TEACHING MULTIVARIABLE DIFFERENTIAL CALCULUS USING GEOGEBRA AND QUIZZES

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Abstract

This paper explains the design of nine multiple-choice quizzes used as formative e-assessment for teaching multivariable differential calculus, including GeoGebra and problem solving tools. It was a preliminary experiment conducted in a second course of calculus for about 200 students in Managerial and Electronical Engineering at University of Udine. Several GeoGebra applets have also been provided to facilitate the graphical representations of the concepts. From the Psychometric Analysis we found out that the quizzes were well chosen, and they discriminated adequately between higher and lower student subject knowledge. A survey was also administered to evaluate the appreciation of the students about the quizzes and GeoGebra. The results were also positive: we found that the quizzes helped students to find gaps in their preparation and to keep up with the lessons, furthermore there was a fair use of GeoGebra and the relative applets.

Keywords: Formative Assessment, Multiple Choice Quizzes, GeoGebra, Multivariable Differential Calculus, Survey

1 INTRODUCTION

Formative assessment is defined “as the iterative processes of establishing what, how much and how well students are learning in relation to the learning goals and expected outcomes in order to inform tailored formative feedback and support further learning, a pedagogical strategy that is more productive when role is shared among the teacher, peers and the individual learner” ([6]).

One advantage of regular quizzes is that they encourage students to keep up with the course content as it is presented ([12], [2]). Additionally, online quizzes provide immediate feedback to students about their level of learning ([7]), and help the tutor identify parts of the course content that students are struggling with and that require further explanation in class. Angus and Watson ([2]) found out that regular, low-stakes, online quizzes improved student learning, as evidenced in the final exam. Research has established that well-designed assessments, including multiple choice questions, enables an assessment of higher cognitive functions, such as critical thinking and analysis skills ([4], [11]). It has been brought to light that students learning for a multiple-choice assessment concentrate on understanding and comprehension, whereas when preparing for a long-answer-type assessment they concentrate on recollection of facts to replicate in their answers.

GeoGebra is a software that allows to create different interactive applications that can be used as teaching tools in math classes. The graphics in books, as well as the pictures drawn on the blackboard, are static and require the students to possess an adequately trained imagination. Nowadays, the existence of free programs with versatile capabilities and interactive representations helps to improve the presentation of contents taught in this area of knowledge, allowing dynamic visualizations. This is extremely important, since only those students able to perform conversions do not confuse a mathematical object with its representation and can transfer their mathematical knowledge to other contexts different from the one of learning ([5]).

There have been several attempts to improve students’ learning and the teaching of mathematics in basic calculus and multivariable calculus through moving away from remedial classes towards teaching to increase understanding. Improving students’ learning through the enhancement of their problem solving and mathematical thinking skills as well as through using technological tools to support conceptual understanding and problem solving methods are now thought to be more appropriate. It enables them to cope with the mathematical needed for solving problems in their fields of study.

Tall in many researches shows that computer can provide an environment to take students into the realms of calculus in 3-dimensions for applications and solutions of real-world problems. He states that

there are not only three distinct types of mathematics worlds; there are actually three significantly different worlds of mathematical thinking as: conceptual-embodied, proceptual-symbolic, axiomatic formal ([14], [15], [16]). Moreover, Tall defined the concepts of generic organiser as programmed on a computer and organizing agent that can be as a teacher to build an embodied approach to mathematics for improving students' difficulties in calculus. In fact, the combination of a human teacher as guide and mentor using a computer environment for teaching, pupil exploration, and discussion can support students' mathematical knowledge construction (see also [1], [8]).

Multivariable calculus is one of the most important parts of mathematics syllabus for engineering students. It is offered as a prerequisite course to other courses. However, multivariable calculus is one of the most difficult courses for most engineering students to study in their field. Various problematic areas have been identified in basic calculus and multivariable calculus. Some of these were, the difficulty of learning some specific mathematical topics, the difficulty in coordinating procedures and manipulating concepts, the particular events that the students experienced in the past, poor problem solving skills, the inability to select and use appropriate mathematical representations, the translation of real-world problems into calculus formulations, absorbing complex new ideas in a limited time, the students' beliefs and their learning styles ([10]).

2 METHODOLOGY

Six quizzes were assigned through the e-learning page of the class, every two or three weeks, one for each chapter of the course. The students did not have a time limit to answer the questions, but it was mandatory for them to reach a certain level to enter the midterm test. Quiz questions were not randomly generated, so all students got the same questions and naturally, students worked together to the solutions shared the answers with each other. To avoid unfairness, it was strongly emphasized that quizzes were important to students, to allow them to test themselves and to get feedback on their level of understanding.

The following nine quizzes were designed for multivariable differential calculus:

1. Recognize the function of two variables from its domain (graphically expressed).
   The purpose of the quiz is to familiarize with the concept from a visual point of view. The various answers were chosen in order to (try to) force the students to reason on the relationship between the variation of the constant and the distance between the various level curves.
2. Recognize the graph of a function of two variables from the graph of its level curves.
   The purpose of the quiz is to practice the study of inequalities in two variables; to understand that domain and sign are studied in two variables in a similar way to the real case (only the shape of the solutions changes); the wrong answers have been prepared to see who confused the conditions of existence of roots, logarithms, fractions (therefore a check on the prerequisites)
3. Recognize the equations of the tangent plane and of the Taylor series at the second order of a function of two variables.
   The purpose of the quiz is to review the formula and the partial derivatives. The wrong answers were written to see who messed with exponentials or partial derivatives
4. Understand for which function the origin is a relative minimum point.
   The purpose of the quiz is to do a sort of function study in two variables, trying to get out of the dry question "calculate this". The answers (right and wrong) were designed to push the kids not to rely only on the differential calculation for the study of max and min but also on theorems (Weierstrass) and reasoning (limits and sign for example).
5. Recognize the truthfulness of some statements (about extreme) concerning functions in two variables. The case of interest is when the Hessian matrix is null, and there are infinite minimum and supremum points.
6. As in 5 but the critical point (the origin) is neither a maximum nor a minimum.
   The purpose of the quiz is to solve an exercise where the classic gradient method did not work and needed a more creative approach. In a nutshell, here too I am trying to make students understand that in analysis one must avoid thinking too algorithmically (for question x I use the solution method y). The wrong answers are obviously written by exclusion.

7. Maximum and minimum with equality constraints.
   The goal is obviously to force students to try this method too (thus avoiding that someone learns just one hoping it will always work). The presence of the exponential was used to reward those who reasoned on the fact that it was enough to search max / min on the polynomial; in the wrong answers I put errors related to inventing non-existent symmetries or forgetting the "e" on the street
8. Evaluate, with the help of GeoGebra, the minimum for the function in two variables subject to inequality constraints.
   The purpose of the quiz is to take the hand with a new method. The wrong answers were written to highlight comprehension errors on the elements of the theorem. The use of GeoGebra served to ensure that the error was not due to mistakes in the graph and then to make the use of the level lines for the determination of the extremal point more visible.
9. Determine the correct formalization of some problems that is possible to solve with the Kuhn-Tucker method. Determine the position of a nuclear waste landfill so that:
   - it is as close as possible to the power plant that produces them (positioned in A = (1,0)).
   - and at least at a distance 2 from a city that is in B = (0,0)
   - either outside a park that is to the right of the line x = 3/2.

They may be classified in different requested levels of mastery of knowledge. MATH taxonomy (Mathematical Assessment Task Hierarchy) for the structuring of assessment tasks turns out to be more appropriate for mathematical tasks than Bloom's taxonomy ([13]). We agree with Smith in that "the aim of the descriptors is to assist with writing examination questions, and to allow the examiner's judgment, objectives and experience to determine the final evaluation of an assessment task". The MATH taxonomy uses eight different descriptors: Factual knowledge (quiz 4); Comprehension (recognition of formulas and situations) (quiz 3); Routine use of procedure or algorithms (quiz 7); Information transfer (classification of math objects) (quiz 2); Application in new situations (planning work, selection of methods) (quiz 9); Justifying, proof, reasoning and interpreting (quiz 5); Implications, making conjectures, comparisons and finding patterns (quiz 1); Evaluation (quiz 6).

2.1. THE PSYCOMETRIC ANALYSIS

Moodle offers a wide range of resources to carry out a psychometric analysis of quizzes, we utilized the Facility Index (FI), the Discrimination Index (DI) and the Discrimination Coefficient (DC) ([9], [3]). FI describes the overall difficulty of the questions, it represents the ratio of users that answer the question correctly. In principle, a very high or low FI suggests that the question is not useful as an instrument of measurement. There are two descriptors to measure effectiveness, DI and DC, both ranging from -1 to +1. The DI index provides a rough indicator of the performance of each item to separate high scores vs. scorers. The DC is a correlation coefficient between scores at the item and at the whole quiz. In both cases, positive values indicate items that discriminate proficient learners, whereas negative indices mark items that are answered best by those with lowest grades, hence not helping to discern between the good and the bad performers. In short, these coefficients can be used as powerful methods of evaluating the effectiveness of the quiz when assessing differentiation of learners. The advantage of using DC over DI is that the former uses information from the whole population of learners, and not just the extreme upper and lower thirds as DI. Thus, this parameter may be more sensitive to detect item performance.

2.2. THE SURVEY

An anonymous and volunteer survey was also proposed to evaluate the appreciation of the students about the quizzes and GeoGebra. We gave the following questions:

- Q1: Do you think the use of quizzes on Moodle has helped you to find gaps in your preparation?
- Q2: Do you think the quizzes on Moodle have helped you to keep up with the lessons?
- Q3: How do you tackle quizzes on Moodle? (respond with the utmost sincerity)
- Q4: How did you deal with the doubts that emerged during the study?
- Q5: Do you think the use of GeoGebra has allowed you to better understand the theoretical notions of the course?
- Q6: Did you use the GeoGebra applets made available on Moodle by the teachers?

3 RESULTS

3.1. THE PSYCOMETRIC ANALYSIS

In this section we analyze, through the psychometric quantity FI, DI and DC, the quality of the assessments. They help to answer whether the questions were appropriate, well chosen to demonstrate concepts, of a suitable level of difficulty and whether they discriminate well between higher and lower mathematical abilities. The results were summarized in the following table:

Table 1. Psychometric results

	Questions	FI	DI	DC
1	Recognize the function from its domain	91.46%	53.25%	71.25%
2	Recognizing the graph from its level curves	92.68%	37.89%	51.68%
3	Recognize the tangent plane and Taylor expansion	84.96%	32.57%	41.05%
4	Recognizing the function with a minimum point	84.15%	42.35%	53.77%
5	Extreme point with null Hessian	64.38%	40.03%	44.40%
6	As in 5 but the critical point is neither a max nor a min	91.67%	38.64%	53.62%
7	Maximum and minimum with equality constraints	80.95%	27.98%	34.74%
8	The minimum with inequality constraints (with GeoGebra)	74.19%	43.48%	55.49%
9	Problem solving with Kuhn-Tucker	69.41%	32.90%	36.03%

From the Psychometric Analysis of the quizzes, we found out that they were well chosen, and they discriminated adequately between higher and lower student subject knowledge. For the students the most difficult question was quiz 5, where to answer correctly is necessary to have a good conceptual knowledge of the subject, followed by quiz 9, which requires a capacity for problem solving and modelling, and 8 that becomes simple if the graphic interpretation of the problem is used. The question with the lowest discrimination index, but acceptable, was quiz 7 followed by quiz 9 and 3 this means that they help less than the others to discern between the good and the bad performances.

3.1. THE SURVEY

148 out of 168 students answered the survey. The results of the survey show, see Figure 1., that the students have appreciated the use of the quizzes: they have considered them useful to highlight their gaps and to keep up with the study of the course.

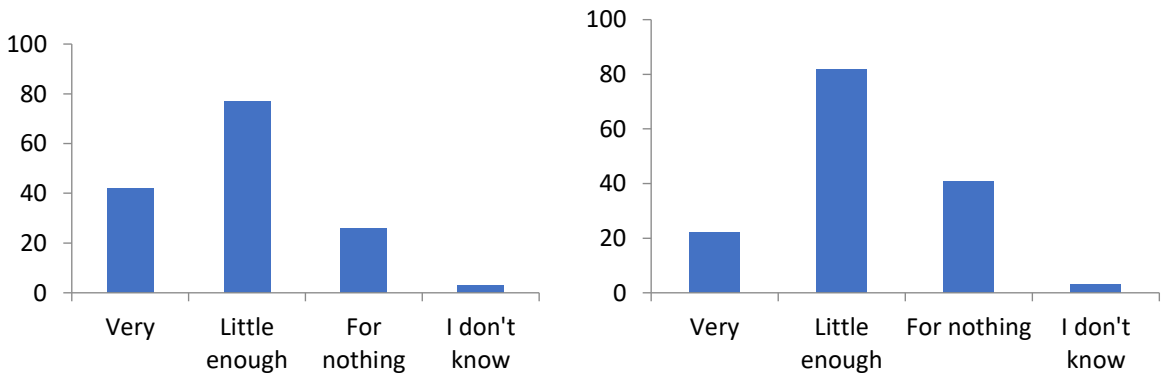

Figure 1: On the left Q1: Do you think the use of quizzes on Moodle has helped you to find gaps in your preparation? On the right Q2: Do you think the quizzes on Moodle have helped you to keep up with the lessons?

In addition, as Figure 2. shown, many students have preferred to cooperate with colleagues, for fear of making mistakes and therefore losing the possibility to participate to the midterm test. The comparison with the colleagues has been also widely used by students to resolve their doubts, followed by consulting online material and then the use of books. Nonetheless asking to professors has been the least chosen alternative (there have been frequent requests for clarification especially at the end of the lessons principally about the subject of the lesson just taught).

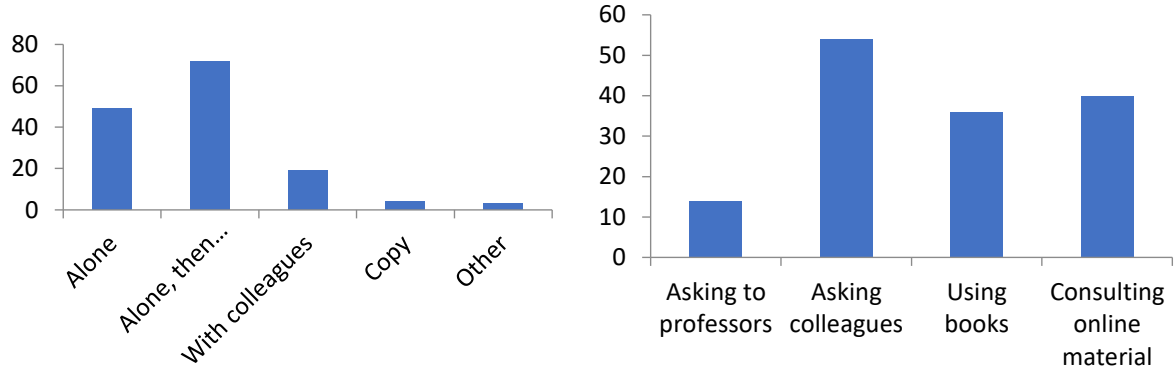

Figure 2: On the left Q3: How do you tackle quizzes on Moodle?
On the right Q4: How did you deal with the doubts that emerged during the study?

Figure 3. shows that the 68% of the students has thought that the use of GeoGebra has allowed them to better understand the theoretical notions of the course Very or Little enough. Just the 29% has Never used the applets made available on Moodle. The 68% of the students has thought that the use of GeoGebra was Very/Sufficiently helpful in understanding the theoretical notions of the course. The 29% on the other said they Never used the applets made available on Moodle. We emphasize that stimulating students to cooperate during the first university year is extremely important for their attitude throughout their student career and it is sometimes more important than the fairness in the results of the quizzes.

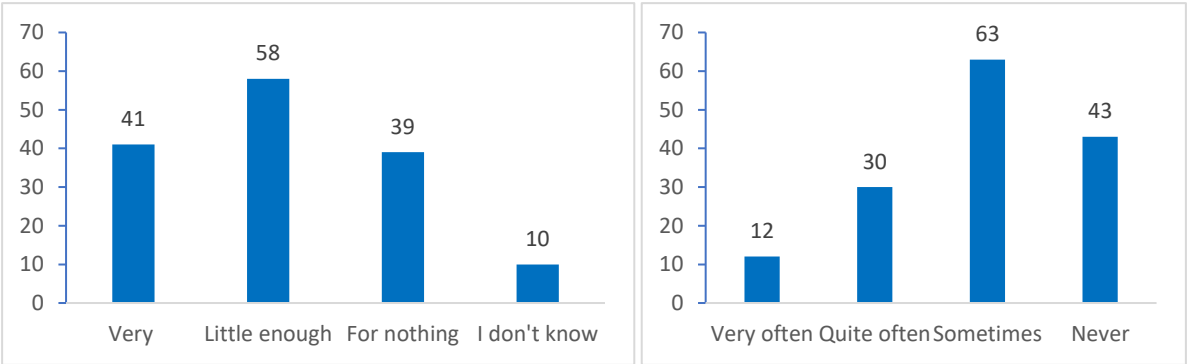

Figure 3: On the left Q5: Do you think the use of GeoGebra has allowed you to better understand the theoretical notions of the course?
On the right Q6: Did you use the GeoGebra applets made available on Moodle by the teachers?

4 CONCLUSIONS

Nine multiple-choice quizzes were designed as formative e-assessment for teaching multivariable differential calculus, including GeoGebra and problem solving tools. From the Psychometric Analysis we found out that the quizzes were well chosen, and they discriminated adequately between higher and lower student subject knowledge. For the students the most difficult question was that one in which it is necessary to have a good conceptual knowledge of the subject to answer correctly followed by the quiz which requires a problem solving and modelling capacity and by the one who becomes simple if the graphic interpretation of the problem is used. From the survey we found that the quizzes helped students to find the gaps in their preparation and to keep up with the lessons, furthermore there was a fair use of GeoGebra and the relative applets.

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