to which they are usually applied to, as per Fig. 4, is also reported in each subsection thanks to identified remarkable examples.

3.1. Multibody system analysis

The analysis of multibody systems is concerned with the kinematics and dynamics of mutually interconnected bodies. Multibody systems analyzed in this field are defined as systems of rigid or flexible mechanical parts, interconnected by rigid or elastic joints, subjected to a set of forces which make them perform large displacements and are typically associated with control systems [124]. Purpose of this section is therefore to list and describe analysis techniques for multibody mechanical systems from an analytical point of view, concluding with a subsection related to numerical simulation with Computational Multibody Dynamics.

3.1.1. Kinematic analysis

The purpose of kinematic analysis is to define the orientation and position of the bodies constituting the multibody system and how those bodies move as a system. In kinematics, the geometrical aspects of the motion of the bodies regardless the forces that produce the motion are of primary interest. As a result, kinematic analysis focus on determining the positions and orientations of the bodies of a multibody system as well as the related velocities and accelerations [125]. An abundant collection of differing notations and techniques for describing the kinematics of a multibody system has been identified in the poll of contributions revised for this work.

In the realm of robotics applications, the Denavit and Hartenberg (DH) convention is a widely employed framework for selecting reference frames [126]. Under this convention, coordinate frames are affixed to the joints connecting two links in the robot's structure. One transformation, denoted as $[\boldsymbol{Z}]$, is associated with the joint, while another, $[\boldsymbol{X}]$, is linked to the respective link. These sequential coordinate transformations along the series of n links within a robotic system collectively give rise to the robot's kinematic equations:

$$[\boldsymbol{T}] = [\boldsymbol{Z}_1]\,[\boldsymbol{X}_1]\,[\boldsymbol{Z}_2]\,[\boldsymbol{X}_2]\ldots[\boldsymbol{Z}_{n-1}]\,[\boldsymbol{X}_{n-1}]\,[\boldsymbol{Z}_n]\,[\boldsymbol{X}_n] \tag{1}$$

where $[T]$ represents the transformation matrix that locates the end-link.

To establish the coordinate transformations $[\boldsymbol{Z}]$ and $[\boldsymbol{X}]$, the joints that link the robot's links are modeled as either hinged or sliding joints. Each of these joints is characterized by a unique line S in space, which defines the joint axis and the relative movement between the connected links. A standard serial robot typically consists of a sequence of six lines, denoted as S_i (i = 1, 2, ..., 6), one for each joint. As a result, in this convention the z-axes are aligned with joint axes, while x-axes are parallel to the common normal between the z-axis of one joint with respect to the former one:

$$\boldsymbol{x}_n = \boldsymbol{z}_n \times \boldsymbol{z}_{n-1} \tag{2}$$

The y-axes are defined according the rules for right-handed coordinates systems. As aforementioned, DH notation is one of the most used approaches to describe the kinematics of a rigid-bodies system composed of links and joints. This indeed identifies DH notation as the most diffused convention for robotic arms, manipulators and robotic applications in general [127]. DH convention can be exploited both for Forward Kinematics (FK) and for Inverse Kinematics (IK) [128]. In FK, the position and orientation of the end effector, normally attached to the end of the last link of the kinematic chain, are computed starting from the joint variables. The approach of FK is therefore based on translating joint variables into Cartesian coordinates, which is usually addressed as a more linear and simplified approach with respect to IK. FK requires indeed just to evaluate a nonlinear function, while IK requires to invert a nonlinear relationship.

Many remarkable examples are present in literature, whose many have been revised for this review: in [67], for instance, a straightforward kinematic solution for a Selective Compliance Assembly Robot Arm (SCARA) robot based on DH parameters is presented. Also in [35] the forward kinematic problem for a standard 6-DOFs robotic arm is solved based on DH parameters. [41] present DH parameters based modeling for a modular robotic manipulator, converting modular desired configurations of the arm into the DH parameters for the FK computation. Also in [69] DH notation is exploited, in this case to include the kinematics of both a human and a robot into their DTs, which are then put into communication via a protocol with the purpose of dynamic scheduling. Even if IK could sometimes result to be more complex and time-consuming than FK, this approach is widely exploited as well. In IK, the transition occurs from the Cartesian space to joint variables, meaning that from the position and orientation of the end effector it is possible to derive positions and orientations of the joints. A representative case is provided by [86], in which IK solution of a marine crane isolates the joint variables to be obtained by maximizing the function form of the matrix components, computing joint variables by using the geometric characteristics of the system. Finally, in [114], scholars exploit the IK method with the goal of motion reconstruction of a virtual human, to estimate bone poses starting from hands and feet.

Additionally with respect to DH notation, also instantaneous screw theory is referred as a common technique for kinematic analysis of multibody systems. In first-order rigid-body kinematics, indeed, instantaneous screw theory usually studies the motion of rigid bodies as combinations of rotations and translations along a common axis, called a screw axis (l). An instantaneous screw motion can be represented by a six-dimensional vector, called a twist, that consists of an angular velocity $\boldsymbol{\omega}$ and a linear velocity $\boldsymbol{v}_{//}$ along the screw axis [79]. The most general twist is an instantaneous helicoidal motion around the body origin O, described by the equation:

$$\boldsymbol{v}_{\boldsymbol{O}} = \boldsymbol{v}_{//} + \boldsymbol{\omega} \times \boldsymbol{r} \tag{3}$$

where r identifies the distance between the origin and the screw axis. Since the pair $(\boldsymbol{\omega}, \boldsymbol{v}_{\boldsymbol{O}})$ completely defines the body velocity field, the body twist with respect to the origin is defined by the triplet $(\omega, \boldsymbol{\sigma}, l)$, respectively by the amplitude, the screw and the screw axis:

$$\xi = \begin{bmatrix} \boldsymbol{\omega} \\ \boldsymbol{v}_{\boldsymbol{O}} \end{bmatrix} = \omega\boldsymbol{\sigma} \tag{4}$$

where $\boldsymbol{\sigma}$ is called a screw of pitch h along line l [129]. By this, the twist is completely defined by the triplet (ω, h, l), namely by the screw $\boldsymbol{\sigma}$ and the amplitude ω. Some examples of instantaneous screw theory application are present in literature. It is worth mentioning the work done by [40], in which the adoption of the screw theory to establish the FK of a five-axis machine tool is described. In conclusion, a good example of screw theory application in the field of applied mechanics is presented by [66], whose goal is the prototyping of a reconfigurable parallel mechanism.

3.1.2. Dynamic analysis

The purpose of dynamic analysis is to study and analyze the forces and torques occurring during the system motion, enabling e.g. the evaluation of energies in play, etc. An analytical model is predominantly characterized by its quantitative essence, describing hence a system through a collection of mathematical equations that define the relationships between parameters and their corresponding values over time, space, or other system variables [130]. Thanks to analytical methods, it is indeed possible to describe the behavior of simple and complex systems in terms of mathematical equations. Generally speaking, the analysis of a system's dynamics encompasses the fundamental step of deriving its equations of motion [131]. The derivation of equations of motion can follow different approaches, too. The most typical distinction comes from the difference between Newtonian mechanics and Lagrangian mechanics. Both methods describe the combined rotational and translational dynamics of a rigid body but, while in the former the equations of motion are derived by the equilibrium of forces and is based on vector notation, in the latter the equations of motion are derived in terms of energies [132].

As described in details by [125], equations of motion of a rigid body in Newton-Euler's notation in the special case of planar motion reduce to three scalar equations. In three-dimensional space, the equations of motion can be expressed by six scalar equations that can be expressed for the unconstrained motion of the i^{th} body of a multibody system as follows:

$$\begin{pmatrix} \boldsymbol{f}^i \\ \boldsymbol{\tau}^i \end{pmatrix} = \begin{pmatrix} m^i \boldsymbol{I}_3 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}^i_{cm} \end{pmatrix} \begin{pmatrix} \boldsymbol{a}^i_{cm} \\ \boldsymbol{\alpha}^i \end{pmatrix} + \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{\omega}^i \times \boldsymbol{I}^i_{cm} \boldsymbol{\omega}^i \end{pmatrix} \tag{5}$$

where $\boldsymbol{f}^i$ is the vector of forces acting on the center of mass; $\boldsymbol{\tau}^i$ is the vector of torques acting about the center of mass; m^i is the mass of the body; $\boldsymbol{I}_3$ is a 3×3 identity matrix; $\boldsymbol{I}^i_{cm}$ is the matrix of mass moments of inertia about the center of mass; $\boldsymbol{a}^i_{cm}$ is the vector that defines the absolute accelerations of the center of mass of the body; $\boldsymbol{\alpha}^i$ is the vector of the angular accelerations of the body; $\boldsymbol{\omega}^i$ is the vector of the angular velocities of the body. It needs to be noted that these equations are formulated with respect to a coordinate frame whose origin coincides with the body's center of mass for torque and an inertial frame of reference for force. Broadly said, Newtonian equations of motion can be used to derive the differential equations of multibody systems in a systematic way, where forces at interconnections between rigid bodies are of primary interest. Some remarkable examples are presented by [11,18,25,49,56,67,87], but not only.

Differently from Newton-Euler's approach, Lagrange's equations of motion are derived by considering the energies acting on the system in exam. Lagrangian mechanics is also usually based on the choice of a complete set of independent coordinates, necessary to describe the system's motion at any instant. The generalized equation of motion for a rigid body in Lagrange's notation and formulated for one single coordinate resembles the following formulation [133]:

$$\frac{d}{dt}\left(\frac{\delta T}{\delta \dot{q}_i}\right) - \frac{\delta T}{\delta q_i} + \frac{\delta R}{\delta \dot{q}_i} + \frac{\delta U}{\delta q_i} = Q_i \tag{6}$$

where T is the kinetic energy in terms of system mass, inertia, linear and angular velocities; R is the energy dissipation produced by viscous friction; U is the potential energy; Q_i represents the corresponding load calculated for each independent coordinate. It needs to be noted that this equation is formulated with respect to a single independent generalized coordinate (q_i), as before stated. Broadly said, Lagrangian equations of motion do not consider constraint forces as in the case of Newton-Euler's approach. Some remarkable examples are presented by [72,75,81,86], but not limited to.

3.1.3. Computational multibody dynamics

Computational Multibody Dynamics Analyzes (MBDAs), which are based on Newton-Euler's approach or Lagrange's approach, are used to solve the dynamics of multibody systems in a numerical way. Equations of motion of multibody systems are generally non-linear differential equations, ordinary or algebraic. Generally, an analytical solution of these equations of motion does not exist. Thus, these equations require solutions from numerical integrators, which can be Ordinary Differential Equations (ODE) solvers or Differential Algebraic Equations (DAE) solvers. The common approach utilized for the resolution of MBDAs involves different steps:

- topology definition (bodies, joints, forces, etc.);
- solid modeling (geometry, shapes, etc.), usually performed via a Computer-Aided-Design (CAD) software;
- definition of inertial data (masses, moments of inertia, etc.);
- modeling of kinematic and dynamic phenomena involved in the multibody system;
- definition of outputs (measures, requests, objectives, etc.).

An example of a MBDA is provided by [45], in which scholars address a complex problem involving the study of a planetary gearbox. The purpose of their work is to exploit a DT for the fault diagnosis of a planetary gearbox set, since they want to include the influence of non-mechanical factors on fault causes. To do so, they develop a rigid-flexible coupled multi-body dynamics model by the use of ADAMS software. Another significant contribution is given by [106] with the proposal of an integrated methodology that combines control and MBDA capabilities in a nonlinear solver called SAMCEF Mecano. The authors develop indeed a virtual machine tool concept to model the dynamic behavior of a five-axes machine, to then validate it via the aforementioned solver. Another worth to be cited example is provided by [26], in which Simscape Multibody and Simulink are used for a co-simulation of the mechanical and hydraulic phenomena affecting the rear hitch subsystem of an agricultural tractor. Successful practices and applications of co-simulation are published in [28] and [31], too.

It needs to be specified that MBDA solvers and software are based on the equations of motion presented in the section before. However, the nonlinear equations of motion of multibody systems can generally not be solved analytically and require numerical methods to arrive to the solution.

3.2. Finite element method (FEM)

As aforementioned, the birth of numerical methods leaked from the need to solve large order analytical and nonlinear equations by introducing large systems of equations with many degrees of freedom and accepting the compromise of an approximated solution due to the discretization of continuous systems [134] and have become increasingly popular due to the ease of analyzing constraints and unknowns in the solution, differently from what could occur with analytical methods [124]. However, it should be considered that a good numerical solution is reached with a consistent number of iterations and that a proper initial estimation of unknown variables in the problem is required.

The Finite Element Method (FEM) has emerged as one of the most commonly employed techniques for solving partial differential equations. This approach necessitates extensive computer usage and is versatile enough to tackle a wide range of practical problems, encompassing both steady and transient scenarios within linear and nonlinear regions across one, two, or three-dimensional domains. Additionally, it can be adapted for application in heterogeneous environments and complexly shaped domains frequently encountered by engineers [135].

Generally speaking, FEM involves simplifying the representation of unknown variables to convert partial differential equations into algebraic equations. It draws upon three fundamental domains: a) the field of engineering sciences, which formulates physical laws expressed through partial differential equations; b) numerical methods, responsible for the development and resolution of algebraic equations; c) computational tools, that enable efficient execution of the required calculations using computers.

Practically speaking, to solve partial differential equations in two- or three-dimensional space, the FEM approach subdivides a large system into smaller, simpler parts called finite elements. This is achieved by a particular discretization in the space dimensions, which is implemented by the construction of a mesh of the object [136]. A good example is presented by [44], in which scholars utilize FEM to model the links of a robotic arm utilized for a material removal process. This is done to determine the impact of this load on the dynamic response of the robotic arm, in order to use it as an input for the dynamic simulation of the cell in which the process occurs. From the poll of contributions analyzed for this review, it is also worth mentioning the work done by [46], with the development of a fully furnished DM based on the FEM model of a rolling mill roll system, and the study from [14], which create a high-fidelity DM of a permanent magnet synchronous motor (PMSM) for generating fault data starting from a detailed FEM analysis of vibrations and contact forces.

3.3. Computational fluid dynamics (CFD)

Computational Fluid Dynamics (CFD) is a widely adopted numerical approach used to simulate fluid-flow phenomena by solving the governing conservation laws of mass, momentum, and energy [137]. Various modeling approaches exist within CFD, each with different levels of complexity and accuracy. Among these, Reynolds-Averaged Navier-Stokes (RANS) models, such as the commonly used k-ϵ turbulence model, are employed to simulate mean flow characteristics in turbulent conditions, which are prevalent in many industrial and natural settings. The $k-\epsilon$ model, a two-equation model, provides a general description of turbulence by solving transport equations for both the turbulent kinetic energy (k) and its dissipation rate (ϵ), offering an efficient and widely applicable solution for moderate to high Reynolds number flows [138].

Beyond RANS models, Large Eddy Simulation (LES) and Direct Numerical Simulation (DNS) represent alternative approaches for turbulence modeling. LES resolves larger turbulent structures explicitly while modeling smaller-scale turbulence, making it more computationally demanding but offering increased accuracy compared to RANS. DNS, on the other hand, solves the Navier-Stokes equations without turbulence modeling, capturing all scales of motion with the highest level of precision but at an extremely high computational cost, limiting its use to small-scale problems and fundamental research.

CFD is widely used as a numerical method for the analysis of complex systems involving the movement of flow. CFD is also suitable to study combined problems, in which flows interact with e.g. mechanical parts, such as moving gears and lubricants, turbines with water flows, etc. A pure and linear example of CFD methodology application is given by [77], in which a shell and tube heat exchanger is analyzed. The CFD model of the heated fluid flow is indeed studied in terms of pressure and velocity to establish a full-furnished DM. Similarly, in [20], a 1D dynamic model for an air energy storage plant is developed to control the start-up process in the virtual environment. A similar process is followed by [80], in which however the scholars focused on the CFD modeling error of a centrifugal compressor. Given the fact that solutions found by numerical methods have to be considered as approximated solutions, computational errors can indeed affect the outcome of a simulation.

4. From digital models to digital twins

In order to answer to the main RQ proposed by this work, different steps were required. While in the former section an overview of modeling techniques of mechanical systems for DMs was presented, this section is intended to in-depth analyze how scholars have developed their DMs into DSs and into DTs. As a result, three subsections are included, representing the three main ingredients of a fully-furnished DT, as stated in the introduction. Subsection 1 gives an overview of simulation-related technical aspects such as the typology of machine in exam, the DOFs, the utilized technology, etc., together with some reasoning about approaches to reduce the model complexity towards real-time integration. While in Subsection 2 the level of integration of DMs is analyzed with a focus on the Information Technologies (IT) infrastructures commonly utilized, Subsection 3 explores the role of DTs for prediction and optimization purposes in the revised poll of contributions.

4.1. Simulation

As already introduced, simulation plays a fundamental role in DMs, providing the basis for their evolution into DSs and DTs [139]. The revised contributions highlight that the primary focus lies on systems exhibiting motion, forces, and energy exchanges, making physics-based simulation techniques essential. Depending on the complexity of the system, scholars have employed kinematic, dynamic, and multibody simulations, with Multibody Dynamics Analysis (MBDA) emerging as a predominant approach, particularly for multicomponent mechanical systems. An overview of the disciplines addressed by the revised contributions is shown, see Fig. 5.

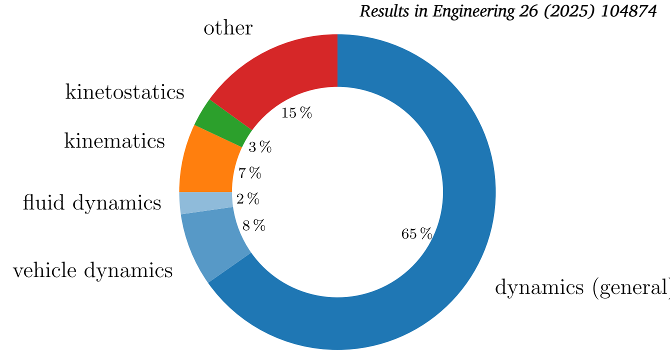

Fig. 5. Distribution of papers' areas of interest.

From a comprehensive review of the software and applications commonly employed by scholars to develop DMs, it emerged that software applications such as Dassault Systems (SolidWorks, Catia V5, LogoCAD, SIMPACK) or Autodesk (AutoCAD, Inventor, 3ds Max) are generally popular, due to the trivial evidence that they constitute the most reliable and widely used way to graphically construct a digital counterpart of a physical entity. Thanks to the increased accessibility and user-friendly interfaces, it is indeed nowadays somewhat simple to access these applications and use the features that they offer. What also emerges is the diversity and plurality of applications, reflecting the wide variety of tools that the present market offers, including some open-source software (e.g., OpenModelica, ROS), but also commercial software, which offer, however, great possibilities in the field of multibody modeling (e.g. Hexagon ADAMS, Siemens SAMCEF Mecano, Ansys tools, modeFRONTIER, Mathworks applications such as Matlab, Simulink, SIMSCAPE, etc.).

As before stated, a DT should rely on real-time optimization and synchronization, ingredients that might be troublesome to achieve for peculiar case studies. However, what actually emerged showed that the majority of case studies stopped at the level of DMs; only 14% of the papers' poll created in fact a full-furnished DT. For example, flexibility is not commonly addressed when dealing with DTs. Flexible bodies usually involve a higher number of variables to be modeled, resulting in a higher number of degrees of freedom, requiring higher computational effort and making it difficult to run the models in real time. This information emerged also from quantifying the number of degrees of freedom of the mechanical systems modeled in papers. What emerged that many works dealt with a restricted number of degrees of freedom. The high computational burden associated with complex physics-based models, particularly when dealing with flexible bodies or systems with a high number of DOFs, is indeed of great importance for DT applications. While high-fidelity simulations offer accurate representations, they often come at the cost of long computation times, making real-time synchronization with the physical counterpart difficult.

To overcome this limitation, Reduced Order Modeling (ROM) techniques have gained traction as a means to simplify complex models while retaining essential dynamics [140]. ROM techniques aim to reduce computational complexity by identifying dominant system behaviors and eliminating redundant DOFs, ensuring that real-time execution remains feasible. Some common ROM approaches found in the literature include:

- Proper Orthogonal Decomposition (POD): Extracts dominant modes from simulation data to construct a lower-dimensional representation of the system [141].
- Krylov Subspace Methods: Reduce large-scale models while preserving critical frequency and time-domain properties.

- Surrogate Modeling: Replaces full-order simulations with data-driven approximations, leveraging techniques such as polynomial regression or Gaussian processes [142].

Additionally, in recent literature, there is a clear shift towards Artificial Intelligence (AI) and Machine Learning (ML)-based modeling techniques. These data-driven approaches are being increasingly employed in DT applications as they provide adaptability and predictive capabilities [92]. Unlike traditional physics-based simulations, AI-driven techniques rely on historical and real-time data to identify patterns, optimize system performance, and forecast future behaviors. While these methods introduce computational overhead and data dependency, they also enable faster response times and self-learning capabilities, which are critical for adaptive control and process optimization. While these techniques offer benefits like improved data handling and automation, they also present challenges such as increased computational complexity and the need for high-quality data. This transition reflects a cautious trend towards leveraging AI to manage more intricate systems. Among the most prominent AI-driven modeling techniques identified, it is worth citing:

- the Bond-Graph method, which models energy transfer between components using a graphical approach [43];
- the Neuro-Fuzzy systems, combining fuzzy logic with neural networks for better learning and adaptability [37];
- Federated Learning, which enables multiple actors to build shared ML models while protecting data privacy [96,55];
- Markov models, often used for stochastic systems, focusing on modeling state changes based only on the current condition, useful for handling uncertainty [58];
- Regression-based algorithms (e.g. Gradient Boosting and Linear Regression), becoming mostly popular for applications dealing with large amount of data [18,21,24,27].

The growing adoption of these data-driven approaches highlights the increasing demand for accurate, flexible, and privacy-conscious modeling techniques. Integrating AI and ROM methods into Digital Twin architectures can leverage more practical and efficient solutions, but also requires a careful balance with model accuracy and computational efficiency to ensure reliable real-time performance in industrial settings.

4.2. Synchronization

Achieving real-time synchronization between the physical and digital counterparts is a critical challenge in DT development. Many contributions in the revised literature employ IT and Internet of Things (IoT) infrastructures to enable a continuous data exchange, supporting real-time updates and adaptive control mechanisms. The level of synchronization determines whether a system operates as a DM, DS, or a fully developed DT. Moreover, a key enabler of real-time synchronization is the choice of communication protocols, which define the efficiency, security, and scalability of DT implementations.

Several technological solutions have been proposed to enhance bidirectional communication between the virtual and physical worlds. For example, ROS-based architectures have been widely used to facilitate real-time updates in robotic systems, as demonstrated in [41], where Application Program Interfaces (APIs) branded Kinova for automated model generation and trajectory planning are integrated within a modular robotic configuration for a custom n-DOF serial manipulator. A peculiar typology of APIs, called REpresentational State Transfer (REST)-APIs, are employed in [39] for real-time path tracking of a vacuum cleaner robot. The DT infrastructure leverages the real-time trajectory adjustments of the robot based on a continuously updated RGB map.

In addition to the previously mentioned IT architectures, also data-driven synchronization solutions play a crucial role. In [100], for instance, the multi-domain DM of a generic six-axis robot is integrated in a peculiar framework, which continuously synchronizes the status of the DM, while concurrently executing event simulation. The real-time system then retrieves simulation-related data and transmits predetermined commands to the physical robot, providing dynamic status data back to the server and allowing a constant update loop of the entire system. The system used Structured Query Language (SQL) databases to ensure structured data storage and retrieval, supporting real-time task execution.

Similarly, sensor-driven 3D models in [56] synchronize robotic joint movements by integrating Gazebo, the dynamic model of the joint developed in MATLAB and real-time sensor data through ROS nodes to compute motion both on the DM and on the physical joint. Message queuing telemetry transport (MQTT) was used as a lightweight messaging protocol to enable efficient real-time communication between MATLAB and the physical actuators. Also in the work by [59] a sensor-based DT synchronization approach is employed in the context of a Human-Robot-Collaboration (HRC) assembly. The approach leverages 3D kinematic modeling, AI-driven task planning and HoloLens visualization to provide operators with real-time status updates, while the DT continuously triggers robotic tasks based on assembly logic and dynamic conditions. The system used OPC Unified Architecture (OPC UA), an industrial communication protocol that enables secure, scalable, and platform-independent communication between sensors and digital models.

Furthermore, multi-physics and multi-domain DTs require more sophisticated real-time synchronization mechanisms. Work such as [76] exploits interfaces of Grasshopper and KukaPrc to manage robot localization and motion planning, ensuring that dynamic conditions in the physical world are immediately reflected in the DT. The system adopted Ethernet-based communication protocols to allow low-latency robot control and status updates. Still related to multi-physics simulation, some papers address the complexity of simulating large and complex dynamic systems, focusing peculiarly on the problem of allocating dynamic responsibilities. In [102], the authors strongly rely on the definition of triggers to perform real-time data flow. Triggers have indeed the role to transfer the responsibility for simulating the robot positioning from the DM to the main physical setup and vice-versa. The same concept is presented by [42], where tasks assignment is indeed triggered in the virtual replica of a mechatronic line integrated with robotic systems. In both these works, MQTT and WebSockets were used to achieve low-latency communication. The authors also mention Industrial IoT (IIoT) protocols, including AMQP (Advanced Message Queuing Protocol) and DDS (Data Distribution Service), as an alternative solution to manage distributed real-time data exchanges efficiently.

Despite advancements in real-time synchronization, major challenges remain in DT implementations, such as:

1. Scalability and Latency: Many contributions rely on lightweight protocols (MQTT, OPC UA, REST-APIs) to minimize latency, but synchronization of large-scale industrial systems remains computationally demanding.
2. Security and Interoperability: Ensuring secure, standardized data exchange across heterogeneous industrial environments is a significant challenge. Future DT implementations could integrate blockchain-based communication layers for secure transaction logging.
3. Bidirectional Synchronization: Many studies implement one-way real-time monitoring (Digital Shadows) rather than full DT synchronization. Achieving true bidirectional integration between simulation tools and physical setups remains an open research challenge.

Many revised contributions highlighted the struggle to achieve full bidirectional integration and overcome communication challenges. Addressing these challenges is key to advancing fully functional DTs beyond simple real-time monitoring (DSs) and into true real-time operational control.

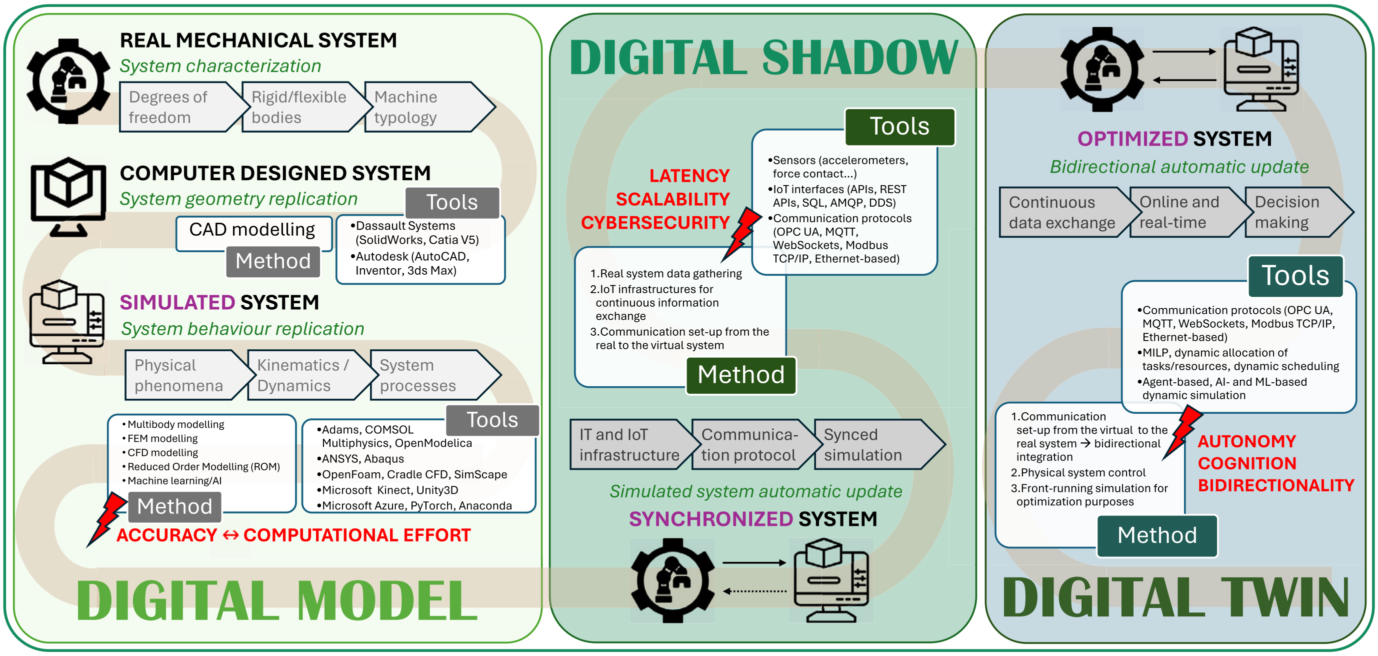

Fig. 6. Framework for DT creation of a mechanical system.

4.3. Optimization

One of the most significant advantages of Digital Twins is their ability to serve as optimization tools. The revised literature highlights that DTs are often developed with the aim of improving system performance, whether in terms of energy efficiency, productivity, process control, or predictive maintenance. For example, DT-based adaptive process control is demonstrated in [16], where 4D (3D + time) Digital Twin enables monitoring and optimization of a robotic additive manufacturing process. The system continuously detects deviations, adjusts printing parameters, and improves overall output quality with an algorithm for real-time reconstruction of images thanks to three Microsoft Kinect Azure cameras.

In manufacturing and industrial automation, DTs have proven particularly useful in energy optimization. For instance, a DT-driven dynamic scheduling approach using a Mixed Integer Linear Program (MILP) is proposed by [32], minimizing energy consumption and operational costs. Similarly, a dynamic allocating problem is addressed by [22] via an agent-based simulation model of a part feeding system, using a fleet of autonomous mobile robots (AMRs) handled with a Python routine to ensure a less energy-consuming resource allocation.

From this in-depth overview of relevant contributions which developed a fully-furnished DT, it is clear that the industry and academic sector of robotics is evidently advanced in this field. A possible reason for these advancements, as stated by [143], might be that the growth of robotic applications in the industry sector has experienced a significant increase over the recent years. Beyond energy and logistics, indeed, robotic and mechatronic systems leverage DT-driven predictive maintenance and fault detection. In [59], a DT system uses real-time sensor data and AI-based planning to proactively detect failures and optimize robotic assembly tasks, reducing downtime and improving efficiency. Likewise, [102] explores trigger-based DTs that dynamically allocate simulation and control responsibilities, ensuring optimized decision-making for real-time production systems.

Despite the clear advantages of DT-driven optimization, many revised contributions indicate that fully autonomous decision-making DTs are still rare. The majority of works rely on real-time monitoring (DSs) or operator-assisted optimizations, rather than complete self-optimizing DT architectures. The challenge lies in integrating AI-driven reasoning layers that can autonomously select optimal scenarios based on real-time data and predictive simulations. Future research must focus on enhancing the intelligence layer of DT architectures to unlock their full potential in autonomous system optimization.

5. A theoretical framework

As a follow-up to the findings of this literature review and the conclusions drawn in the previous section, this chapter presents a reference framework summarizing the key elements required for the structures development of a fully-furnished DT of a mechanical system. The framework is proposed in Fig. 6, delineating the progressive evolution of a DM into a DS and, ultimately, a fully integrated DT.

The framework begins with the development of a DM, which represents the system's geometry and behavior in a virtual environment. This process entails defining the machine typology, the degrees of freedom and the 3D representation of the system in the virtual world, which is commonly done via CAD software. The DM of a mechanical system is then completed with the simulation of the physical phenomena, movements, and processes performed by the machine parts, utilizing methodologies such as Multibody Dynamics Analysis (MBDA), Finite Element Method (FEM), Computational Fluid Dynamics (CFD), Reduced Order Modeling (ROM), and AI-driven approaches. A balance must be maintained between model accuracy and computational effort, as excessive complexity can hinder the subsequently needed real-time feasibility in DT architectures.

The transition of a DM to a DS is enabled by real-time data collection and the use of IT and IoT infrastructures for continuous synchronization. As a result, the system becomes connected and synced thanks to the constantly updated simulation with real-time data. The primary challenges in this stage include latency, scalability, and cybersecurity. The revised contributions indicate that various communication protocols, such as OPC UA, MQTT, WebSockets, Modbus TCP/IP, and Ethernet-based protocols, are used to facilitate data exchange between physical and virtual systems. Additionally, IoT interfaces, sensors, and APIs (e.g., REST APIs, SQL, AMQP, DDS) play a crucial role in enabling real-time data streaming.

The final step in the evolution from a DS to a DT is characterized by the establishment of bidirectional communication, allowing real-time updates and optimizations from the virtual environment to the physical system. The middle layer within the DT architecture leverages AI-based decision-making, MILP (Mixed-Integer Linear Programming), dynamic task/resource allocation, and agent-based simulations to achieve real-time system control and front-running optimization. The key challenges in this phase involve ensuring autonomy, cognition, and bi-directionality, as a fully functional DT must not only mirror the physical system but also reason, predict, and optimize operational parameters dynamically.

This structured approach provides a clear roadmap for researchers and practitioners aiming to develop DT architectures for mechanical systems. While advancements in real-time synchronization and optimization continue to emerge, future research efforts should focus on refining these methodologies to enhance scalability, efficiency, and interoperability across diverse industrial applications.

6. Conclusions

The virtualization of complex mechanical systems for the sake of prediction, control and optimization has been a critical research topic for decades. With the birth of DT technology, which enables real-time information exchange between a physical entity and its digital counterpart, the potential for improved decision making and automation has expanded significantly. However, the transition from Digital Models to fully operational Digital Twins remains a challenge due to limitations in modeling strategies, synchronization methods, and bidirectional data exchange.

This study has systematically analyzed the common and best practices for kinematic and dynamic models of mechanical systems towards their integration into DT architectures. The findings highlight that while there are a variety of methodologies and software tools available to construct accurate DMs, their evolution to DSs and DTs requires a more structured approach. The lack of standardization in building a fully operational DT architecture, combined with generalized incongruities in the definition of 'Digital Twin', often results in misclassifications of DMs and DSs as DTs. A more precise conceptual framework is needed to establish clear criteria for defining and implementing fully operational DTs. To address this gap, a theoretical framework was introduced in Section 5, outlining the progression from DMs to DSs and DTs. The proposed framework emphasizes key challenges at each stage: balancing accuracy and computational efficiency in DMs, addressing latency, scalability, and cybersecurity in DSs, and ensuring autonomy and bidirectionality in DTs. The findings suggest that while research in DTs is growing, a limited number of studies have successfully demonstrated fully integrated DT implementations, particularly outside of robotics and industrial automation.

6.1. Limitations and future work

Despite the advancements in DT research, several critical challenges remain unaddressed:

- Standardization Issues: There is no universally accepted framework or methodology for transitioning from DMs to DTs, leading to inconsistencies in implementation across different domains.
- Real-Time Bidirectional Synchronization: the distinction between an offline DT, which results to be a mere model, from an online DT, which updates real-time with respect to its physical counterpart, and vice-versa, still lacks of peculiar specification in the field.
- Cybersecurity and Interoperability: The integration of DTs into industrial environments raises concerns regarding secure data exchange and compatibility with existing IT infrastructures.
- Scalability and Computational Load: High-fidelity simulations introduce computational burdens that make real-time optimization impractical for complex mechanical systems.

Future research should focus on developing standardized DT architectures that ensure interoperability across different platforms and industries, exploring potential applications beyond robotics, including fluid-structure interactions, energy systems, and large-scale industrial processes. Another keypoint would be the enhancement of real-time control capabilities through advanced AI-driven optimization and simulation-based reasoning. Finally, addressing cybersecurity challenges in DT implementations, particularly for applications in manufacturing and industrial automation, would strengthen the development of fully operational DT infrastructures.

In the era of Industry 5.0, the trend is to emphasize human-centric, sustainable and resilient manufacturing, further driving the need for scalable and adaptive DTs. By refining synchronization techniques, improving computational efficiency and ensuring seamless integration with real-world industrial applications, future research can unlock the full potential of DT technology across diverse domains.

CRediT authorship contribution statement

Chiara Nezzi: Writing – original draft, Methodology, Formal analysis, Data curation, Conceptualization. **Veit Gufler:** Writing – review & editing, Methodology. **Renato Vidoni:** Writing – review & editing, Supervision, Methodology, Conceptualization. **Erwin Rauch:** Supervision, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

No data was used for the research described in the article.

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