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Minimum waiting time scheduling of power supply assignation to variable rate requests ⋆

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Abstract
The paper deals with a novel scheduling strategy for the assignation of a power resource. More precisely, a set of tasks, characterized by power requests with variable power rate, such as in the domestic electric appliances, is considered and the strategy aims at minimizing the average waiting time. The main result is that to determine the assignation strategy only the information on the maximum needed power and on the duration of the tasks is required. During the implementation of the strategy, the scheduler needs to periodically obtain, from the appliances, information on the maximum power needed to complete the task. In the case of two tasks, the strategy is shown, both analytically and with simulations, to perform better than a non-interruptible strategy.

Keywords: Scheduling problem, optimal control, source assignment

1. INTRODUCTION

The problem of scheduling the access of users to a common source of limited capacity is widely known and treated. In this paper, in particular, the case of an electric power supplier that has to provide energy to different users is considered. The problem is interesting when the sum of all the requested powers is larger than the maximum available power so that a scheduling strategy must be chosen in order to assign the resource to the users. Several quality indices can be considered as objective function and many results are available in the literature: demand-energy methods (Fan et al. (2023); Pi et al. (2021); Shafie-khah et al. (2019)), heuristics (Banga and Rana (2017); Gupta and Singh (2012)), predictive energy management (Shakeri et al. (2018); Shareef et al. (2018)), smart charging (Mukherjee and Gupta (2015); Wang et al. (2016)). Herein we propose a scheduling scheme aiming at minimizing the waiting time of the users.

The main contribution of the paper can be summarized as follows.

- We show that a supervised scheduling based on the information of all the future consumption profile of all the users may render the average waiting time minimum.
- We define a scheduling method based on the information on (i) the total amount of time the user needs the resource and (ii) the maximum power requested.
- We prove that a linear approximation of the future consumption profile may lead, in more that half of the instances, to the optimal solution.
- We show result of simulations that confirm this result.

2. MOTIVATING EXAMPLE

Consider the case of two appliances (e.g. a washing machine and an oven) that has to perform two tasks, T1 and T2, having the power consumption profile shown in Figure 1. The task performed by the first appliance requires 2 KW for the first 20 minutes and then 0.2 KW for the remaining 30 minutes; the other task requires 0.5 KW for the first 50 minutes and then 2.5 KW for the remaining 40 minutes.

2.1 Minimum information strategy

In a typical non-interruptible (NI) scheduling strategy, as one of those described in Rosset et al. (2022), the power profile of the two appliances must be considered constant and, obviously, when computing the assignation strategy the value of each of them needs to be set to the maximum value. This methodology, that will be referred to as "minimum information" (MI) has the advantage of requiring only the information concerning the maximum value of the required power for each appliance and the duration of each of the two processes. Nevertheless, as expected, it does not optimize the use of the total power. In

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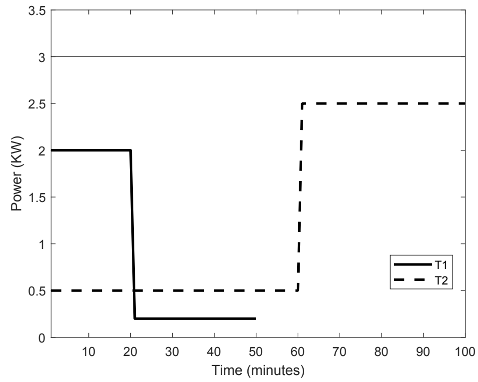

Figure 1. Power consumption profiles of two domestic appliances connected to the same power supplier having a maximum capacity of 3 KW.

this numerical example, for instance, since the sum of the two maximal values of the power profiles is larger than the maximum available power, the two tasks cannot overlap and have to be performed subsequently. The minimum waiting time for "same time requests" is then[1] $\Delta^M_{ST,\min} = \min\{50, 100\} = 50$ which is obtained when the scheduler assigns the resource first to T1 while T2 begins after 50 time units, namely when T1 has finished. However, if the request for task T1 arrives after the request for T2, then T1 has to wait until time T2 has finished so that the waiting time can be up to $\Delta^M_{\sup} = 100$.

As a matter of fact, the *relative* waiting time, namely the ratio between the waiting time associated with a task and the duration of the task, could be a more informative measure. In this scenario we have

$$\Delta^{M,r}_{ST,\min} = \min\left\{\frac{\tau_1}{\tau_2}, \frac{\tau_2}{\tau_1}\right\} = \frac{1}{2}$$

and

$$\Delta^{M,r}_{\sup} = \max\left\{\frac{\tau_1}{\tau_2}, \frac{\tau_2}{\tau_1}\right\} = 2\,.$$

2.2 All information strategy

When the information on the whole power profile is available, a more efficient strategy can be designed. In the above example, for instance, this information would allow one to know that the two tasks can indeed be performed at the same time, since at any time-instant the sum of the powers required by the two tasks is less than the maximum available power. As a consequence, the minimum waiting time for "same time requests" is[2] $\Delta^A_{ST,\min} = 0$. On the other hand, if the request for T1 arrives, for instance, after T2 has completed the first 45 seconds, then the power request for the two tasks is 2.5 kW (for T2) plus 2 kW (for T1) which is larger than the maximum available power (3 kW). As a consequence, T1 has to wait until T2 has finished so that the waiting time can be up to $\Delta^A_{\sup} = 60$. As far the relative waiting time are concerned, we have $\Delta^{A,r}_{ST,\min} = 0$ and $\Delta^{A,r}_{\sup} = 60/50 = 1.2$. Both the absolute and the relative waiting times are less than in the previous strategy. Unfortunately, keeping information on the power profile of both the appliances (and, in a general situation, of all the possible appliances that may request access to the same power supply), requires an additional resource.

[1] The index "M" stays for "minimum information" while the index "ST" stays for "same time".

[2] Here the index "A" stays for "all information".

2.3 Future values information strategy

There is (at least) a third strategy which lies in between the two described above and has two advantages: (i) few information is required to design the scheduling and (ii) it yields a sub-optimal solution to the problem of minimizing the waiting time. This strategy can be described as follows.

- Given a power consumption profile $p_k : [0, \tau_k] \to [0, P_{\max}]$, we associate with p_k a non-increasing function of time, $p_{k,F} : [0, \tau] \to [0, P_{\max}]$, whose value in t is the maximum power needed from t until the end of the task:
$$p_{k,F}(t) = \max_{\sigma\in[t,\tau_k]} p_k(\sigma)\,. \quad (1)$$
A pictorial example is shown in Figure 2).

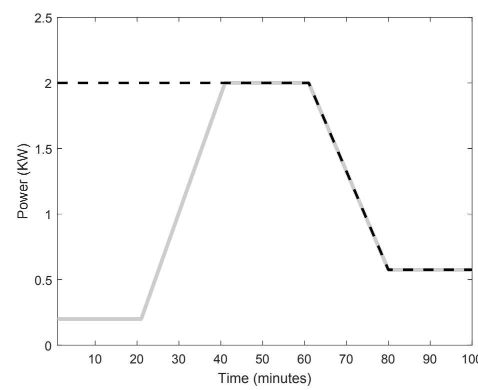

Figure 2. A power consumption profile (grey solid line) and its maximum future value (MFV) (black dashed line) defined by $p_{k,F}(t)$ as in (1).

- Periodically, say every T time units, the scheduler asks to each appliance the current value of the MFV for the remaining part of its task.
- A new task, with power profile p_n, waiting to be admitted to the power resource, can be admitted at the time-instant $t^* = hT$ such that
$$p_{n,F}(0) + \sum_{i=1}^{n-1} p_{k,F}(t^*) < P_{\max}\,. \quad (2)$$
In this way, the constraint on the maximum supply power, fulfilled at time t^*, is fulfilled for all $t > t^*$.
- When several tasks are all admissible at time t^*, the scheduler decides which one is to be admitted first, according to some rule (see Section 3 where a rule minimizing the average waiting time is described).

It is not difficult to see how this strategy works in the case of the above considered appliances (see again Figure 1). First of all, $p_{1,F}$ coincides with the power consumption profile (which is always the case when the profile is a non-increasing function of time, as the profile of T1) while $p_{2,F}$ is a constant function taking the value 2.5. Moreover, when the two power requests arrive at the same time two scenarios are possible: if the scheduler assigns first the resource to T1, then, since $p_{2,F}(0) = 2.5$, T2 has to wait at least 20 minutes; on the other hand, if the scheduler assigns first the resource to T2, then T1 has to wait that T2 terminates, i.e. 100 minutes. As a consequence, this strategy leads to[3] $\Delta^F_{ST,\min} = 20$. It is also easy to see that $\Delta^F_{\sup} = 100$, $\Delta^{F,r}_{ST,\min} = 20/100 = 0.2$ and $\Delta^{F,r}_{\sup} = 100/50 = 2$.

2.4 Formulation of the general problem

Before formalizing the scheduling problem, some comments concerning the quantities introduced above are suitable.

Proposition 1. Regardless the number of tasks that require power at the same time,

$$\Delta^M_{ST,\min} \geq \Delta^F_{ST,\min} \geq \Delta^A_{ST,\min}\,, \tag{3}$$

$$\Delta^M_{\sup} \geq \Delta^F_{\sup} \geq \Delta^A_{\sup}\,. \tag{4}$$

Proof. When the power consumption profile is completely known, also the information on the future maximum value can be recovered, so that the strategy based on the future values can also be applied. Hence the last inequalities in (3) and (4) hold. An analogous reasoning allows to prove the first inequalities in both the equations. ■

Remark 1. It is important to highlight that the strategy described in Section 2.3 is characterized by two features:

- the function $p_{k,F}$ which is evaluated by the appliance performing the task (and not by the scheduler);
- the strategy minimizing the average waiting time which is based on the current value of the $p_{k,F}$ communicated by the appliance to the scheduler.

While the first of these features is deterministically computable once the power demand profile is known, the second one can have several realizations and shall be designed in order to minimize the (average) waiting time.

In the following, the waiting time for task i is denoted by t^*_i while a generic solution for the scheduling of n tasks is denoted by $(i_1, \ldots, i_n)$.

The problem can, now, be formalized as follows.

Problem 1. Suppose that the maximum value of the power that can be supplied is $P_{\max}$ and that at time $t = 0$ requests for tasks $T_1, \ldots, T_n$, with maximum requested power $p_{1,F}(0) = P_1(0), \ldots, p_{n,F}(0) = P_n(0)$ and durations $\tau_1, \ldots, \tau_n$ are active. Suppose that, for all $j = 1, \ldots, n$,

$$P_j(0) < P_{\max}$$

and that

$$\sum_{j=1}^{n} P_j(0) > P_{\max}\,.$$

Find the scheduling $i_1, \ldots i_n$ that minimizes the total waiting time

$$\min_{i_1,\ldots,i_n} \sum_{i=1}^{n} t^*_i\,.$$

Remark 2. In Problem 1 the total waiting time can be replaced by the total *relative* waiting time defined by $\min_{i_1,\ldots,i_n} \sum_{i=1}^{n} t^*_i/\tau_i$.

[3] Here the index "F" stays for "future information".

3. MAIN RESULT

In this section we show how a sub-optimal strategy[4] can be designed on the basis of the MFV. We begin with the simple example of two tasks, characterized by the maximum future value functions depicted in Figure 3. Since $p_{1,F}(0) + p_{2,F}(0) > P_{\max}$ the two tasks cannot begin

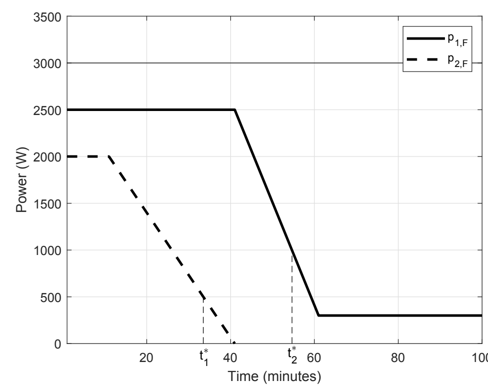

Figure 3. Time history of the MFV functions of two tasks. Given the value of the maximum supplying power (3KW) the waiting time is t^*_1 if the scheduling is "first T_1 and then T_2" and t^*_2 if the scheduling is "first T_2 and then T_1".

at the same time, hence a scheduling strategy must be designed. In particular, if T_1 starts first, then T_2 will have access to the energy source at t^*_2, namely when $p_{1,F}$ decreases down to $P_{\max} - p_{2,F}(0) = 3000 - 2000 = 1000$; if T_2 starts first, then T_1 will have access to the energy source at t^*_1, namely when $p_{2,F}$ decreases down to $3000 - p_{1,F}(0) = 3000 - 2500 = 500$. This reasoning can be repeated in a generic case, leading to the following result, the proof of which is immediate.

Proposition 2. Given two tasks, T_1 and T_2, let t^*_1 and t^*_2 be defined by

$$p_{2,F}(t^*_1) + P_1(0) = P_{\max}\,,\;\; p_{1,F}(t^*_2) + P_2(0) = P_{\max}\,. \tag{5}$$

The scheduling strategy solving Problem 1 is:

- first T_1 (at 0) and then T_2 (at t^*_2) if $t^*_2 \leq t^*_1$;
- first T_2 (at 0) and then T_1 (at t^*_1) if $t^*_2 > t^*_1$. □

Remark 3. A result analogous to Proposition 2 holds when considering the relative waiting times.

The strategy underlying the claim of Proposition 2 is based on the knowledge of t^*_1 and t^*_2 which, in turn, can be obtained from $p_{2,F}$ and $p_{1,F}$. Unfortunately, as mentioned above, this information is not available to the decision maker. However an approximated strategy that *in the average* behaves as the optimal one can be designed. The reasoning leading to this approximated strategy lies in the answer to the following question: if the only available information concerning task T_j consists in the values $P_j(0)$ and τ_j, what is the function that best approximates $p_{j,F}(t)$? More formally, considering $p_{j,F}(t)$ as a stochastic process, what is the function $q_j : [0, \tau_j] \to [0, P_j(0)]$ such that

[4] The strategy is called "sub-optimal" with respect to the "all information" one.

$$\mathcal{E}\left\{\int_0^{\tau_j} |p_{j,F}(t) - q_j(t)|dt\right\}$$

(where $\mathcal{E}$ denotes the expectation) is minimum? It is not difficult to see that this function is the linear one connecting the points $(0, P_j(0))$ and $(\tau_j, 0)$. In fact, for any possible $p_{j,F}$ (see the solid bold line in Figure 4) it is possible to construct another function $\widetilde{p}_{j,F}$ (see the dashed bold line) associated with the same error with respect to the "diagonal" (the measure of which is the shaded area). More precisely, $\widetilde{p}_{j,F}$ is the symmetric of $p_{j,F}$ with respect

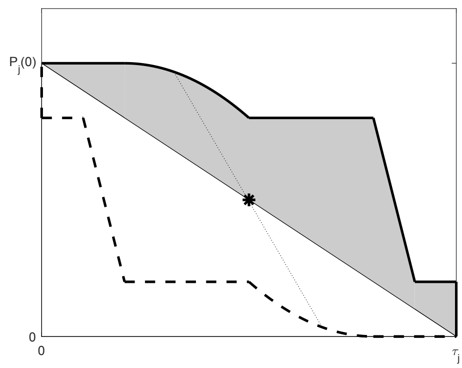

Figure 4. A possible MFV function and its symmetric with respect to the mean point.

to the point $(\tau_j/2, P_j(0))$ (the black star in Figure 4).

Now, approximating the MFV functions with linear functions has two advantages. First, the quantities t_1^* and t_2^* have the simple expressions:

$$\bar{t}_1^* = \beta\frac{\tau_2}{P_2(0)}\,,\ \bar{t}_2^* = \beta\frac{\tau_1}{P_1(0)}\,. \tag{6}$$

with $\beta = P_1(0) + P_2(0) - P_{\max}$. In addition, it is easy to implement the approximated strategy defined as follows.

Definition 1. Given two tasks T_1 and T_2, the *linear* strategy is

- first T_1 (at 0) and then T_2 (at t_2^*) if $\bar{t}_1^* > \bar{t}_2^*$;
- first T_2 (at 0) and then T_1 (at t_1^*) if $\bar{t}_1^* < \bar{t}_2^*$.

Finally, the linear approximation leads to the following reasonable assumption.

Assumption 1. The value $\bar{t}_1^*$ is the median value of t_1^* and the probability distribution is uniform both on the left and on the right:

$$\Pr\{t_1^* < t\} = \begin{cases} 0.5\dfrac{t}{t_1^*}\,, & \text{if } t \in [0, \bar{t}_1^*]\,,\\[2mm] 0.5 + 0.5\dfrac{t - t_1^*}{\tau_2 - t_1^*}\,, & \text{if } t \in (\bar{t}_1^*, \tau_2]\,. \end{cases} \tag{7}$$

An analogous property holds for $\bar{t}_2^*$. ■

Theorem 1. In the case of two tasks, if $P_1(0) = P_2(0) = P_0$, Assumption 1 guarantees that the linear strategy solves Problem 1 *in the average*, meaning that

$$\bar{t}_1^* > \bar{t}_2^* \quad \Rightarrow \quad \Pr\{t_1^* < t_2^*\} < \frac{1}{2}\,.$$

Proof. Without loss of generality, we suppose $\tau_2 > \tau_1$. Two cases have to be considered.

Case (i): $\bar{t}_1^* < \tau_1$ (see Figure 5). The probability that

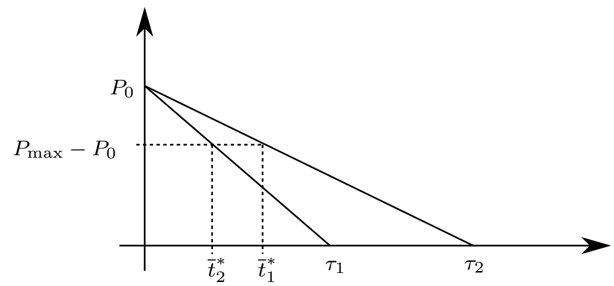

Figure 5. The case of two linear approximation with $P_1(0) = P_2(0) = P_0$ characterized by $\bar{t}_1^* < \tau_1$.

$t_1^* < t_2^*$ can be computed as follows.

$$\begin{aligned}
\Pr\{t_1^* < t_2^*\} &= \int_0^{\tau_1} \Pr\{t_2^* > t\}\Pr\{t_1^* \in [t, t+dt]\} = \\
&= \int_0^{\bar{t}_1^*} \Pr\{t_2^* > t\}\frac{dt}{2\bar{t}_1^*} + \int_{\bar{t}_1^*}^{\tau_1} \Pr\{t_2^* > t\}\frac{dt}{2(\tau_2 - \bar{t}_1^*)} = \\
&= \int_0^{\bar{t}_2^*}\left(1 - \frac{t}{2\bar{t}_2^*}\right)\frac{dt}{2\bar{t}_1^*} + \int_{\bar{t}_2^*}^{\bar{t}_1^*}\frac{\tau_1 - t}{2(\tau_1 - \bar{t}_2^*)}\frac{dt}{2\bar{t}_1^*} + \\
&\quad + \int_{\bar{t}_1^*}^{\tau_1}\frac{\tau_1 - t}{2(\tau_1 - \bar{t}_2^*)}\frac{dt}{2(\tau_2 - \bar{t}_1^*)} = \\
&= \frac{\bar{t}_2^*}{2\bar{t}_1^*} - \frac{(\bar{t}_2^*)^2}{8\bar{t}_1^*\bar{t}_2^*} + \frac{\tau_1(\bar{t}_1^* - \bar{t}_2^*)}{4\bar{t}_1^*(\tau_1 - \bar{t}_2^*)} - \frac{(\bar{t}_1^*)^2 - (\bar{t}_2^*)^2}{8\bar{t}_1^*(\tau_1 - \bar{t}_2^*)} + \\
&\quad + \frac{\tau_1(\tau_1 - \bar{t}_1^*)}{4(\tau_2 - \bar{t}_1^*)(\tau_1 - \bar{t}_2^*)} - \frac{\tau_1^2 - (\bar{t}_1^*)^2}{8(\tau_2 - \bar{t}_1^*)(\tau_1 - \bar{t}_2^*)}\,.
\end{aligned} \tag{8}$$

Substituting the explicit expressions of $\bar{t}_1^*$ and $\bar{t}_2^*$ (6) into (8) and after some computations (see Appendix), we obtain

$$\Pr\{t_1^* < t_2^*\} = \left(\frac{2\alpha^2 - \alpha + 1}{8\alpha^2}\right)\frac{\tau_1}{\tau_2} + \left(\frac{2\alpha^2 - 3\alpha + 1}{8\alpha^2}\right)\frac{\tau_2}{\tau_1} + \frac{2\alpha - 1}{4\alpha^2}\,, \tag{9}$$

where $\alpha = P_{\max}/P_0 - 1$. Now, the right-hand side in (9) can be interpreted as a function of $\tau_1/\tau_2 = x$, parametrized by α:

$$\Pr\{t_1^* < t_2^*\} = a(\alpha)x + \frac{b(\alpha)}{x} + c(\alpha)\,, \tag{10}$$

where

$$a(\alpha) = \frac{2\alpha^2 - \alpha + 1}{8\alpha^2}\,, \quad b(\alpha) = \frac{2\alpha^2 - 3\alpha + 1}{8\alpha^2}\,, \quad c(\alpha) = \frac{2\alpha - 1}{4\alpha^2}\,.$$

We want to show that (10) is an increasing function of x for all α. To this purpose, we compute the first derivative with respect to x:

$$a(\alpha) - \frac{b(\alpha)}{x^2}\,. \tag{11}$$

Two cases are in order.

- If $\alpha \in (0.5, 1)$ then $b(\alpha) < 0$. Hence
$$a(\alpha) - \frac{b(\alpha)}{x^2} > a(\alpha) > 0\,.$$
- If $\alpha \in (0, 0.5)$ then $b(\alpha) > 0$. The condition $\bar{t}_1^* < \tau_1$ (Case (i)) implies $x > 1 - \alpha$ which, substituted into (11) yields

$$a(\alpha) - \frac{b(\alpha)}{x^2} > \frac{2\alpha^2 - \alpha + 1}{8\alpha^2} - \frac{2\alpha^2 - 3\alpha + 1}{8\alpha^2(1-\alpha)^2} = \\ = \frac{2\alpha^2 - 5\alpha + 3}{8(1-\alpha)^2} > 0 \,. \quad (12)$$

As a consequence, since $x \leq 1$, (10) reaches its maximum value for $x = 1$:

$$\Pr\{t_1^* < t_2^*\} \leq \frac{2\alpha^2 - \alpha + 1}{8\alpha^2} + \frac{2\alpha^2 - 3\alpha + 1}{8\alpha^2} + \frac{4\alpha - 2}{8\alpha^2} = \frac{1}{2} \,.$$

Case (ii): $\bar{t}_1^* > \tau_1$ (see Figure 6). In this case the

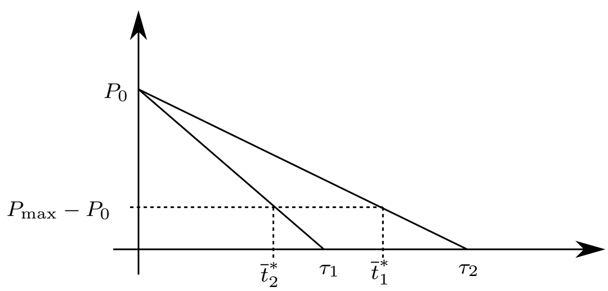

Figure 6. The case of two linear approximation with $P_1(0) = P_2(0) = P_0$ characterized by $\bar{t}_1^* > \tau_1$.

probability that $t_1^* < t_2^*$ is

$$\Pr\{t_1^* < t_2^*\} = \\ = \int_0^{\bar{t}_2^*} \left(1 - \frac{t}{2\bar{t}_2^*}\right) \frac{dt}{2\bar{t}_1^*} + \int_{\bar{t}_2^*}^{\tau_1} \frac{\tau_1 - t}{2(\tau_1 - \bar{t}_2^*)} \frac{dt}{2\bar{t}_1^*} = \\ = \frac{\bar{t}_2^*}{2\bar{t}_1^*} - \frac{\bar{t}_2^*}{8\bar{t}_1^*} + \frac{\tau_1}{4\bar{t}_1^*} - \frac{\tau_1 + \bar{t}_2^*}{8\bar{t}_1^*} = \\ \left(\frac{3}{8} + \frac{1}{4(1-\alpha)} - \frac{2-\alpha}{8}\right) \frac{\tau_1}{\tau_2} = \frac{3-\alpha^2}{8(1-\alpha)} \frac{\tau_1}{\tau_2} \,. \quad (13)$$

The condition $\bar{t}_1^* > \tau_1$ (Case (ii)) implies $x < 1 - \alpha$; hence

$$\Pr\{t_1^* < t_2^*\} < \frac{3-\alpha^2}{8} < \frac{1}{2} \,.$$

■

Theorem 1 guarantees that (if $P_1(0) = P_2(0)$) the scheduling designed according to the value of the average waiting times $\bar{t}_1^*$ and $\bar{t}_2^*$ (the task with smaller average waiting time is performed last) is the optimal one with probability larger than 1/2. Next section shows the outcome of simulations that confirm this result.

4. NUMERICAL SIMULATIONS

To provide a practical confirmation of the claim of Theorem 1 several groups of instances of Problem 1 with two tasks have been carried out for the *equal power* (EP) case, namely the case in which $P_1(0) = P_2(0) = P_0$. Moreover, the behaviour of the linear strategy has been tested also in the *different power* (DP) case, namely the case in which $P_1(0) \neq P_2(0)$, for which no result analogous to Theorem 1 is available. Some details about the simulations are reported next.

- The value of $P_{\max}$ has been set to 20 while the value of P_0 in the EP case, as well as the values of $P_1(0)$ and $P_2(0)$ in the DP case, have been chosen randomly between 10 and 20, with the constraints $2P_0 > P_{\max}$ and $P_1(0) + P_2(0) > P_{\max}$.
- The values of τ_1 and τ_2 have been chosen randomly between 10 and 110. Without loss of generality, their values have been switched when $\tau_2 < \tau_1$ in order to have $\tau_1 < \tau_2$ for all the instances.
- For each choice of P_0 (or $P_1(0)$ and $P_2(0)$), τ_1 and τ_2, 10000 pairs of possible MFV functions have been simulated.
- Results for the EP case are reported in Table 1. The fourth column contains the values of $\Pr\{t_1^* < t_2^*\}$ predicted by (8) (or (13)), namely on the basis of the values of τ_1, τ_2, $P_{\max}$ and P_0, while the fifth column contains the empirical probability measured through the 10000 simulated MFV functions. It can be noted that the two values are quite similar.
- The last column of Table 1 reports the percentage of cases (among the 10000 trials) in which the linear strategy provides the optimal solution. The average is about 80% of success.
- Results for the DP case are reported in Table 2.

P_0	τ_1	τ_2	Predicted	Measured	Success
11.3808	36	71	0.3242	0.3274	67.26%
13.2161	30	100	0.1491	0.1656	83.44%
15.1481	84	105	0.3588	0.3592	64.08%
13.6502	31	93	0.1612	0.1726	82.74%
13.1018	22	61	0.1854	0.1959	80.41%
18.7743	20	24	0.3198	0.2559	74.41%
17.6934	42	65	0.2544	0.1996	80.04%
15.8688	56	106	0.2214	0.2027	79.73%
10.0105	32	106	0.3247	0.1178	88.22%
19.8234	59	92	0.2412	0.0436	95.64%

Table 1. Outcomes of simulations for the EP case. The fourth and the fifth column confirm the claim of Theorem 1 while the last column shows the performance of the linear strategy.

$P_1(0)$	$P_2(0)$	τ_1	τ_2	Measured	Success
17.7839	10.7992	27	109	0.0513	94.87%
15.6531	12.6249	94	100	0.3892	61.08%
12.5727	11.5336	54	57	0.4552	54.48%
13.6172	15.9258	31	75	0.2139	78.61%
15.2119	15.4934	13	16	0.3649	63.51%
15.3711	11.3768	33	83	0.1636	83.64%
17.9618	14.381	16	68	0.0446	95.54%
16.0806	14.4210	58	107	0.2007	79.93%
18.0057	11.1941	63	63	0.3157	68.43%
18.7667	14.4409	37	81	0.0719	92.81%

Table 2. Outcomes of simulations for the DP case.

5. CONCLUSIONS

For tasks concerning power requests with variable rate, a novel scheduling strategy, that uses only the information on the maximum requested power and on the duration of the task, has been defined and studied. The strategy performs better than a non-interruptible strategy, at least in the case of two tasks, where an analytic proof can also be provided. Due to the results obtained in this simple scenario, it is reasonable to assume that it could also work well in the more general case with more than 2 tasks, which may be the topic for future improvements of the strategy. In the case of 3 or more tasks, to possible extensions could be as follows.

(i) The easiest way is to build a list of the tasks ordered according to the values of the quantities $\tau_i/P_i(0)$'s (see equation (6)).

(ii) Another possibility is, for each task i, to interpret the remaining tasks as a single task of duration

$$\tau_{<i>} = \sum_{j\neq i} \tau_j$$

and maximum requested power

$$P_{<i>}(0) = \sum_{j\neq i} P_j(0)$$

and choose the first task to be performed comparing, as done above in the case of two tasks, $\tau_i/P_i(0)$ with $\tau_{<i>}/P_{<i>}(0)$.

Another development could be the application of the strategy to real-world tasks (as the one described in Issi and Kaplan (2018)) and the measure of its performance in more complex power networks.

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APPENDIX

Proposition 1. Equation (8) and (9) are equivalent.

Proof. Note that $P_1(0) = P_2(0) = P_0$ implies

$$\frac{\bar{t}_2^*}{\bar{t}_1^*} = \frac{\tau_1}{\tau_2}\,, \quad \tau_1 - \bar{t}_2^* = \alpha\tau_1\,, \quad \tau_2 - \bar{t}_1^* = \alpha\tau_2\,.$$

Hence equation (8) can be written as

$$\begin{aligned}
\Pr\{t_1^* < t_2^*\} &= \frac{1}{2}\frac{\tau_1}{\tau_2} - \frac{1}{8}\frac{\tau_1}{\tau_2} + \frac{(\bar{t}_1^* - \bar{t}_2^*)(2\tau_1 - \bar{t}_1^* - \bar{t}_2^*)}{8\bar{t}_1^*\alpha\tau_1} + \\
&\quad + \frac{\tau_1^2 - 2\tau_1\bar{t}_1^* + (\bar{t}_1^*)^2}{8\alpha^2\tau_1\tau_2} = \\
&= \frac{3}{8}\frac{\tau_1}{\tau_2} + \frac{\left(1-\frac{\tau_1}{\tau_2}\right)\left(\alpha+1-(1-\alpha)\frac{\tau_2}{\tau_1}\right)}{8\alpha} + \\
&\quad + \frac{1}{8\alpha^2}\frac{\tau_1}{\tau_2} - \frac{1-\alpha}{4\alpha^2} + \frac{(1-\alpha)^2}{8\alpha^2}\frac{\tau_2}{\tau_1} = \\
&= \left[\frac{3\alpha^2 - \alpha^2 - \alpha + 1}{8\alpha^2}\right]\frac{\tau_1}{\tau_2} + \left[\frac{-\alpha + \alpha^2 + (1-\alpha)^2}{8\alpha^2}\right]\frac{\tau_2}{\tau_1} + \\
&\quad + \frac{\alpha^2 + \alpha + \alpha - \alpha^2 - 2(1-\alpha)}{8\alpha^2}
\end{aligned}$$

from which equation (9) easily follows. ■