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Article

Water Distribution Network Partitioning Based on Complex Network Theory: The Udine Case Study

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Abstract: Water Distribution Network Partitioning (WDNP), which is the partitioning of the existing Water distribution Network (WDN) into smaller and more homogeneous portions called District Metered Areas (DMAs), is an effective strategy that allows water utilities to improve network management through water balance, pressure control, water loss detection, and protection from contamination. The partitioning is realized physically, closing the pipes between two different districts, or virtually, installing flow meters which measure the districts inflow and outflow. Pipe closures lead to a considerable network performance worsening, reducing minimum pressure, resilience, and redundancy; on the other hand, flow meters allow us to avoid these issues but involve a higher investing cost. Hence, the DMAs' definition could become a hard task because both network performance and maximum investing cost must be respected. This paper presents the application of an optimization approach, based on complex network theory, coupled with an optimization technique based on genetic algorithms (GA). The methodology, implemented in Python environment, consists of a clustering phase carried out with two different algorithms (Girvan–Newman and spectral clustering) and a dividing phase which defines whether a gate valve or a flow meter should be installed in a pipe. The last phase is fulfilled with the GA which allows us to optimize one or more objectives in order to minimize the cost and maximize the network performance. The methodology has been applied on the Udine water distribution system, whose hydraulic model has been calibrated with a recent measure campaign. The results produced with the different clustering algorithms and objective functions have been compared to show their pros and cons.

Keywords: water network partitioning; district metered areas; complex network; genetic algorithms

Citation: Spizzo, F.; Venaruzzo, G.; Nicolini, M.; Goi, D. Water Distribution Network Partitioning Based on Complex Network Theory: The Udine Case Study. *Water* **2023**, *15*, 1621. https://doi.org/10.3390/w15081621

Academic Editors: Qiting Zuo and Dandy Graeme

Received: 16 March 2023
Revised: 7 April 2023
Accepted: 19 April 2023
Published: 21 April 2023

1. Introduction

Water distribution network partitioning (WDNP) is one of the most efficient techniques used by water utilities to improve water distribution network (WDN) management, simplifying water loss detection, pressure management, and water balance definition [1–3]. WDNP consists in partitioning a WDN in hydraulically independent subsystems called District Metered Areas (DMA), which are created by installing water flow meters or gate valves in boundary pipes.

Planning the number and dimension of different districts is not an easy task because of the intrinsic complexity of the WDN. Moreover, WDNP comes with many disadvantages, in fact, pipe closure can worsen network performance both in qualitative and quantitative terms. Regarding water quality, pipe closure creates more dead-end pipes which lead to an increase in water age and could cause flow reversals which lead to a lifting of the pipe sediments [4]. Further, WDNP reduces pipe redundancy and therefore network resilience, changing the WDN behavior and reducing node heads [5,6].

Before the advent of mathematic models, utilities designed DMAs without a global perspective, considering factors such as the number of inhabitants, main roads, and the economic level of leakage [2]. Today, thanks to mathematical methods, it is possible to create optimal DMAs which consider operational constraints and objectives. The WDNP, as presented by most authors in literature, is usually carried out in two distinct phases [1,7]: clustering and dividing.

The clustering phase consists of determining the shape and size of the districts and is based on the network connectivity and topology. Clustering aims at forming feasible DMAs, balancing the number of nodes within each cluster and minimizing the number of boundary pipes (i.e., pipe cuts where gate valves or flow meters will be installed). Several methods are used to cluster the WDN: graph theory algorithms such as depth-first search (DFS) and breadth-first search (BFS) [3,5,6,8,9], community structure [7,10–13], modularity-based procedures [14–17], multilevel partitioning [7,18,19], spectral approach [20–22], and multi-agent approach [23–25]. The main features of the clustering methods are well described by Bui et al. [26].

The dividing phase consists of optimizing the state of the boundary pipes, namely, the position and number of flow meters and gate valves required to achieve reliable DMA operation. Moreover, in this phase, the optimal solution which guarantees the best trade-offs between the investing cost and the indicators of the benefits of WDNP is investigated [3]. In fact, installing a flow meter is often more expensive than placing a gate valve, and closing a valve modifies network behavior and reduces its reliability.

Many algorithms and heuristic procedures have been proposed for this phase. Single-objective optimization is based on objective functions which consider the total power of a WDN [3,13,18,22], the system resilience [11,19,27,28], or the cost [29]; these optimization problems are generally solved with genetic algorithms (GA) [3,13,20,30–32]. Multiple-objective optimization considers simultaneously two or more objective functions, such as number of boundary pipes, network pressure uniformity, water age uniformity, total costs, elevation uniformity, and network resilience [15,33–36]. To solve multi-objective optimization, several optimization algorithms have been applied: NSGA-II [33–35,37], which generates the Pareto front with the optimal solutions, multi-objective GA [14], agent-swarm optimization [38–40], and the iterative approach, which generates a sequence of solutions until the initial set of criteria is met [10,14,21,41].

In recent years, several methodologies have been defined coupling or modifying clustering and dividing algorithms in order to achieve specific objectives with the partitioning. Liu and Lansey [42] proposed a multiphase DMA design methodology which incorporates different metrics (main and secondary feed pipes optimization, availability, water quality, and leakage), directly into the DMA design optimization searching process. Bui et al. [43] proposed an approach for the optimal design of DMAs in the multiple-criteria decision analysis (MCDA) framework based on the outcome of a coupled model comprising a self-organizing map (SOM) and a community structure algorithm (CSA). Bianchotti et al. [44] presented a two-stage approach for optimal DMAs' design in which three different performance indices are analyzed and compared (standard deviation, Gini coefficient, and loss of resilience). Zhou et al. [45] compared two existing partitioning methods, which differ in their consideration of main trunks, as well as in terms of economy, water quality, and leakage control.

In this paper, a WDNP methodology based on complex network theory coupled with optimization technique based on GA is applied and assessed on the WDN of Udine. This WDN is a large real-world WDN with 6 reservoirs and 6466 pipes. In particular, for the clustering phase, community structure and spectral algorithms are used, and their results compared. For the dividing phase, single-objective optimization and multiple-objective optimization are used. The optimization functions consider costs and network resilience. The different WDNP solutions generated from the objective functions have been evaluated and compared with a set of metrics and performance indices that reflect WDNP requirements. In particular, minimum and mean WDN pressure, investing cost, resilience index, and water age have been used.

Having a wide range of possible solutions is the recommended way to face the challenge; in fact, the problem is that practical and non-measurable factors, such as main roads position, impossible access to a certain pipe, etc., intervene in the final choice, which is generally made by the water utility.

This work allows us to assess the results arising from the application of a proposed methodology on a complex real-life-calibrated network. In fact, the Udine WDN is highly looped, is supplied by several points, and has several tanks and pump stations, features that make the decisional issue of the DMAs' definition a difficult task.

2. Materials and Methods

As described above, WDNP is carried out in two different phases—water network clustering and dividing [1,7]—which have to be analyzed separately.

2.1. Clustering

Clustering, which consists of assigning each network's node to a district, is based on graph theory and complex networks theory. A generic WDN can be considered as simple a graph $G = (V; E)$, where V is the set of n vertices v_i, and E is the set of m edges e_i. A k-way graph clustering problem consists of partitioning V vertices of G into k subsets, $P_1, P_2, \ldots, P_k$, such that $\bigcup_{i}^{k} P_i = V$ (the union of all clusters, P_i, must contain all the vertices v_i), $P_i \cap P_j = \varnothing$ (each vertex can belong to only one cluster P_i), $\varnothing \sqsubset P_i \sqsubset V$ (at least one vertex must belong to a cluster and no cluster can contain all vertices), and $1 < k < n$ (the number, k, of clusters must be different from one and from the number, n, of vertices).

In the literature, various algorithms that define the network's districts were proposed: graph partitioning algorithms such as the Kernighan–Lin algorithm [46] and the spectral bisection method [47]; k-means [48]; Markov cluster algorithm [49]; spectral methods [50,51]; hierarchical clustering [52]; multi-level-recursive algorithm [53], divisive algorithms [54]; and some other methods described in detail by Fortunato [55].

In this work, the spectral clustering algorithm and the Girvan–Newman algorithm have been chosen.

Spectral-based graph clustering has been implemented in many fields over the last decade, especially in computer sciences, bioinformatics, and data analysis. Recently, in the field of WDN management, spectral graph theory has been used for various purposes including the definition of an optimal cluster configuration. The core idea of spectral clustering is the Laplacian matrix whose spectrum in eigenvectors is used to cluster groups of points into communities [22,56]. There are three types of Laplacian matrix, defined by different equations. The first is the non-normalized Laplacian relationship, which solves a relaxed version of the ratiocut problem proposed by Von Luxburg [57]:

$$L = D - A \tag{1}$$

where D is a diagonal matrix of nodal degrees k_i, $D = \text{diag}(d)$, in which $d = [k_1, k_2, \ldots, k_n]^{\text{T}}$, and A is the adjacency matrix.

The other two matrices are normalized graph Laplacians, which are closely related and can be defined as

$$L_{sym} = D^{-1/2} L D^{-1/2} \tag{2}$$

$$L_{rw} = D^{-1} L, \tag{3}$$

where L_{sym} is a symmetric matrix proposed to solve the NCut problem [57] and L_{rw} is closely related to a random walk, which can be used to solve the same problem. $D^{-1/2}$ is a diagonal matrix in which the values on the diagonal are the inverse of D square values.

The aim of spectral graph partitioning is to divide graph G into $p \leq n$ subgraphs G_1, $G_2, \ldots, G_p$. Then,

$$V = V_1 \cup V_2 \cup \cdots \cup V_p, \text{ where } V_i \cap V_j = \emptyset, i \neq j. \tag{4}$$

Let $G_k = (V_k, E_k)$ represent for subgraph k, in which $k = 1, \ldots, p$, and V_k is the set of vertices of subgraph G_k. From Equation (4), an edge that has its endpoints in different vertex subsets is not contained in any of the formed subgraphs, G_k, and is called an intercluster edge. Let a set of the intercluster edges with one endpoint in V_k be denoted as Equation (5).

$$\partial(V_k) := \{ij : i \in V_k \text{ and } j \notin V_k\} \tag{5}$$

Two different sets of edges can thus be distinguished as follows:

- Intracluster edges: $E_1 \cup E_2 \cup \cdots \cup E_p$;
- Intercluster edges: $\partial(V_1) \cup \partial(V_2) \cup \ldots \cup \partial(V_p)$

From the optimal bipartitioning of a graph point of view, minimizing the cut values are objective functions. Von Luxburg [57] and Shi and Malik [51] proposed these functions to optimize cut value, called the ratiocut method and normalized cut method, in Equations (6) and (7), respectively.

$$\min_{V_1, V_2, \ldots, V_p} \sum_{k=1}^{p} \frac{vol(\partial(V_k))}{|V_k|} \tag{6}$$

$$\min_{V_1, V_2, \ldots, V_p} \sum_{k=1}^{p} \frac{vol(\partial(V_k))}{vol(V_k)} \tag{7}$$

where $vol(\partial(V_k))$ is the sum of the weights on all intercluster edges in $\partial(V_k)$; $|V_k|$ is the number of vertices in V_k; and $vol(V_k)$ is the sum of the weights on the vertices in V_k. Equations (6) and (7) are NP-complete problems; however, they can be relaxed to find approximate solutions and reformed as Equations (8) and (9) following:

$$LU = U\Phi \quad \text{for ratio cut,} \tag{8}$$

$$LU = DU\Phi \quad \text{for normalized cut,} \tag{9}$$

where $\Phi := \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_p) \in \mathrm{R}^{pxp}$ and $U := [u_1, u_2, \ldots, u_p] \in \mathrm{R}^{nxp}$.

Equations (8) and (9) are eigenvalue problems for p smallest eigenvalues $\lambda_1 = 0 \leq \lambda_2 \leq \cdots \leq \lambda_p$ of the Laplacian matrix L and their corresponding eigenvectors $u_1, u_2, \ldots, u_p$.

The Girvan–Newman algorithm is a community structure detection algorithm and, in particular, a bottom-up hierarchical approach based on graph theory [58]. It uses greedy optimization of a quantity known as modularity (Q), which is defined in Equation (10). The algorithm uses the quality measure of network density to define the clusters, assuming that the density of a network division was effective if there were many edges within communities (intraclusters) and only a few between them (interclusters). Modularity index is a network property used as an indicator to quantify the quality of graph division in the community. The clustering method is based on maximizing the modularity index. Higher values of that metric are related to a community structure of the network, which is significant if $Q \geq 0.3$ [58]:

$$Q = \frac{1}{2m} \sum_{ij} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \delta(C_i C_j), \tag{10}$$

where $\delta(C_i C_j)$ is the Kronecker delta coefficient, and $\delta(C_i C_j) = 1$ if vertices i and j are the same community; otherwise $\delta(C_i C_j) = 0$.

If we assume that the fraction of pipes that have both start and end nodes belonging to the same community is e_{ii}, and a_i is the portion of pipes with at least one end node in the community i, then the modularity can be formulated as:

$$Q = \sum_c e_{ii} - a_i^2. \quad (11)$$

The change in the two communities i and j to increase modularity can be computed by [58]:

$$\Delta Q = 2\left(e_{ij} - a_i a_j\right). \quad (12)$$

In this work, spectral graph clustering has been implemented with the Python module scikit-learn [59], which uses the Normalized-Cut method [51]. This method allows us to minimize the edge-cuts (pipes between different DMA); hence, it is economically convenient. The second one has been implemented with the NetworkX code [60]. This hierarchical algorithm forms clusters through division, creating a tree structure where subgroups of clusters are gradually defined.

The clustering result can be evaluated with several metrics. In this work, two kinds of metrics are used to compare the quality of the first phase: clustering quality indices and hydraulic indices [22].

- Clustering quality indices:

N_{EC}: number of edge-cuts, represents the total number of boundary pipes; hence, it is an indication of the intervention dimension;

I_B: balance index, expresses how homogeneous the clusters are. If the index is close to one, then the districts have the same number of nodes.

$$I_B = \frac{k \cdot max(sizeDMA)}{N} \quad (13)$$

where k is the number of clusters, $max(sizeDMA)$ is the number of nodes of the larger DMA, and N is the number of network nodes.

- Hydraulic indices:

C_{EC}: represents the sum of the ratios d/l of all boundary pipes and can be considered as a proxy of pipe conductance;

R_{EC}: represents the sum of the hydraulic resistances l/d5 of all boundary pipes and represents a proxy of the pipe's hydraulic resistance.

2.2. Water Network Dividing

After forming DMAs, the layout of the flow meters and gate valves among the boundary pipes is optimized in the dividing phase. The position of these devices is important because closing a valve changes network hydraulic behavior and reduces its reliability, but installing flow meters is often more expensive [3,18,21]. Many algorithms and heuristic procedures have been proposed to find the optimal solution: single-objective optimization approach, multiple-objective optimization approach, iterative approach, and adaptive sectorization for dynamic DMAs that are well described by Bui et al. [26].

In this work, the optimization is carried out with two single-objective and one multi-objective genetic algorithm (GA).

Defined a set of boundary pipes in the clustering phase, the first objective is to determine how many flow meters and gate valves have to be inserted. As reported before, their position influences network properties such as resilience, water quality, hydraulic performance, and leakage rate, and the decision is also tied to the cost of interventions. Therefore, dividing should be considered as a multi-objective optimization problem. To simplify computational demand, some hypotheses have been proposed to convert the problem into a single-objective and apply evolutionary algorithms to achieve feasible or optimal solutions. There are different objective functions that can be used; in this work, a minimizing investing cost function and a maximizing resilience index function have been used.

Minimizing the investment cost and maximizing the profits is one of the main goals for water utilities. Using an objective function that minimizes costs, the GA will find that the best solution is the layout with only gate valves in the boundary pipes. This solution is not acceptable because gate valves would prevent water from reaching the DMAs, which would become isolated. Therefore, for this particular case, it is necessary to introduce hydraulic constraints that guarantee the required level of service. The investing cost function is based on the flow meters and boundary valves installation cost.

As seen before, guaranteeing the network performance in terms of pressures and resilience is the main objective of the water utilities and installing boundary valves due to the WDNP being in contrast with this. Therefore, maximizing performance during the dividing phase is a good strategy. However, it could lead to a layout with only flow meters in the boundary pipes, which is not sustainable. In the literature, several indices that quantify network resilience are proposed. Todini [61] proposed an index to measure system resilience when redesigning a system. Several studies [11,19,27] have used this index as an objective function for dividing optimization; the function to maximize in order to indicate a greater surplus of available power is the following:

$$I_r = \frac{\sum_{j=1}^{N} q_j \cdot \left(h_j - h_j^*\right)}{\left(\sum_{r=1}^{N_r} Q_r \cdot H_r + \sum_{p=1}^{N_p} P_p\right) - \sum_{j=1}^{N} q_j \cdot h_j^*}, \quad (14)$$

where N is the junctions' number, q_j is the j-th node demand, h_j is the j-th node head, h_j^* is the admissible head at the j-th node, N_r is the number of reservoirs, Q_r is the flow that the r-th reservoir enters in the system, H_r is the r-th reservoir's head, N_p is the number of pumps, and P_p is the power introduced in the system by the p-th pump.

Some authors proposed different variants of the Todini's index due to the fact that according to Equation (14), real losses contribute positively to the resilience of the network. Prasad et al. [62] combined Todini's index with the uniformity of pipe diameters, as minimal diameter changes result in more reliable water circulation. Saldarriaga et al. [63] proposed and tested the unitary power measure, which leads to very similar results as Todini's index, defined as the product of the flow rate in each pipe and the piezometric head difference between the pipe's initial and terminal nodes. Creaco et al. [64] proposed a generalized resilience index, which uses both demand and pressure-driven modeling approaches in order to account for network leakages. Jayaram and Srinivasan [65] demonstrated that if the network analyzed has more than one reservoir, the Todini's index does not adequately represent its resilience. In fact, a network with a large demand surplus is characterized by a high immitted power which causes an excessive decrease of the resilience index. Therefore, they proposed the Modified Resilience Index (MRI) that we have adopted in this work:

$$\mathrm{MRI} = \frac{\sum_{j=1}^{N} q_j \cdot \left(h_j - h_j^*\right)}{\sum_{j=1}^{N} q_j \cdot h_j^*} \quad (15)$$

Actually, Todini's index, like many others, could have invariance problems [66]. To avoid this problem, a set of local energetic indices is proposed by Caldarola et al. in [67] and applied to a WDN in [68]. Their invariant implementation is defined in [66].

To solve these optimization problems, the GA [69] has been applied as in other works of Di Nardo et al. [3,13,20,30,31]. Optimizing one of the previous functions, a solution which satisfies a single objective is obtained. Nevertheless, as seen before, minimizing or maximizing a single objective function usually leads to a non-applicable solution. This issue can be overcome using a multi-objective function, which allows us to find the best compromise between different objectives. In this study the multi-objective function considers both costs and MRI and is solved with NSGA-II [37]. GA are here implemented with the DEAP pack [70]. To assess and compare the WDNP results, cost, MRI, water age, WDN mean pressure, DMAs minimum and maximum pressure, and number of flow reversals are calculated.

2.3. Methodology

The described procedure has been developed in Python and is completely automatic once the input network file is defined. It starts with the creation and calibration of the WDN model, which allows us to extrapolate the input file (in this work EPANET 2.2. [71] is used). After that, the hydraulic simulation starts and the network's parameters (minimum, maximum and mean pressure, resilience index, and water age) are calculated and the graph of the network is created. Using the network graph, the clusters are defined with the spectral clustering algorithm or the Girvan–Newman algorithm. There follows a comparison between the two clustering methods with the quality and hydraulic clustering indices described above. Defined the boundary pipes, the network is divided using a single objective function, which minimizes cost or resilience reduction, or a multi-objective function, which considers both. The outputs of the single-objective functions are visualized in QGIS [72] and compared in terms of cost, MRI, and water age. Instead, the output of the multi-objective function is the Pareto frontier; hence, the solution is chosen based directly on the sought MRI and cost values.

The procedure has been validated with an existing partitioned water distribution network (EPA Net3 example network). The flow chart in Figure 1 represents the Python code.

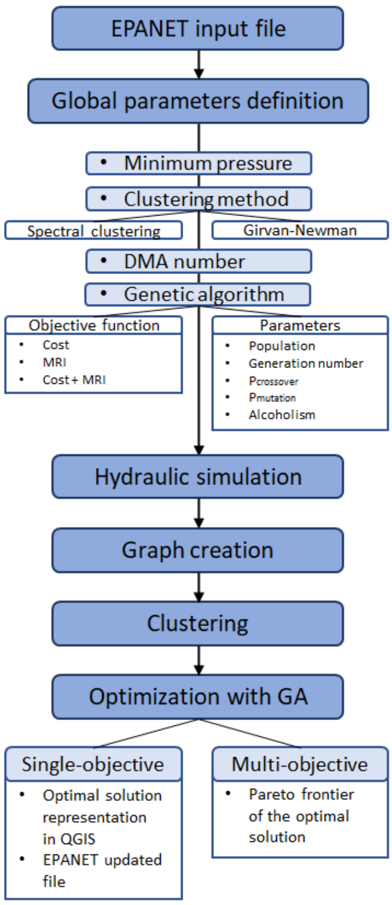


Figure 1. Python code flow chart.

3. Results

The described procedure has been applied on the WDN of Udine, an Italian town of 100,000 inhabitants sited in northeast Italy. The network was modelled and calibrated in EPANET 2.2 (Figure 2) with a demand driven approach and has the following characteristics:

- n = 5793 nodes;
- m = 6466 pipes;
- Six sources;
- Five tanks;
- Five pumps.

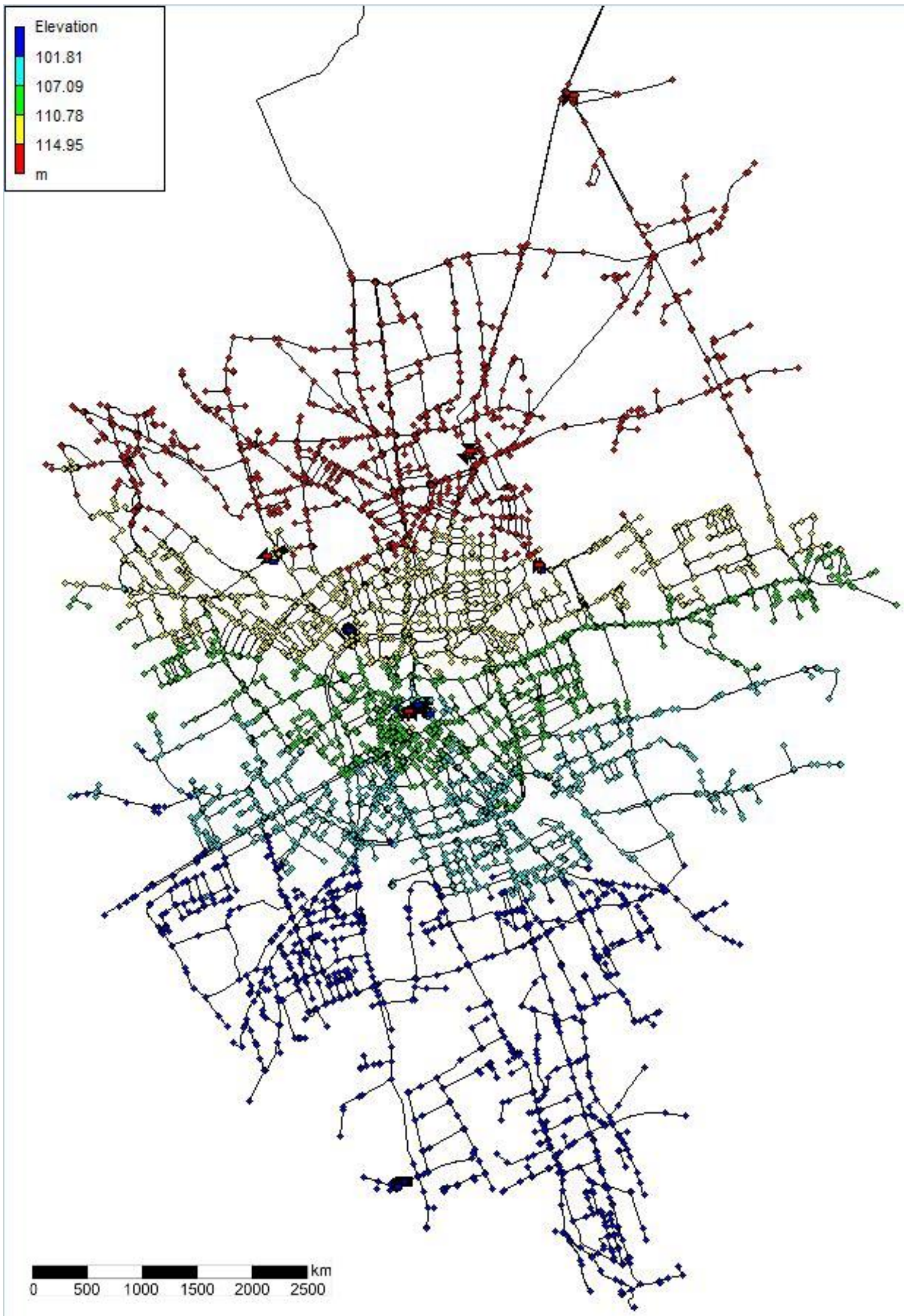


Figure 2. Hydraulic model of the Udine water distribution network.

The system is mainly supplied by two sources located north of Udine and three city wells that are activated when needed. The sources and the wells load five tanks whose characteristics are reported in Table 1. The network has a total pipe length of 406.6 km and the pipes, with a diameter range that varies between 150 mm and 600 mm, are made of iron and cast iron.

Table 1. Udine WDN tanks main characteristics.

Name	Volume [m^3]	Min Level [m]	Max Level [m]
San Bernardo	400	158.0	163.0
Vat	1750	150.5	156.0
Don Bosco	2200	145.0	153.5
Cotonificio	2200	145.0	153.5
Castello	4570	136.5	139.2

Before WDNP, the following parameters have been computed for the Udine WDN: MRI = 0.188; water age = 13.42 h; mean pressure = 38.66 m; and minimum pressure = 18.46 m. During the optimization phase, the goal is to create the DMA without worsening these parameters, which represent the WDN performance.

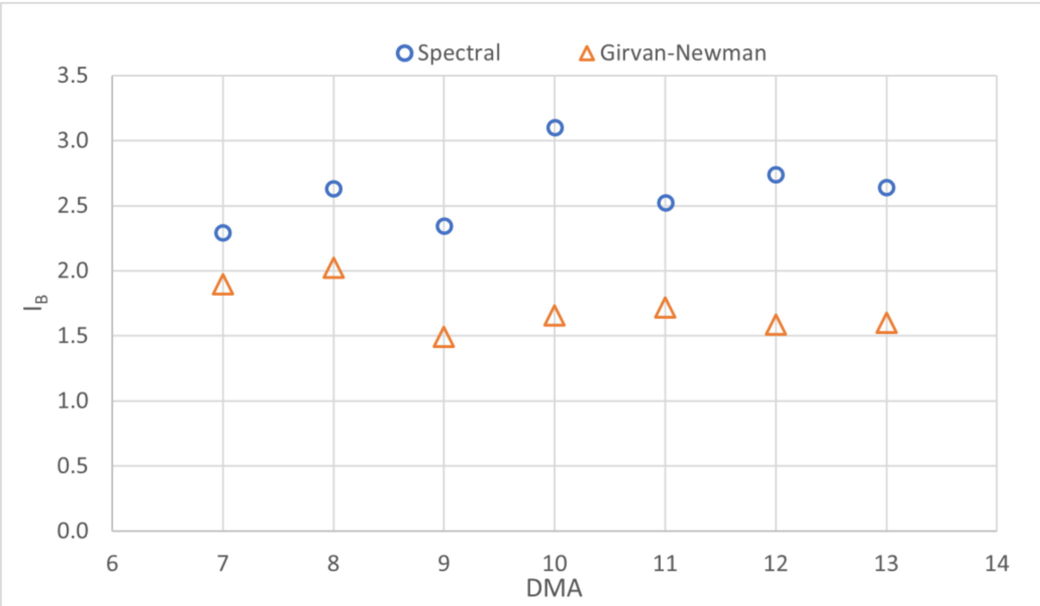
3.1. Clustering Analysis

Due to its dimensions, partitioning the Udine WDN is a complex issue; hence, an experience-based design methodology cannot handle it. The only way to correctly determine the districts and their shape and position is to use numerical methods. To define the best districts' number, the clustering algorithm has been applied for a variable range of DMAs' number. For every result obtained for both spectral and Girvan–Newman methods the balance index, the number of edge-cuts, the pipe conductance, and the pipe resistance have been calculated in order to compare the results (Table 2) (Figure 3).

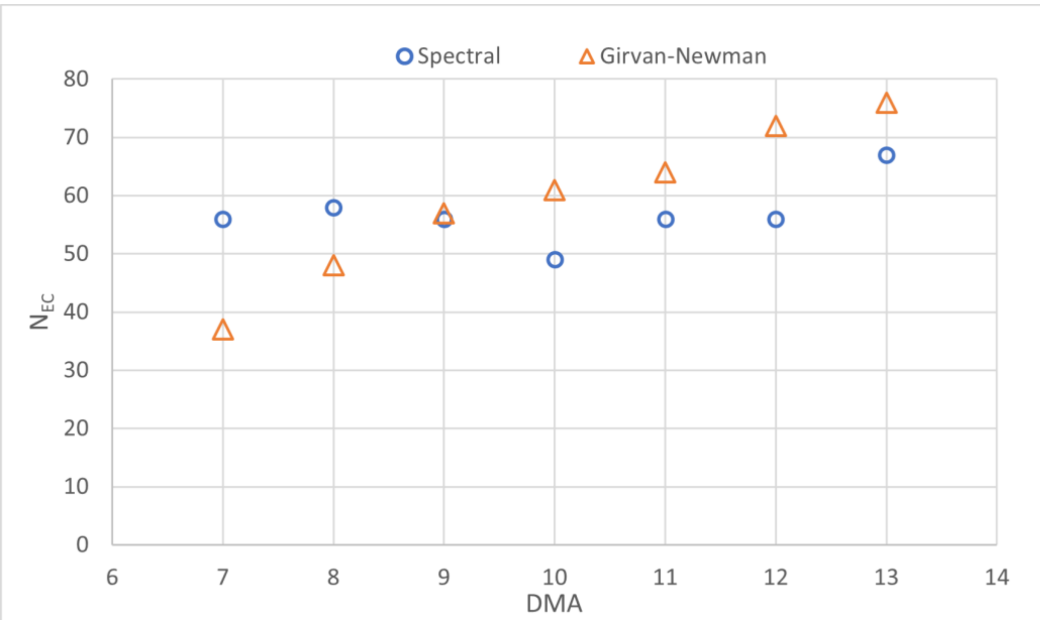
Table 2. Number of edge-cuts, clustering, and hydraulic indices for spectral clustering and Girvan–Newman algorithm.

	Spectral Clustering				Girvan–Newman			
DMAs	I_B [-]	N_{EC} [-]	C_{EC} [-]	R_{EC} [m^{-4}]	I_B [-]	N_{EC} [-]	C_{EC} [-]	R_{EC} [m^{-4}]
7	2.295	56	0.774	7.39×10^8	1.895	37	0.605	1.83×10^8
8	2.632	58	0.821	9.08×10^8	2.026	48	0.892	2.02×10^8
9	2.345	56	1.119	4.25×10^8	1.493	57	0.955	2.59×10^8
10	3.102	49	0.801	7.33×10^8	1.659	61	1.017	2.70×10^8
11	2.523	56	1.280	2.58×10^8	1.718	64	1.082	2.77×10^8
12	2.740	56	1.135	4.19×10^8	1.588	72	1.462	2.35×10^8
13	2.640	67	0.756	9.97×10^8	1.603	76	1.694	3.49×10^8

The chosen range of DMAs number varies between 7 and 13 districts because a lower value would not bring the relevant benefits and a higher value would cause an excessive number of edge-cuts. The results show that the hydraulic parameters C_{EC} and R_{EC} do not differ so much between the clustering methods. This indicates that the boundary pipes diameters are quite the same. Instead, the index that differs the most between the two methods is the balance index, which is higher in the spectral method. As predicted by the index, the clusters obtained with this algorithm do not have a similar number of nodes and, as a consequence, there are big and small clusters. This could represent an issue to consider in the design phase because, though determining the water balance of a small DMA is an easy task, recognizing and identifying leaks in a larger DMA could be difficult. On the contrary, the spectral method presents lower values of N_{EC} against the Girvan–Newman algorithm which can determine a considerable reduction of the costs. Moreover, while with the Girvan–Newman algorithm the number of edge-cuts and, consequently, the costs grow with the number of DMAs, with the spectral method, the number of DMAs is quite similar and it is possible to choose a higher number of districts without influencing the budget. After analyzing the pros and cons of the solutions obtained, the WDNP has been applied to the case of 12 DMAs for both spectral and Girvan–Newman algorithms. The DMAs features are reported in Figure 4 and Table 3. The configuration, obtained with the Girvan–Newman algorithm, has been chosen because it represents a good compromise between a uniform clusters division and a small number of edge-cuts (N_{EC} = 72).

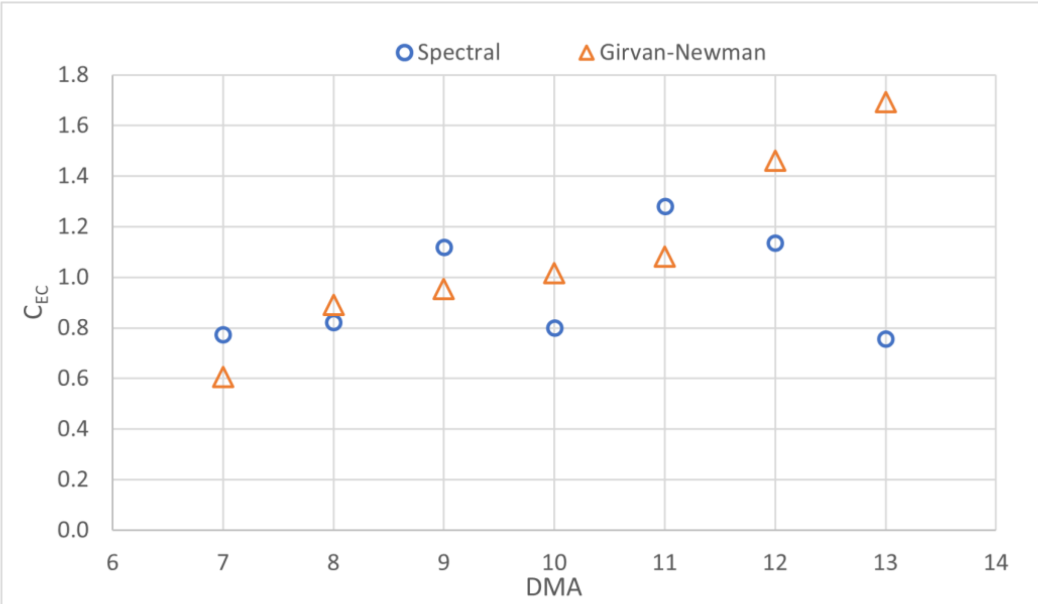


(**a**)



(**b**)

Figure 3. *Cont*.



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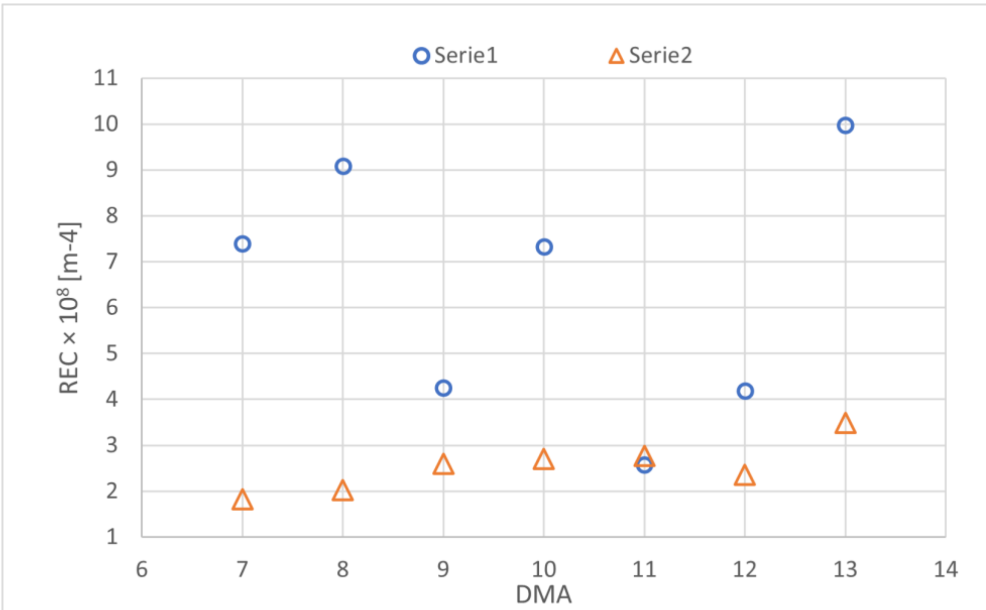


(**d**)

Figure 3. (**a**) Number of DMA-balance index relation for clustering algorithms; (**b**) number of DMA-number of edge-cuts relation for clustering algorithms; (**c**) number of DMA-conductance relation for clustering algorithms; (**d**) number of DMA-resistance relation for clustering algorithms.

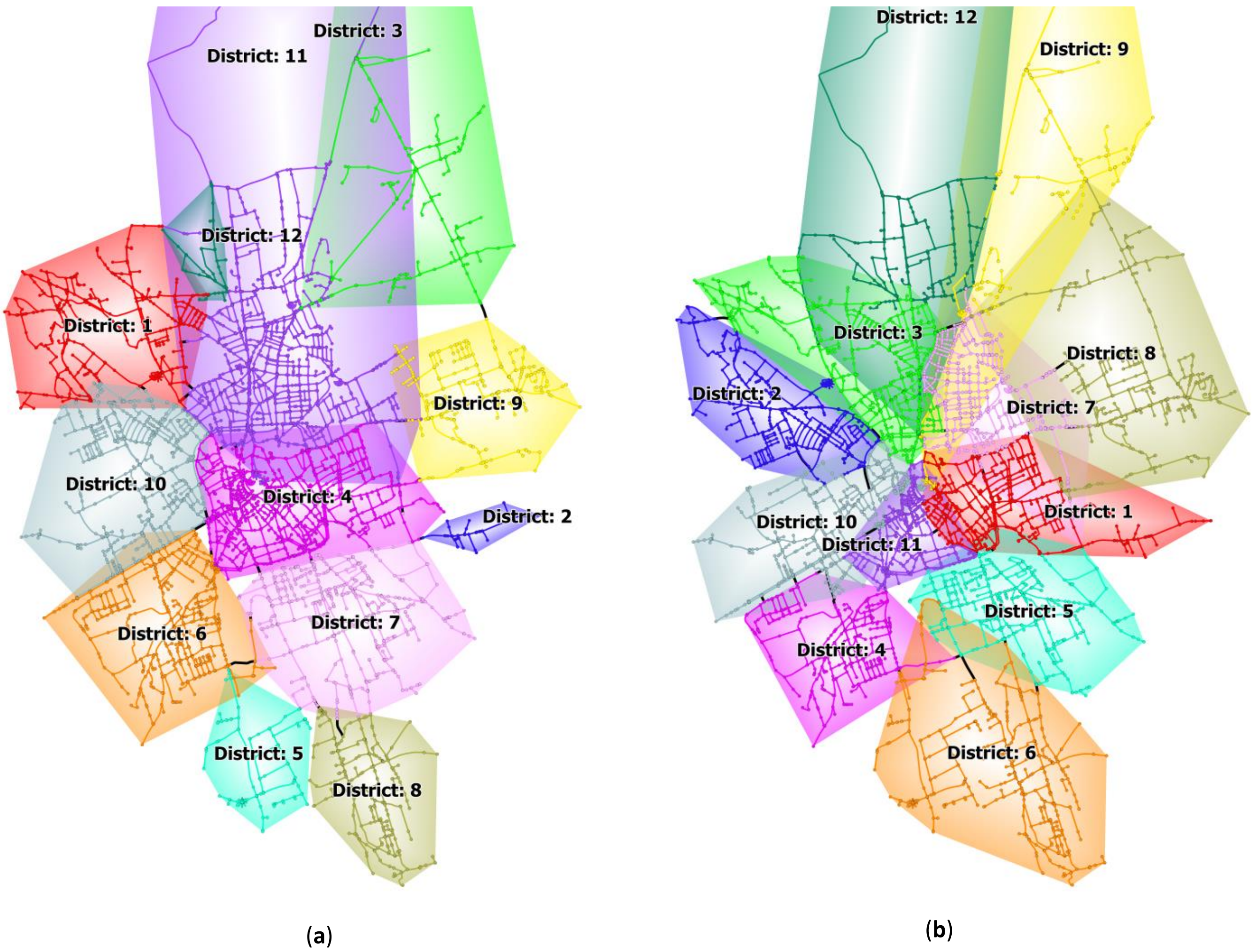


Figure 4. (**a**) Udine WDN clustering: spectral clustering algorithm—12 DMAs; (**b**) Udine WDN clustering: Girvan–Newman algorithm—12 DMAs.

Table 3. Udine: DMAs features for spectral clustering and the Girvan–Newman algorithms.

	Spectral Clustering			Girvan–Newman		
DMA	**Dimension**	**Length [km]**	**Mean Demand [L/s]**	**Dimension**	**Length [km]**	**Mean Demand [L/s]**
1	476	34.07	27.8	724	38.15	60.85
2	53	2.94	3.57	472	30.35	30.46
3	185	27.42	17.84	765	47.04	52.89
4	1287	60.57	107.99	374	26.04	26.12
5	109	8.98	4.27	517	31.78	28.63
6	546	37.19	38.74	454	35.52	28.26
7	577	37.49	33.42	613	36.86	40.01
8	276	19.99	19.12	402	29.27	34.19
9	295	21.23	24.58	192	38.22	13.17
10	601	36.32	38.57	505	28.49	33.17
11	1320	107.69	87.91	517	20.78	42.92
12	56	5.57	5.69	246	35.48	18.84

3.2. Dividing Phase

Once the network nodes have been assigned to the clusters, GAs have been applied to decide what to insert in the boundary pipes (gate valve or flow meter). The chosen general parameters of the genetic algorithms are the following:

- Individuals = 150;
- Generations = 150.

Using these values, a complete genetic evolution lasts between 4 h and 7 h due to the network dimension that increases the simulation computational time. The "biological" operators used are:

- Tournament selection with tournament dimension equal to two for the single-objective optimization;
- Elitist non-dominated sorting genetic algorithm (NSGA-II) for the multi-objective function;
- Two-point crossover with crossover probability $(p_c) = 0.9$;
- Flip-bit mutation with individual mutation probability $p_m = 0.2$ and gene mutation probability $p_{mutation} = 4\%$.

The mutation parameter value means that one in five individuals can mutate; this is conducted to avoid an evolution convergence around a local optimum. Elitism has also been applied not to lose the best individuals over the generations.

The two clustered Udine WDN have been divided using two single-objective functions—cost and performance—and a multi-objective function considering both cost and performance. Due to the high number of reservoirs and tanks, the Todini resilience index would not represent the system performance. Hence, the performance parameter suitable for the Udine network and used in the optimization phase is the MRI. Due to the wide range of results (Tables 4 and 5), a detailed analysis is necessary to better understand the Udine WDNP. The solution obtained by minimizing the investing cost with the spectral method (Figure 5a) tends to leave only one alimentation point for every DMA, closing all the unnecessary pipes for the pressure bond fulfillment.

Table 4. DMAs minimum and mean pressure after different WDNP.

Spectral Clustering Algorithm						Girvan–Newman Algorithm					
Objective Function: Investing Cost			Objective Function: MRI			Objective Function: Investing Cost			Objective Function: MRI		
DMA	Mean Pressure [m]	Minimum Pressure [m]	DMA	Mean Pressure [m]	Minimum Pressure [m]	DMA	Mean Pressure [m]	Minimum Pressure [m]	DMA	Mean Pressure [m]	Minimum Pressure [m]
1	33.93	22.53	1	31.43	18.67	1	32.03	16.9	1	37.34	27.04
2	28.48	15.12	2	40.51	30.74	2	34.06	22.91	2	34.8	23.84
3	30.63	20.8	3	30.69	20.96	3	30.57	18.16	3	31.85	17.13
4	33.35	20.36	4	37.15	26.79	4	37.8	15.12	4	47.62	36.46
5	54.68	33.7	5	62.16	47.82	5	35.36	19.75	5	43.63	31.24
6	35.2	16.17	6	46.21	33.9	6	80.3	29.2	6	80.3	29.2
7	38.17	22.29	7	44.68	29.94	7	31.78	21.99	7	33.76	25.7
8	47.11	23.78	8	55.87	40.99	8	33.06	18.92	8	33.23	18.6
9	34.96	26	9	36.38	26.58	9	37.67	18.85	9	38.24	18.54
10	30.58	15.92	10	37.48	28.31	10	32.61	17.19	10	38.99	24.48
11	33.21	22.21	11	34.63	22.55	11	33.58	22.1	11	39.04	26.75
12	43.31	30.16	12	43.09	29.95	12	43.24	26.6	12	43.86	23.27

Table 5. Cost, MRI, water age, number of boundary valves, and flow meters for the different clustering algorithms and objective functions.

	Spectral Clustering		Girvan–Newman	
	Investing Cost	MRI	Investing Cost	MRI
Cost [€]	85,382.4	114,434	97,981.3	154,057.3
MRI [-]	0.15724	0.19035	0.17781	0.20621
Water age [h]	13.01	13.54	11.83	13.49
Mean pressure [m]	34.84	38.96	37.36	40.95
boundary valves	40	27	56	20
flow meters	16	29	16	52

Even if this solution reduces the investing cost, it causes the collapse of the resilience index, reducing the mean pressure from 38.6 m to 34.8 m and the minimum pressure from 18.46 m to 15.12 m. The hydraulic inefficiency due to low pressures is not acceptable for the water utility; hence, this solution would probably be discarded. The solution obtained with the same objective function and the Girvan–Newman algorithm (Figure 5b) could represent a good result since the mean pressure is higher (37.7 m) and the investing cost does not increase considerably. Instead, the solutions found by maximizing the resilience index (Figure 5c,d) involve a higher investing cost—nearly doubled—since, by only installing a lot of flow meters, it is possible to preserve the meshed paths that guarantee network redundancy and resilience. In this case, with the Girvan–Newman algorithm, the mean pressure reaches 40.95 m.

These solutions, though preferable due to the increasing of the hydraulic performance of the WDN, are rarely feasible and often rejected for their high cost. The results obtained with the single-objective function, as we can see from Figure 6, show that the WDNP causes a worsening of the MRI index, except when the index represents the objective function. In fact, network performance can be increased by closing only certain flow paths. However, in this case, the economic burden would become considerable and would lead to the installation of few boundary valves and a lot of flow meters.

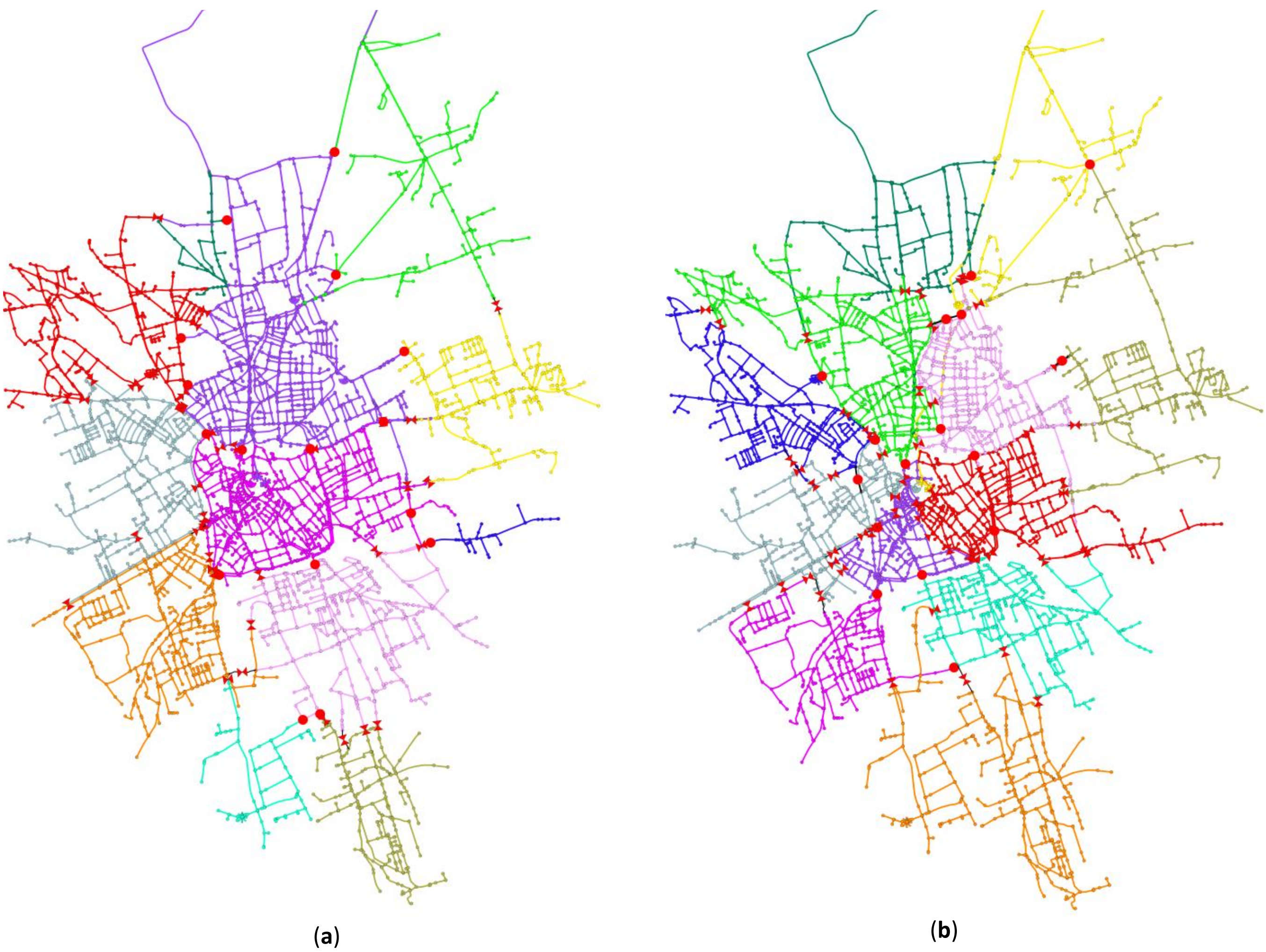
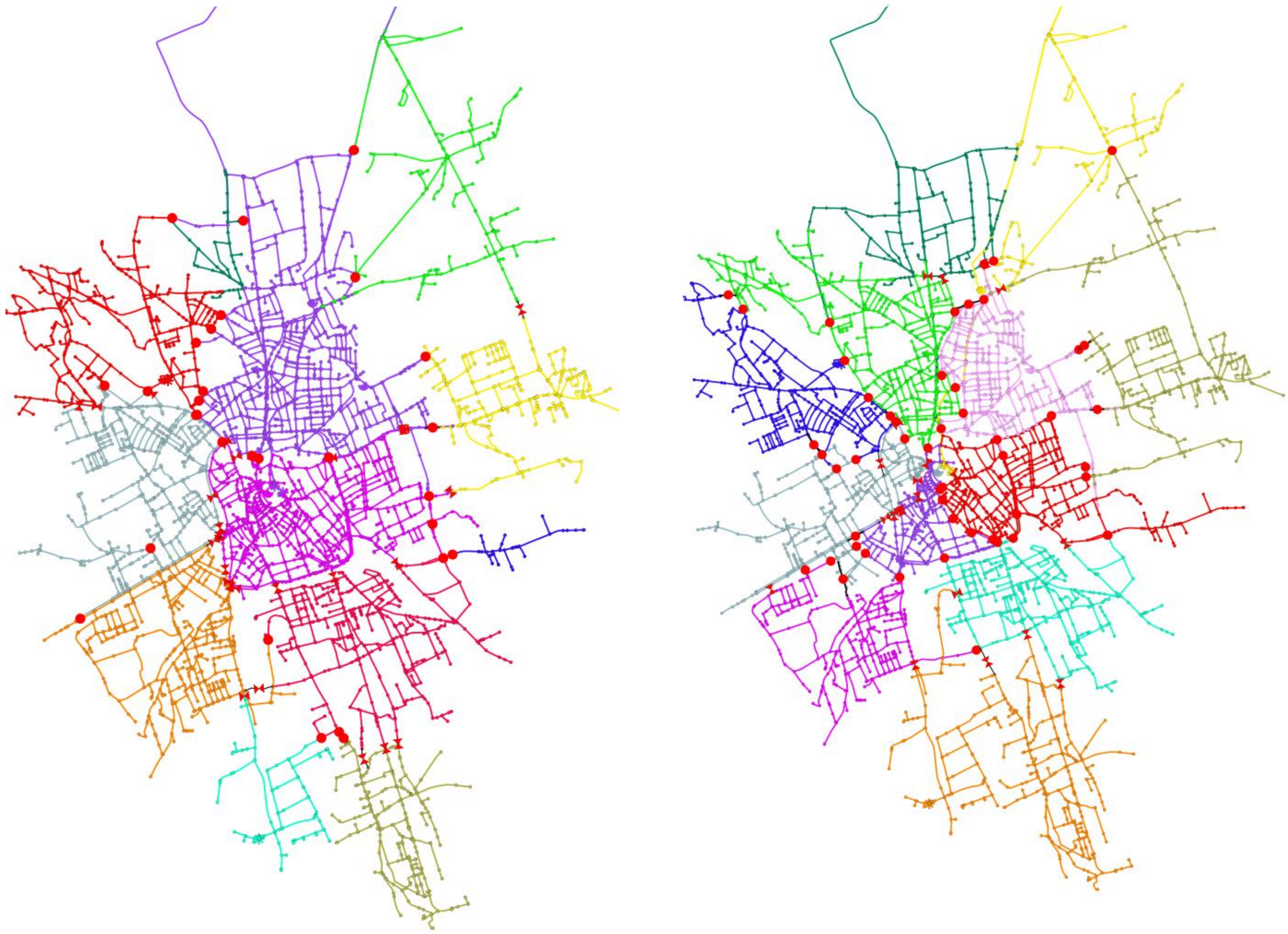


Figure 5. *Cont.*



(**c**) (**d**)

Figure 5. (**a**) Udine WDNP: Spectral clustering algorithm and investing cost objective function; (**b**) Udine WDNP: Girvan–Newman algorithm and investing cost objective function; (**c**) Udine WDNP: Spectral clustering algorithm and MRI objective function; (**d**) Udine WDNP: Girvan–Newman algorithm and MRI objective function.

Nevertheless, the most relevant results for the WDNP are achieved with multi-objective optimization. These results, represented with the Pareto frontier, allow the water utility to define the acceptable compromise between two opposing objectives. The Pareto frontier obtained with the multi-objective function considering both the investing cost and the MRI index presents a trend shown in Figure 7a. Initially, the MRI index increases fast with the cost and then remains constant. In general, the solutions obtained from the Girvan–Newman algorithm tend to be more expensive than the spectral one. In fact, spectral clustering allows us to support the investing cost due to the lower edge-cuts number, but with MRI index values lower than the Girvan–Newman ones. On the other hand, with the Girvan–Newman algorithm (Figure 7b), it is possible to reach MRI values greater than 0.2 with obvious advantages in terms of service—e.g., mean pressure—provided that greater investment is made. Moreover, WDNP, achieved with Girvan–Newman, guarantees water ages lower than 11 h.

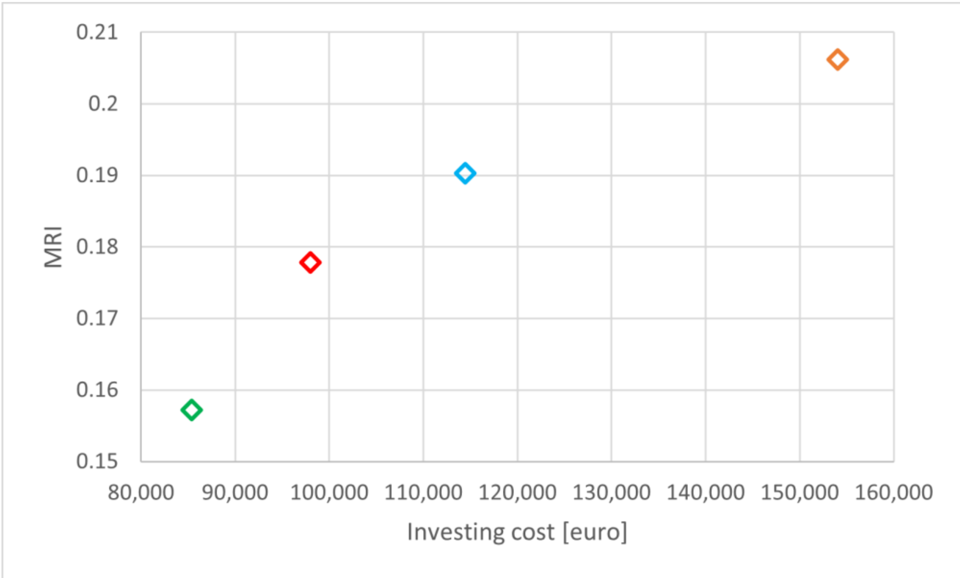
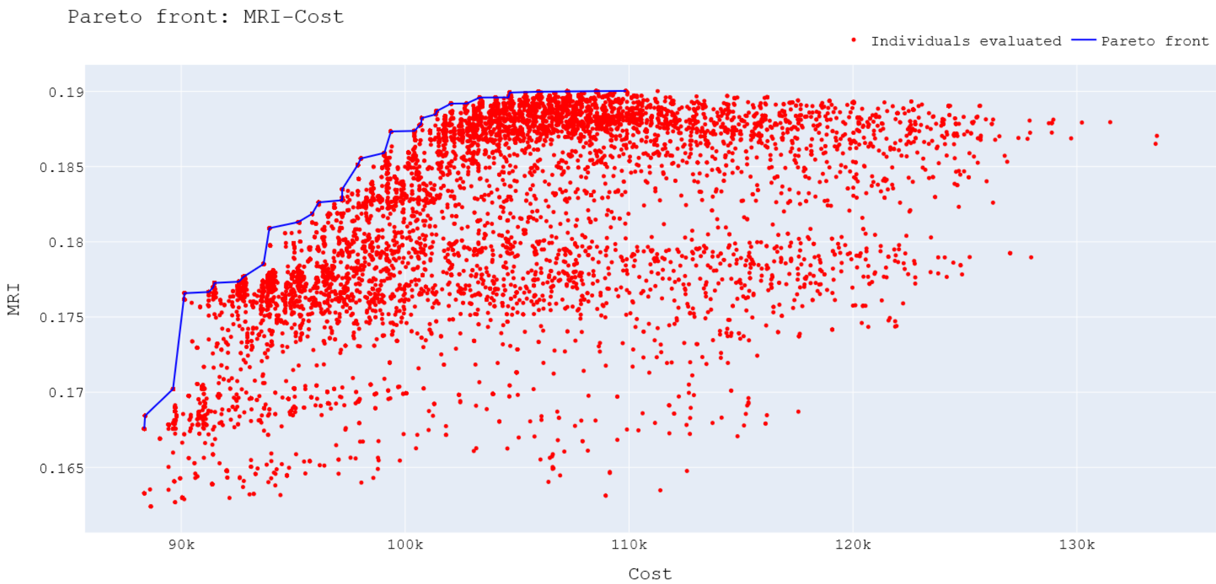


Figure 6. MRI and investing cost values of the single-objective function.

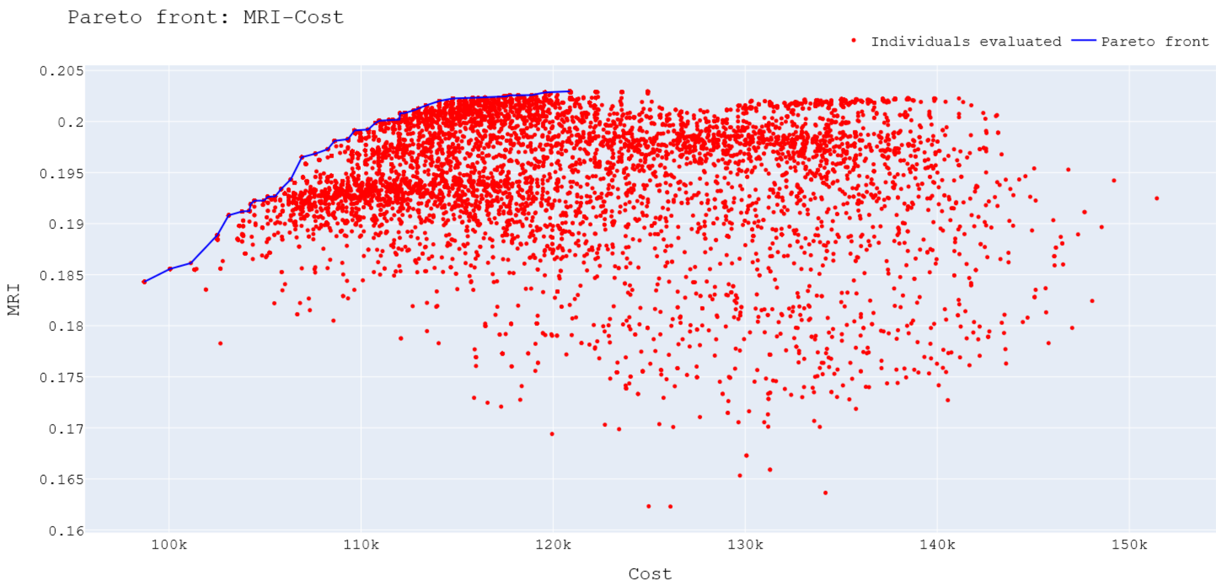
Finally, considering the GA behavior, optimal objective function trends are shown, which, in a few generations, settle around the minimum or maximum. Furthermore, it is possible to declare that the Pareto frontier represents fundamental information for the correct DMAs design because, afterwards, it allows us to define which is the best option among the possible and equally optimal solutions.

The previous thoughts would not have been understood if only a single-objective function had been considered. Hence, the single-objective function results are secondary compared to an overall view given from the Pareto frontier.



(**a**)

Figure 7. *Cont*.



(**b**)

Figure 7. (**a**) Pareto frontier: Spectral clustering algorithm and MRI + investing cost objective function; (**b**) Pareto frontier: Girvan–Newman algorithm and MRI + investing cost objective function.

With particular reference to the Udine WDN, it is believed that both the WDNP analyzed constitute a good result for the network; in particular. the spectral method WDNP allows us to obtain a satisfying and suitable DMAs' configuration, a global low investing cost, and a good solutions distribution along the Pareto frontier, which gives to the water utilities a wide range of choice. The Girvan–Newman method WDNP guarantees better results in terms of both resilience and water quality in the face of a higher investment cost. Hence, the better solution choice for the Udine WDN depends especially on the water utility priorities.

4. Conclusions

The adopted WDNP methodology, based on complex network theory, represents an effective methodology to define the DMAs' configuration and the optimal location of flow meters and gate valves on boundary pipes. While both the adopted clustering and dividing algorithms have already been used and validated in experimental or virtual networks, they have never been applied on a real-life calibrated network with the complexity of the Udine WDN, in which the high loop number and the presence of several supply points made the partitioning a difficult task. For this reason, the feasible solutions obtained allow us to confirm that the approach is an efficient tool for water utilities, able to define optimal water network partitioning by evaluating network resilience and cost. So, it is reasonable to assume that the methodology is applicable to a wide range of networks with similar or less complexity.

Despite the good results, some limits can be recognized in the proposed methodology. The greater limit is that the DMAs' determination is the result of an unsupervised clustering, i.e., the designer does not control the final algorithm product. Therefore, the DMA boundary definition does not consider the possibility of including, if in a sufficient number, a tank or a reservoir for every district, nor does it consider the nodes elevation to have the same total head within a cluster. It is possible to consider these aspects by modifying the algorithms' codes. The investing cost objective function does not consider other spending due to leakage or particular situations during interventions or device maintenance. Despite this, the calculation performed does not consider the presence of all the interception valves already inserted in the network. Hence, the predicted investing cost represents an optimal indicator of the partitioning cost, and an expression development could be achieved with a complete and updated GIS portal that defines the existing boundary valves' position. Another limit derives from the hydraulic model, which uses a demand-driven approach (DDA), and which assumes that the required demand is always fully satisfied no matter the existing pressure. However, in scenarios of pressure-deficient conditions, the DDA results are not accurate, and a pressure-driven approach (PDA) is needed. In particular, hydraulic losses could modify the analysis results and should be considered by outflow equations which make the model more realistic.

In addition to the described limits, several improvements could be implemented: the practical district fulfillment requires particular attention to avoid unexpected effects due to hydraulic uncertainty. Hence, progressive partitioning—where DMAs are gradually divided into smaller ones—could be suitable and achievable by assigning as an input file the previously partitioning output file. Different WDN performance, resilience, and vulnerability indices, such as network energy and network resilience index (NRI), could be considered to compare several solutions. To improve the partitioning, it could be useful to introduce and consider pressure management with PRV valves, which allow us to govern the network pressure and act directly on the hydraulic losses.

Author Contributions: Methodology, F.S.; Software, G.V.; Writing—original draft, F.S.; Supervision, M.N. and D.G. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The data presented in this study are available on request from the corresponding author. The data are not publicly available because they are water utility's sensitive data.

Conflicts of Interest: The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

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